Fluctuations in 1D point patterns

Rosalba Garcia-Millan,[1,*] George Cantwell,[2,†] Konstantinos Aronis,[3,‡] and Carlos Marcelo[4,§]

[1] *Imperial College London*
[2] *University of Michigan*
[3] *Johns Hopkins School of Medicine*
[4] *University of Chicago*

Random point patterns with similar apparent characteristics (such as density of points) may display very different behaviours when it comes to the way these points are distributed in space. Strongly negatively correlated points (e.g. due to repulsive forces) show a high degree of order, whereas strongly positively correlated points (e.g. due to attractive forces) show clustering and hence, a high degree of disorder. These two extreme cases define a spectrum where point patterns are classified according to their fluctuations from small fluctuations (order, *hyperuniformity*) to large fluctuations (disorder, clustering). Point patterns with weak correlations show the same trend in their fluctuations as the typical uncorrelated case and are said to display Poisson-like behaviour.

In this report, we show numerical results on different one-dimensional point patterns both computer generated and experimentally obtained (heartbeat recordings). Furthermore, we regard the case of anomalously suppressed fluctuations (*hyperuniformity*) as a case of special interest and we try to derive the underlying mechanisms that give rise to this behaviour. We study the Dirichlet distribution as a model that can be tuned to display any behaviour from a periodic signal to a clustered signal.

This report is a summary of early work that we have done during the Complex Systems Summer School 2018 organised by the Santa Fe Institute, New Mexico. Throughout the report we discuss a number of directions that we can take to further research about the questions that we pose here.

* garciamillan16@imperial.ac.uk
† gcant@umich.edu
‡ karonis1@jhmi.edu
§ cmarceloservan@uchicago.edu

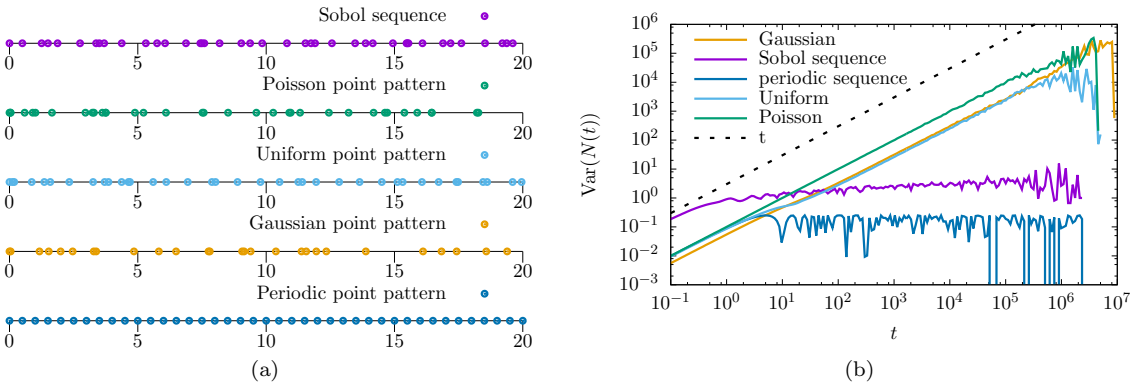

FIG. 1. (a) Sample of the considered 1D point patterns (b) Curves $\mathrm{Var}(N(t))$ for the different point patterns shown in Fig. 1(a), obtained from drawing 10^6 waiting times in each case and using Eq. (2). We observe that the point patterns "periodic" and "Sobol" show anomalous suppression of fluctuations (hyperuniformity) and "Gaussian", "uniform" and "Poisson" show the typical Poisson-like behaviour. The dashed line indicates the typical Poisson-like behaviour in 1D point patterns, namely the scaling form $\mathrm{Var}(N(t)) \propto t$.