

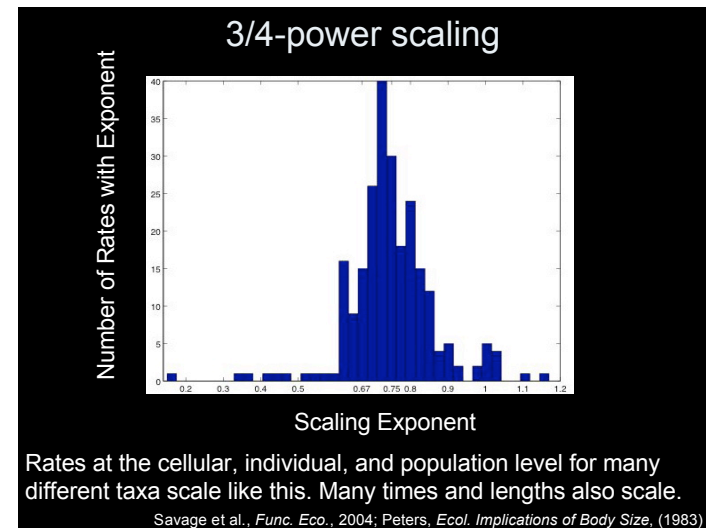
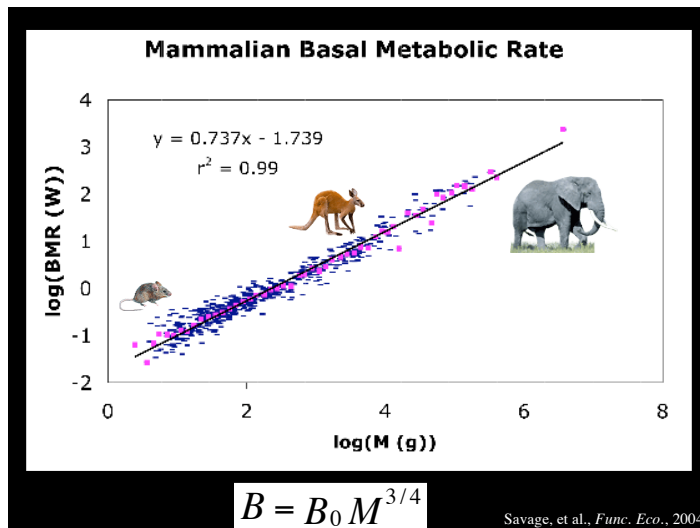
Biological scaling theory and effects on populations

By
Van Savage
Department of Systems Biology
Harvard Medical School

Beijing CSSS, 2007

Outline

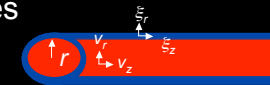
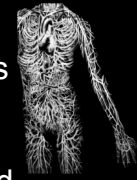
1. Theory for dependence of biological rates, times, and lengths on body size
2. Dependence for biological rates on body temperature
3. Scaling or population growth
4. Scaling of species interactions (predator-prey)
5. Conclusions



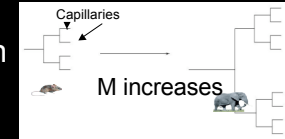
Theory for body mass scaling

Theory has three assumptions

- i. Branching, hierarchical network that is space filling to feed all cells
- ii. Minimization of energy to pump blood from the heart to the capillaries



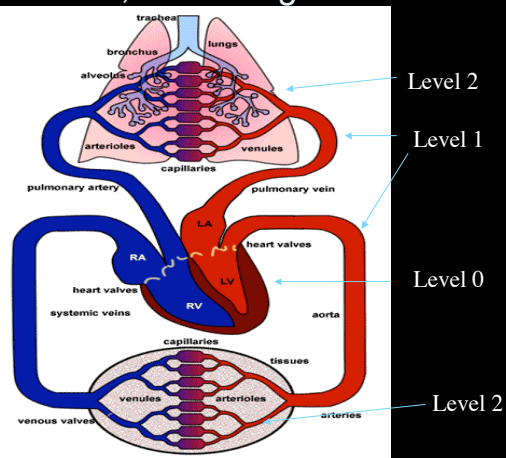
- iii. Capillaries are invariant in size



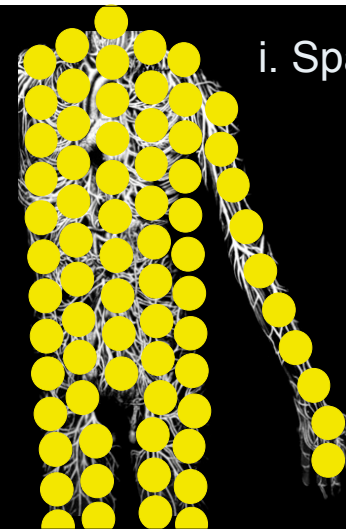
West et al. *Science* (1997)

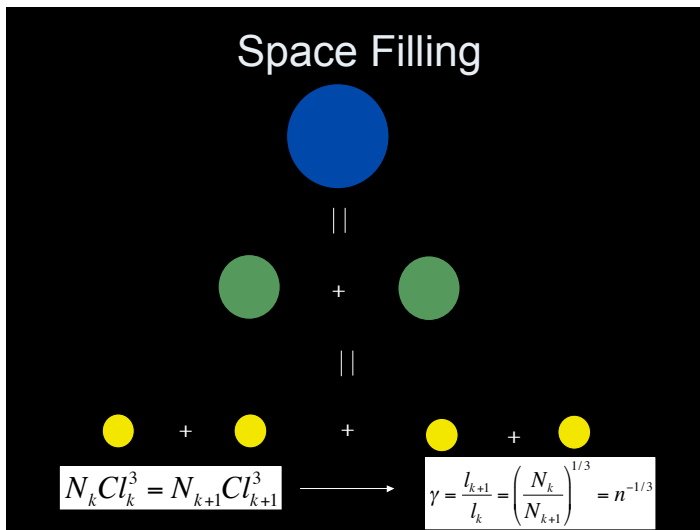
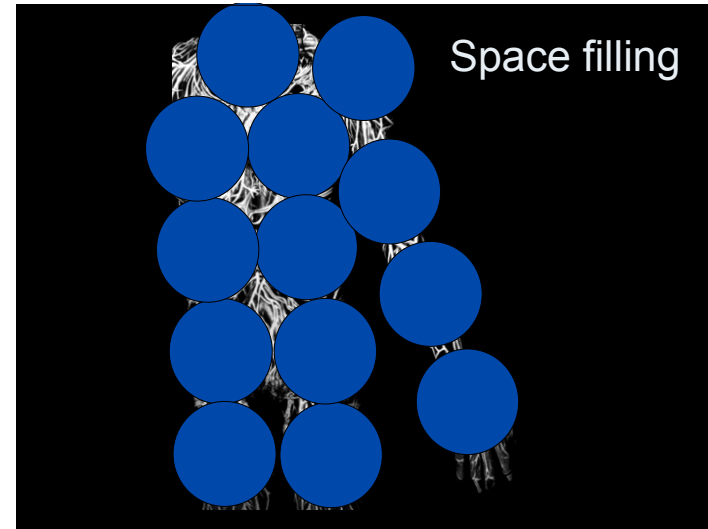
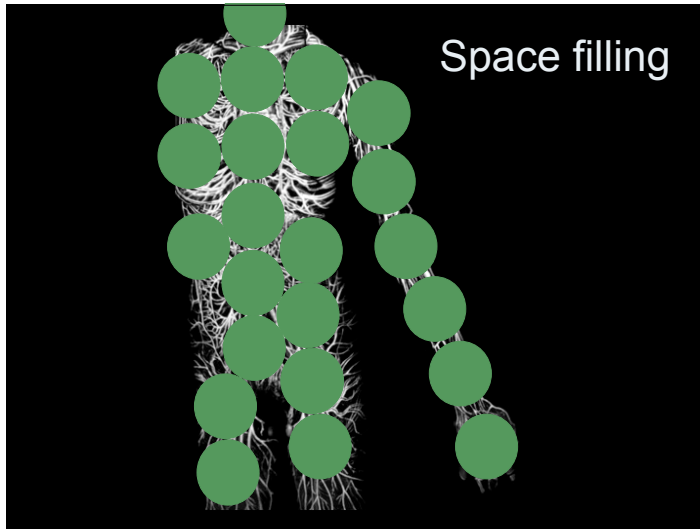
Hierarchical, Branching Network

$$n = 2$$



i. Space filling





ii. Minimize energy loss through selection

ρ_w -wall density
 E -Young's modulus
 ρ -blood density
 μ -blood viscosity

Blood Flow: $\rho \frac{\partial \mathbf{v}}{\partial t} = \mu \nabla^2 \mathbf{v} - \nabla p$
 Vessel Wall: $\rho_w \frac{\partial^2 \xi}{\partial t^2} = E \nabla^2 \xi - \nabla p$

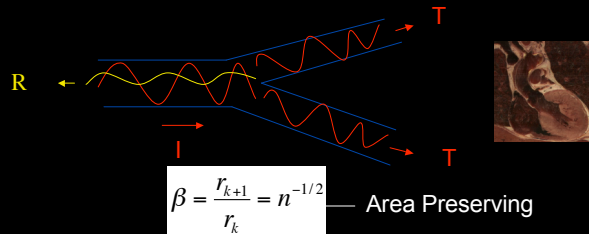
Can derive total impedance to flow

$$Z \sim \frac{c_0 \rho i}{\pi r^2} \sqrt{\frac{J_0 \left(i^{3/2} \sqrt{\frac{\omega \rho}{\mu}} r \right)}{J_2 \left(i^{3/2} \sqrt{\frac{\omega \rho}{\mu}} r \right)}}$$

ω -angular frequency of wave
 c_0 -Korteweg-Moens velocity
 $c_0 = \sqrt{\frac{Eh}{2\rho r}}$

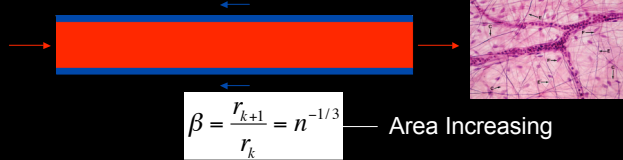
ii. Minimize energy loss (selection)

Reflection at junctions (Important for larger vessels, pulsatile flow)



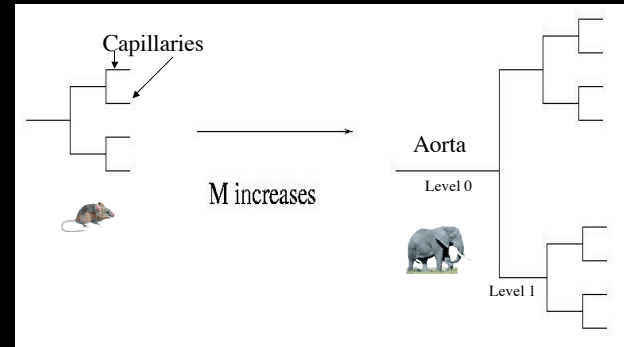
$$\beta = \frac{r_{k+1}}{r_k} = n^{-1/2} \quad \text{Area Preserving}$$

Dissipation (Important for small vessels, Poiseuille flow)



$$\beta = \frac{r_{k+1}}{r_k} = n^{-1/3} \quad \text{Area Increasing}$$

Body Size Changes Network Size



Terminal units are invariant.

Metabolic Rate, B , and Body Mass, M

$$\text{Mass} \rightarrow M \propto V_b = \sum_{\text{levels}} n_k \pi r_k^2 l_k \propto N_T^{4/3} V_T \propto B^{4/3} \leftarrow \text{Met Rate}$$

Follows from Energy Min.

Use scale factors to relate each level to terminal units.

Invariance of terminal units
 $B = N_T B_T$

$$B \propto M^{3/4}$$

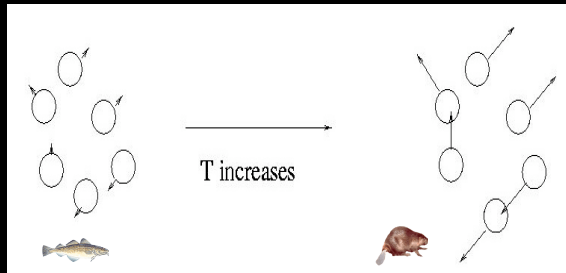
West et al. *Science* (1997)

Theory has three assumptions

- Branching, hierarchical network that is space filling to feed all cells \rightarrow relates vessel lengths across levels of cardiovascular system
- Minimization of energy to send vital resources to the terminal units (pump blood from the heart to the capillaries) \rightarrow relates vessel radii across levels of cardiovascular system and connects blood volume to body size
- Capillaries are invariant in size \rightarrow sets overall scale for cardiovascular system

Together these determine the scaling for the network.

Theory for body temperature: biochemical reaction kinetics



Changes energy of impact and frequency of collisions,

Characteristic Biological Rates and Times

$$B = B_0 M^{3/4} e^{-E/kT}$$

$$\bar{B} = \frac{B}{M} = B_0 M^{-1/4} e^{-E/kT}$$

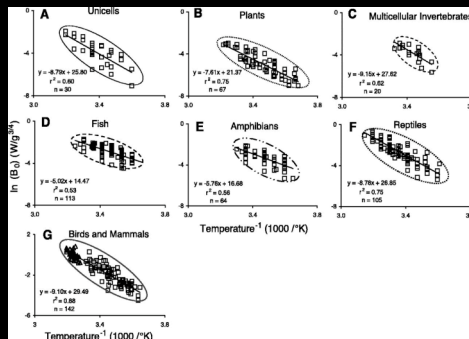
$$t_{BIO} \propto M^{1/4} e^{E/kT}$$

15 orders of magnitude collapses to 1 order of magnitude variation!

Zeroth-order model for quantifying differences between organisms and identifying other relevant parameters.

Gillooly, Charnov, West, Savage, and Brown *Nature* (2002).

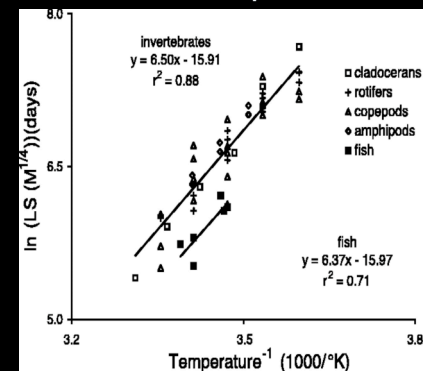
Temperature Dependence of Metabolic Rate



Similar slopes reflect similar activation energies and shared biochemistry through evolution. M and T explain dominant variation.

Gillooly, Brown, West, Savage, and Charnov *Science* (2001)

Mass and Temperature Dependence of Lifespan



Gillooly, West, Brown, Savage, and Charnov *Science* (2001)

An Anacreontick

Busy, curious, thirsty Fly,
Gently drink, and drink as I;
Freely welcome to my Cup,
Could'st thou sip, and sip it up;
Make the most of Life you may,
Life is short and wears away.

Just alike, both mine and thine,
Hasten quick to their Decline;
Thine's a Summer, mine's no more,
Though repeated to threescore;
Threescore Summers when they're gone,
Will appear as short as one.

By William Oldys

Population Level

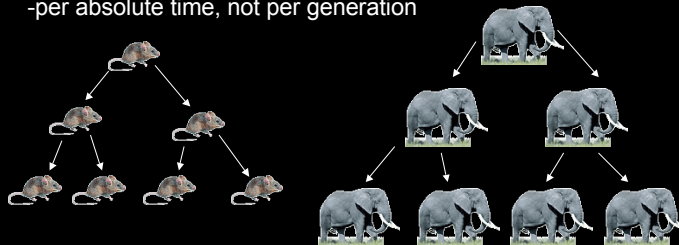
Population Growth: From Individuals To Populations

$$\frac{dN}{dt} = rN$$

or

$$N(t) = N_0 e^{rt}$$

-per absolute time, not per generation



Euler's Equation (for positive growth)

-Survivorship and fecundity are also important,
and can make that explicit!

$$B(t) = \int_0^{\infty} B(t-x)l(x)b(x)dx$$

->

$$1 = \int_0^{\infty} e^{-rx}l(x)b(x)dx$$

$N(t)$ is population size at time t
 r is rate of increase, fundamental variable
 $B(t)$ is number of births at time t
 $l(x)$ is probability of survivorship up to age x
 $b(x)$ is fecundity rate at age x

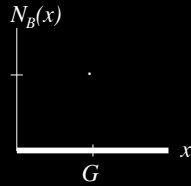
Savage et al. *Am. Nat.* 2004

Discrete Generations

Organisms reproduce once in their lifetime

$N_B(x)$ is cumulative number of offspring at age x

G is the generation time and is the average age at which an organism has offspring



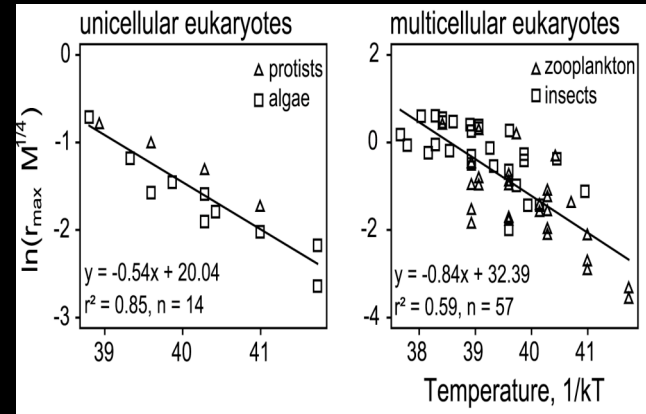
$$r = \frac{1}{G} \ln[l(G) N_B(G)] \propto M^{-1/4} e^{-E/kT}$$

Univoltine insects follow this exactly

Many other unicells, insects, and zooplankton are a good approximation to this Savage et al., *Am. Nat.*, 2004.

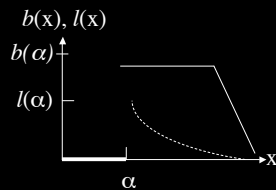
Empirical M dependence: Fenchel (1973), Southwood et al. (1974), May (1976), Blueweiss et al. (1978)
Empirical T dependence: Monod (1942), Birch (1948), Hinshelwood (1966), Droop (1968), Eppley (1972)

Intrinsic Rate of Increase



Savage et al. *Am. Nat.* 2004

Overlapping Generations



Mortality Rate

$$l(x) = l(\alpha) e^{-Z(x-\alpha)}$$

Age at 1st Reproduction

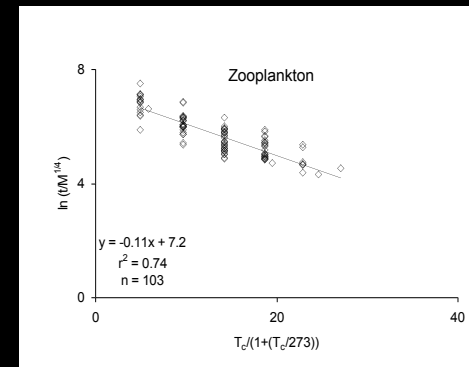
$$Z \propto r \propto \alpha^{-1} \propto M^{-1/4} e^{-E/kT}$$

$$l(\alpha) = e^{-\bar{Z}\alpha} \propto M^0 T^0$$

Size and temperature do not affect the percentage of offspring that survive to the age of first reproduction.

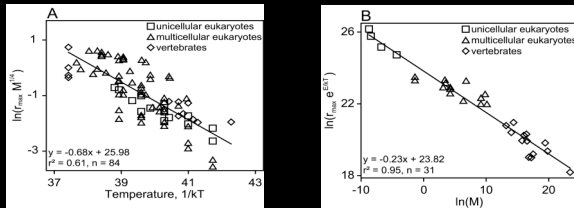
Savage et al., *Am. Nat.*, 2004

Time to First Reproduction, α Measure of Development



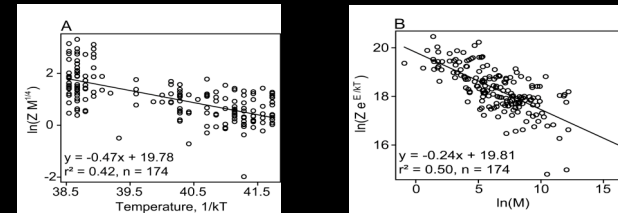
Gillooly, Charnov, West, Savage, and Brown *Nature* (2002).

Little Variation Across Broad Assortment of Organisms



Savage et al. *Am. Nat.* 2004

Mortality Rates in the Field



Data from Pauly (1980)

Savage et al. *Am. Nat.* 2004

Scaling of Population Size

Assume fixed amount of resources P

At steady state, $dN/dt=0$, population is at carrying capacity for environment, $K(M,T)$

$$P \propto B(M,T)K(M,T)$$

$$K(M,T) \propto \frac{1}{B(M,T)} \propto M^{-3/4} e^{E/kT}$$

With energy equivalence, Damuth, *Bio. J. Lin. Soc.* (1987), this holds for multiple populations in a shared environment

Savage et al. *Am. Nat.* 2004

Physics [Science] is mathematical not because we know so much about the physical world, but because we know so little; it is only its mathematical properties that we can discover.
~Bertrand Russell

Conclusions

1. Power laws are common in biology (and elsewhere)
2. Dynamical model based on distribution of resources makes many predictions that match data.
3. Temperature also affects biological rates and times and can include this effect
4. Can build up from one level to the next to understand population growth (but ants...)
5. Species interactions and competition can begin to be incorporated into both scaling and models of effects of climate change on biological systems.

Collaborators

Geoff West (Santa Fe Institute and Los Alamos)
James Brown (U of New Mexico)
James Gillooly (U of New Mexico)
Woody Woodruff (Los Alamos)
Drew Allen (U of New Mexico)
Eric Charnov (U of New Mexico)
Alex Herman (UC San Francisco)
Tony Dell (James Cook University)
Jon Norberg (Stockholm University)
Colleen Webb (Colorado State University)
Eric Deeds (Harvard Medical School)