NONEXTENSIVE STATISTICAL MECHANICS

AND THERMODYNAMICS

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Rio de Janeiro, Brazil

Santa Fe Institute, New Mexico, USA

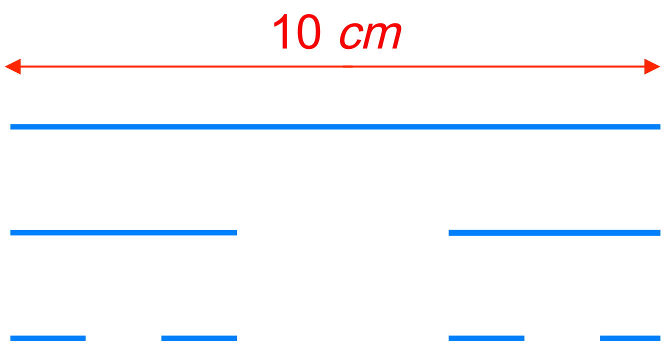
Complexity Science Hub Vienna, Austria

Santa Fe, June 2017

World champion of complexity!

(Photo taken in London by Luciano Pietronero)

TRIADIC CANTOR SET:



$$d_F = \frac{\ln 2}{\ln 3} = 0.6309...$$

Hence the interesting measure is

$$(10\ cm)^{0.6309...} \cong 4.275\ cm^{0.6309}$$

It is the natural (or artificial or social) system itself which, through its geometrical-dynamical properties, mandates the specific informational tool --- entropy --- to be meaningfully used for the study of its thermostatistical and thermodynamical properties.

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q = 1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable (**unique** trace form; Enciso-Tempesta)

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J. Stat. Phys. **52**, 479 (1988)

$$DEFINITIONS: \; q-logarithm: \qquad \ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\; \ln_1 x = \ln x)$$

$$q-exponential: \qquad e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\,\ln W$	$k\sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\,\ln_q W$	$k\sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

6198 articles by 12672 scientists (96 countries)

[26 June 2017]

CONTRIBUTORS (6198 MANUSCRIPTS)

USA	2225	HUNGARY	89	SLOVENIA	19	BOLIVIA	4
ITALY	1034	PORTUGAL	89	SAUDI ARABIA	17	INDONESIA	4
GERMANY	959	AUSTRALIA	88	IRELAND	14	NIGERIA	4
BRAZIL	872	ROMANIA	80	VENEZUELA	14	SENEGAL	4
UNITED KINGDOM	778	NORWAY	75	LITHUANIA	12	VIETNAM	4
FRANCE	682	SWEDEN	69	CYPRUS	11	BELARUS	3
CHINA	670	TAIWAN	63	MOROCCO	11	MACEDONIA	3
SWITZERLAND	638	UKRAINE	57	ARMENIA	10	MOLDOVA	2
RUSSIA	536	EGYPT	47	BELARUS	10	NORTH CYPRUS	2
JAPAN	484	PAKISTAN	46	PUERTO RICO	10	PANAMA	2
INDIA	385	SLOVAKIA	45	SINGAPORE	10	PHILIPINES	2
SPAIN	360	DENMARK	43	NEW ZEALAND	9	UN. ARAB EMIRATES	2
POLAND	202	SERBIA	36	FRENCH GUIANA	8	BAHRAIN	1
GREECE	187	FINLAND	34	GEORGIA	8	BENIN	1
CANADA	177	SOUTH AFRICA	33	KHAZHAKSTAN	8	ECUADOR	1
NETHERLANDS	147	BELGIUM	32	ESTONIA	7	ETHIOPIA	1
CZECH REPUBLIC	140	CHILE	31	ALBANIA	6	IRAQ	1
ARGENTINA	132	BANGLADESH	30	AZERBAIJAN	6	LUXENBOURG	1
IRAN	126	COLOMBIA	29	CAMEROON	6	OMAN	1
TURKEY	125	ALGERIA	26	JORDAN	6	QATAR	1
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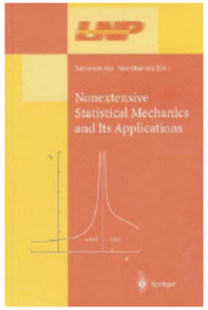
ISRAEL	109	BULGARIA	20	PERU	5	TUNISIA	1
MEXICO	107	CUBA	19	THAILAND	5	UZBEKISTAN	1
AUSTRIA	93	MALAYSIA	19	URUGUAY	5	YEMEN	1

12672 SCIENTISTS **96 COUNTRIES**

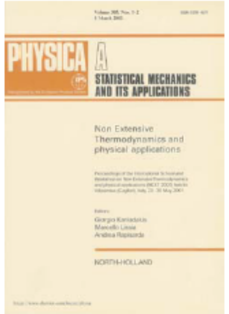
[Updated 26 June 2017]

BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

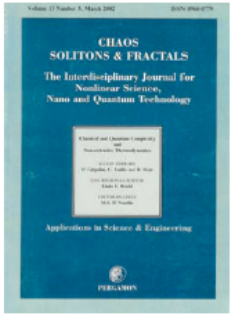
1999



2001



2002



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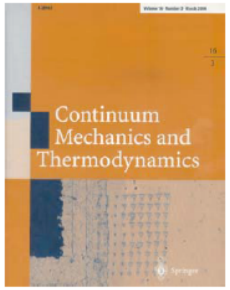

2004

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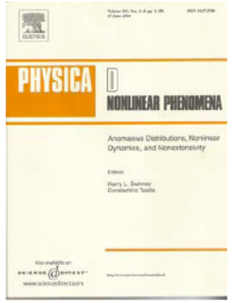

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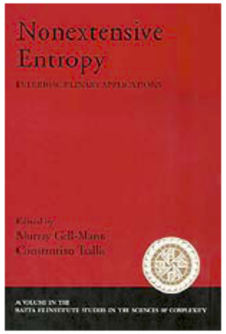

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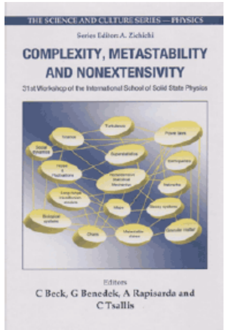

2005

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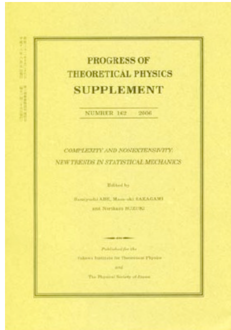
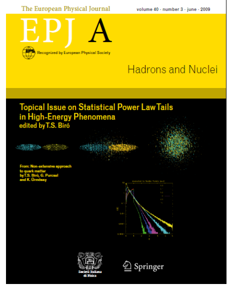
2006



2006

2007



2009

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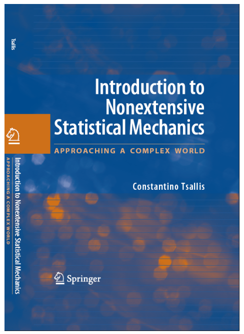
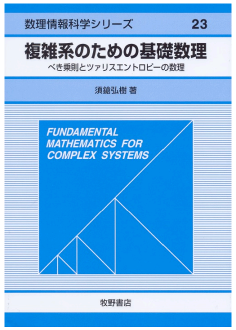
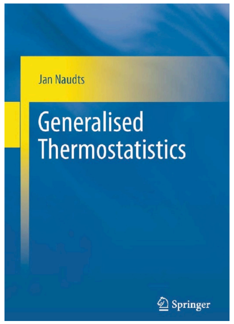


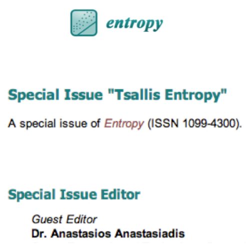
2009



2010

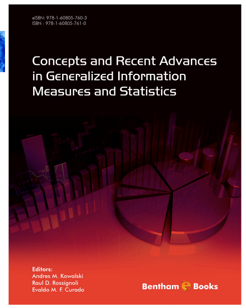


2011

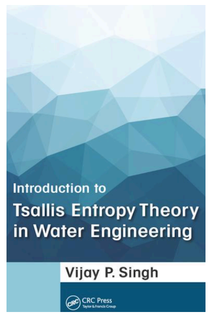


2011

2012



2013

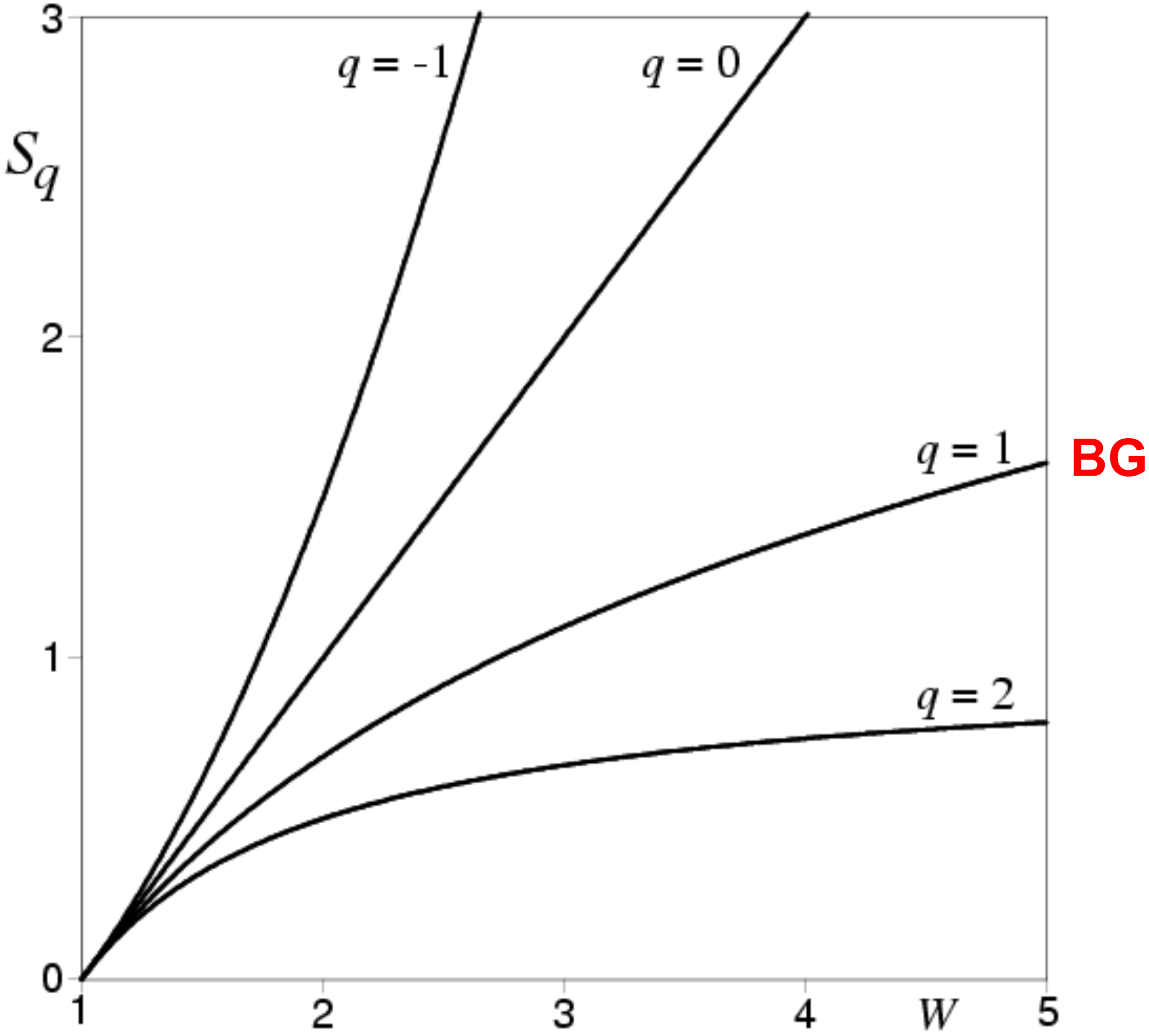


2016

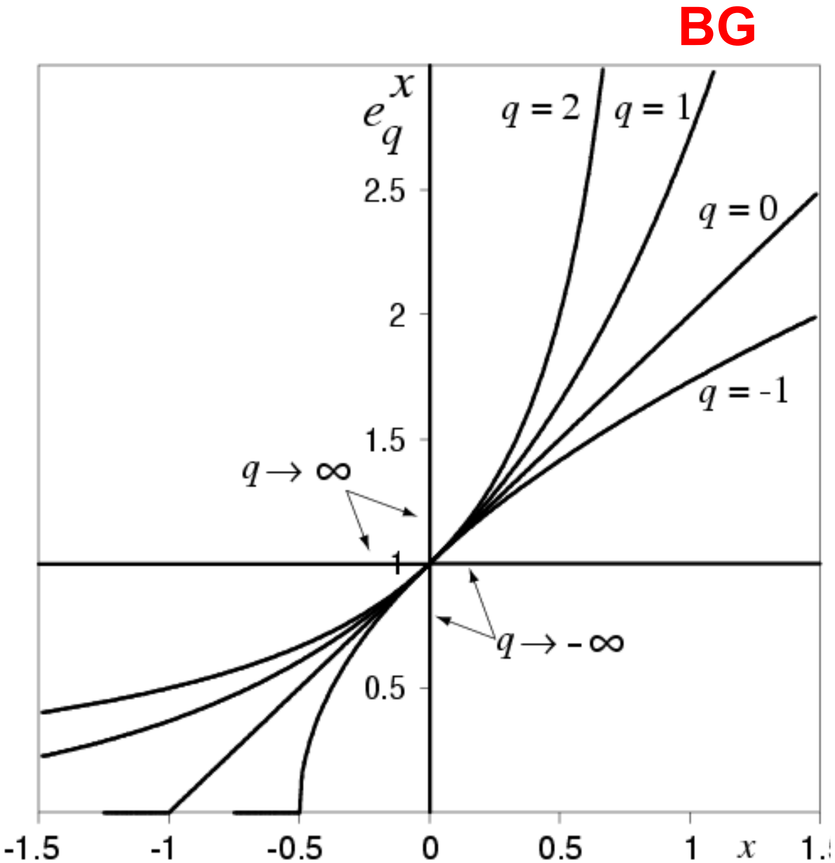
Generalized Entropies:
C.T., E.M.F. Curado and F.D. Nobre, Cambridge Univ. Press, in preparation

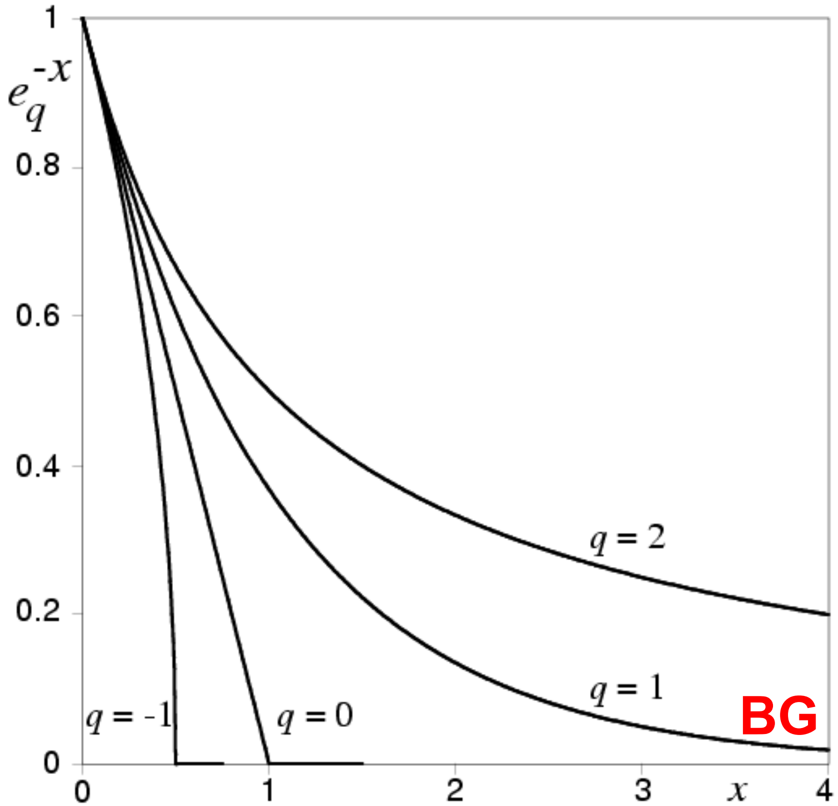
~2017

$$S_q = k \ln_q W \equiv k \frac{W^{1-q} - 1}{1-q}$$



$$e_q^x = [1 + (1-q)x]^{\frac{1}{1-q}}$$





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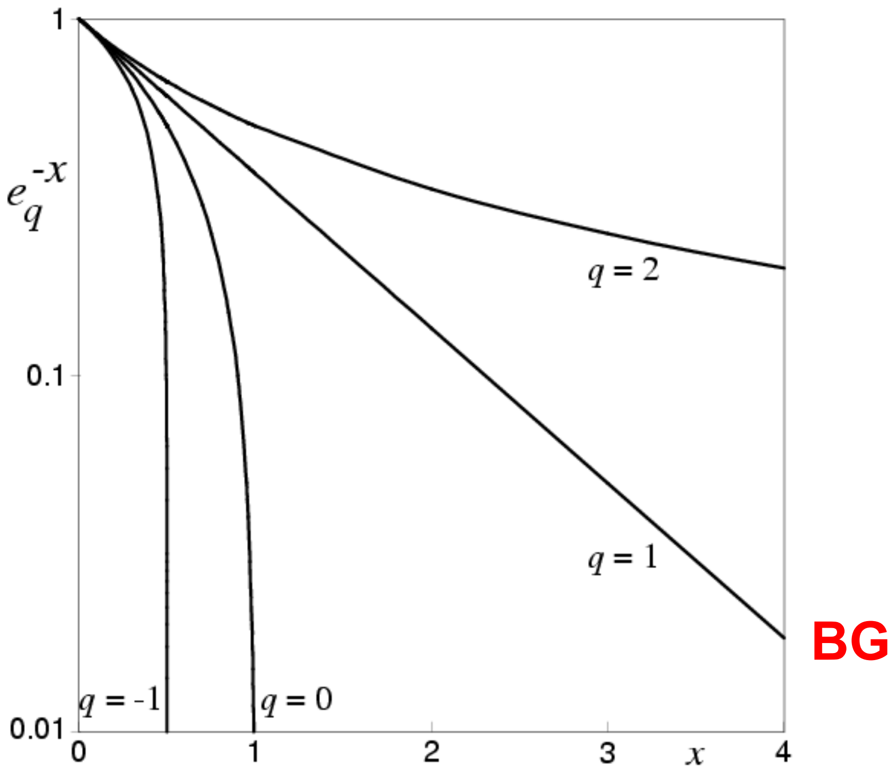
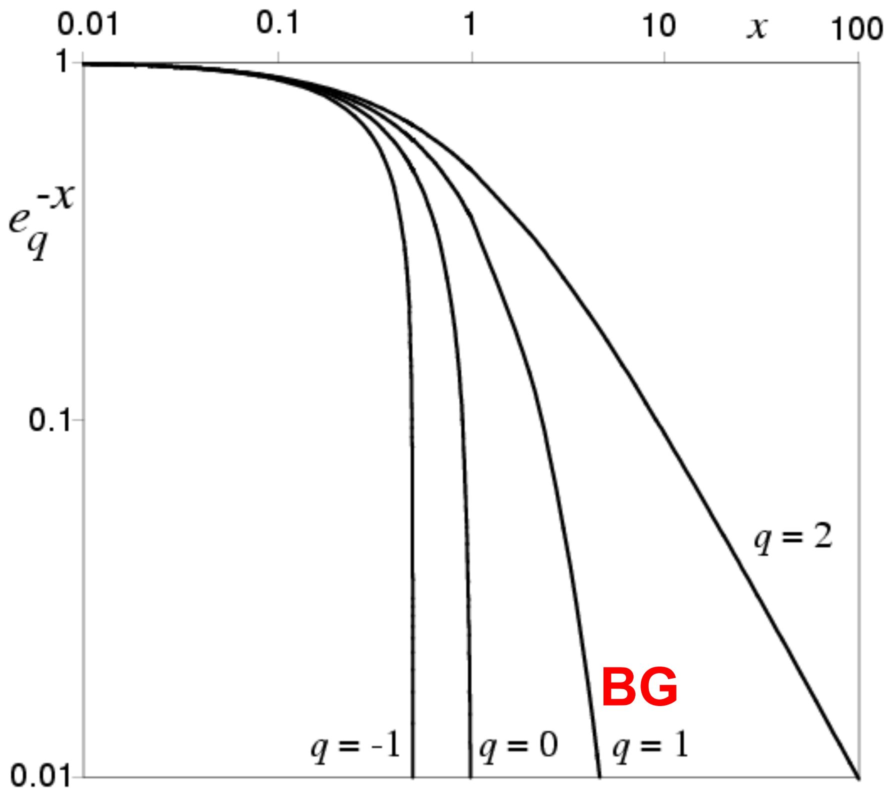
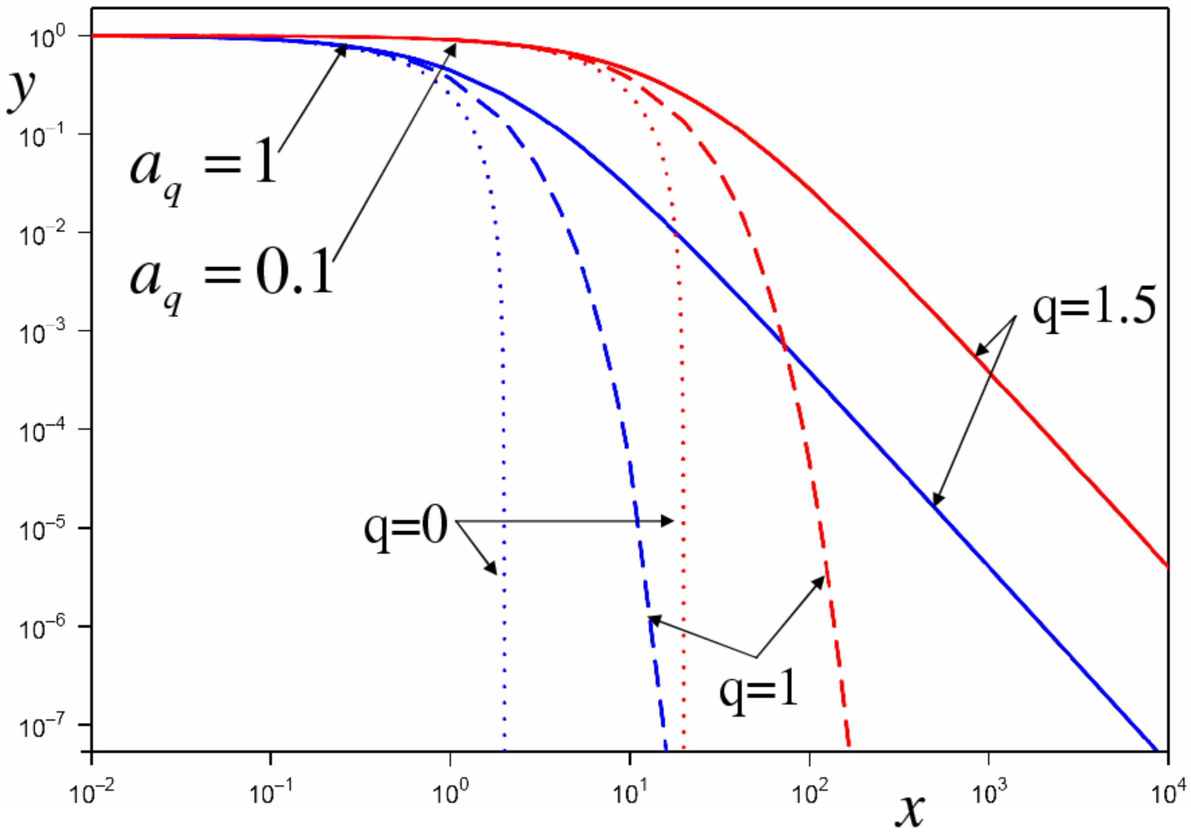
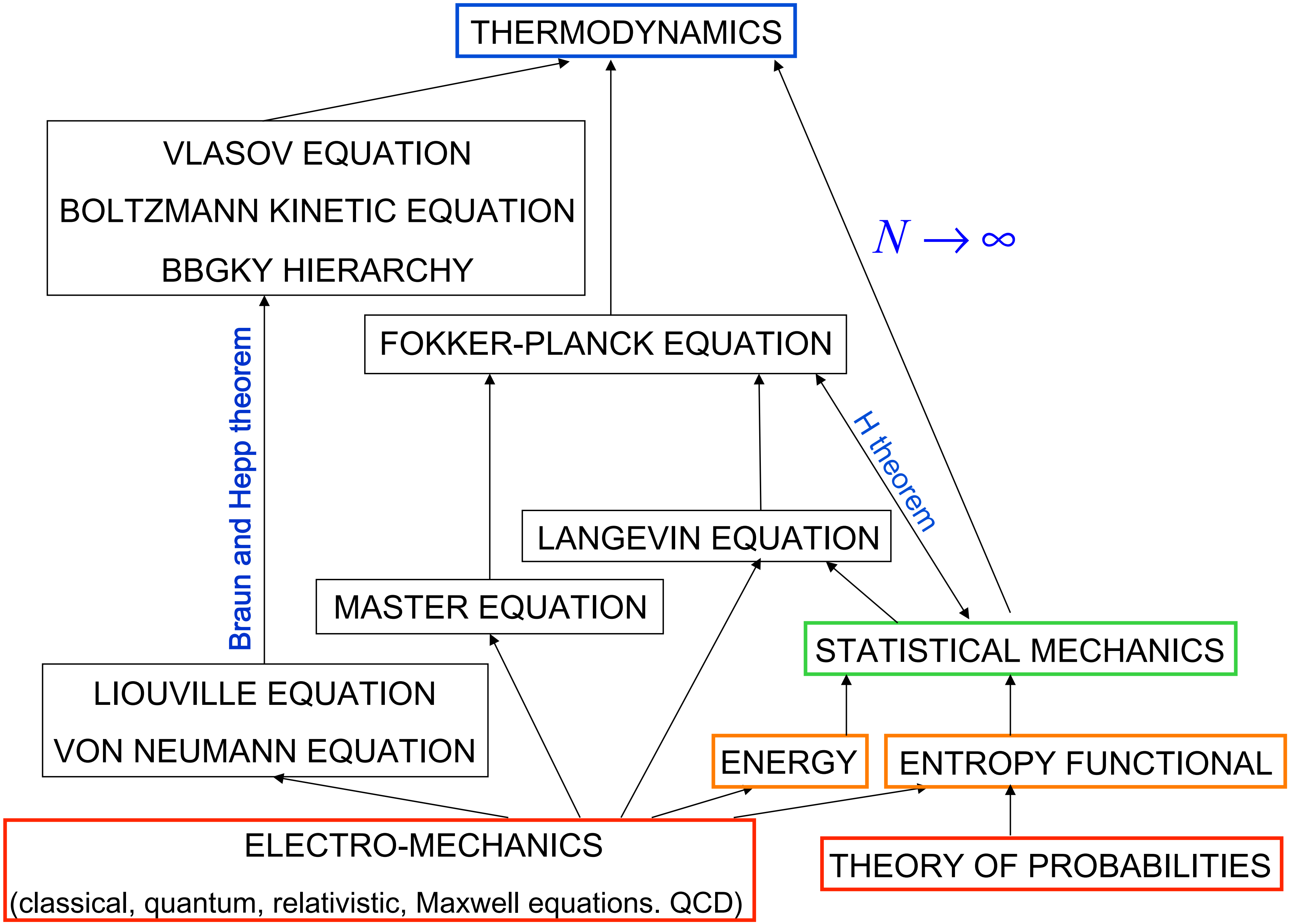




$$\frac{dy}{dx} = -a_q y^q \quad \text{with } y(0) = 1$$

$$\Rightarrow y = e_q^{-a_q x} \equiv \frac{1}{\left[1 + (q-1)a_q x\right]^{\frac{1}{q-1}}}$$




THERMODYNAMICS
VLASOV EQUATION
BOLTZMANN KINETIC EQUATION
BBGKY HIERARCHY
$N \to \infty$
FOKKER-PLANCK EQUATION
Braun and Hepp theorem
H theorem
LANGEVIN EQUATION
MASTER EQUATION
STATISTICAL MECHANICS
LIOUVILLE EQUATION
VON NEUMANN EQUATION
ENERGY
ENTROPY FUNCTIONAL
ELECTRO-MECHANICS
(classical, quantum, relativistic, Maxwell equations. QCD)
THEORY OF PROBABILITIES

TYPICAL SIMPLE SYSTEMS:

$$W(N) \propto \mu^N \;\; (\mu > 1)$$

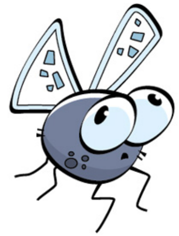
Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear and homogeneous Fokker-Planck equations, Gausssians



→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

$$\text{e.g.,} \;\; W(N) \propto N^{\rho} \;\; (\rho > 0)$$

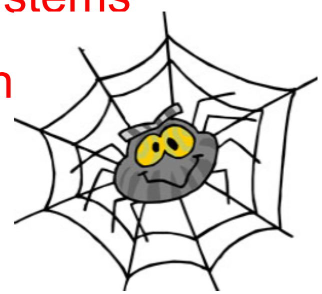
Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

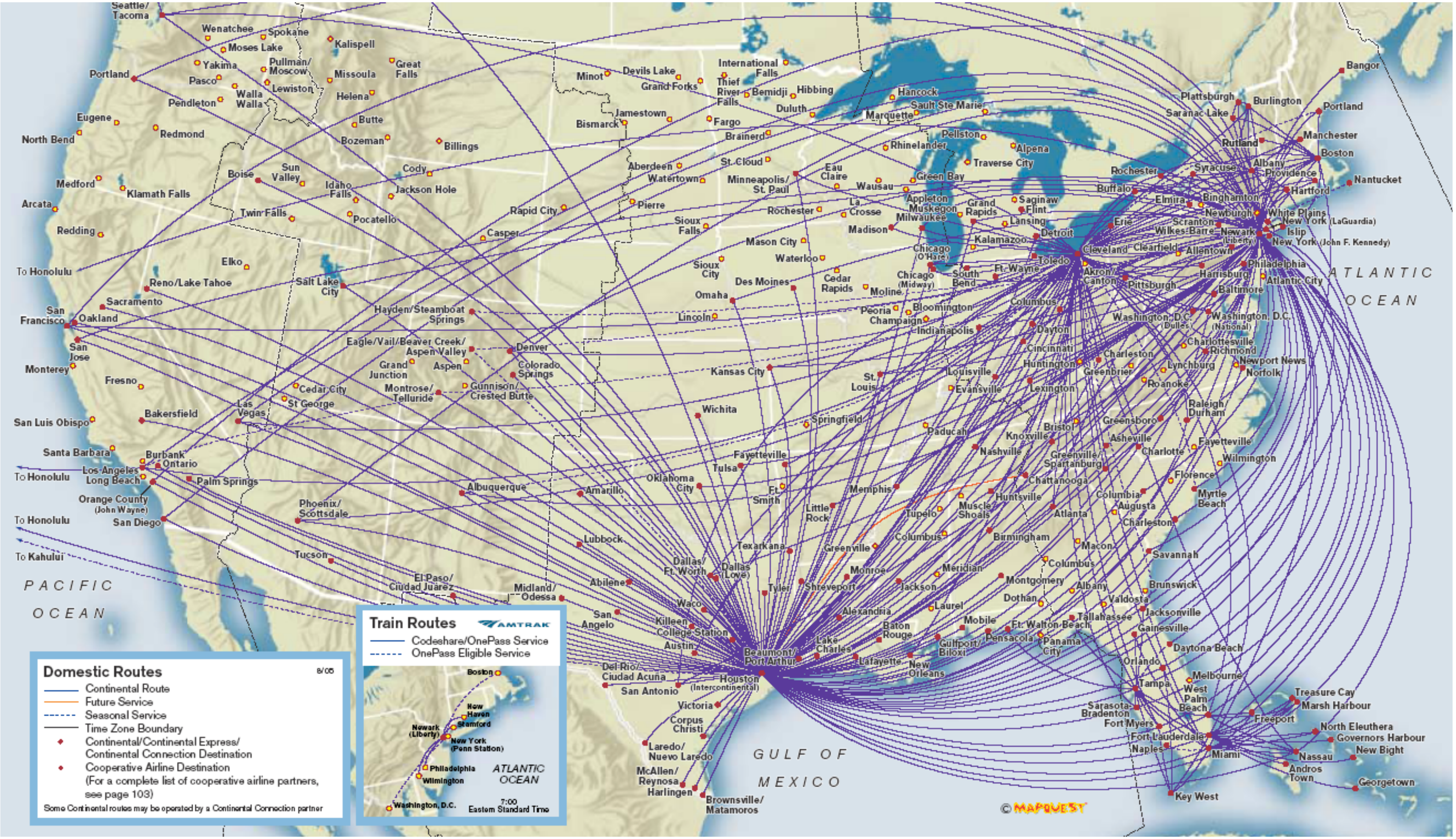
Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear and/or inhomogeneous Fokker-Planck equations, *q*-Gaussian



→ **Entropy S_q (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**



Continental Airlines

ADDITIVITY:

O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\,\frac{S_q(A)}{k}\,\frac{S_q(B)}{k}\ ,$$

S_{BG} and $S_q^{Renyi}\,(\forall q)$ are additive, and $S_q\ (\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and $A_2,\ \ldots,\ A_N$.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty\ ,\ \textit{i.e.,}\ \ S(N) \propto N\ \ (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

$W \equiv$ total number of possibilities with nonzero probability, assumed to be equally probable

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

IMPORTANT: $\boxed{\mu^N >> \nu^{N^\gamma} >> N^\rho \quad \text{if} \quad N >> 1}$

All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy (*Anna Karenina*, 1875-1877)

SYSTEMS $W(N)$ *(equiprobable)*	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$e.g., \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$e.g., N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE $(q = 1 - 1/\rho)$	NONEXTENSIVE
$e.g., \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\delta = 1/\gamma)$

King Thutmosis I
18^{th} Dynasty
circa 1500 BC

A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown.

Albert Einstein (1949)

THERMODYNAMICS:

$$G(V,T,p,\mu,H,\ldots)=U(V,T,p,\mu,H,\ldots)-TS(V,T,p,\mu,H,\ldots)$$
$$+pV-\mu N(V,T,p,\mu,H,\ldots)-HM(V,T,p,\mu,H,\ldots)-\ldots$$

where $S,V,N,M,\ldots$ scale like $V\equiv L^d$ (extensivity)

$T,p,\mu,H,\ldots$ scale like L^θ

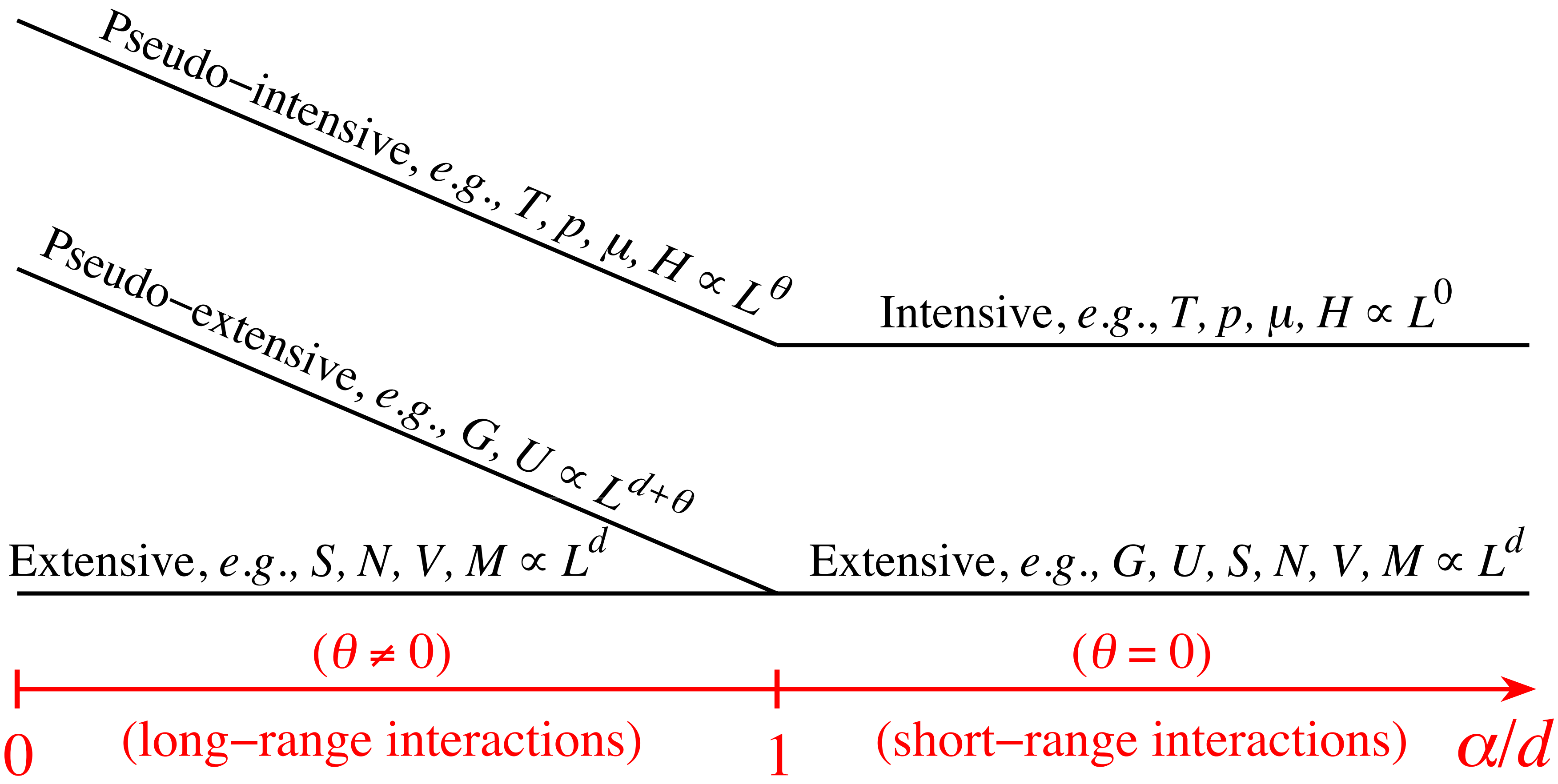
$G,U,\ldots$ scale like L^ε

hence $\varepsilon=\theta+\mathrm{d}$

Dividing by $L^{\theta+d}$ we obtain

$$g\left(\frac{T}{L^\theta},\frac{p}{L^\theta},\frac{\mu}{L^\theta},\frac{H}{L^\theta},\ldots\right)=u\left(\frac{T}{L^\theta},\frac{p}{L^\theta},\frac{\mu}{L^\theta},\frac{H}{L^\theta},\ldots\right)-\frac{T}{L^\theta}s\left(\frac{T}{L^\theta},\frac{p}{L^\theta},\frac{\mu}{L^\theta},\frac{H}{L^\theta},\ldots\right)$$
$$+\frac{p}{L^\theta}-\frac{\mu}{L^\theta}n\left(\frac{T}{L^\theta},\frac{p}{L^\theta},\frac{\mu}{L^\theta},\frac{H}{L^\theta},\ldots\right)-\frac{H}{L^\theta}m\left(\frac{T}{L^\theta},\frac{p}{L^\theta},\frac{\mu}{L^\theta},\frac{H}{L^\theta},\ldots\right)-\ldots$$

where all variables are intensive.



–Classical short-range-interacting many-body Hamiltonian systems

(i.e., $\alpha > d$) $\Rightarrow \theta = 0 \Rightarrow \varepsilon = d$ (textbooks)

–Classical long-range-interacting many-body Hamiltonian systems

(i.e., $0 \leq \alpha < d$) $\Rightarrow \theta = d - \alpha \Rightarrow \varepsilon = 2d - \alpha$ (widely verified in the literature)

–Schwarzschild (3+1)-dimensional black hole $\Rightarrow M_{bh} \propto L \Rightarrow \varepsilon = 1 \Rightarrow \theta = 1 - d$

–Banados-Teitelboim-Zanelli (2+1)-dimensional black hole $\Rightarrow \varepsilon = 2 \Rightarrow \theta = 2 - d$

Annalen der Physik 33 (1910)

11. *Theorie der Opaleszenz von homogenen Flüssigkeiten und Flüssigkeitsgemischen in der Nähe des kritischen Zustandes;*

von A. Einstein.

$$S = \frac{R}{N} \lg W + \text{konst.} \tag{1}$$

Daß die zwischen S und W in Gleichung (1) gegebene Beziehung die einzig mögliche ist, kann bekanntlich aus dem Satze abgeleitet werden, daß die Entropie eines aus Teilsystemen bestehenden Gesamtsystems gleich ist der Summe der Entropien der Teilsysteme.

The relation between *S* and *W* given in Eq. (1) is the only reasonable given the proposition that the entropy of a system consisting of subsystems is equal to the sum of entropies of the subsystems.

(Free translation by Tobias Micklitz)

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

May 2015

EPL, **110** (2015) 30005 www.epljournal.org
doi: 10.1209/0295-5075/110/30005

Boltzmann-Gibbs entropy is sufficient but not necessary for the likelihood factorization required by Einstein

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[2] *Santa Fe Institute - 1399 Hyde Park Road, Santa Fe, NM 87501, USA*
[3] *Office for Outer Space Affairs, United Nations - P.O. Box 500, A-1400 Vienna, Austria*

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published online 21 May 2015

IOPscience

Highlights-of-2015

(21 March 2016: 624 downloads)

Einstein 1910 (reversal of Boltzmann formula):

For any two independent systems A and B,

the likelihood function should satisfy

$$\Omega(A+B)=\Omega(A)\,\Omega(B) \quad \text{(Einstein principle)}$$

$q=1$: $\quad S_{BG}=k_B \ln W$ hence $\Omega(\{p_i\}) \propto e^{S_{BG}(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto e^{S_{BG}(A+B)/k_B} = e^{S_{BG}(A)/k_B+S_{BG}(B)/k_B} = e^{S_{BG}(A)/k_B} e^{S_{BG}(B)/k_B} \propto \Omega(A)\,\Omega(B)$$

OK!

$\forall q$: $\quad S_q=k_B \ln_q W$ hence $\Omega(\{p_i\}) \propto e_q^{S_q(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto \mathrm{e}_q^{S_q(A+B)/k_B} = \mathrm{e}_q^{S_q(A)/k_B+S_q(B)/k_B+(1-q)[S_q(A)/k_B][S_q(B)/k_B]}$$

$$= e_q^{S_q(A)/k_B} e_q^{S_q(B)/k_B} \propto \Omega(A)\,\Omega(B)$$

OK $\forall q$!

COMPOSITION OF VELOCITIES OF INERTIAL SYSTEMS (d=1)

$$v_{13} = v_{12} + v_{23} \qquad \text{(Galileo)}$$

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \qquad \text{(Einstein)}$$

Newton mechanics:
It satisfies Galilean additivity **but** violates Lorentz invariance (hence mechanics can not be unified with Maxwell electromagnetism)

Einstein mechanics (Special relativity):
It satisfies Lorentz invariance (hence mechanics is unified with Maxwell electromagnetism) **but** violates Galilean additivity

Question: which is physically more fundamental, the additive composition of velocities **or** the unification of mechanics and electromagnetism?

Special relativity recovers Newtonian/Galilean mechanics as particular case:

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \sim v_{12} + v_{23}$$

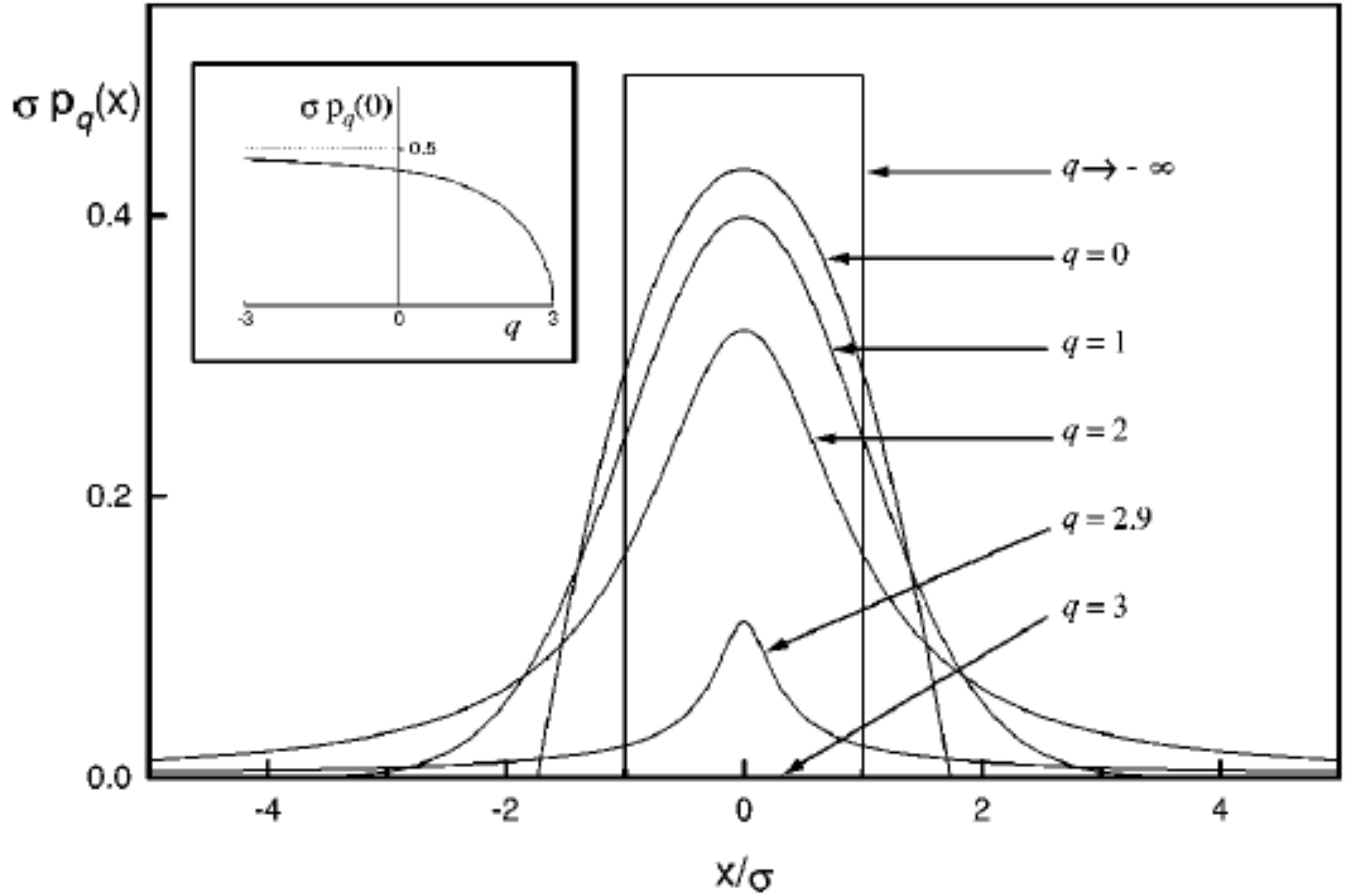
if $\quad 1/c \to 0 \quad$ or $\quad \forall\ 1/\mathrm{c} \neq 0$ with $v/c \to 0$

q-statistics recovers Boltzmann-Gibbs statistics as particular case:

$$\mathrm{e}_q^{-\beta E} \equiv \frac{1}{\left[1+(q-1)\beta E\right]^{\frac{1}{q-1}}} \sim e^{-\beta E} = e^{-E/kT}$$

if $\quad (q\text{-}1) \to 0 \quad$ or $\quad \forall\ (\mathrm{q}\text{-}1) \neq 0$ with $\beta\mathrm{E} = \frac{E}{kT} \to 0$

q-GAUSSIANS: $p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}}$ $\quad (q < 3)$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

Milan j. math. 76 (2008), 307–328
© 2008 Birkhäuser Verlag Basel/Switzerland
1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
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See also: H.J. Hilhorst, JSTAT P10023 (2010)

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A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)

A. Plastino and M.C. Rocca, Physica A **392**, 3952 (2013)

S. Umarov and C. T., J Phys A **49**, 415204 (2016)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ - *scaled attractor* $\mathcal{F}(x)$ *when summing* $N\to\infty$ q-*independent identical random variables with symmetric distribution* $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \dfrac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$\mathcal{F}(x) = $ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$\mathcal{F}(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x\rvert \ll x_c(q,2) \\ f(x) \sim C_q / \lvert x\rvert^{2/(q-1)} & \text{if } \lvert x\rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q\to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0<\alpha<2)$	$\mathcal{F}(x) = $ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x\rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \quad \text{if } \lvert x\rvert \ll x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x\rvert^{1+\alpha} \quad \text{if } \lvert x\rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha\to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$\mathcal{F}(x) = L_{q,\alpha}$, *with same* $\lvert x\rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\,\alpha}(x) \sim C^*_{q,\alpha} / \lvert x\rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \text{(intermediate regime)} \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\,2}(x) \sim C^L_{q,\alpha} / \lvert x\rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \text{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

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SCIENTIFIC REPORTS

OPEN

The standard map: From Boltzmann-Gibbs statistics to Tsallis statistics

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Accepted: 09 March 2016
Published: 23 March 2016

Ugur Tirnakli[1,*] & Ernesto P. Borges[2,3,*]

As well known, Boltzmann-Gibbs statistics is the correct way of thermostatistically approaching ergodic systems. On the other hand, nontrivial ergodicity breakdown and strong correlations typically drag the system into out-of-equilibrium states where Boltzmann-Gibbs statistics fails. For a wide class of such systems, it has been shown in recent years that the correct approach is to use Tsallis statistics instead. Here we show how the dynamics of the paradigmatic conservative (area-preserving) standard map exhibits, in an exceptionally clear manner, the crossing from one statistics to the other. Our results unambiguously illustrate the domains of validity of both Boltzmann-Gibbs and Tsallis statistical distributions. Since various important physical systems from particle confinement in magnetic traps to autoionization of molecular Rydberg states, through particle dynamics in accelerators and comet dynamics, can be reduced to the standard map, our results are expected to enlighten and enable an improved interpretation of diverse experimental and observational results.

STANDARD MAP (Chirikov 1969)

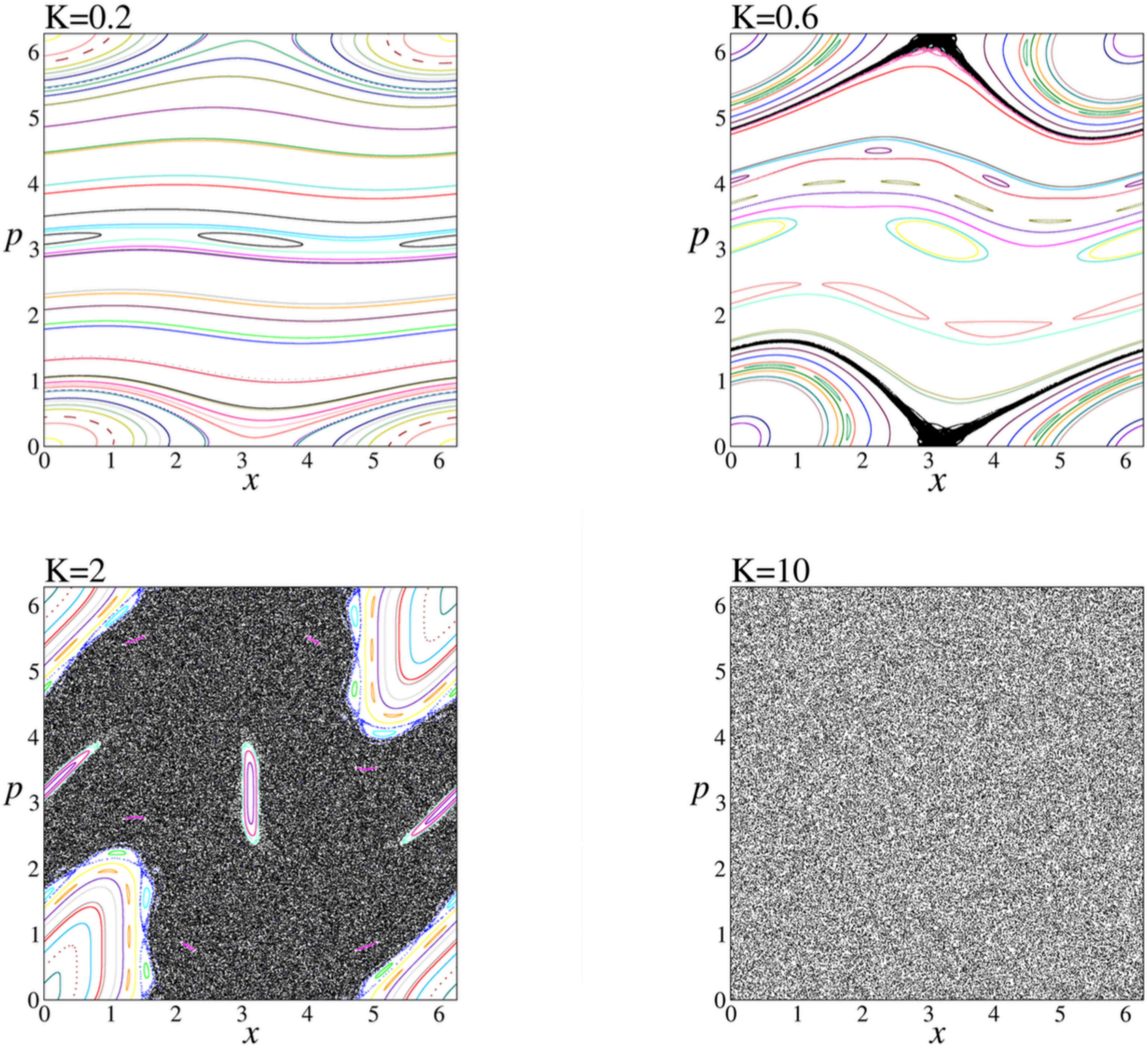
$$p_{i+1} = p_i - K \sin x_i \quad (\mathrm{mod}\ 2\pi)$$

$$x_{i+1} = x_i + p_{i+1} \quad (\mathrm{mod}\ 2\pi)$$

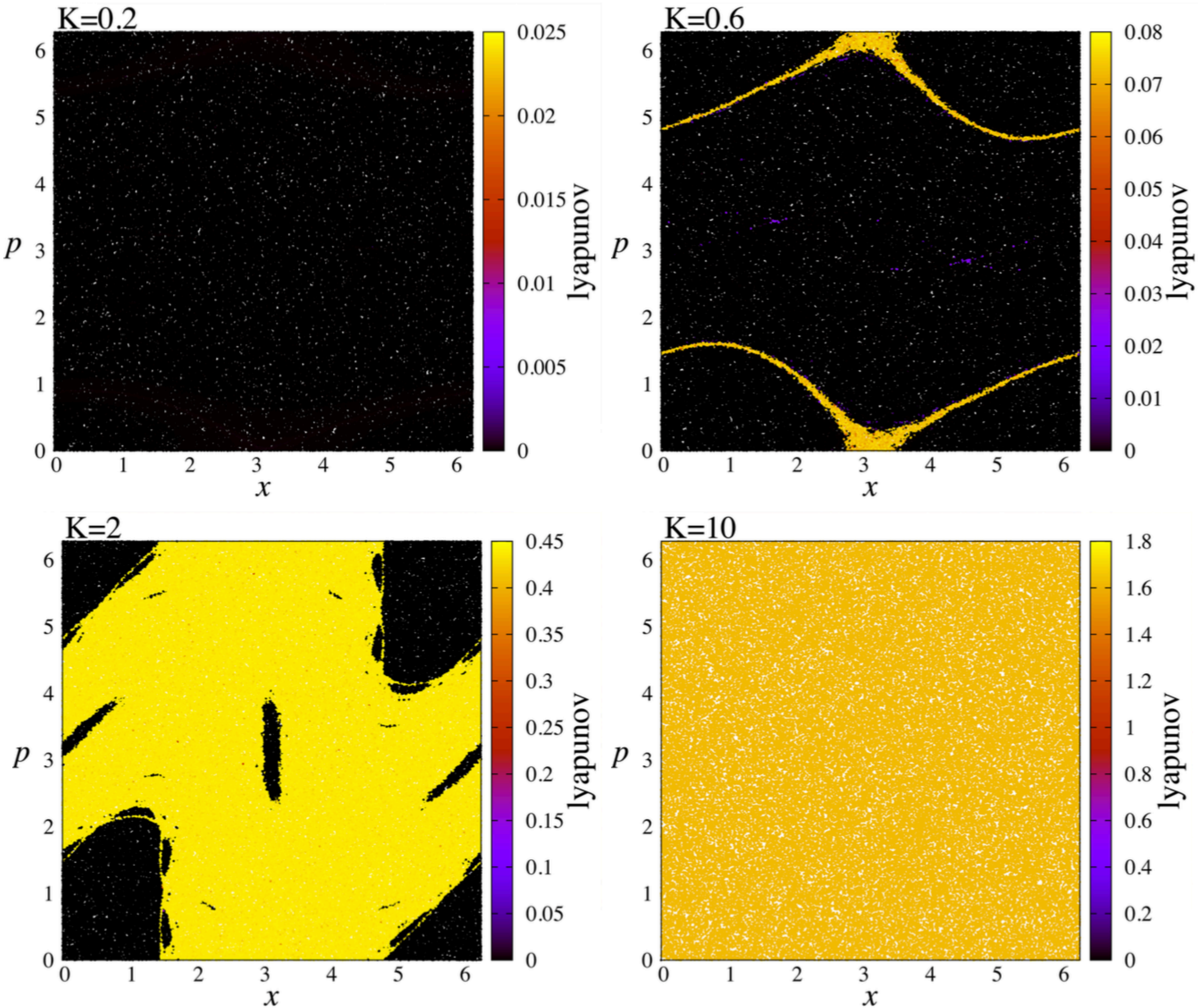
$$(i = 0,1,2,\ldots)$$

(area-preserving)

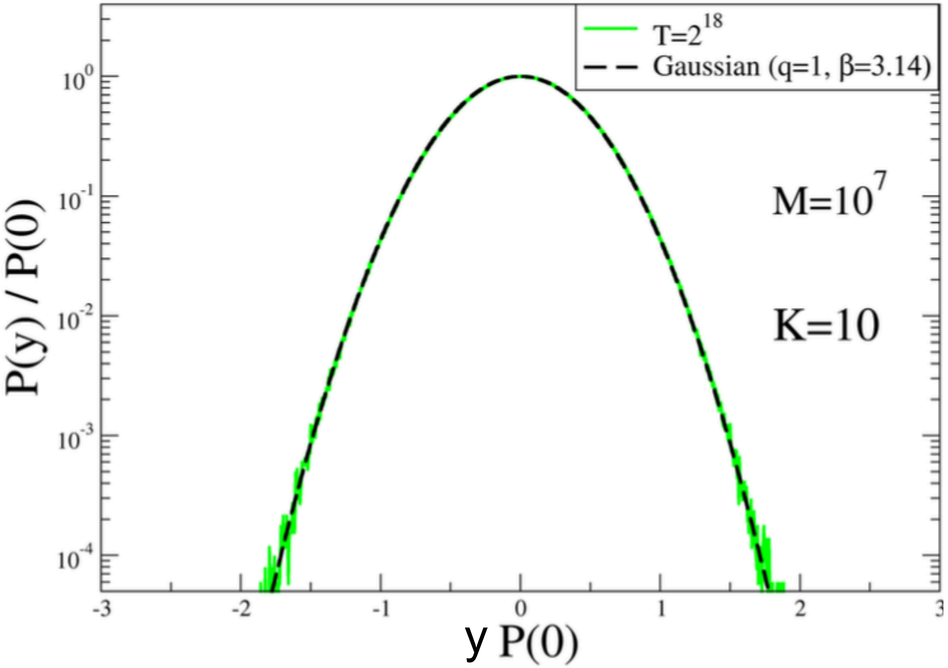
Particle confinement in magnetic traps,
particle dynamics in accelerators,
comet dynamics,
ionization of Rydberg atoms,
electron magneto-transport

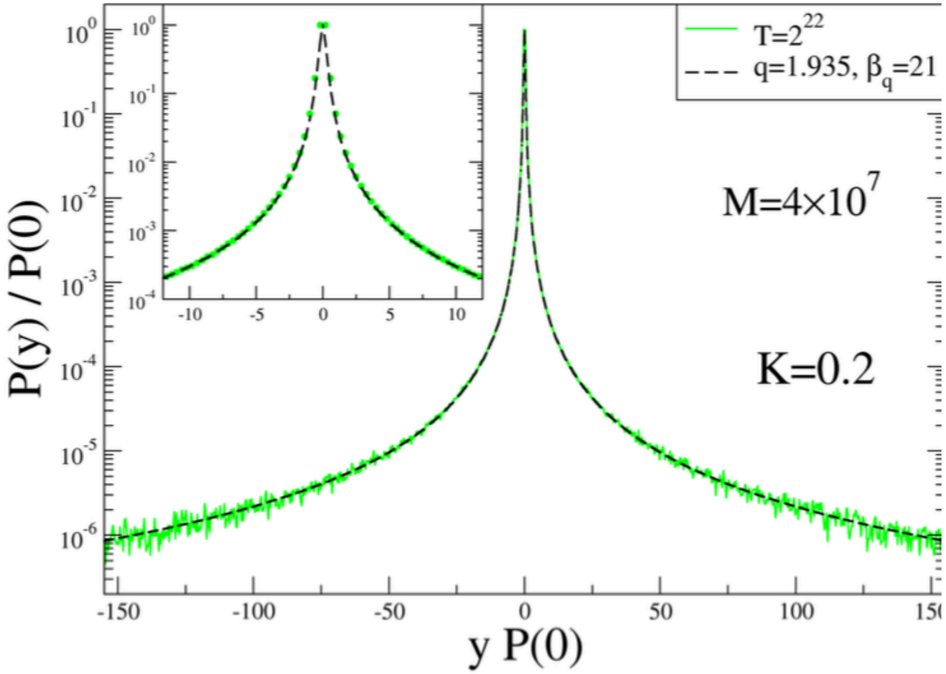


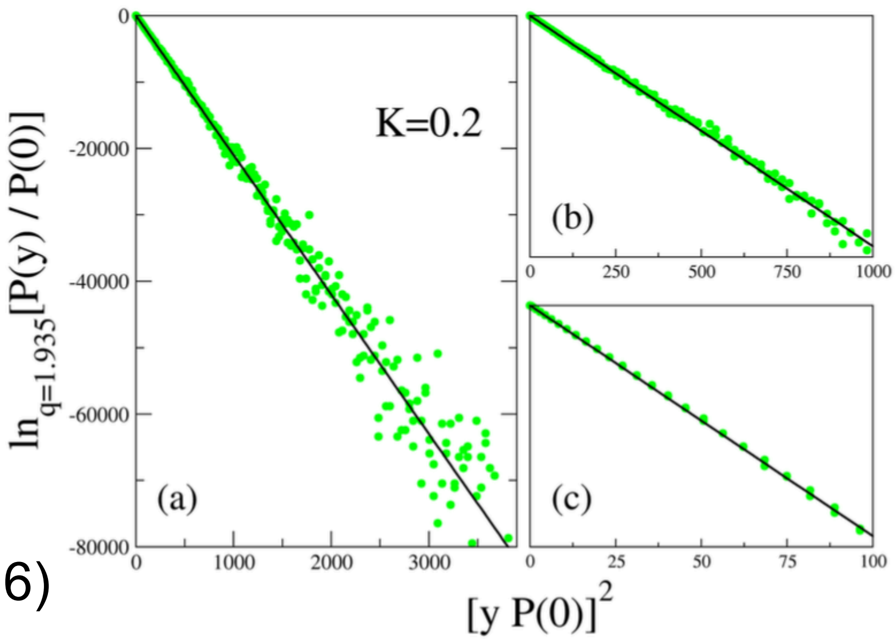
Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)



Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)







Tirnakli and Borges

Nature / Scientific Reports **6**, 23644 (2016)

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The standard map: From Boltzmann-Gibbs statistics to Tsallis statistics

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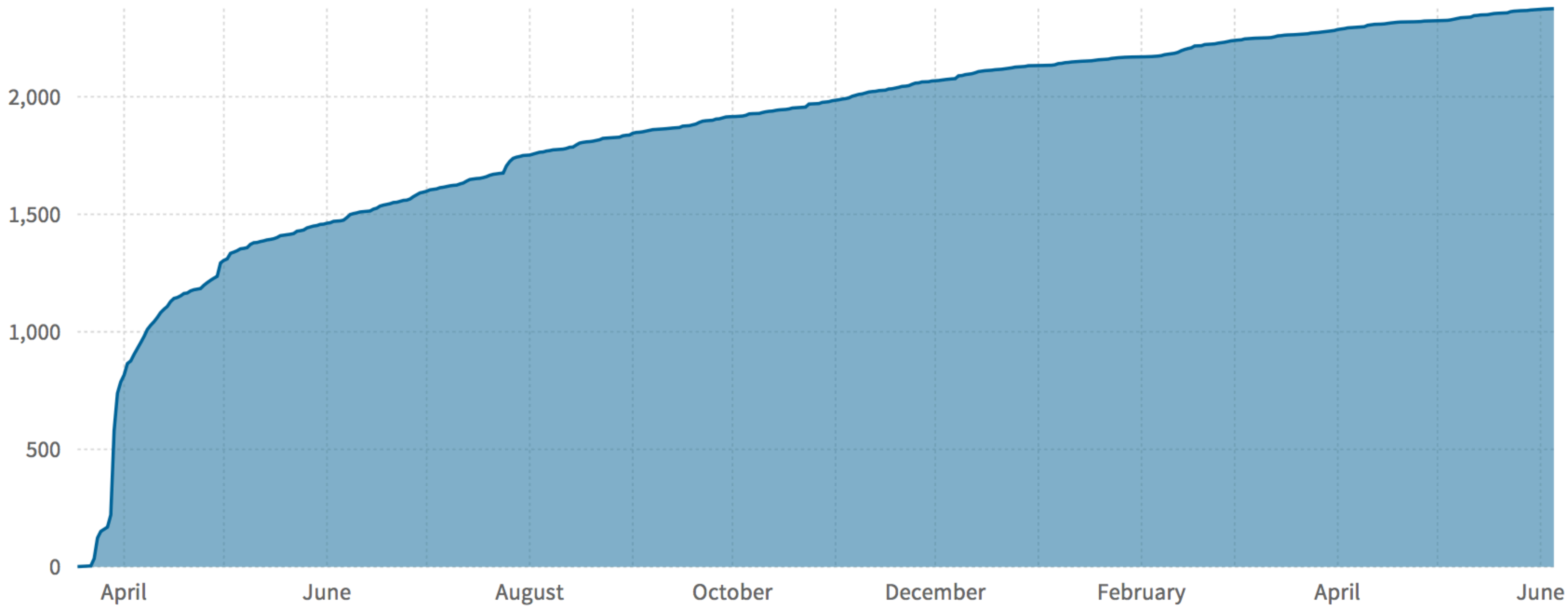

PLOS ONE | DOI:10.1371/journal.pone.0112534 December 1, 2014

RESEARCH ARTICLE

Universal Internucleotide Statistics in Full Genomes: A Footprint of the DNA Structure and Packaging?

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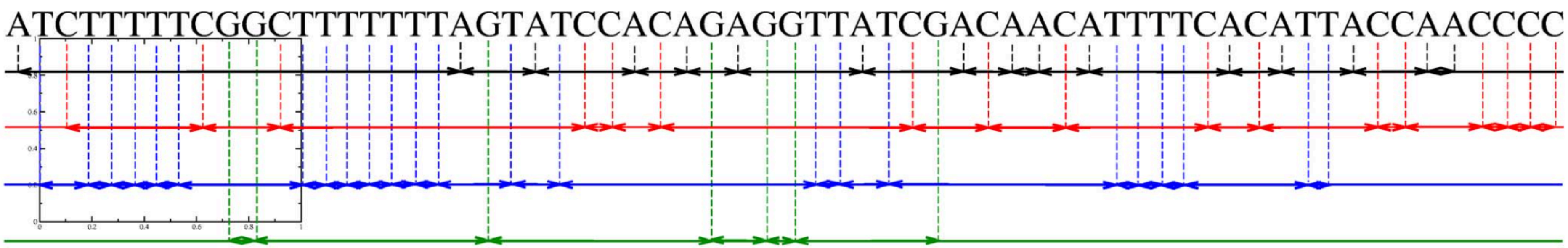
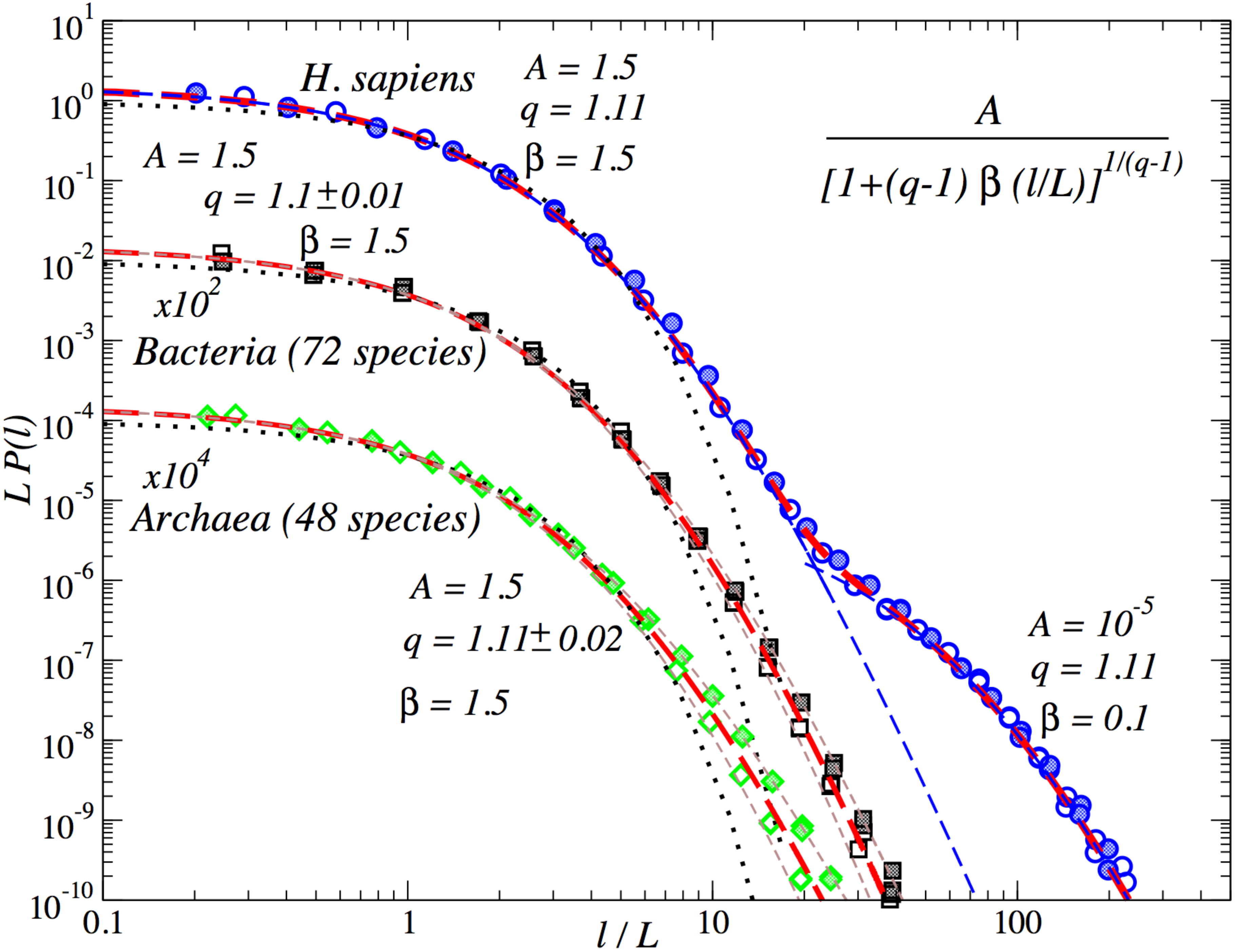


Figure 1. The procedure of the assessment of the four internucleotide interval sequences from the DNA primary sequence.



H. sapiens
A = 1.5
q = 1.11
β = 1.5
A = 1.5
q = 1.1±0.01
β = 1.5
x10^2
Bacteria (72 species)
x10^4
Archaea (48 species)
A = 1.5
q = 1.11± 0.02
β = 1.5
A = 10^-5
q = 1.11
β = 0.1
A / [1+(q-1) β (l/L)]^{1/(q-1)}
L P(l)
l / L

Computers and Mathematics with Applications 60 (2010) 2426–2432

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A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

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Department of Electrical Engineering, IIT Kanpur, UP-208016, India

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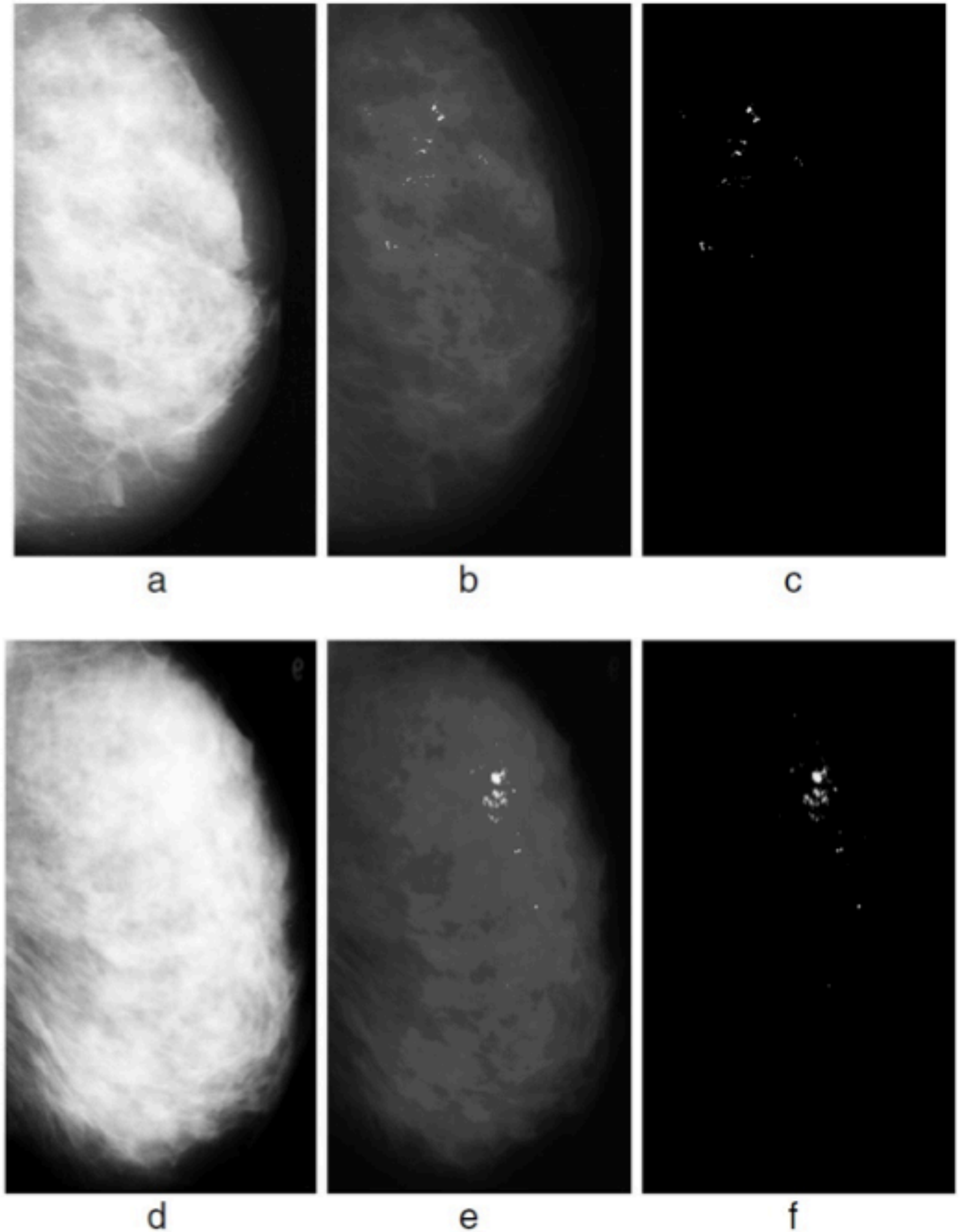
Type II fuzzy index
Shannon entropy
Mammograms
Microcalcification

ABSTRACT

This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter 'q', which depends on the non-extensiveness of a mammogram. In previous studies, 'q' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of 'q'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.


a
b
c
d
e
f

Brazilian Journal of Medical and Biological Research (2010) 43: 77-84
ISSN 0100-879X

Brain tissue segmentation using q-entropy in multiple sclerosis magnetic resonance images

P.R.B. Diniz[1], L.O. Murta-Junior[3], D.G. Brum[1], D.B. de Araújo[3] and A.C. Santos[1,2]

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[3]Departamento de Física e Matemática, Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto, Universidade de São Paulo, Ribeirão Preto, SP, Brasil

Abstract

The loss of brain volume has been used as a marker of tissue destruction and can be used as an index of the progression of neurodegenerative diseases, such as multiple sclerosis. In the present study, we tested a new method for tissue segmentation based on pixel intensity threshold using generalized Tsallis entropy to determine a statistical segmentation parameter for each single class of brain tissue. We compared the performance of this method using a range of different q parameters and found a different optimal q parameter for white matter, gray matter, and cerebrospinal fluid. Our results support the conclusion that the differences in structural correlations and scale invariant similarities present in each tissue class can be accessed by generalized Tsallis entropy, obtaining the intensity limits for these tissue class separations. In order to test this method, we used it for analysis of brain magnetic resonance images of 43 patients and 10 healthy controls matched for gender and age. The values found for the entropic q index were 0.2 for cerebrospinal fluid, 0.1 for white matter and 1.5 for gray matter. With this algorithm, we could detect an annual loss of 0.98% for the patients, in agreement with literature data. Thus, we can conclude that the entropy of Tsallis adds advantages to the process of automatic target segmentation of tissue classes, which had not been demonstrated previously.

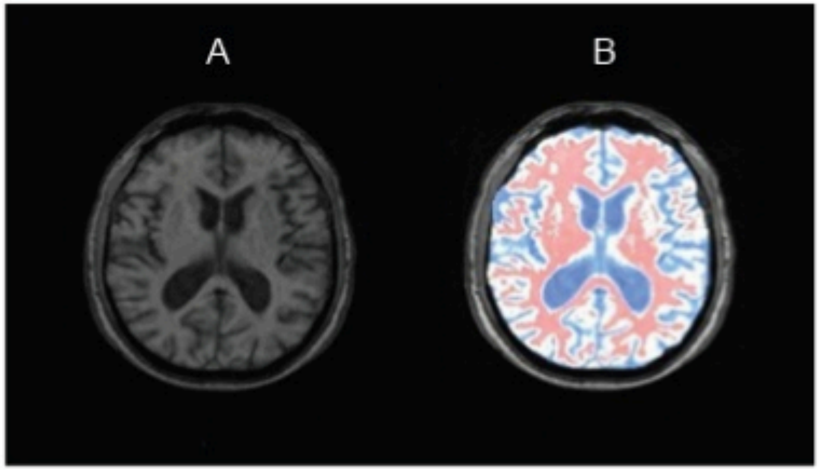


Figure 3. Maximum entropy segmentation example. *A*, Original image; *B*, image with the segmentation masks. Blue indicates cerebrospinal fluid, white indicates the gray matter, and red indicates the white matter.

The ideal q values for the segmentation of the classes are: CSF = 0.2, WM = 0.1, GM = 1.5, which have not been shown previously.

These characteristics allow its application to clinical routine.

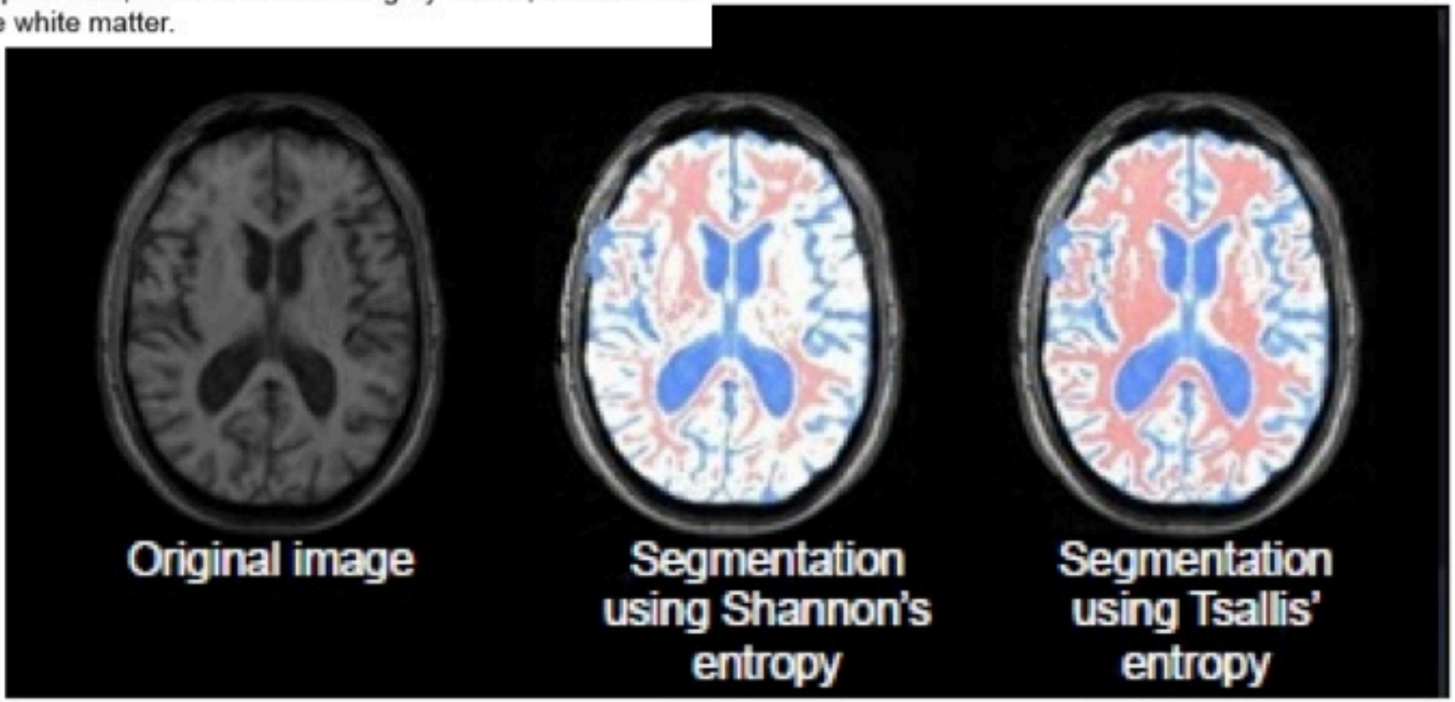


Figure 6. Segmentation using Shannon and Tsallis entropies.

PRB Diniz, LO Murta Jr., DG Brum, DB de Araujo and AC Santos
Braz. J. Med. Bio. Res. **43**, 77 (2010)

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

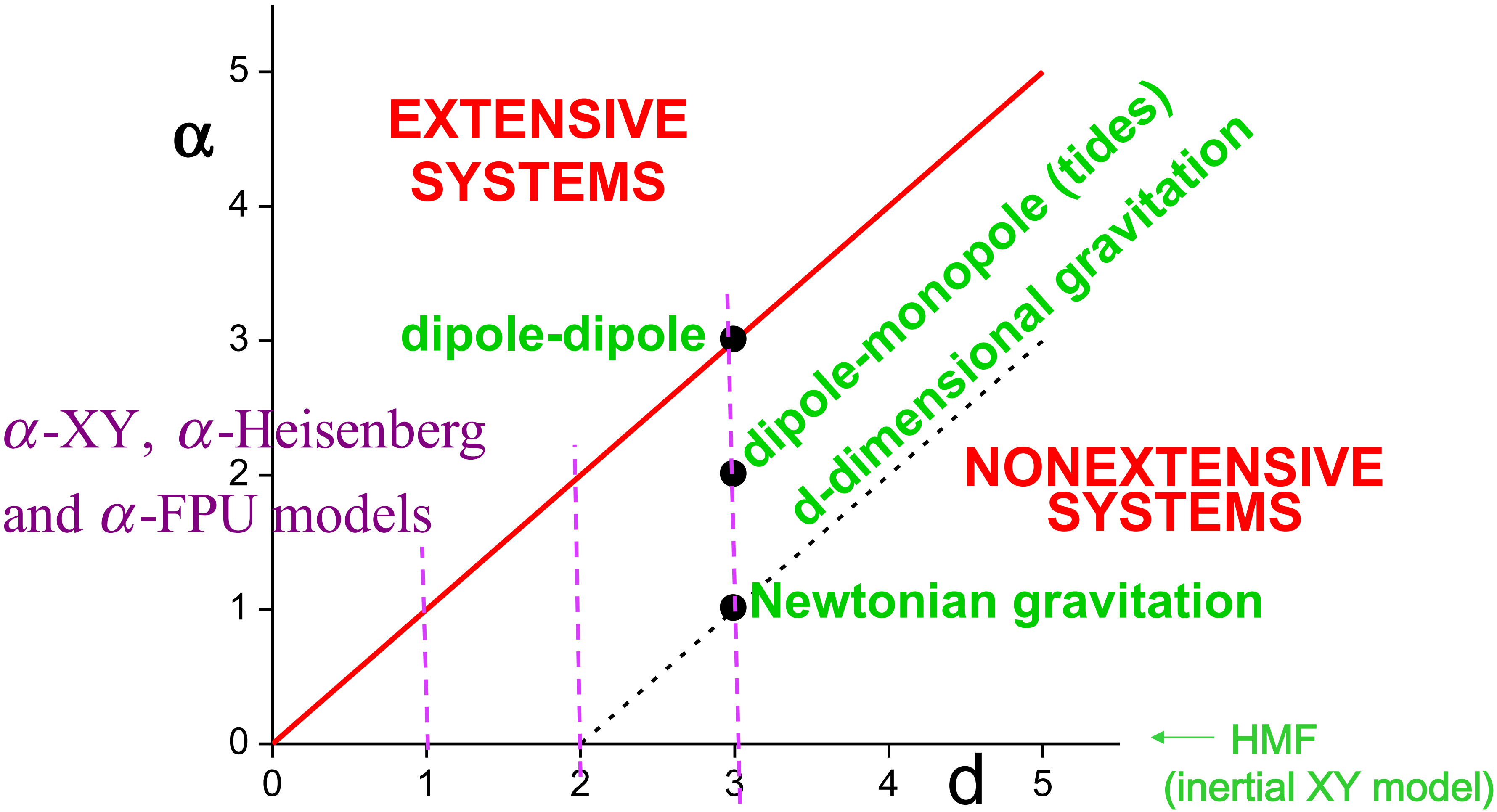
In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0,\ \ \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \qquad (\textit{short-ranged})$$

$$\textit{non-integrable if} \quad 0 \leq \alpha / d \leq 1 \qquad (\textit{long-ranged})$$



Physica A 393 (2014) 286–296

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Influence of the interaction range on the thermostatistics of a classical many-body system

Leonardo J.L. Cirto [a,*], Vladimir R.V. Assis [b], Constantino Tsallis [a,c]

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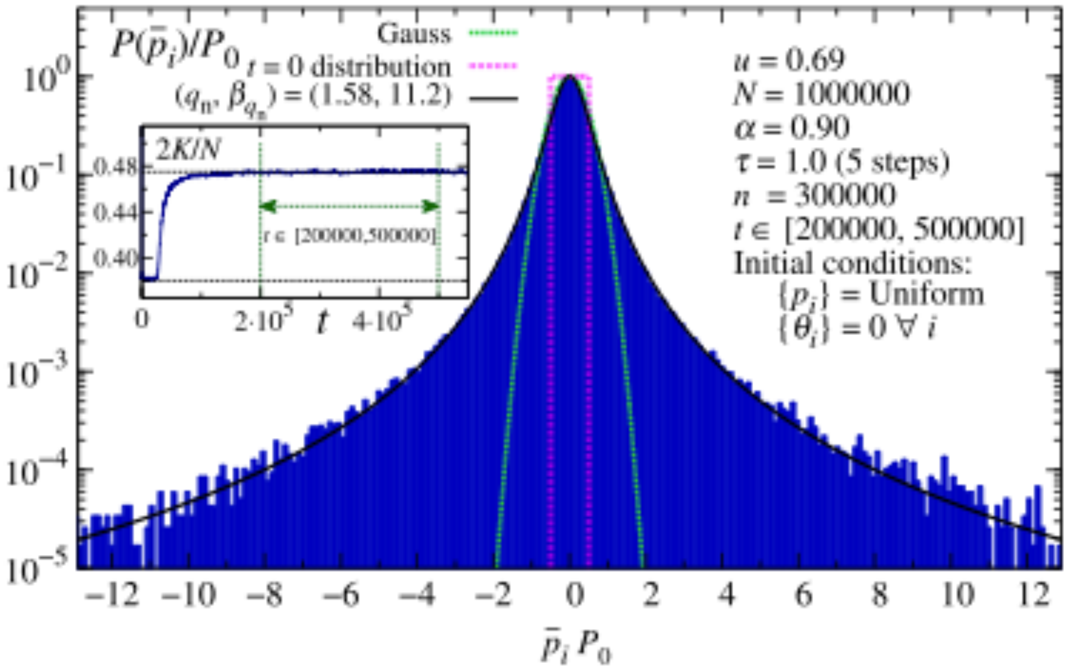
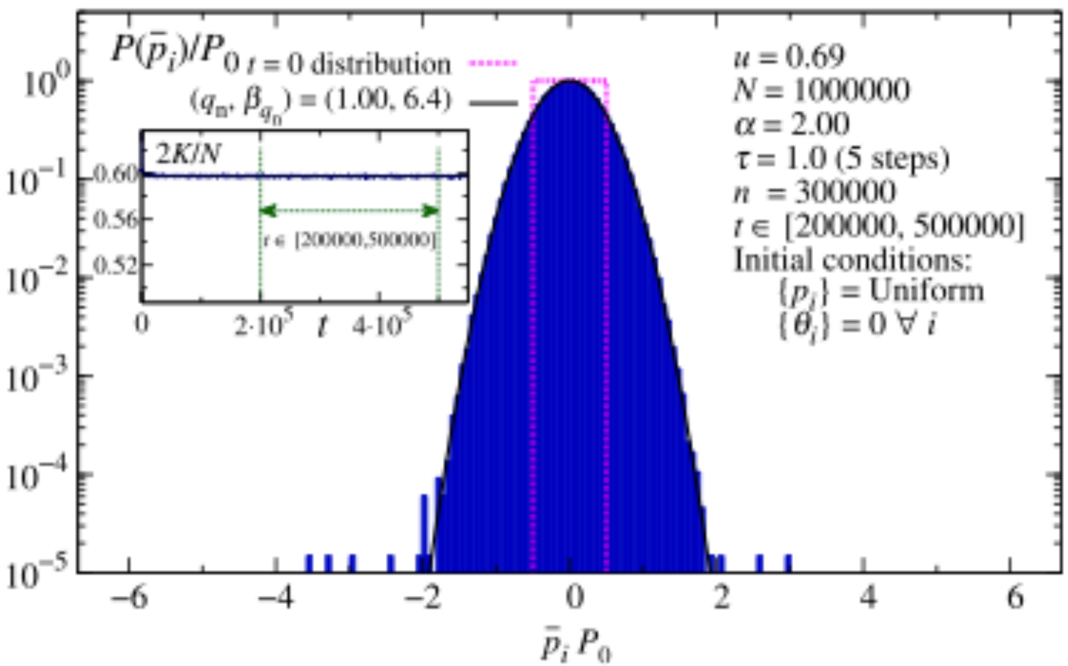
[c] Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA



$\alpha = 0.9$

yields

$q = 1.58$



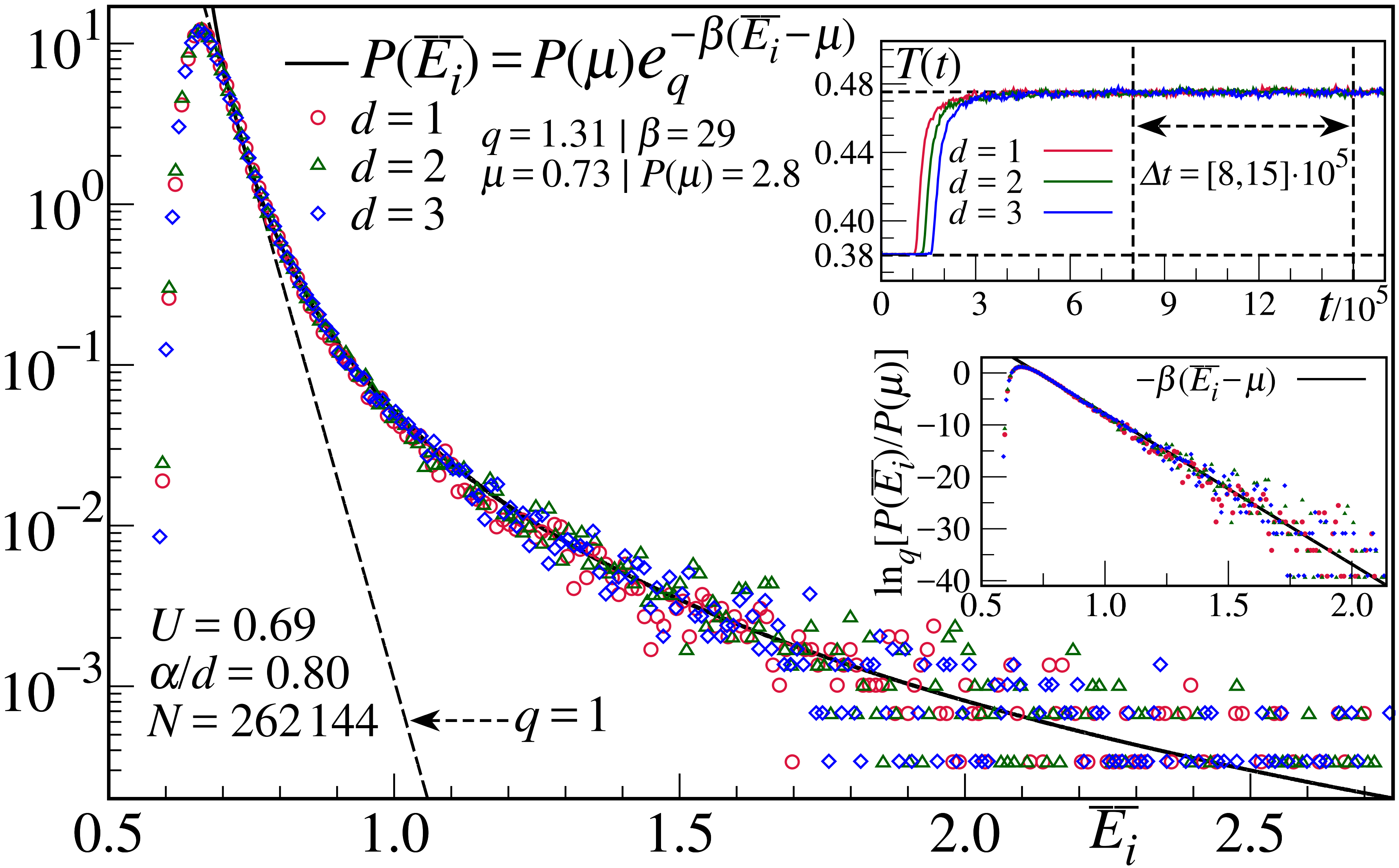
$\alpha = 2$

yields

$q = 1$

L.J.L. Cirto, V.R.V. Assis and C. T., Physica A **393**, 286 (2014)

ONE-BODY ENERGY DISTRIBUTION



L.J.L. Cirto, A. Rodriguez, F.D. Nobre and C. T. (2017)

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

November 2014

EPL, **108** (2014) 40006
doi: 10.1209/0295-5075/108/40006

www.epljournal.org

Fermi-Pasta-Ulam model with long-range interactions: Dynamics and thermostatistics

H. CHRISTODOULIDI[1], C. TSALLIS[2,3] and T. BOUNTIS[1]

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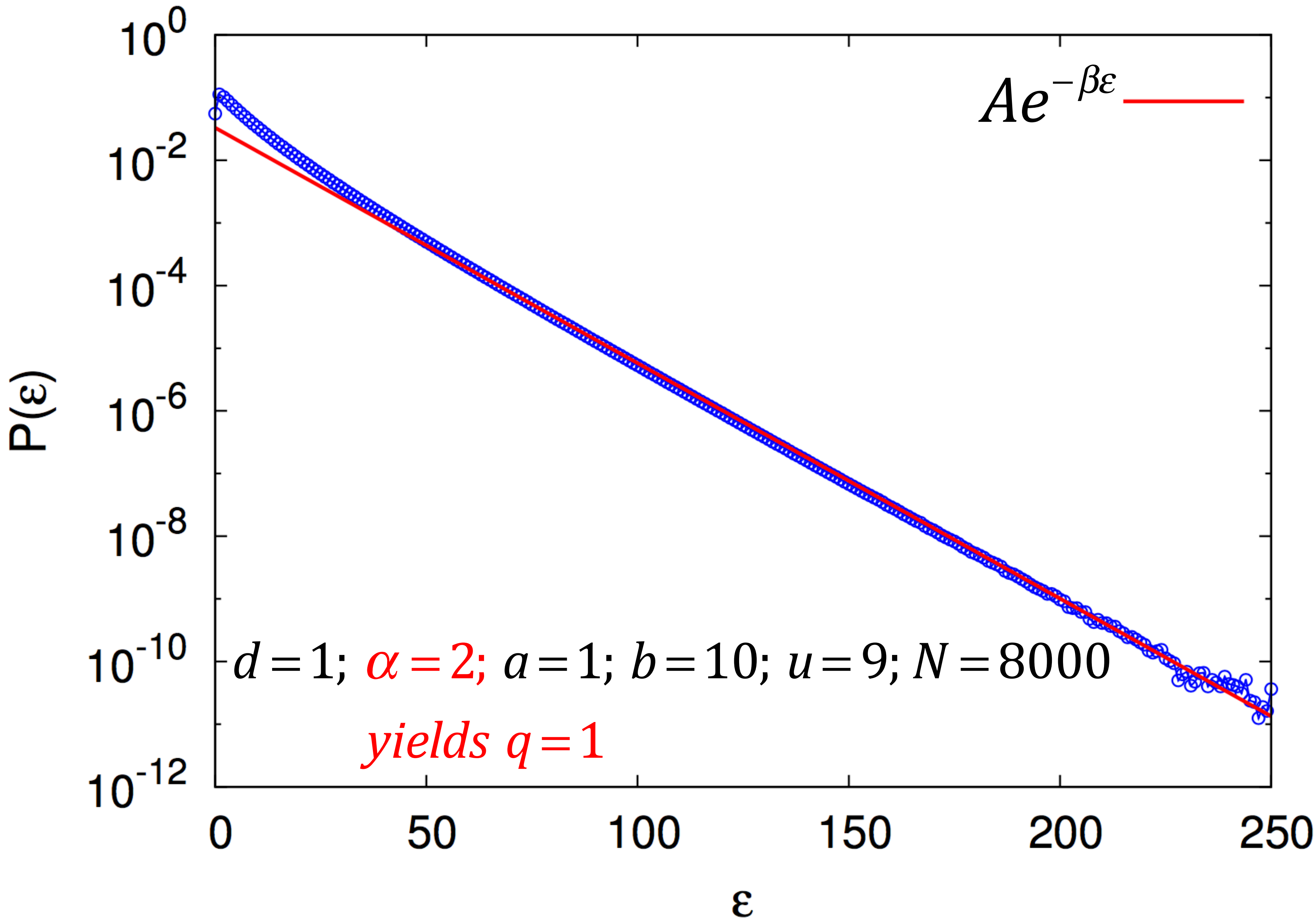
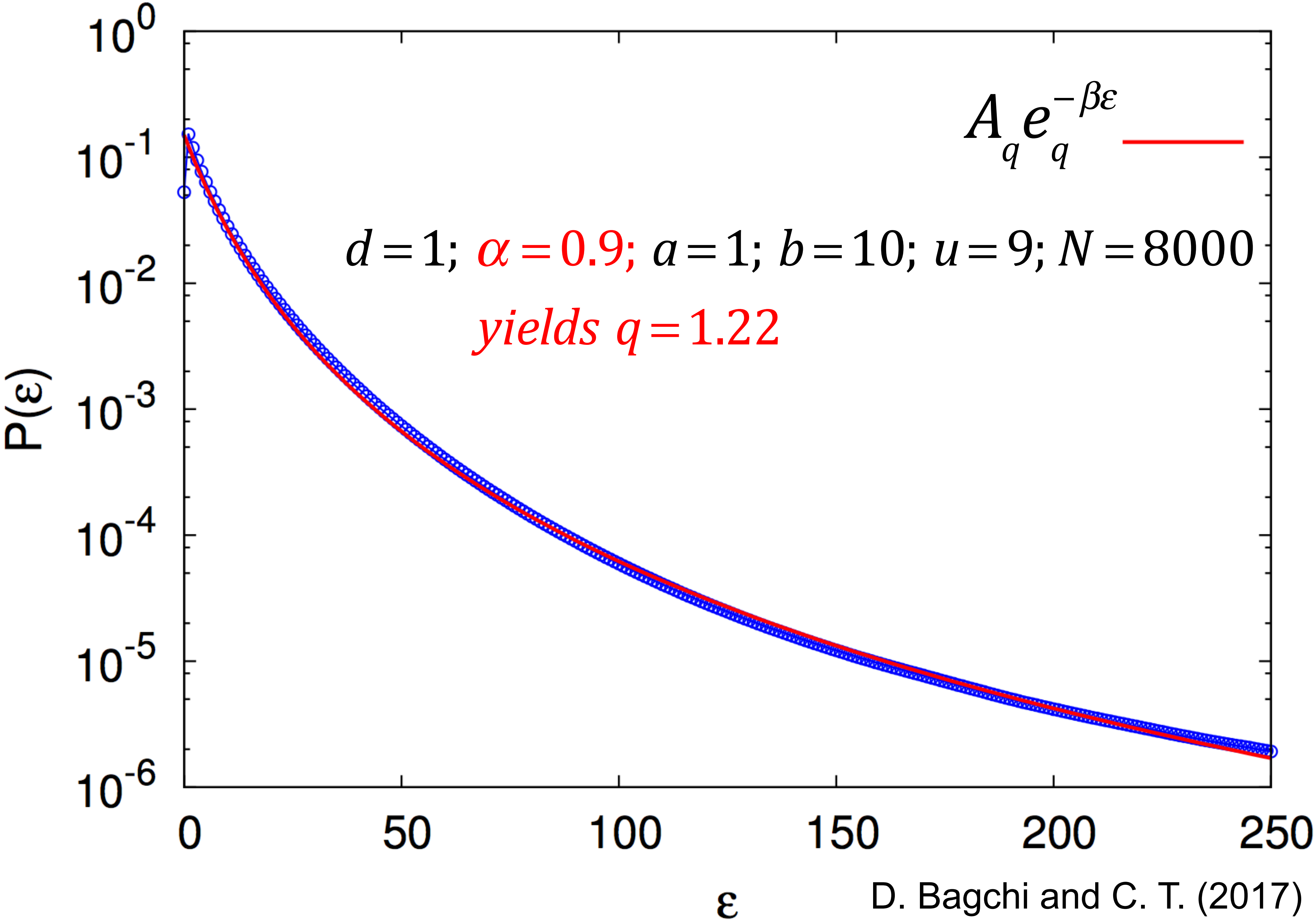
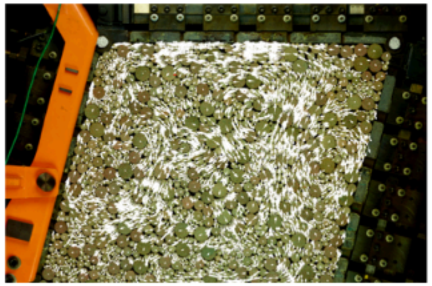
[3] *Santa Fe Institute - 1399 Hyde Park Road, Santa Fe, NM 87501, USA*

$$\mathcal{H} = \frac{1}{2}\sum_{n=1}^{N} p_n^2 + \frac{1}{2}\sum_{n=0}^{N}(x_{n+1} - x_n)^2 + \frac{b}{4\widetilde{N}}\sum_{n=0}^{N}\sum_{m=n+1}^{N+1}\frac{(x_n - x_m)^4}{|n-m|^\alpha} \quad (b > 0;\ \alpha \geq 0)$$



D. Bagchi and C. T. (2017)


$A_q e_q^{-\beta\varepsilon}$
$d=1$; $\alpha=0.9$; $a=1$; $b=10$; $u=9$; $N=8000$
yields $q=1.22$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
P(ε)
0
50
100
150
200
250
ε
D. Bagchi and C. T. (2017)



EDITORS' SUGGESTION

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

The velocity distribution of sheared granular media shows unexpected similarities with turbulent fluid flows.

Gaël Combe, Vincent Richefeu, Marta Stasiak, and Allbens P.F. Atman

Phys. Rev. Lett. **115**, 238301 (2015)

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

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Avenida Amazonas 7675, 30510-000 Belo Horizonte-MG, Brazil

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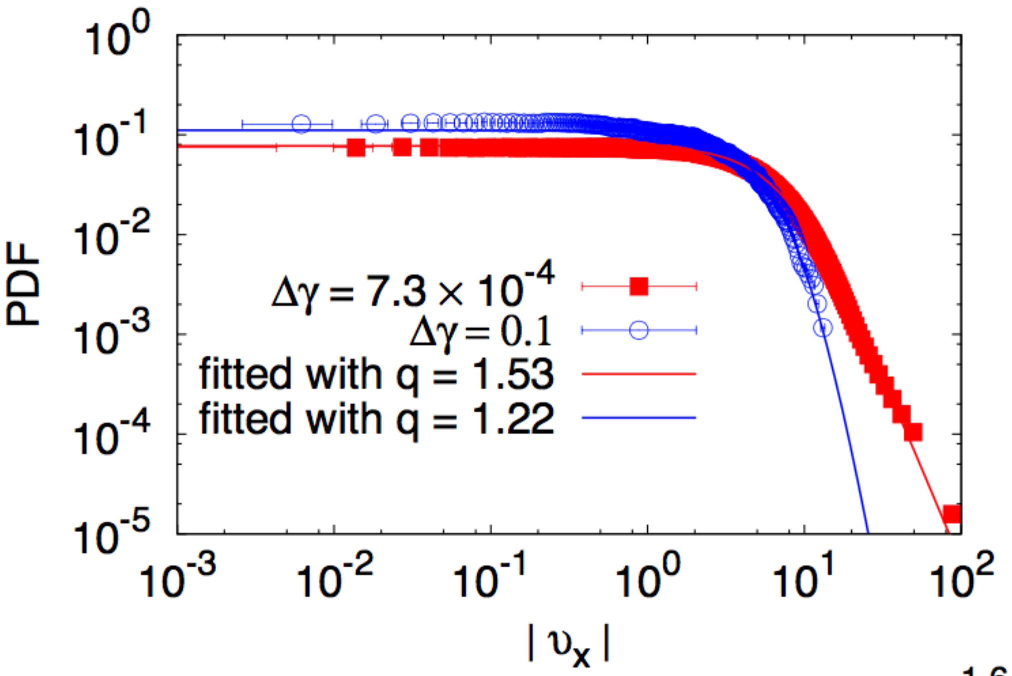
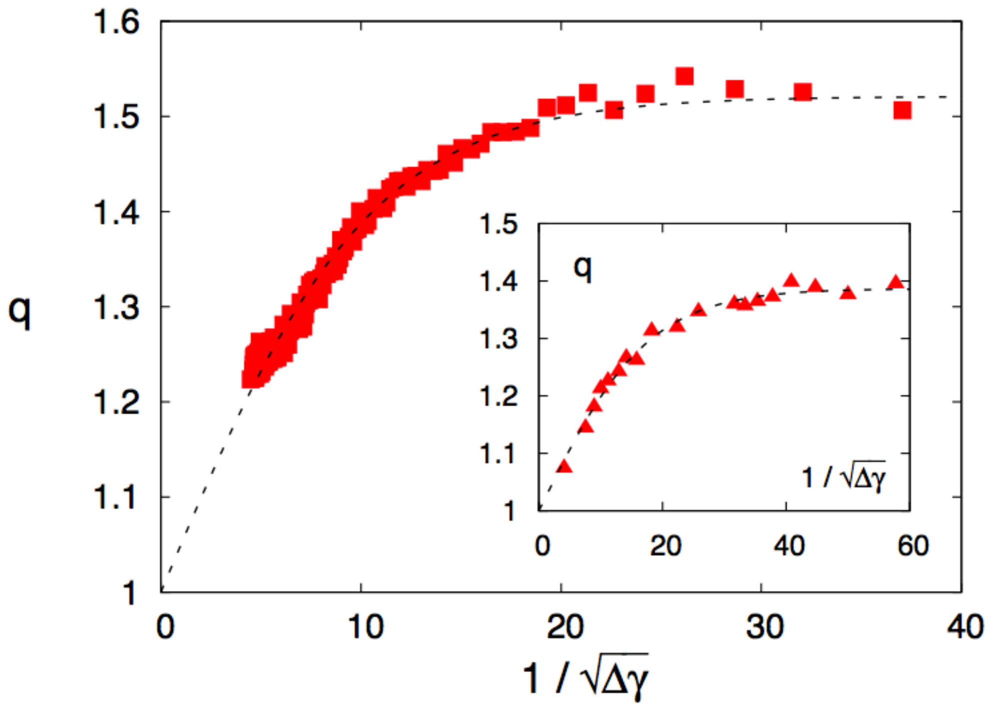




Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

$$\langle x^2 \rangle \propto t^\alpha$$

Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

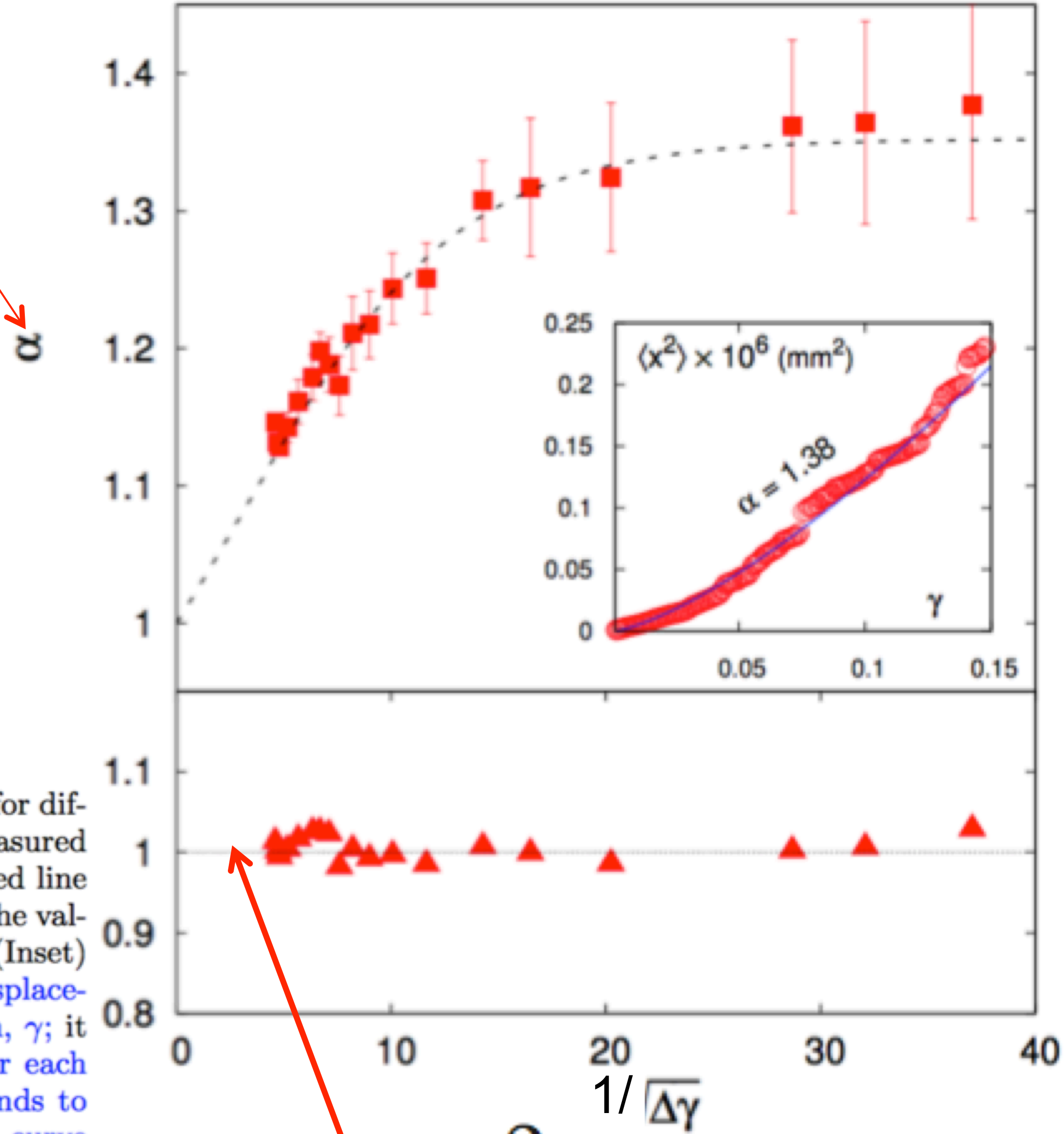


FIG. 4. Verification of the Tsallis-Bukman scaling law for different regimes of diffusion. (top) Evolution of the measured diffusion exponent α as a function of $1/\sqrt{\Delta\gamma}$ the dashed line is a direct application of the scaling law from the fit of the values shown in Fig. 3, $\alpha(1/\sqrt{\Delta\gamma}) = 2/[3 - q(1/\sqrt{\Delta\gamma})]$. (Inset) a typical diffusion curve showing the mean square displacement fluctuations, $\langle x^2 \rangle$, in function of the shear strain, γ; it allows the assessment of the diffusion exponent, α, for each strain window tested. In the case shown, it corresponds to the smallest strain window, the rightmost point in the curve at the main panel. Note that for a constant strain rate, γ is proportional to time. (Bottom) Measure of the deviation of the data relative to the scaling law prediction, as a function of $1/\sqrt{\Delta\gamma}$, showing an agreement on the order of $\pm 2\%$.

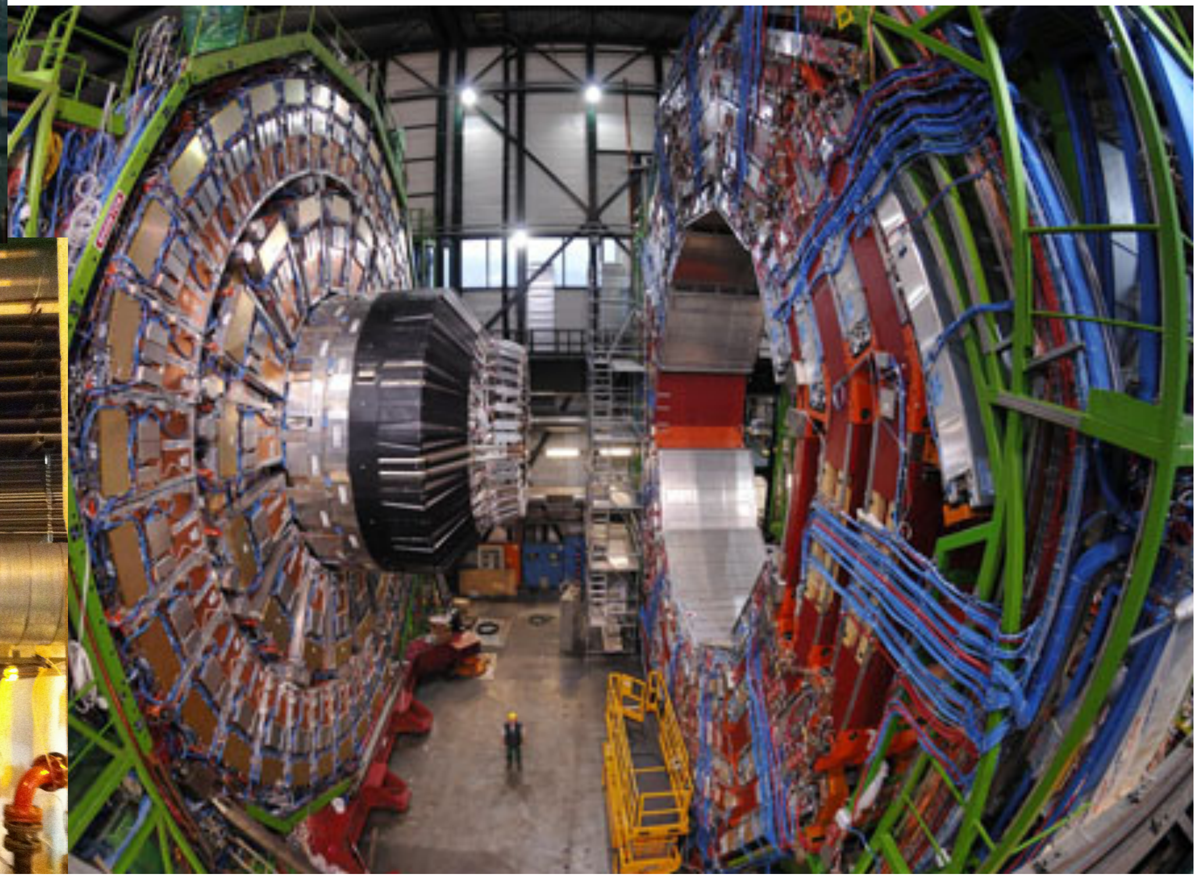
$$\alpha = \frac{2}{3-q}$$

CT and DJ Bukman, PRE **54** (1996) R2197

LHC (Large Hadron Collider)

CMS, ALICE, ATLAS and LHCb detectors

~ 4000 scientists/engineers from ~ 200 institutions of ~ 50 countries



PHYSICAL REVIEW D **87,** 114007 (2013)

Tsallis fits to p_T spectra and multiple hard scattering in pp collisions at the LHC

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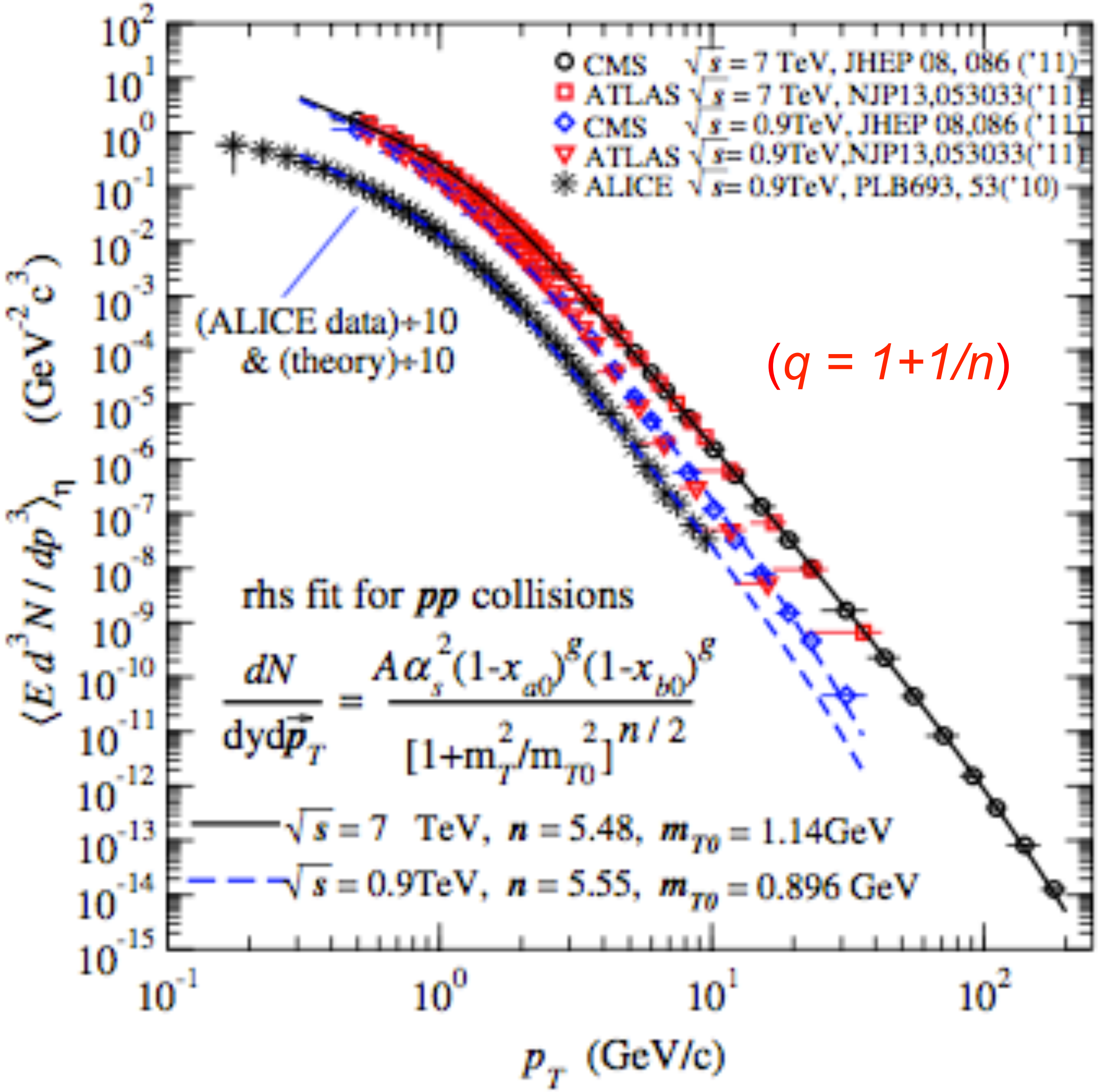
Grzegorz Wilk
National Centre for Nuclear Research, Warsaw 00-681, Poland
(Received 12 May 2013; published 5 June 2013)

Phenomenological Tsallis fits to the CMS, ATLAS, and ALICE transverse momentum spectra of hadrons for pp collisions at LHC were recently found to extend over a large range of the transverse momentum. We investigate whether the few degrees of freedom in the Tsallis parametrization may arise from the relativistic parton-parton hard-scattering and related processes. The effects of the multiple hard-scattering and parton showering processes on the power law are discussed. We find empirically that whereas the transverse spectra of both hadrons and jets exhibit power-law behavior of $1/p_T^n$ at high p_T, the power indices n for hadrons are systematically greater than those for jets, for which $n \sim 4$–5.



CMS $\sqrt{s}$ = 7 TeV, JHEP 08, 086 ('11)
ATLAS $\sqrt{s}$ = 7 TeV, NJP13,053033('11)
CMS $\sqrt{s}$ = 0.9TeV, JHEP 08,086 ('11)
ATLAS $\sqrt{s}$= 0.9TeV,NJP13,053033('11)
ALICE $\sqrt{s}$= 0.9TeV, PLB693, 53('10)
(ALICE data)÷10
& (theory)÷10
rhs fit for *pp* collisions
$$\frac{dN}{dyd\vec{p}_T} = \frac{A\alpha_s^2(1-x_{a0})^g(1-x_{b0})^g}{[1+m_T^2/m_{T0}^2]^{n/2}}$$
$\sqrt{s}$ = 7 TeV, n = 5.48, m_{T0} = 1.14GeV
$\sqrt{s}$ = 0.9TeV, n = 5.55, m_{T0} = 0.896 GeV
$\langle E\, d^3N/dp^3\rangle_\eta$ $(\mathrm{GeV}^{-2}c^3)$
p_T (GeV/c)

($q = 1+1/n$)

SIMPLE APROACH: TWO-DIMENSIONAL SINGLE RELATIVISTIC FREE PARTICLE

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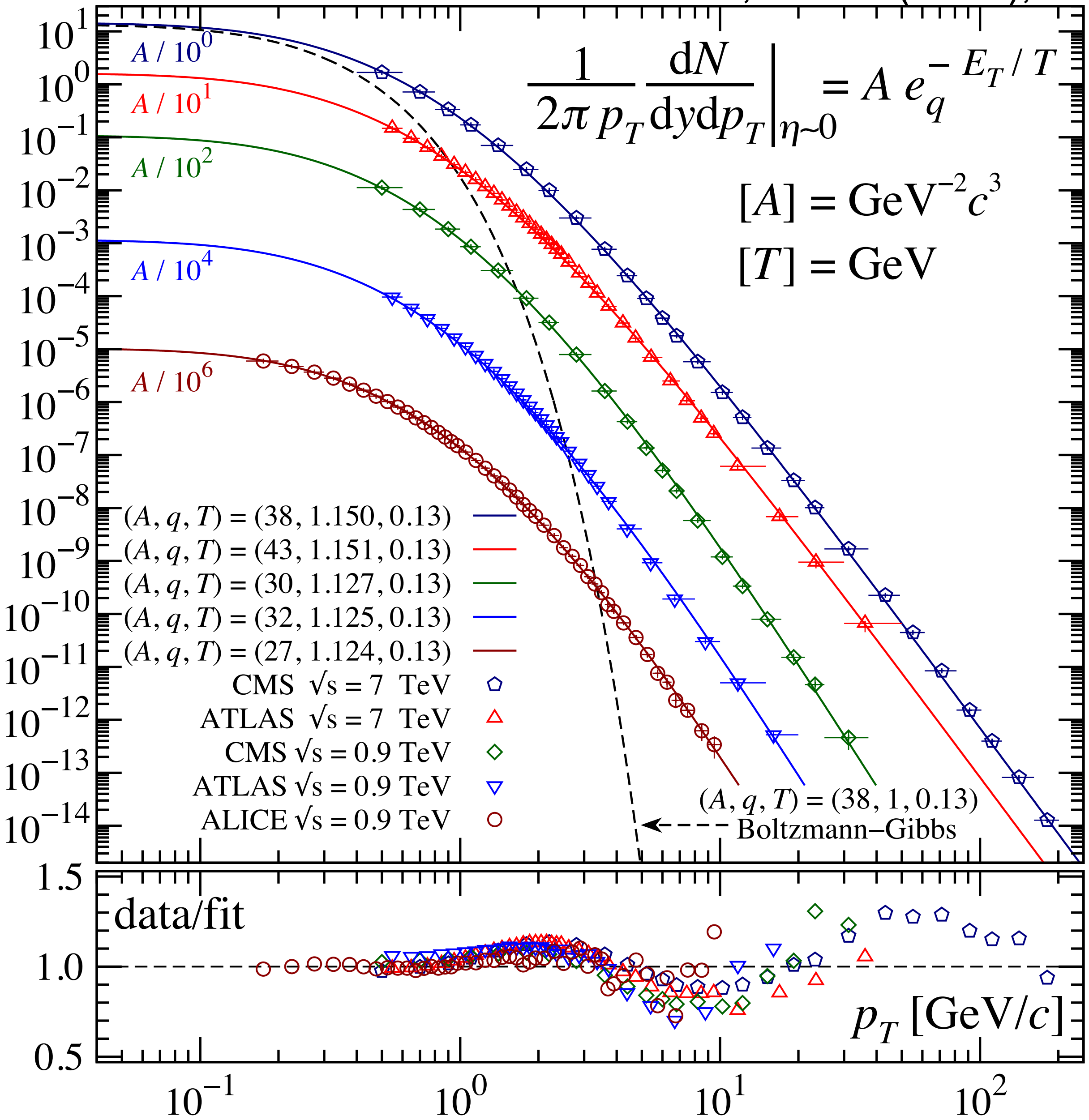
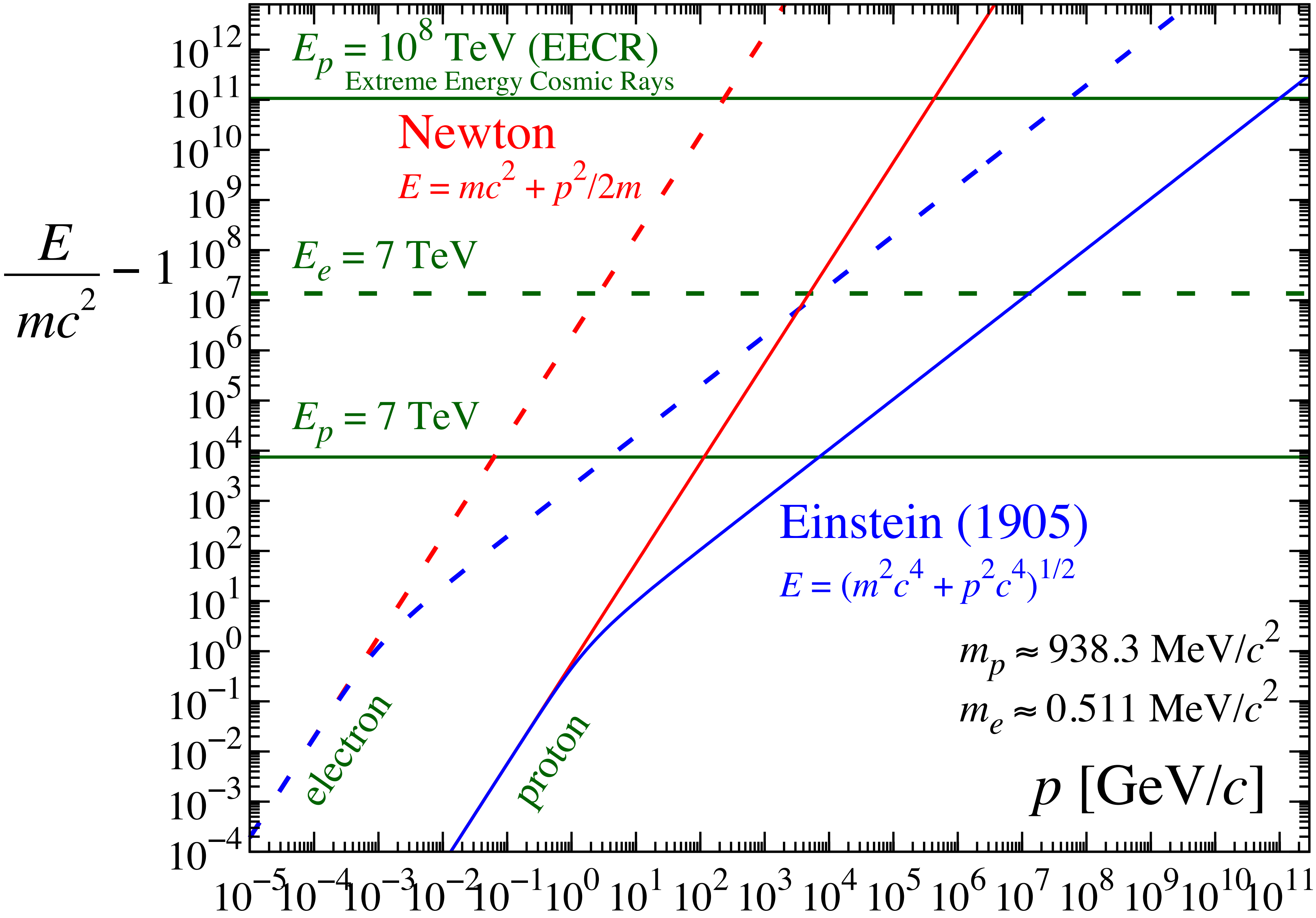
EPJ Web of Conferences **90**, 04002 (2015), and PRD **91**, 114027 (2015)



$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$



$E_p = 10^8$ TeV (EECR)
Extreme Energy Cosmic Rays
Newton
$E = mc^2 + p^2/2m$
$E_e = 7$ TeV
$E_p = 7$ TeV
Einstein (1905)
$E = (m^2c^4 + p^2c^4)^{1/2}$
$m_p \approx 938.3$ MeV/c^2
$m_e \approx 0.511$ MeV/c^2
electron
proton
p [GeV/c]
$\frac{E}{mc^2} - 1$

Tout le monde savait que c’ était impossible.

Il y avait un qui ne le savait pas.

Alors il est allé et il l’a fait.

Jean Cocteau (Marcel Pagnol, Winston Churchill, Mark Twain ...)

Si l’action n’a quelque splendeur de liberté, elle n’a point de grâce ni d’honneur.

Montaigne