Evolutionary dynamics of fitness driven walkers on a graph
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The availability of empirical data on humans and animal mobility has had a crucial impact on many dynamical processes found in both nature and society. Here we take a closer look at the influences of both mobility and non-trivial network architecture on how interactions among individuals progress. Conventional models that involve spatial games are limited to representing players as nodes in the network with interactions steered by the linkages to other nodes, apart from the game rules. In this work, instead of using nodes to represent individual agents, we use the nodes to represent communities where players are situated. Two general processes are involved in this study namely, the inter-node and intra-node dynamics, where each is described by its own spatio-temporal scale. Inside the nodes, individual players evolve under the context of the Ultimatum Game. This paradigm provides a measure, which is the average fitness of individuals inside a node, that can be interpreted as a mechanistic drive behind the mobility of agents on a network.

I. INTRODUCTION

Understanding the dynamics behind migration and/or mobility has recently sparked the interests of researchers in various fields such as evolutionary biology, anthropology, epidemiology, geography, and genetics, to name a few [1–9]. Migration in broad terms has been defined as "an adaptive response to changes in resource availability, to escape from competition, and/or to reach newer habitats" [3]. In population-genetics for example, geographical patterns of humans have been found to support and, to a limited extent, explain heterozygosity [1]. From a geographical point of view, on the other hand, the nature of mobility (and stability) of individuals are of great interests as it sheds light on certain sociological issues such as community attachment and participation, socio-economic contributions and investments [4]. In addition, the availability of empirical data on how humans move has proven that mobility plays a crucial role in the study of many dynamical processes including, but not limited to, examples such as epidemics spreading [5–10] or social interactions [11, 12]. Similar evidence has been observed in the study of animals [13, 14], insects or even colonies of bacteria [15, 16]: mobility and distance cannot be neglected in the analysis of interactions and collective behavior.

Meanwhile, attention has been increasingly focused on how topology of networks affect dynamical processes [17–19]. In fact, patterns of interactions in our world are far from being mean-field or random, thus leading sometimes to unexpected and dramatic outcomes in dynamical processes. As a consequence, evolutionary graph theory has been keen on implementing game theoretic models on networks with varying topological properties [20, 21].

With the underlying assumption that individuals make decisions that are geared towards maximizing their utility function, we deem it appropriate to tackle the problem in an evolutionary game theoretic framework. Here we study the role of both mobility and non-trivial network topology by looking at how interactions among individuals evolve. Traditionally, studies involving spatial games are limited to representing players as nodes in the network and where their interactions are guided by their connectivity to other nodes, apart from the game rules [20–22].

Wright [23] was the first to postulate a model in which

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evolution was considered on multiple islands. In Wright's island model is big population is subdivided into smaller "islands". The individuals migrate between these islands with a uniform migration rate and the number of individuals on an island remains constant over time. In this manuscript we couple the two ideas together. The island model is represented by a specific network type. Each node harbours a population which evolves according an evolutionary game. Migration disturbs this evolution. We assess the impact of the network structure on the evolution of the strategies and discuss the importance of the structure and mobility dynamics on the maintenance or extinction of strategies in a population.

In this work, we take it a step further where instead of using nodes to represent individual agents, we use the nodes to represent communities that contain the individual players that interact with each other in the context of the Ultimatum Game. This paradigm also provides a measure, namely fitness, that can be interpreted as a force driving the movement of agents on a network.

II. MODEL

In this model, we consider a network of N nodes. M agents are then distributed among the network nodes, where $M \gg N$. Agents can interact with each other only when they are on the same node.

Two scales are taken into consideration: spatial and temporal, which are further broken down into two, the inter-node and intra-node dynamics. For the intra-node dynamics, the population in each node is allowed to evolve for an average population generation time given by M/N time-steps. Within that period, the residents of the node play the Ultimatum game from which individual payoffs are calculated. For each complete generation, the average fitness of the population is determined; this is interpreted as the fitness of the node. After one generation, individuals can decide either to migrate with a probability p to a neighbouring node or remain in their current location with a probability $1-p$. The way individuals choose the target node to migrate to is biased by the fitness of the target node. Also a small probability q to choose neighbours at random is included, taking into account the propensity of individuals to make errors when actually executing the decision or exploration of strategies [24].

Summing up, our model is made up of two kinds of dynamics, taking place at two different timescales: (i) an intra-node dynamics, which lasts for M/N timesteps, during which agents accumulate fitness and evolve according to a Moran process, and (ii) an inter-node dynamics, taking place at the end of an intra-node generation where agents can migrate.

A. Intra-Node Dynamics

Here we consider the dynamics of the players within a node. The players on a node are in a well mixed state, meaning that they can interact with all the others, and play the so-called Ultimatum game. In an Ultimatum game, two players are chosen at random from the population. One of them acts as proposer while the other acts as responder and these roles are assigned randomly. The goal is to split a given reward amongst themselves. The proposer proposes a way to split the amount and the responder can decide whether to accept the offer or not. If the responder is amenable to the proposition, a deal is sealed; otherwise, none gets anything.

Intuitively, a proposer should offer to split the reward such as that the responder gets the minimum amount; at the same time, a responder would be better off accepting whatever offer than getting nothing. However, experimental studies have shown quite a different scenario, where proposers actually offer 40 to 50 percent of the amount to be split and the responders also decline offers which are less than 30 percent [25–27]. The experiments were performed worldwide indiscriminate of societies and with widely varying valuables at stake.

Inherently, the Ultimatum game has a number of different strategies which could be adopted; however for this particularly study, we take inspiration from the work by Nowak et. al. [28] using the mini-ultimatum game. Since each individual can assume either the role of a proposer and of a responder, we consider a set of two actions per strategy. An individual can offer an amount α when acting as a proposer and can aspire to get at least β when a responder. Hence (α, β) make up the strategy space. If one assumes the amount to be split to be 1, α and β lie between 0 and 1. Since the proposer expects a payoff of $1-\alpha$, this value should be larger than the aspiration level β, i.e. $\alpha+\beta \leq 1$ [28]. Without loss of generality, we limit our study to two outlooks taken from an individual's point of view. Each individual has two roles, being a proposer and an acceptor. Being a proposer the agreed amount to split can be high h or low l. Similarly when an acceptor the response could be to accept a high offer h or a ow offer l. By the earlier argument we are bound to $0 < l < h < 1/2$ [28, 29]. Thus there are four possible strategies, LL, LH, HL or HH, where the first letter is for proposing level and the second for the acceptance level.

We summarize the payoffs for the different possible strategies in the Ultimatum game.

$$\begin{array}{c} \\ LL \\ HL \\ HH \\ LH \end{array} \begin{array}{c} \begin{array}{cccc} LL & HL & HH & LH \end{array} \\ \begin{pmatrix} 1 & 1-l+h & h & l \\ 1-h+l & 1 & 1 & 1-h+l \\ 1-h & 1 & 1 & 1-h \\ 1-l & 1-l+h & h & 0 \end{pmatrix} \end{array}. \quad (1)$$

In a finite population size of $m_i(t)$ individuals on a node i. The population size changes over time due to the inter-node dynamics imposed on the system. For the time being however, we will focus on describing the intra-node dynamics and therefore dropping the time component t in the population count m_i.

Now we consider the number of individuals playing different strategies to be a, b, c and d for the four strategies LL, HL, HH, LH, respectively where of course $a + b + c + d = m_i$, the total population of the node i. The fitnesses are then given by

$$\begin{aligned} f_{LL} &= \frac{1}{m_i}\left[a + (1 - l + h)b + (h)c + (l)d\right] \\ f_{HL} &= \frac{1}{m_i}\left[(1 - h + l)a + b + c + (1 - h + l)d\right] \\ f_{HH} &= \frac{1}{m_i}\left[(1 - h)a + b + c + (1 - h)d\right] \\ f_{LH} &= \frac{1}{m_i}\left[(1 - l)a + (1 - l + h)b + (h)c + (0)d\right] \end{aligned} \quad (2)$$

and the average fitness of the population is,

$$\bar{f}_i = \frac{1}{m_i}\left[a\ f_{LL} + b\ f_{HL} + c\ f_{HH} + d\ f_{LH}\right]. \quad (3)$$

which is fitness of node i.

For a large finite population we can safely ignore self interactions. The process by which the dynamics proceeds in the finite population is by virtue of the Moran process. In a Moran process, an individual is chosen for reproduction according to its fitness; simultaneously a random individual is randomly chosen to die. Thus, for each of the strategy $s \in \{LL, HL, HH, LH\}$, we have transition probabilities of either increasing the number of players by one, T_s^+ or decreasing it by the same amount, T_s^-; else, the system remains unchanged with probability, $1 - T_s^+ - T_s^-$, where,

$$\begin{aligned} T_s^+ &= \frac{x_s f_s}{\bar{f}}\left(\frac{\sum_{r\neq s} x_r}{m_i}\right) \\ T_s^- &= \frac{x_s}{m_i}\left(\frac{\sum_{r\neq s} x_r f_r}{\bar{f}}\right), \end{aligned} \quad (4)$$

x_r is the number of players playing strategy r and f_r is the fitness of that particular strategy. Thus, in all, there are 12 transition probabilities, three for each strategy.

B. Inter-Node Dynamics

Inter-node dynamics commences after one complete generation, which is set to be $\frac{M}{N}$ timesteps. This is the part where the movement of agents on the network happens. In the inter-node phase, each individual in a node decides with a probability $p \in [0, 1]$ (mobility parameter) to move to a neighboring node and with $(1-p)$, otherwise. If an individual residing in node i chooses to migrate, it does so by either randomly choosing a node j to migrate to which has a small probability q (random jump parameter) of realizing or by choosing a node j proportional to its average fitness. The fitness of a node is calculated using Eq. 3. Figure II A shows a schematic diagram of the dynamics where individuals inside node i would have a higher probability of moving to node j since it has the highest fitness value compared to all other neighbors of i. Mathematically, the probability that agents in node i migrate to node j is,

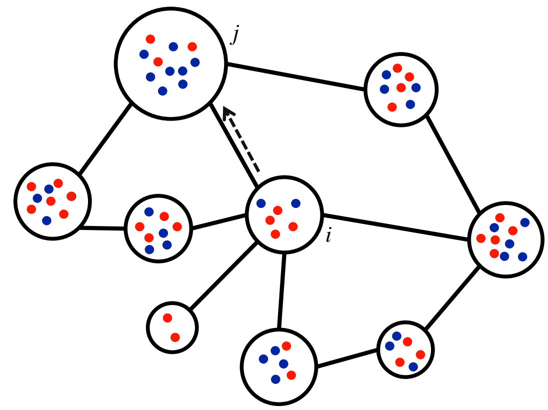


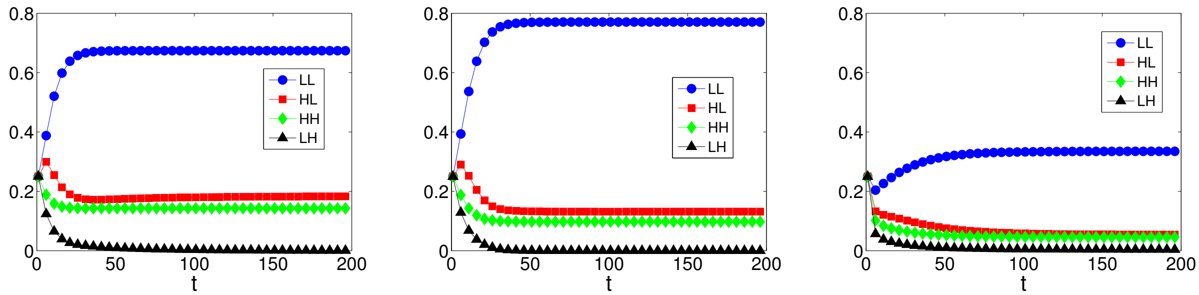
FIG. 1. Schematic diagram of the model. In each node, individuals with different strategies, for simplicity blue and red, are present. Each individual can move to a neighboring node with a probability that is biased by the fitness of that node. The node size in this figure corresponds to the fitness of that particular node, i.e. a bigger node has a greater fitness value than a smaller one. Also a small probability that an individual makes a random jump to a node is considered.

$$\Pi_{i\to j} = p\left[\Theta(\bar{f}_j - \bar{f}_i)\left(q\frac{1}{\sum_l a_{il}} + (1 - q)\frac{a_{ij}(\bar{f}_j - \bar{f}_i)}{\sum_l a_{il}(\bar{f}_l - \bar{f}_i)}\right)\right], \quad (5)$$

where a_{ij} is the element in row i and column j of the adjacency matrix (represents the graph under consideration) and takes either one of two values: 1 if an edge i, j exists or 0, otherwise. $\Theta(x)$ is the Heaviside step function which satisfies $\Theta(x > 0) = 1$ and $\Theta(x \leq 0) = 0$. This makes sure that the individual will move to node j only if the fitness of node j is greater than the fitness of the individual's current node i. Thus the movement of the individuals is governed by a fitness-biased random walk on a graph, with a small probability of agents making random jumps.

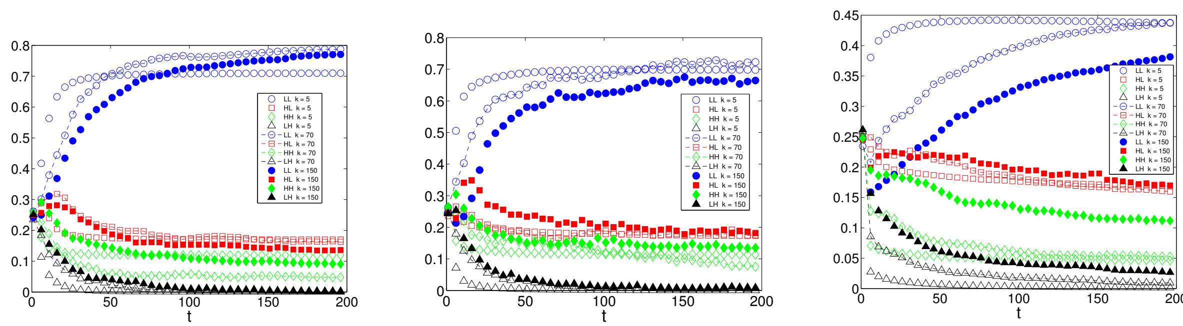
III. RESULTS AND DISCUSSION

The model was implemented on two kinds of graphs: (i) a lattice and (ii) uncorrelated scale-free (SF) graphs. Both graphs have the same number of nodes ($N = 1000$) and the same average degree $\langle k \rangle = 8$. The SF graphs



(a)$p = 0$ (b)$p = 0.1$ (c)$p = 1.0$

FIG. 2. Time evolution of the four strategies LL, HL, HH, LH for a lattice of degree $\langle k \rangle = 8$. The three subplots refer to different values of the mobility parameter p. The random jump parameter q has been set to zero.



(a)$p = 0.1$ (b)$p = 0.5$ (c)$p = 1.0$

FIG. 3. Time evolution of the four strategies LL (blue), HL (red), HH (green), LH (black) for uncorrelated scale-free graphs with $\langle k \rangle = 8$. Results for nodes with $\langle k \rangle = 5$ (empty symbols) and $\langle k \rangle = 150$ (full symbols) are shown. The three subplots refer to different values of the mobility parameter p. The random jump parameter q has been set to zero.

were generated using the configuration model [30, 31] with a degree distribution $P(k) = k^{-\gamma}$, $\gamma = 2.5$. Simulations were averaged over 100 different initial conditions and, in the case of SF graphs, over 100 different network realizations.

A. Role of mobility parameter p

Results of the simulation run on a lattice resemble the case of a well-mixed population. This situation is exactly achieved when both p and q are set to zero. This is expected since there is no mobility on the graph and only individuals on the same node, i.e. in a well-mixed state, interact. In Fig. II B, we show the behavior of the average fraction of each strategy on the nodes of the network as a function of time. Each panel corresponds to different values of p. In all cases, the strategy LH is promptly dominated by the other strategies, which is consistent with the analytical predictions of the well-mixed case. However, it can be noticed that the presence of a certain degree of mobility changes the distribution of the different strategies in the node. In particular, for higher values of p, mobility weakens the dominance of the LL strategy over the others.

In the case of the SF networks, non-trivial results were found. We attribute this to the heterogeneity in the degree distribution. In contrast to a lattice, nodes in a SF graph can play different roles depending on their degree of connectivity. In each panel found in Fig. II B, we show the fraction of strategy for two different degree classes: $\langle k \rangle = 5$ and $\langle k \rangle = 150$.

Results show interesting dynamics inside high degree nodes, where the dominant strategy becomes less and less ruling as the value of p is increased. The dominance of the LL strategy becomes weaker, which we surmise is only strongly supported in the intra-node dynamics. Since a high value of p means greater mobility for agents, the effect of intra-node generations counters that of the fitness-biased mobility. The results found here may shed light on the possible effects of mobility in instigating segregation of strategies by weakening the effects of intra-node dynamics. Moreover, these results for high-degree nodes would eventually approach that of a well-mixed population given an infinite amount of time; but, the more interesting insight from the model is that the evolution is found to have varying time scales.

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