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Collective behavior in complex networks

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Applications

- Percolation, synchronization, Ising, reaction-diffusion…
- Searching (searching in the Web, in P2P)
- Network "tomography" (structure of the Web)
- Failure and robustness
- Propagation (disease, information, fashion, electricity…)
- Ecological (predation, mutualism…)
- Genetic and metabolic regulation
- Neural models
- Communities (clusterization, assortativity…)
- …

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Outline

- Extra short review of main concepts
- Short review of models
- Applications of CN
  - Epidemics
  - Computer virus epidemics
  - Rumors ??
  - Associative memory
  - Community analysis, or how to become a superhero

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Three central concepts of complex networks

Small world phenomenon
We seem to be rather closely connected to one another.
Average path length between a pair of nodes $L \sim \ln N$.

Cliquishness, or clusterization
Overlap of the circle of acquantances, cliques.
Clustering coefficient C.

Inhomogeneous connectivity
Not all the nodes have the same number of edges.
Distribution of the connectivity $P(k)$.

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Two paradigms

Small World Networks
Watts & Strogatz

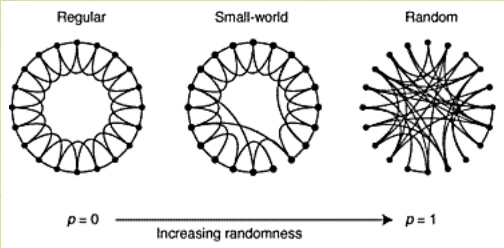
To interpolate with a single parameter between regular lattices and random graphs.

Scale-Free Networks
Albert & Barabási

To capture the highly inhomogeneous structure of the connectivity.

Small World Networks

Watts & Strogatz, Nature (1998)

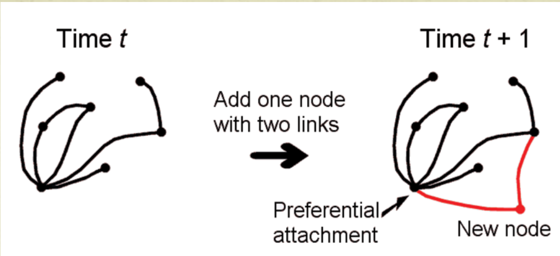


- Small average path length
  (as in random graphs)

- High clusterization
  (as in regular lattices)

Scale Free Networks

Albert & Barabási (1999)



- Small average path length
  (as in random graphs)

- High inhomogeneous connectivity
  (degree distribution $P(k) \sim k^{-3}$)

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How does the dynamics of a propagation phenomenon depend on the structure of the system?

Traditional propagation problems
- Well-mixed population
- Diffusion
- Regular lattice
- Random graphs

Systems with a structure?
- Complex networks

Social networks, public transportation, power grids, communications networks, the WWW, neural networks...

Simple epidemic models

Since 1920's…

S Susceptible
I Infectious
R Removed

$$S \rightarrow I \rightarrow R$$
$$S \rightarrow I \rightarrow R \rightarrow S$$
$$S \rightarrow I \rightarrow S$$
$$S \rightarrow E \rightarrow I \rightarrow R$$
$$\ldots$$

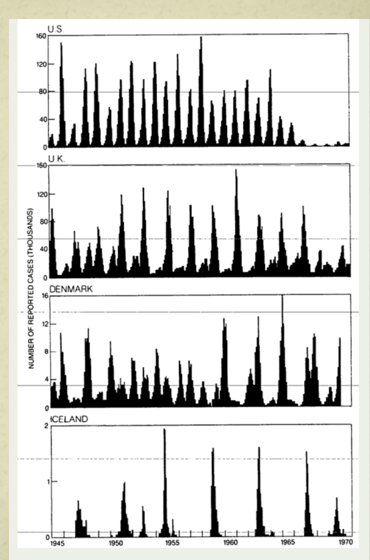
$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Mean field:
well mixed populations!

Transition:
epidemics ↔ no epidemics

Epidemics in a Small World?

Do epidemic waves depend on the *structure of the population?*



1945 1970

Reported measles cases in four countries
Cliff & Haggett, Sci. Am. (1984)

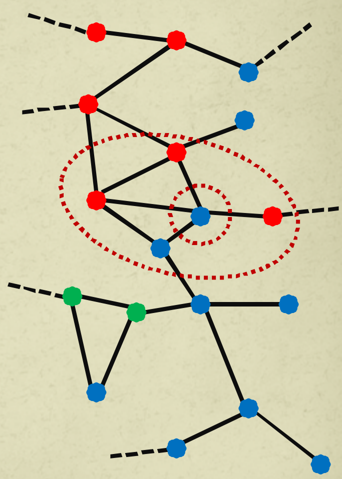
Epidemics in a Small World

Kuperman & Abramson, PRL (2001)

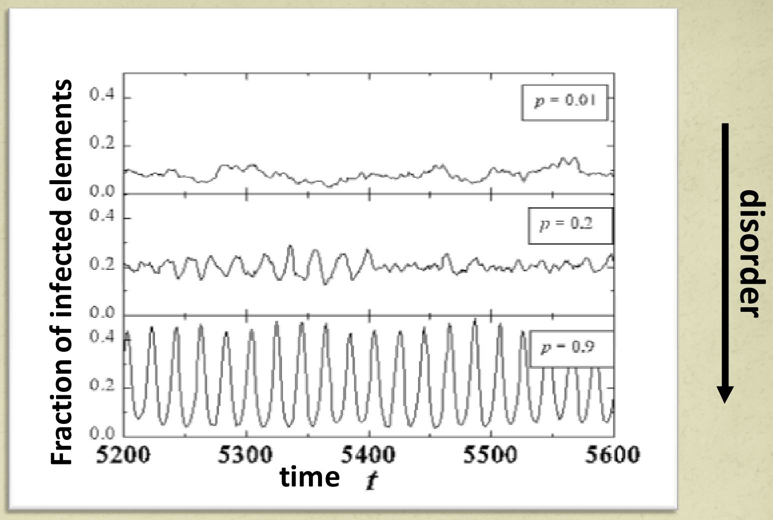
$$S \xrightarrow{\beta} I \xrightarrow{🕒} R \xrightarrow{🕒} S$$

- Stochastic contagion (local)
- Deterministic disease cycle

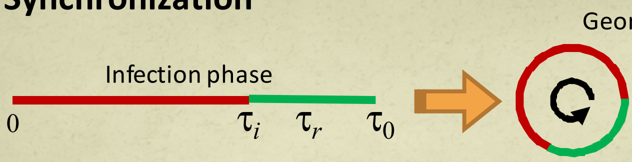
Transition to epidemic waves as a function of the *structure*



SW network



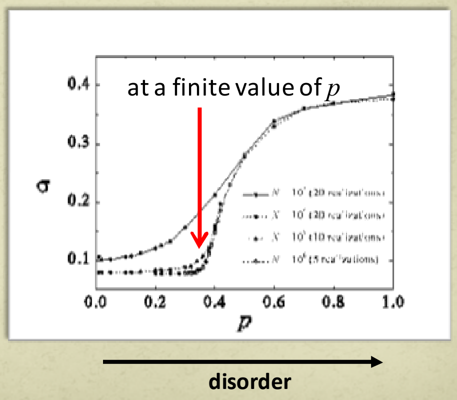
Synchronization



$$\phi_j = 2\pi(\tau_j - 1)/\tau_0$$

Synchronization parameter:

$$\sigma(t) = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j(t)} \right|$$



Computer Virus Epidemics

Pastor-Satorras & Vespignani, PRL (2001)

Regular lattice models:

Critical value of an infectivity parameter λ_c

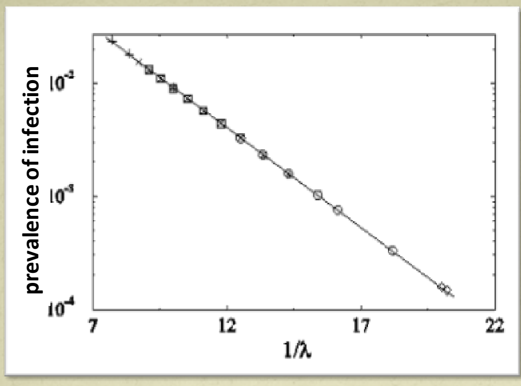
$\lambda < \lambda_c$ infection disappears exponentially

$\lambda \geq \lambda_c$ infection spreads and persits

Computer virus epidemics:

Lack of epidemic threshold

The Internet is scale-free!



smaller infectivity →

$$S \xrightarrow{\lambda} I \xrightarrow{\delta} S$$

Albert-Barábasi network

Stochastic contagion with probability λ

Stochastic recovery at a constant rate δ $(=1)$

The threshold is zero!

Relevance? The network of human sexual contacts might have a scale-free structure
Liljeros et al., Nature (2001)

Mean field model

Density of infected nodes with k links: ρ_k

$$\frac{\partial \rho_k(t)}{\partial t} = -\rho_k + \lambda k[1-\rho_k(t)]\Theta(\lambda)$$

Probability of pointing to an infected node

Stationary value: $\rho_k = \dfrac{k\lambda\Theta(\lambda)}{1+k\lambda\Theta(\lambda)}$ (1)

From preferential attachment

Probability of pointing to an infected node with k links: $\Theta \propto kP(k)\rho_k$

Probability of pointing to an infected node: $\Theta(\lambda) = \sum_k \dfrac{kP(k)\rho_k}{\sum_s sP(s)}$ (2)

(1) & (2): $\rho = \sum_k P(k)\rho_k$, with $P(k) = 2m^2/k^{-3}$

$$\rho \approx 2e^{-1/m\lambda}$$

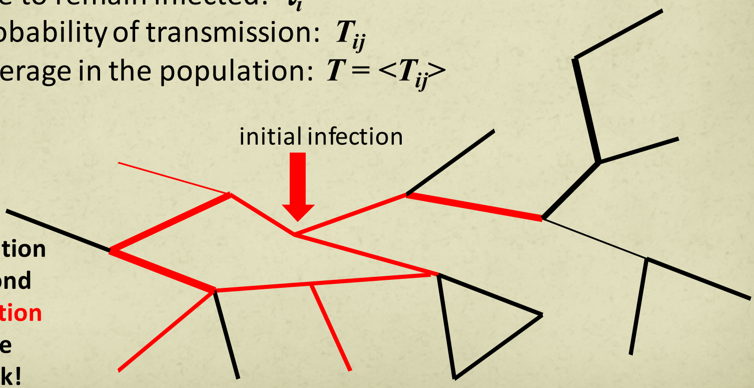
A reduction of transmissivity does not erradicate the infection!

SIR on a network

Newman, PRE (2002)

$S \rightarrow I \rightarrow R$

Rate of contacts between individuals: r_{ij}
+ Time to remain infected: τ_i
$\rightarrow$ Probability of transmission: T_{ij}
$\rightarrow$ Average in the population: $T = \langle T_{ij} \rangle$



Bond percolation with bond occupation *T* on the network!

Everything analytical:

Probability that a vertex has degree k: p_k

Generating function: $G_0(x) = \sum_{k=0}^{\infty} p_k x^k$

Probability of arriving at a node with degree k, following a random edge: $G_1(x)$

Probability distribution of "occupied" edges, for a given T:

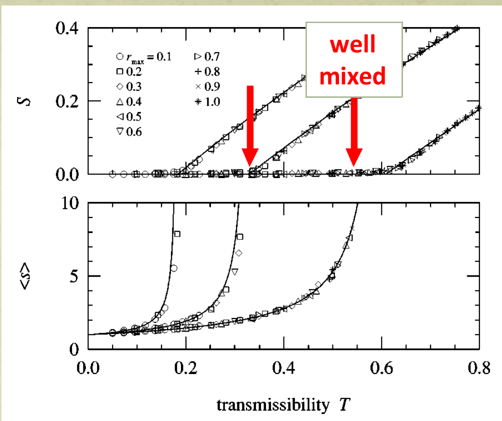
$$G_0(x;T) = G_0(1+(x-1)T)$$
$$G_0(x;T) = G_0(1+(x-1)T)$$

Distribution of the size of outbreaks: $H_0(x,T)$

Example: $p_k \propto k^{-2} e^{-k/\kappa}$

Epidemic size:

Average outbreak size:

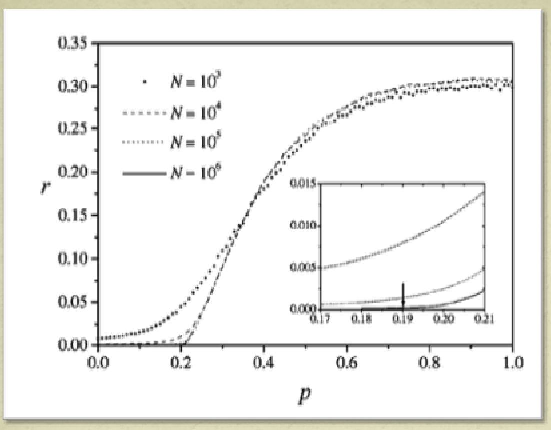


Fully mixed model

All outbreaks above the critical point give rise to epidemics

Network model

The probability that an outbreak becomes and epidemic is $S(T)$



Average fraction $r = \langle N_R/N \rangle$ of R at the end of the evolution, as a function of the disorder p

$$r \approx |p - p_c|^{\gamma}$$

($p_c = 0.19$, $\gamma = 2.2$ from a finite size scaling analysis)

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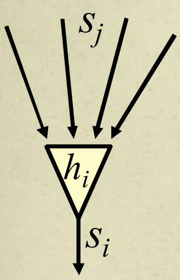
Order kills memory

Morelli, Abramson & Kuperman, EPJB (2004)

Associative memory model

Dynamics of neurons:

$$s_i(t) = \text{sign}[h_i(t)]$$ (state: $s = 1$ or -1, fire or not fire)



Field: $h_i(t) = \sum_{j=1}^{N} w_{ij}\, s_j(t)$

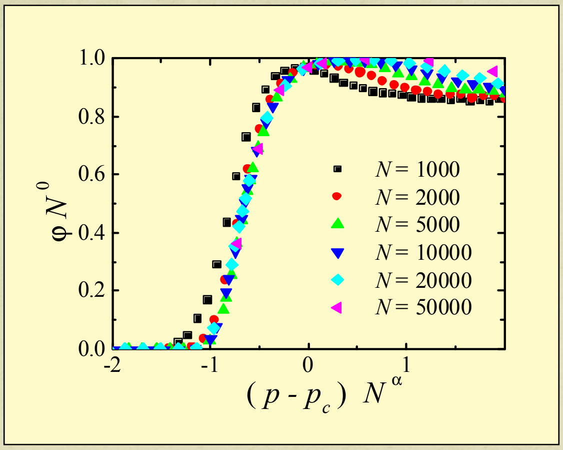
Weights: $w_{ij} = \frac{1}{N} \sum_{\mu=1}^{M} c_{ij}\, \xi_i^\mu \xi_j^\mu$

connectivity

"Stored" patterns: $\xi_i^\mu,\ \mu = 1 \ldots M$

(neural states, recovered as fixed points of the dynamics)

Efficacy φ - The fraction of realizations that, from random initial conditions, retrieve one of the stored patterns (or their reversals).



disorder

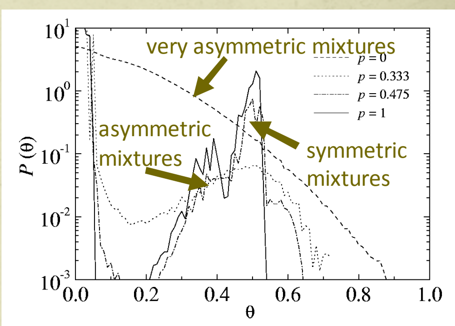
Small *p*: high clusterization, well defined neighborhoods far away from one another.

Greater *p*: neighborhoods disappear until at $p = 1$ the whole system is essentially a single neighborhood.

At *p* = 0 different regions of the network align themselves with different patterns ⇒ completely asymmetric mixture.

Asymmetric mixtures had been observed before, but are very rare in fully connected or randomly diluted networks.

Here they play an essential role in the destruction of the ability to retrieve patterns.



(mixture: a state that has a large overlap with more than one pattern)

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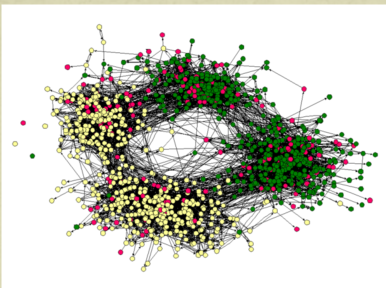
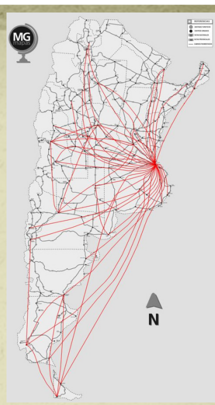
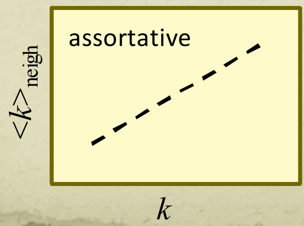
Community structure

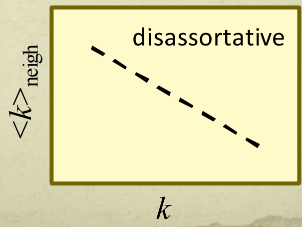


Social networks: communities



Technological networks: hubs

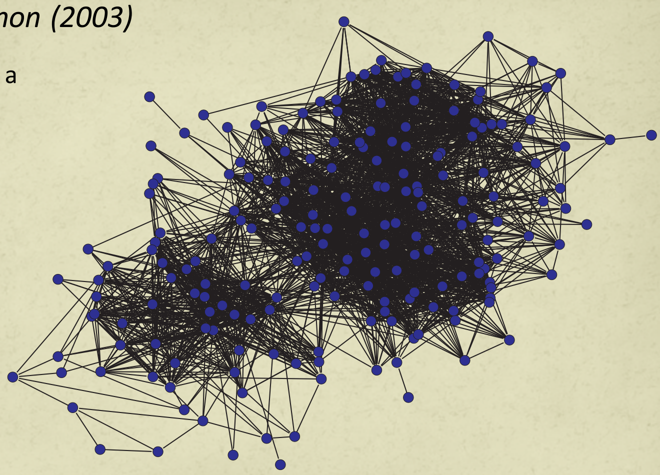




Jazz bands network (1920-1940)

Gleiser & Danon (2003)

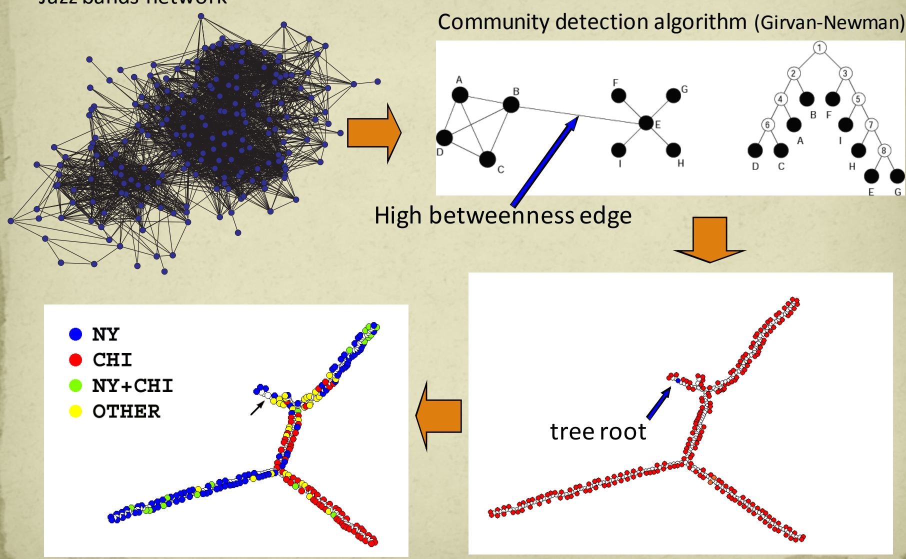
A case study of a collaboration network



Two bands are connected if they share at least one musician

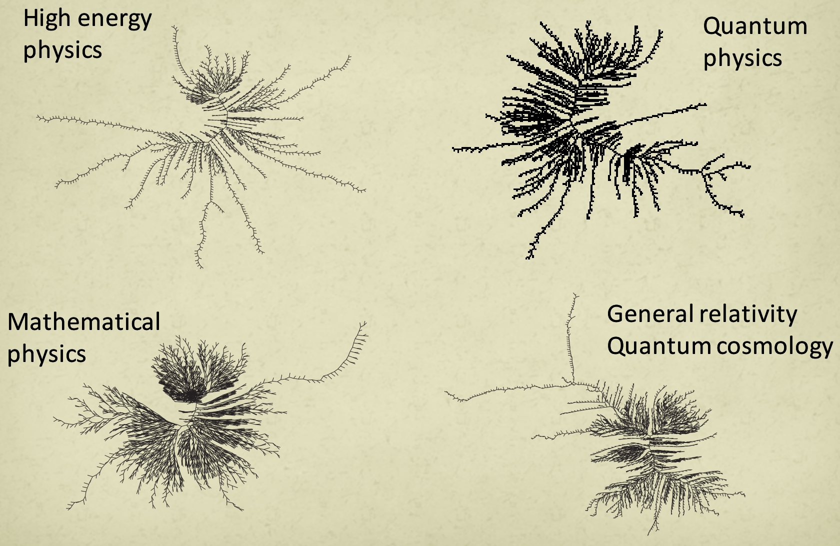
Community detection

Jazz bands network



Other collaboration networks…

Arenas, Danon, Díaz-Guilera, Gleiser & Guimerà, EPJB (2004)



An <u>artificial</u> collaboration network: the MARVEL Universe

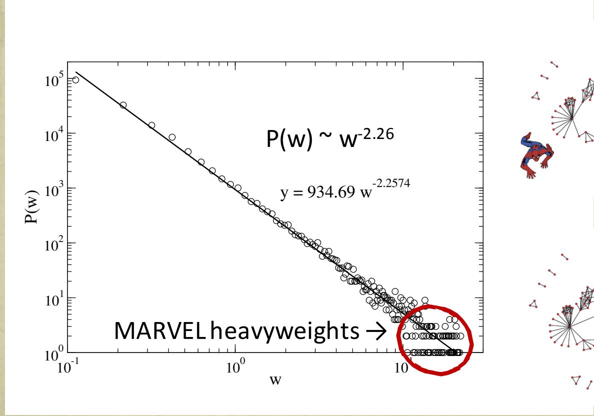
Gleiser, J. Stat. Mech. (2007)

image from MARVEL website

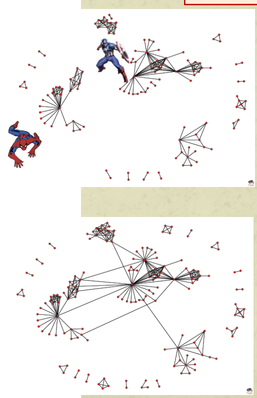
The MARVEL network is ***disassortative***, unlike natural social networks.

Still, ***there are communities***, built around the most popular characters, and connected to one another by them.

$$w_{ij} = \sum_k \frac{\delta_i^k \delta_j^k}{n_k - 1} \qquad \delta_i^k \begin{cases} 1, & \text{if character } i \text{ appears in book } k \\ 0, & \text{otherwise} \end{cases}$$



Addition of links according to weight



Summarizing…

- Complex networks are able to capture some complexity of naturally occurring networks
  Small world phenomenon, cliquishness, inhomogeneous connectivity, communities
- Richness in the dynamics of propagation phenomena in complex networks
- Model epidemics, rumor propagation, memory capacity, etc, depend on the structure of the network substrate
- Model epidemics in scale-free networks have no threshold
- Network concepts can identify community structure both in assortative and disassortative networks

Thank you!

References

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