

# PATTERNS OF SEGREGATION AND INTEGRATION

Jean-Paul Carvalho, Huang Cao, Ryan Muldoon, Carolyn Snyder, Francis Wong\*

September 6, 2006

## Abstract

This paper studies the basic dynamics of segregation and integration. We develop three agent-based models in which diverse agents choose their location in a two-dimensional space. Agents engage in dyadic interactions with all agents in their fixed radius. An agent's location choice is determined by the payoffs from their interactions in the current period. Our first model recovers Schelling's (1971) result in which agents segregate completely according to type, under the standard assumption that individuals prefer to interact with agents of their own type. The second and third models consider two types of deviation from the first model, in order to investigate different conceptions of the costs and benefits to diversity. We find that while we are able to generate different social dynamics between model types, there is never a stable in-

tegrated state. Segregation is an attracting state. From this we conclude that greater agent intelligence is required for agents to be able to take advantage of the benefits of diversity.

## 1 Introduction

Diversity is a hallmark of modern large-scale societies. Individuals differ *inter alia* according to attitudes, beliefs, norms of behavior, social identities and physical characteristics. Nevertheless, we see segregation according to one or more individual traits in various forms of social organization. A celebrated example of this is residential segregation based on race in the United States [U.S. Census Bureau, 2002].

In a classic paper, Thomas Schelling [1971] developed one of the first agent-based models in the social sciences to explain how extreme residential segregation according to race can occur. He showed that when individual agents have only a "mild" preference for locating next to agents of their own type, then *complete* segregation is far more likely to emerge than integrated states in which different types are neighbors.

Segregation has important political, social and economic consequences. William Julius Wilson [1987] has argued that urban poverty, crime and anti-social behavior increased markedly in black communities in the United States after the civil rights movement, as middle class African Americans moved out of predominantly black urban neighborhoods. These neighborhoods lost their so-

---

\*The authors are from, respectively, the University of Oxford, University of Science and Technology of China, University of Pennsylvania, Stanford University, and Chinese University of Hong Kong. The authors' email addresses are, respectively, jean-paul.carvalho@economics.oxford.ac.uk, huangcao@mail.ustc.edu.cn, rmuldoon@sas.upenn.edu, cpsnyder@stanford.edu, and franciswong@cuhk.edu.hk. Contact Author: Ryan Muldoon, Department of Philosophy, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6304. The authors gratefully acknowledge financial support from the Santa Fe Institute and Institute of Theoretical Physics, Beijing, as well as helpful comments from participants at the 2006 Complex Systems Summer School in Beijing. Carvalho is grateful for financial support from the Commonwealth Bank Foundation and General Sir John Monash Foundation. Snyder is grateful for financial support from the National Science Foundation Graduate Fellowship, and the Gabilan Stanford Graduate Fellowship.

cial role models, pools of talented individuals, and individuals with the financial capital to support local community organizations.

This paper investigates the basic dynamics of segregation and integration. Schelling’s simple model of spatial segregation is a natural starting point. Our main departure from Schelling’s model is to introduce rewards for integration. Not only might individuals have a preference for interacting in diverse groups, but the aggregation of diverse opinions might yield better understanding of a problem and more efficient problem-solving [Hong and Page, 2004]. Given costs and benefits of diversity, we investigate: (i) the conditions under which segregation/integration arises; and (ii) the patterns of segregation/integration which emerge.

We develop three agent-based models. Section 2 describes the common set-up of these models. In section 3, we outline the distinct features of each model, and present our results. Our first model recovers Schelling’s result of complete segregation, and serves as a benchmark. The second and third models build in different costs and benefits to diversity, and produce contrasting results. In section 4, we summarize our results, discuss the limitations of these models, and provide some suggestions for future research.

## 2 Model Set-Up

We develop three agent-based models in which diverse agents choose their location on an  $80 \times 80$  torus. An agent’s type is denoted by  $i \in \Omega = \{Red, Green, Blue\}$ . Agents have a fixed radius of interaction, have no memory and are only aware of those agents in their radius. Each period, every agent engages in dyadic interactions with every other agent in their radius. A dyadic interaction either “fails” or “succeeds”. If a dyadic interaction fails, both agents receive a payoff of zero. Both agents in a successful interaction receive a payoff of  $x_{ij}$ , and the probability of a successful interaction is  $p_{ij}$ ,  $i, j \in \Omega$ . These parameters are partic-

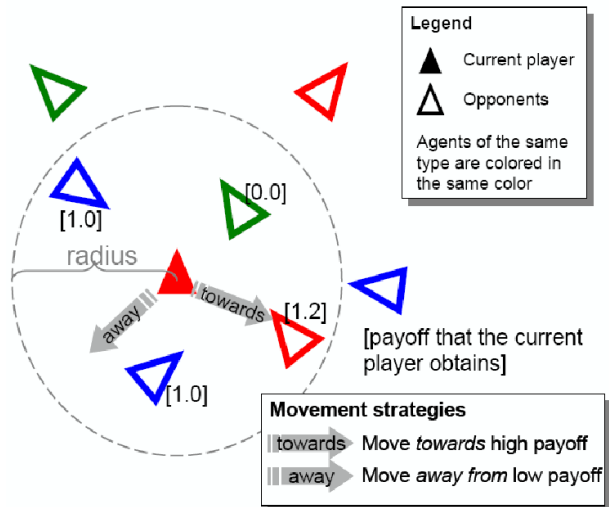


Figure 1: Movement Rules

ular to each model, and are specified in the next section.

After interacting with their neighbors, agents choose where to locate. The *average payoff* to an agent in a given period is the sum of the payoffs received in all the agent’s dyadic interactions in that period, divided by the number of dyadic interactions. As will become clear, an agent’s movement decision is based on several factors including his average payoff, his location relative to the neighbor from whom he receives the maximum or minimum payoff, as well as a parameter determining his “level of satisfaction”.

### 2.1 Movement Rules

Figure 1 summarizes the way in which agents move across the interaction space:

The agent, denoted by the solid triangle, engages in four dyadic interactions, since there are four agents within his radius. We build in “satisficing” behavior by assuming that an agent only moves to a new location if his average payoff (0.8 in the figure) is below a satisfaction level, which is uniform across agents. An agent relocates using one of three movement strategies: (i) relocates to

a random position within his radius with probability 0.1, (ii) moves with a heading directly toward the neighbor who gave him the maximum payoff during that round of interactions (if there are more than one, one of these is selected at random) with probability 0.45, and similarly (iii) moves with a heading directly away from the agent who gave him the lowest payoff, with probability 0.45. Under movement strategies (ii) and (iii), an agent moves a distance  $d$  given by:

$$d = dm * (max - min + \delta) / max$$

where  $dm$  is the distance from the agent who gives him the maximum payoff,  $max$  and  $min$  are the maximum and minimum the agent receives from the series of interactions, and  $\delta$  (set to 0.5) ensures there is movement even when  $max = min$ . When every interaction in which an agent engages fails, i.e. the agent receives a zero payoff in every dyadic interaction, the agent moves away from his current location with a random heading and a constant distance that is equal to his radius.

### 3 Results

In this section, we investigate the dynamic patterns of spatial segregation/integration that emerge in three models which do not admit a segregated or integrated state. Each model shares the basic set up outlined in section 2, but has a different specification of payoffs from dyadic interactions. We shall now explicate the payoff specification for each model, and present the results that follow.

#### 3.1 Model 1: Preference for Own Type

The first model serves as a benchmark. Schelling’s (1971) result of complete segregation is recovered under the standard assumption that individuals prefer to interact with agents of their own type. All dyadic interactions are successful, i.e.  $p_{i,j} = 1$  for all  $i, j \in \Omega \times \Omega$ . However,  $x_{i,j} = 1 + bias$  when

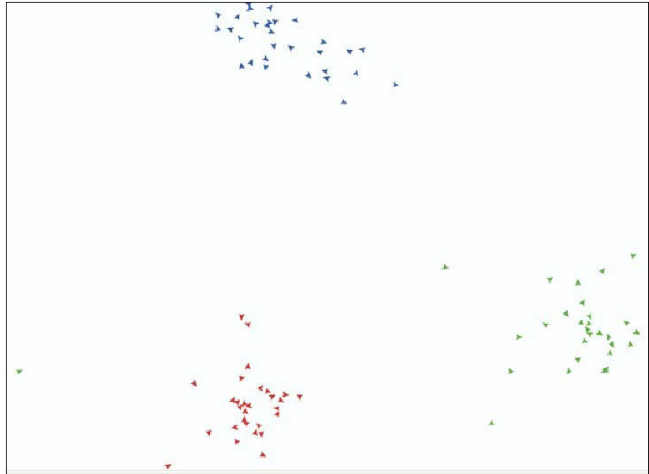


Figure 2: Complete Segregation

$i = j$  and  $x_{i,j} = 1$  when  $i \neq j$ , so that individuals receive a higher payoff when interacting with agents of their own type.

The simulations reveal that when the satisfaction threshold is high, complete segregation according to type occurs very quickly (see figure 2). However, as the satisfaction threshold is lowered below  $1 + bias$ , integrated states can persist, since agents “satisfice” and do not seek out more profitable interactions with their own type.

#### 3.2 Model 2: Risks of Homogeneity

The second model considers diversity as a means of reducing risks of interaction between different agent types. This is an attempt to represent the ecological dangers of monoculture. We maintain the assumption of bias toward an agent’s own type, but we introduce an element of risk. In this model, there is a single probability distribution from which each agent type  $i \in \Omega$  draws with replacement. If in a given round the draw  $p_i$  is greater than  $0 < t < 1$ , then for all  $i, j \in \Omega \times \Omega$ ,  $x_{i,j} = 0$ . So, if an agent is surrounded by only one type of agent, there is perfect correlation of payoffs from each dyadic interaction. If one interaction has zero payoff, then all will. If instead the agent has

neighbors of various types, the risk of getting an average payoff of zero is reduced according to the proportions of agent types within the interaction neighborhood. This design provides agents with an opportunity to manage their risks by increasing integration. In a monoculture, the chance of a negative event is lower, but it has a devastating impact. In integrated environments, negative events can happen with greater frequency, but their impact is smaller. This concept is leveraged in portfolio management, based on the work of Markowitz [1952] and Sharpe [1964].

The simulations show that the segregation effects that appeared so quickly under model 1 arise under fewer conditions. As the number of agents in the average interaction neighborhood grows (either by increasing the number of agents in the simulation or by increasing the size of agents' interaction radius), segregation gives way to "swarming" behavior. This swarming behavior is characterized by groups of the same agent type which move around the interaction space, temporarily interacting with other groups as they go. This looks much like bird flocking behavior as described by Toner and Tu [1998].<sup>1</sup> As the probability of failure increases (the value of  $t$  decreases), swarming behavior becomes less stable, and breaks down into an unstable integrated state. In this state, there is very little structure to the movement patterns of agents. We find this result because of the high incidence of dyadic interactions with a zero payoff. As the average payoff drops, agent movement increases, and so there is little chance for stability.

### 3.3 Model 3: Risks of Diversity

The third model is an attempt to capture the tension between results like Markowitz [1952] and Sharpe [1964] found in economics and the results in political science and social choice, such as Arrow

<sup>1</sup>Unfortunately, quantitative analysis of this behavior has proven difficult. The bird flocking literature has studied how to generate flocks, but not how to identify which birds are in a flock.

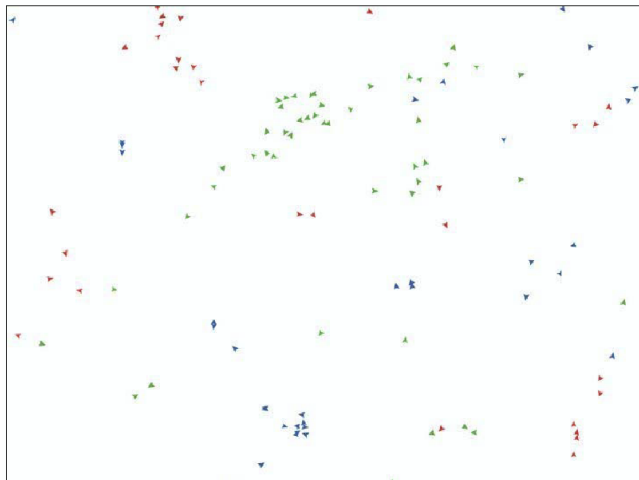


Figure 3: Swarming Behavior

[1951]. As Page (2003) points out, this represents a fundamental disconnect that has not yet been understood. On one hand, we find that diversity is desired for hedging risk, while on the other we find that diversity leads to a failure of consensus and social strife.

To represent this, we increase both the risk and the return of interaction when agents are of a different type as opposed to the same type. That is, dyadic interactions are always successful only between agents of the same type, i.e.  $p_{i,j} < 1$  for all  $i, j \in \Omega \times \Omega$  where  $i \neq j$ , while  $p_{i,i} = 1$  for all  $i \in \Omega$ . However,  $x_{i,j} = 1 + advantage$  when  $i \neq j$  and  $x_{i,j} = 1$  when  $i = j$ , so that individuals receive a higher payoff when interacting with a different type.<sup>2</sup>

Simulation shows that this model is the most interesting to consider. First, we find that when the satisfaction threshold is at or below 1, segregation emerges. This is the opposite of the effect of the satisfaction threshold in model 1, because of the change from a bias for one's own type to an advantage to interacting with agents of other types. In the cases in which the satisfaction threshold is greater than 1, unstable integration is dominant. Agents

<sup>2</sup> $advantage > 0$

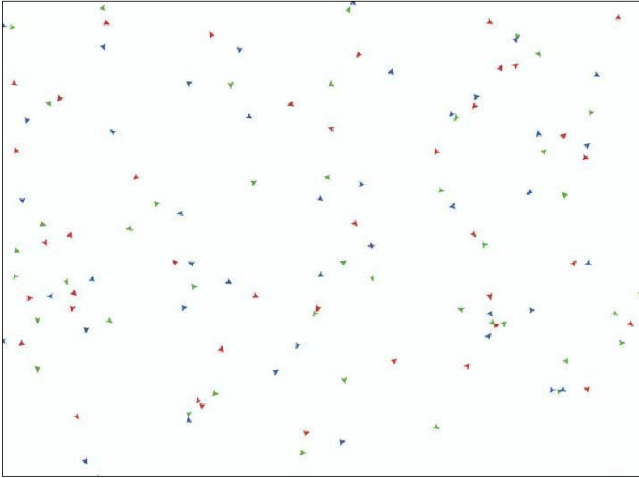


Figure 4: Unstable Integration

seek out agents of other types, and because of our definition of the movement rules, agents will move unless their average payoff is greater than their satisfaction threshold. Because of the diversity of agents found in integration, agents will generally have a reason to move, as not every interaction will offer the same payoff. As the probability of failure increases, more swarming behavior emerges. This is in effect a stochastic version of lowering the satisfaction threshold. Agents will seek out other agents of their type to claim the guaranteed interaction payoff more often than they will seek out the higher potential payoffs of interactions with other agent types, as there will be too many payoffs of zero.

## 4 Conclusions

The segregation dynamic from Schelling’s classic (1971) provided a compelling starting point to better understand the dynamics of segregation and integration. We were able to recreate that dynamic, and investigate the effects of a systematic modification of the modeling assumptions. In our modified models with no absorbing segregating or integrating state, we find a number of instances of seg-

regation emerging, but no instances of “stable integration”. However, we have far fewer cases of segregation than Schelling’s original model, and have uncovered the effects of additional model parameters. The introduction of risk produced most of the results, but these effects were not straightforward. The level of risk interacted with satisfaction thresholds: satisficers will segregate with higher levels of risk, and will integrate with lower levels of risk.

There is a distinct limitation to these models, which can carry over to how useful they are for the study of social dynamics. Agents in the models never find a stable integrative state in part because they are not smart enough. They have no memory, so they cannot calculate expected values of interactions. Because of this, the agents have no way to determine optimal hedging strategies. Further, in integrated environments, individuals will have neighbors that provide different interaction payoffs. Given the way the movement rules are defined, individuals will constantly move to increase their average payoff. By using these simple models, we were able to research extensions of an existing and popular model, but we reached the limits of what the models could tell us.

In future research we plan to use game-theoretic models with agents that have memory. This leaves open the opportunity to look into the effects of learning rules of different complexities, and would allow us to explore the dynamics with more realistic agents. With this increase in agent sophistication, new areas of inquiry become possible. Just two possibilities are to consider agents with heterogeneous preferences for diversity, as well as agents who can perform higher-order discrimination over preferences.

## References

- Kenneth Arrow. *Social Choice and Individual Values*. Wiley, New York, 1951.
- U.S. Census Bureau. Racial and ethnic segregation in the united states: 1980-2000. 2002.

Lu Hong and Scott E. Page. Diversity trumps ability. *Proceedings of the National Academy of Sciences*, 101(46):16385–16389, 2004.

Harry M. Markowitz. Portfolio selection. *Journal of Finance*, 7:77–91, 1952.

Scott Page. Diversity and robustness, 2003.  
URL <http://www.cscs.umich.edu/~spage/robust.htm>.

Thomas Schelling. Dynamic models of segregation. *Journal of Mathematical Sociology*, 1(1): 143–186, 1971.

William F. Sharpe. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3):425–442, 1964.

John Toner and Yuhai Tu. Flocks, herds, and schools: A quantitative theory of flocking. *Physical Review E*, 58(4):4828–4858, 1998.

William Julius Wilson. *The Truly Disadvantaged*. University of Chicago Press, Chicago, IL, 1987.