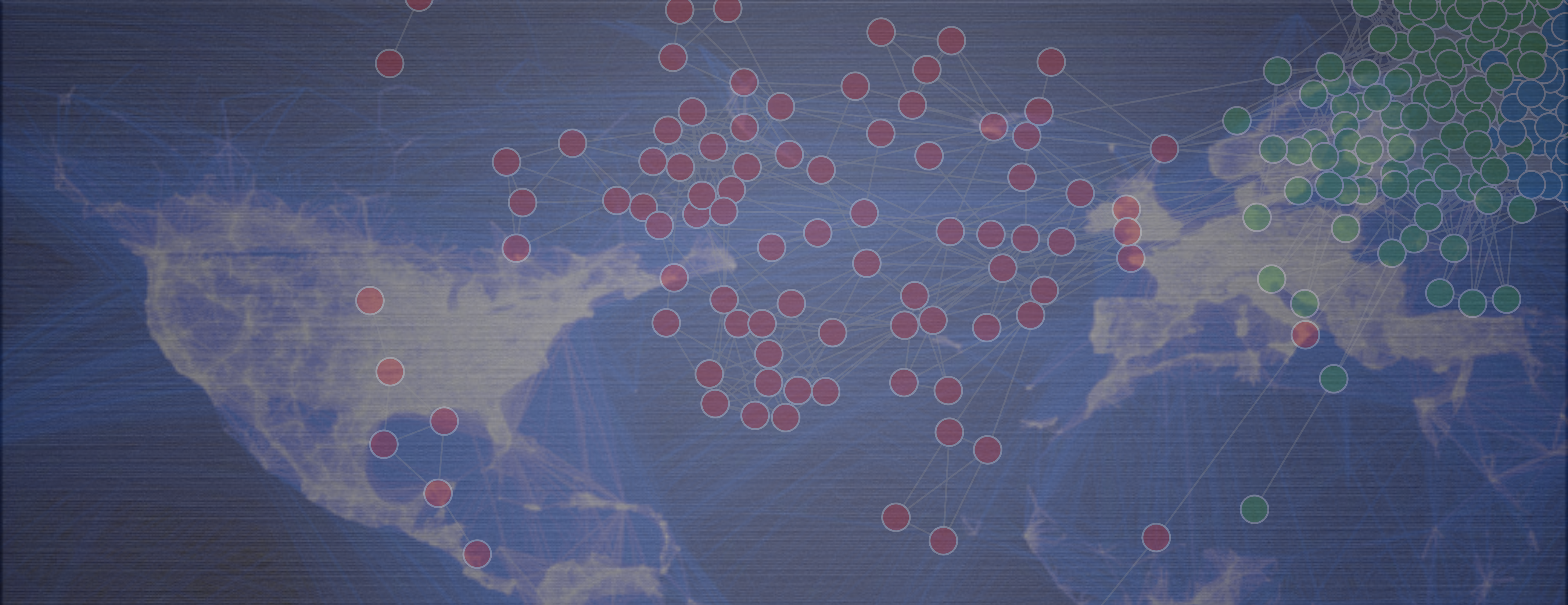

Five Lectures on Networks

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University of Colorado Boulder

External Faculty, Santa Fe Institute

lecture 3

University of Colorado **Boulder**

Network Analysis and Modeling

Instructor: Aaron Clauset

This graduate-level course will examine modern techniques for analyzing and modeling the structure and dynamics of complex networks. The focus will be on statistical algorithms and methods, and both lectures and assignments will emphasize model interpretability and understanding the processes that generate real data. Applications will be drawn from computational biology and computational social science. No biological or social science training is required. (Note: this is not a scientific computing course, but there will be plenty of computing for science.)

Full lectures notes online (~150 pages in PDF)

http://santafe.edu/~aaronc/courses/5352/

Software

R
Python
Matlab
NetworkX [python]
graph-tool [python, c++]
GraphLab [python, c++]

Standalone editors

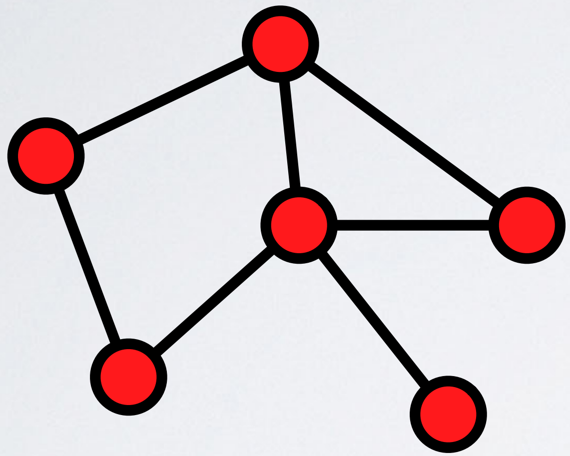
UCI-Net
NodeXL
Gephi
Pajek
Network Workbench
Cytoscape
yEd graph editor
Graphviz

Data sets

Mark Newman's network data sets
Stanford Network Analysis Project
Carnegie Mellon CASOS data sets
NCEAS food web data sets
UCI NET data sets
Pajek data sets
Linkgroup's list of network data sets
Barabasi lab data sets
Jake Hofman's online network data sets
Alex Arenas's data sets

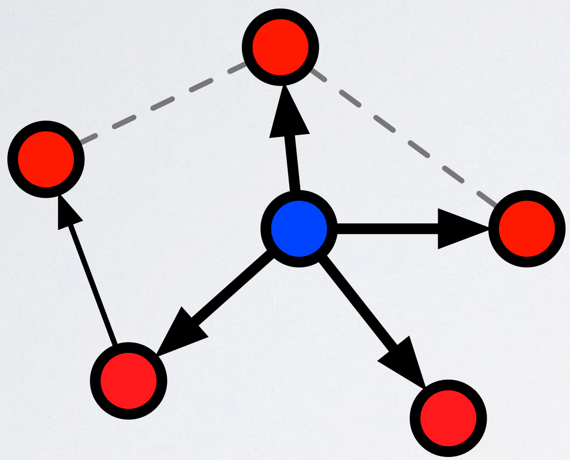
1. defining a network
2. **describing a network**
3. **null models for networks**
4. statistical inference

describing networks



position

describing networks

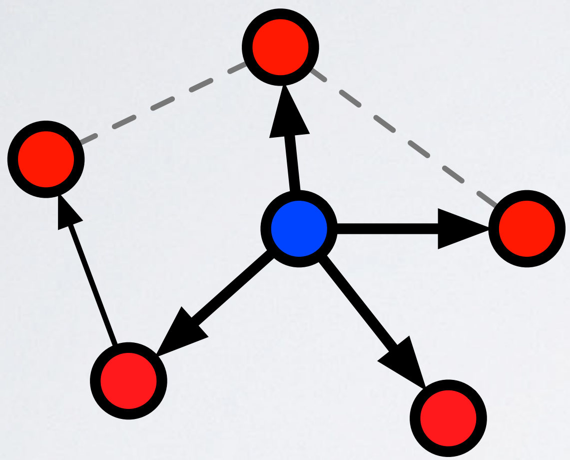


position = centrality:
measure of positional "importance"

geometric	harmonic centrality
	closeness centrality
	betweenness centrality
connectivity	degree centrality
	eigenvector centrality
	PageRank
	Katz centrality
	many many more…

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

describing networks



position = centrality:
harmonic, closeness centrality

importance = being in "center" of the network

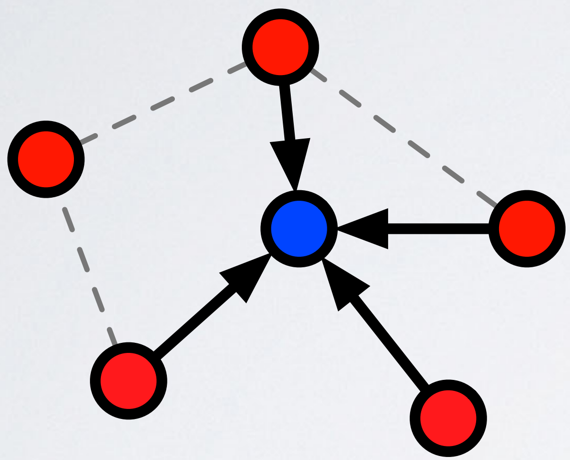
harmonic $$c_i = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

length of shortest path

distance: $$d_{ij} = \begin{cases} \boxed{\ell_{ij}} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

describing networks



position = centrality:
PageRank, Katz, eigenvector centrality

importance = sum of importances* of nodes that point at you

$$I_i = \sum_{j \to i} \frac{I_j}{k_j}$$

or, the left eigenvector of

$$\mathbf{A}\mathbf{x} = \lambda\,\mathbf{x}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

*modulo several technical details

network position

an example

Giovanni de Medici

network position

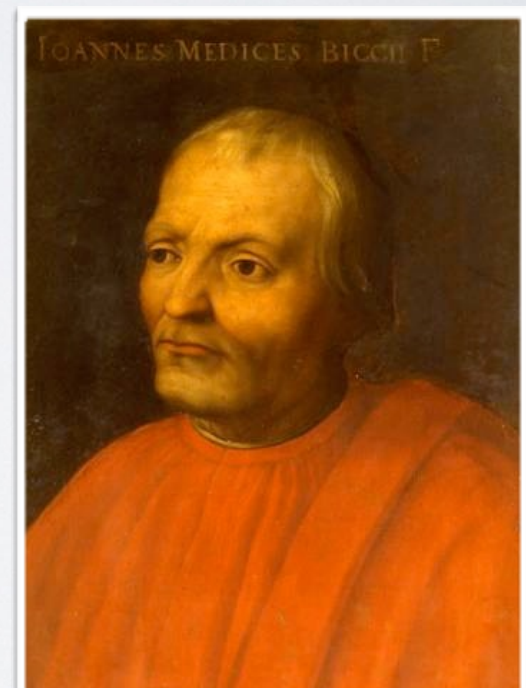
Robust Action and the Rise of the Medici, 1400–1434[1]

John F. Padgett and Christopher K. Ansell

1993

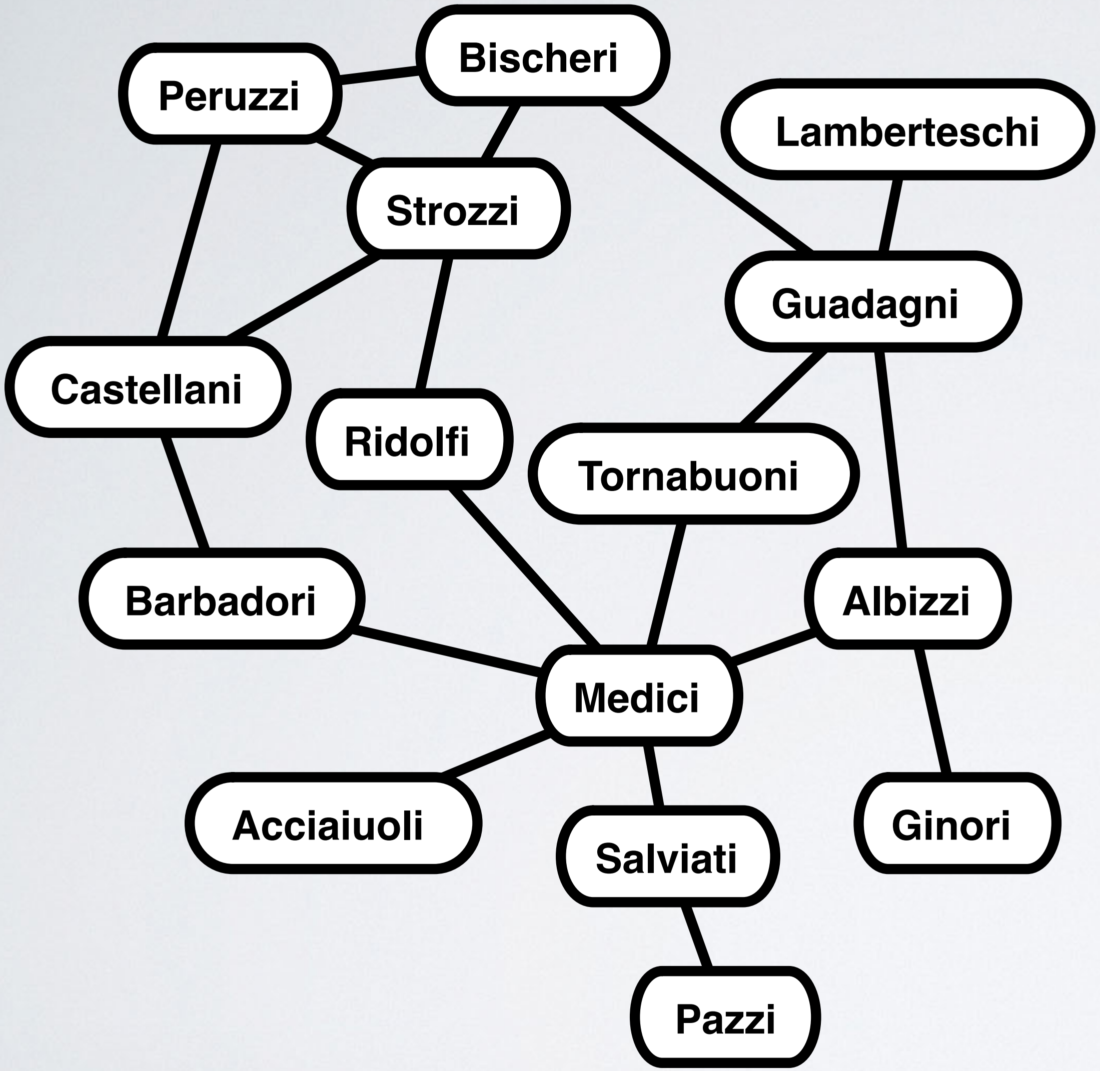
Duomo

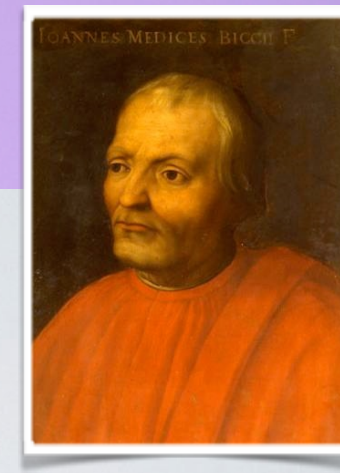
Palazzo Medici



Giovanni de Medici

network position: closeness

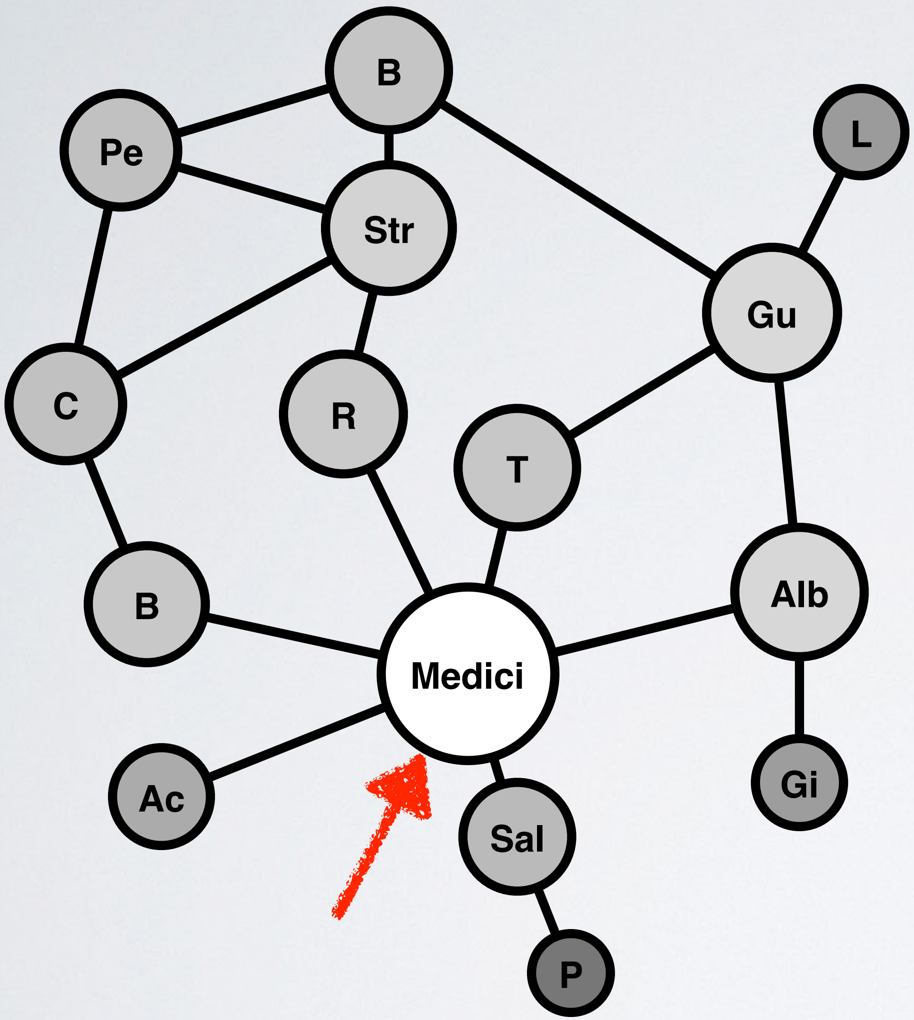




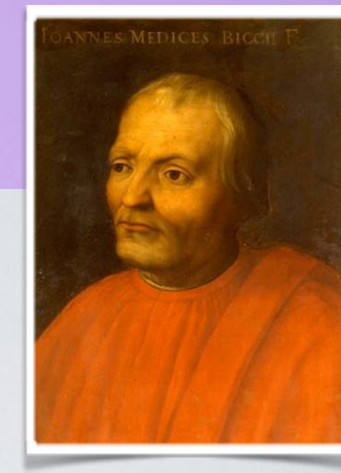
nodes: Florence families
edges: inter-family marriages

which family is most central?

network position: closeness

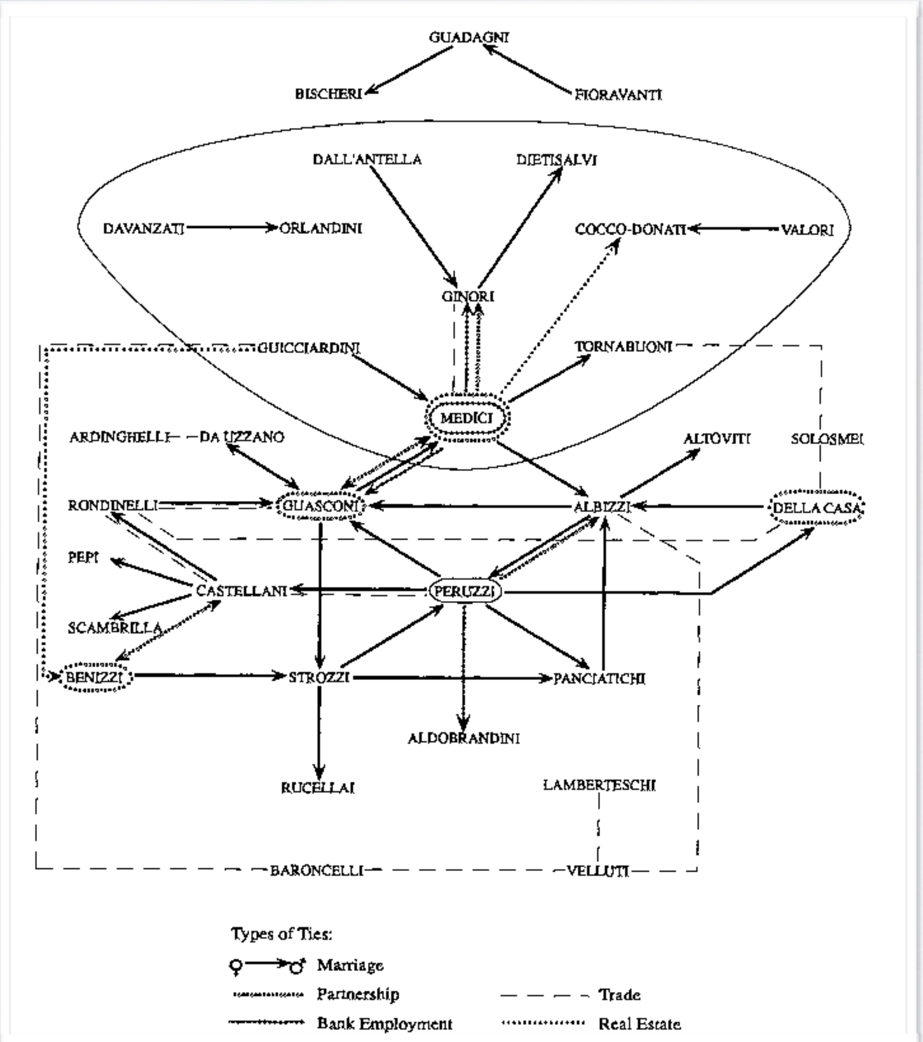


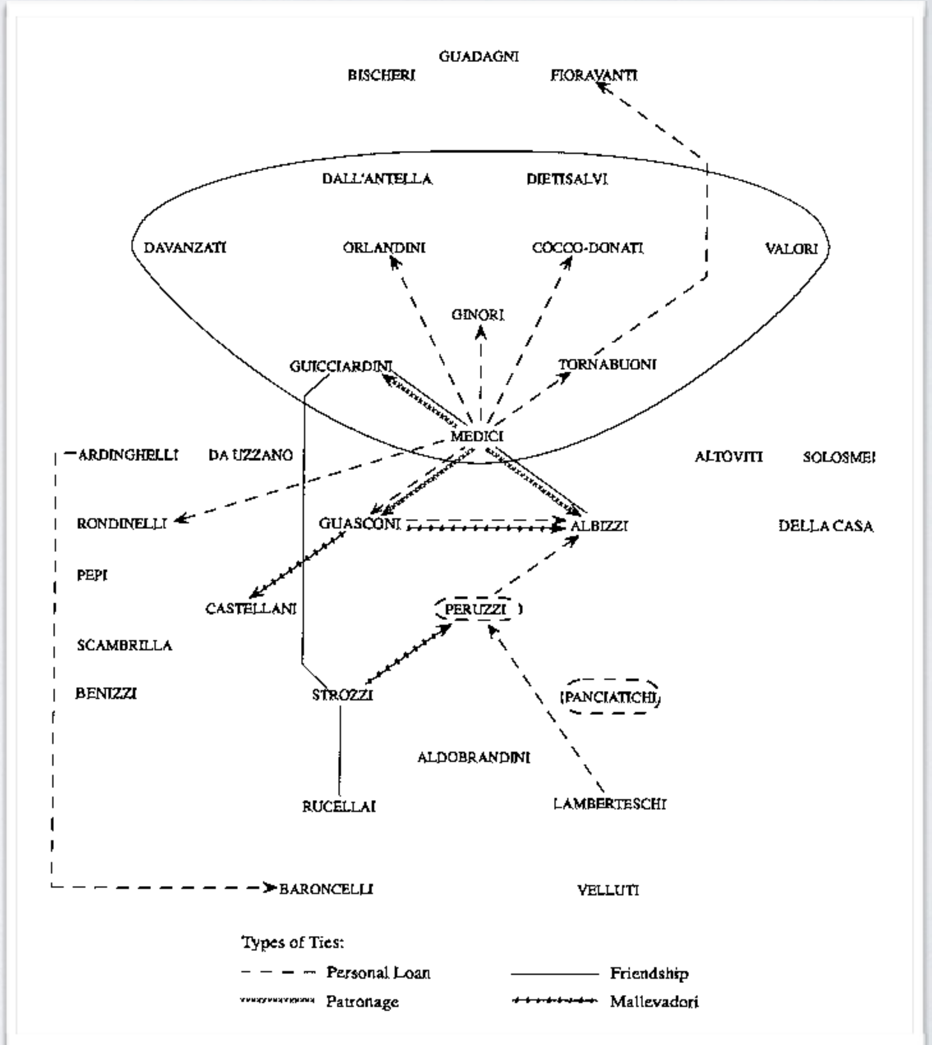
Medici	9.5
Guadagni	7.92
Albizzi	7.83
Strozzi	7.67
Ridolfi	7.25
Bischeri	7.2
Tornabuoni	7.17
Barbadori	7.08
Peruzzi	6.87
Castellani	6.87
Salviati	6.58
Acciaiuoli	5.92
Ginori	5.33
Lamberteschi	5.28
Pazzi	4.77



network position

actually, it's complicated...

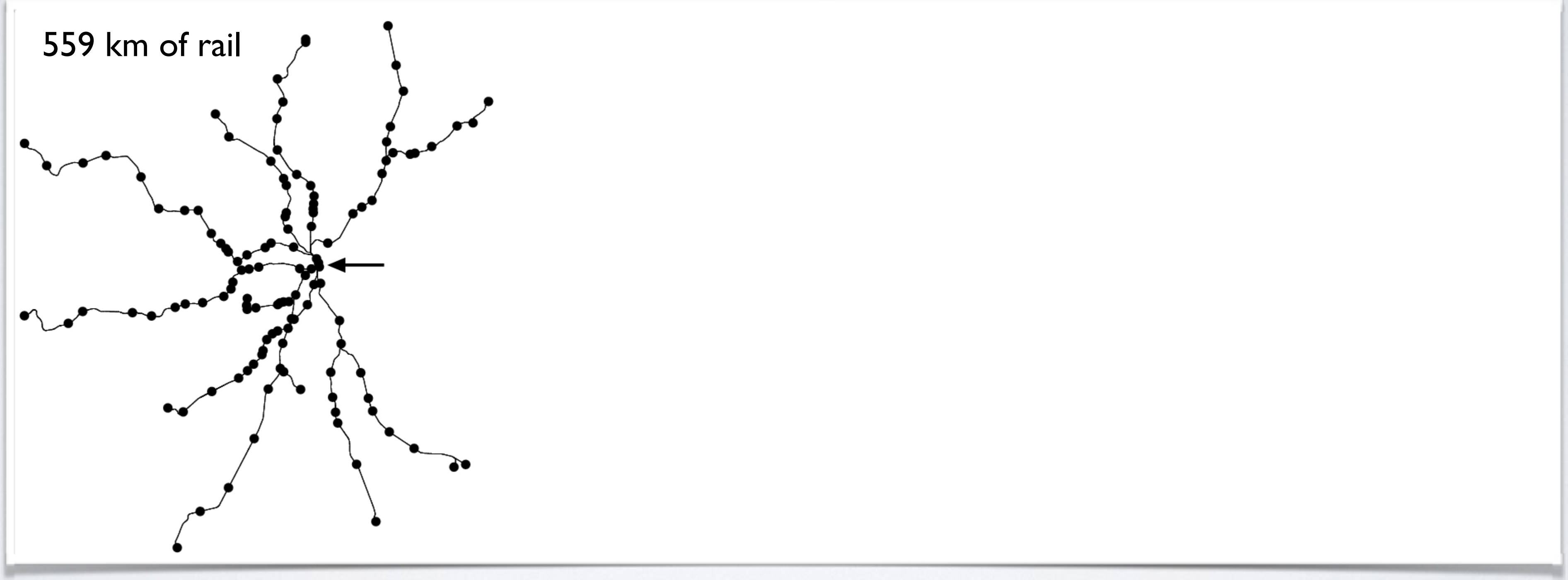




network position

an example

Boston commuter rail



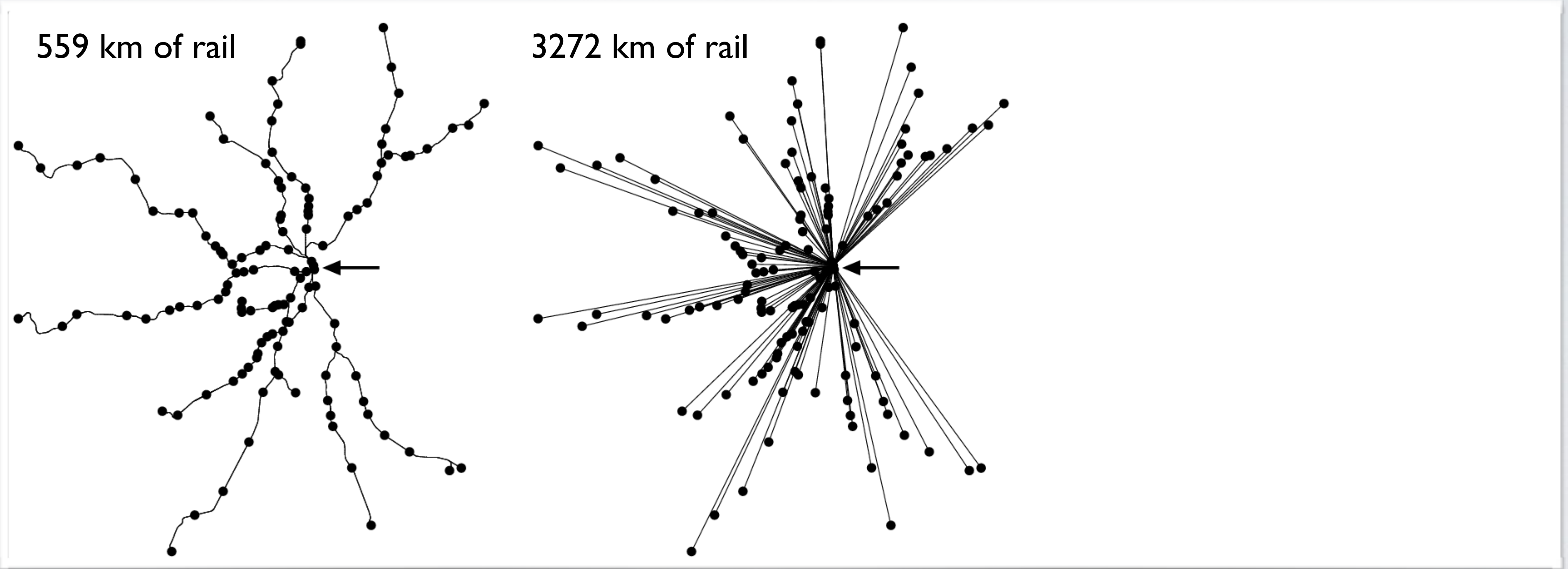
data

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position

an example

Boston commuter rail



data

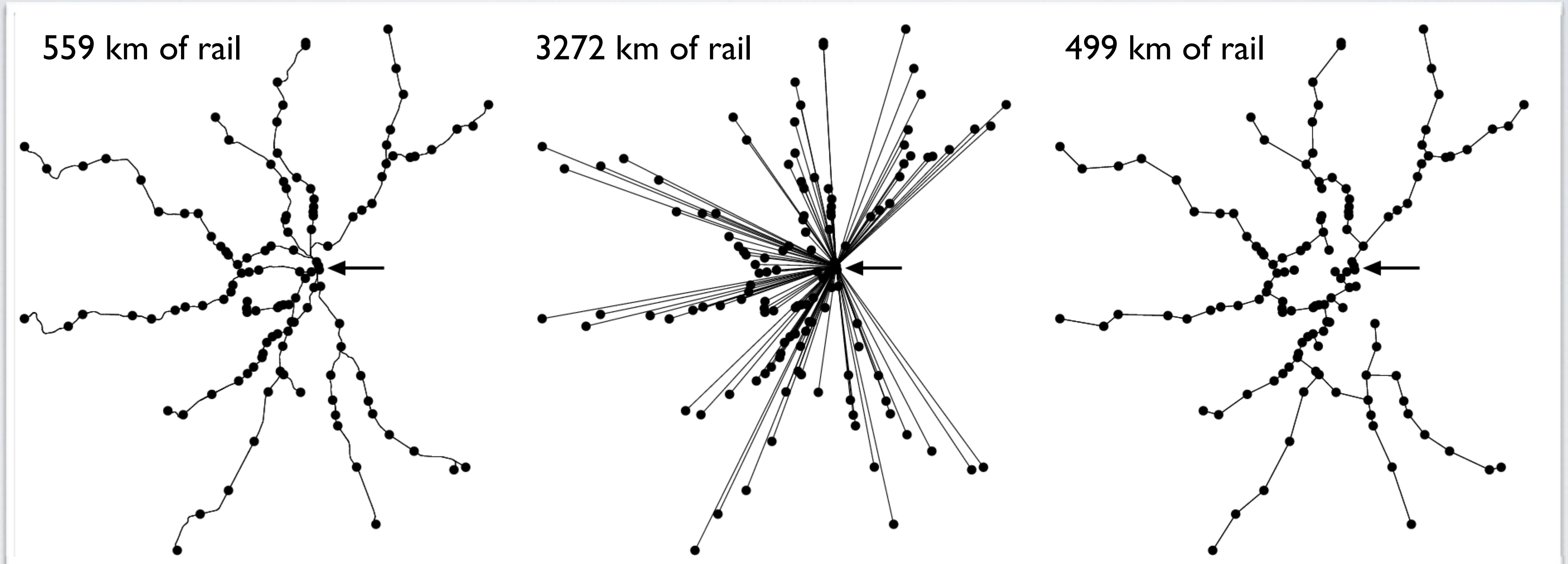
minimum travel time

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position

an example

Boston commuter rail



data minimum travel time minimum weight tree

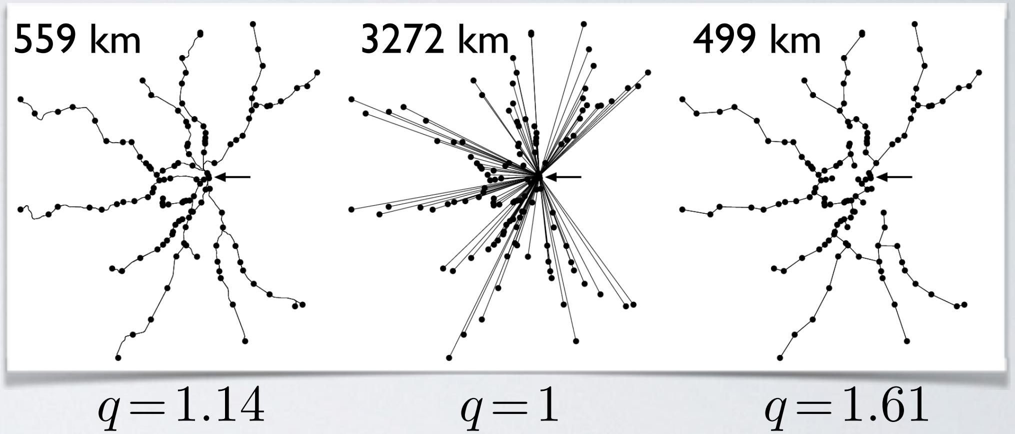
Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position

route factor

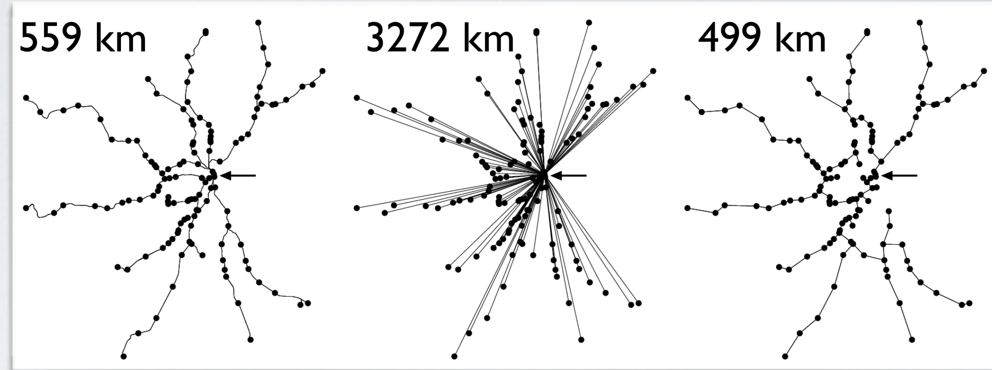
$$q = \frac{1}{n} \sum_{i=1}^{n} \frac{\ell_{i0}}{d_{i0}}$$

mean ratio of distance along edges ℓ_{i0} to direct Euclidean distance d_{i0} to root 0



Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

network position



$q = 1.14$ $q = 1$ $q = 1.61$

a simple model

embed n vertices in a plane

until all vertices connected

add edge (i, j) with minimum value for

$$w_{ij} = d_{ij} + \beta \ell_{j0}$$

distance from i to j

parameter

route length to root

$\beta = 0 \longrightarrow$ minimum spanning tree*

$\beta > 0 \longrightarrow$ prefer shorter paths to root

Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

*this is exactly Prim's algorithm for MSTs

network position

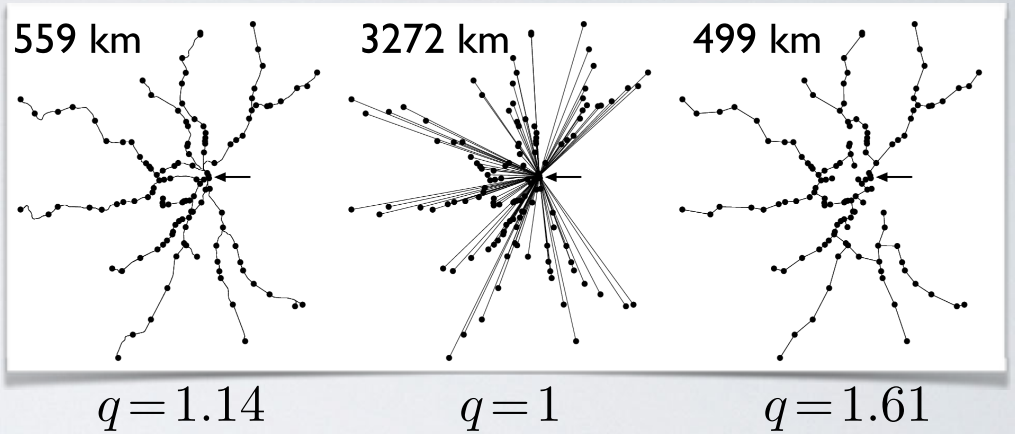
a simple model

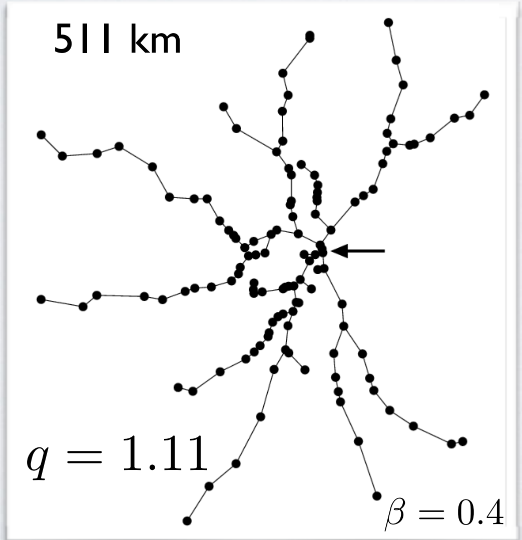
embed n vertices in a plane

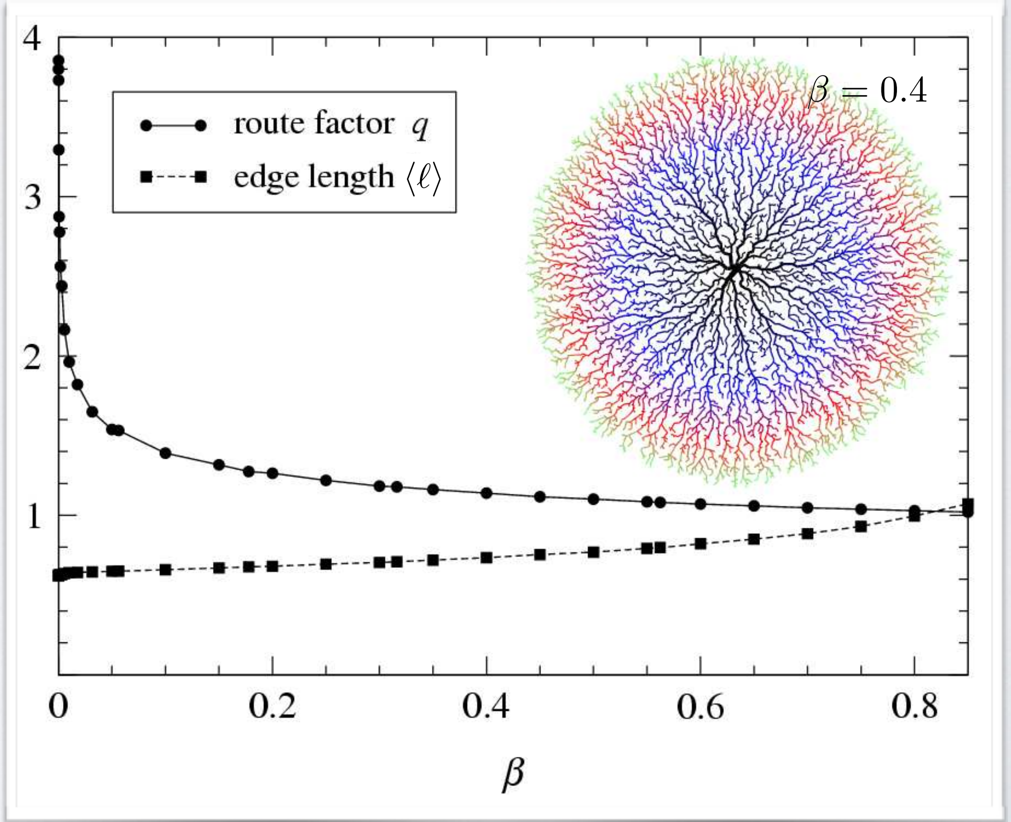
until all vertices connected

add edge (i, j) with minimum value for

$$w_{ij} = d_{ij} + \beta \ell_{j0}$$







Gastner & Newman, *J. Stat. Mech.* P01015 (2006)

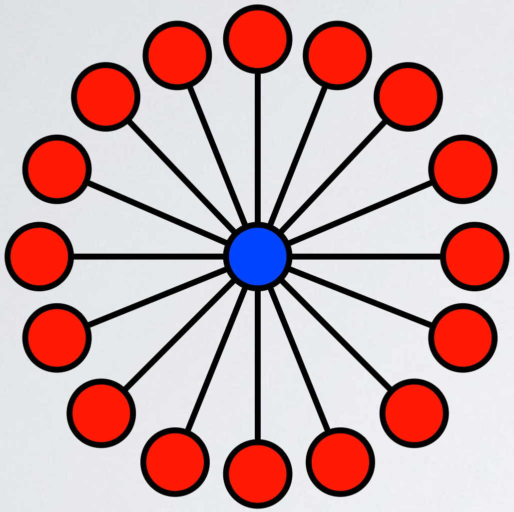
network position

most
centralized

vast wilderness
of in-between

most
decentralized

network position

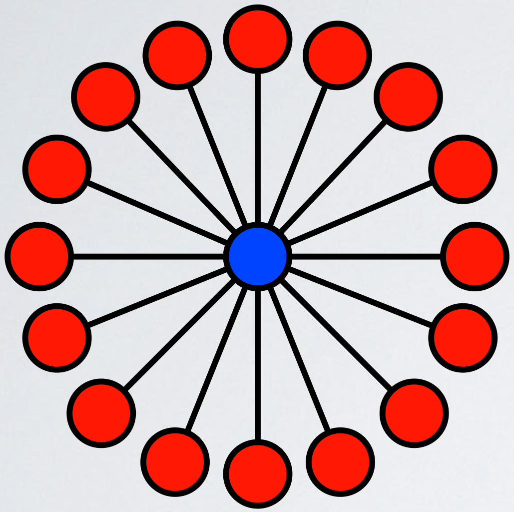


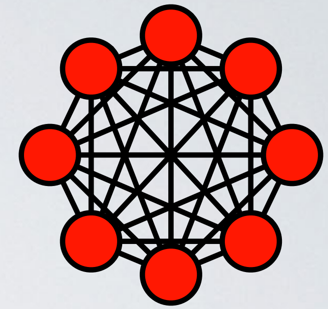
most
centralized

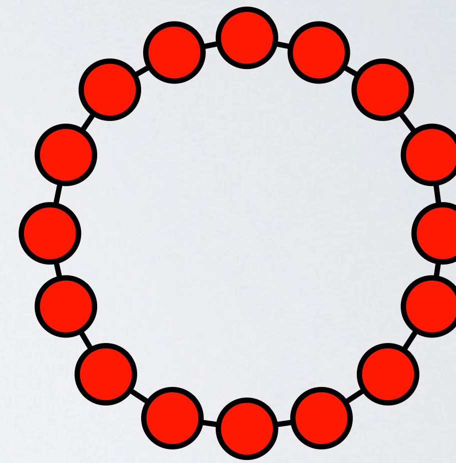
vast wilderness
of in-between

most
decentralized

network position





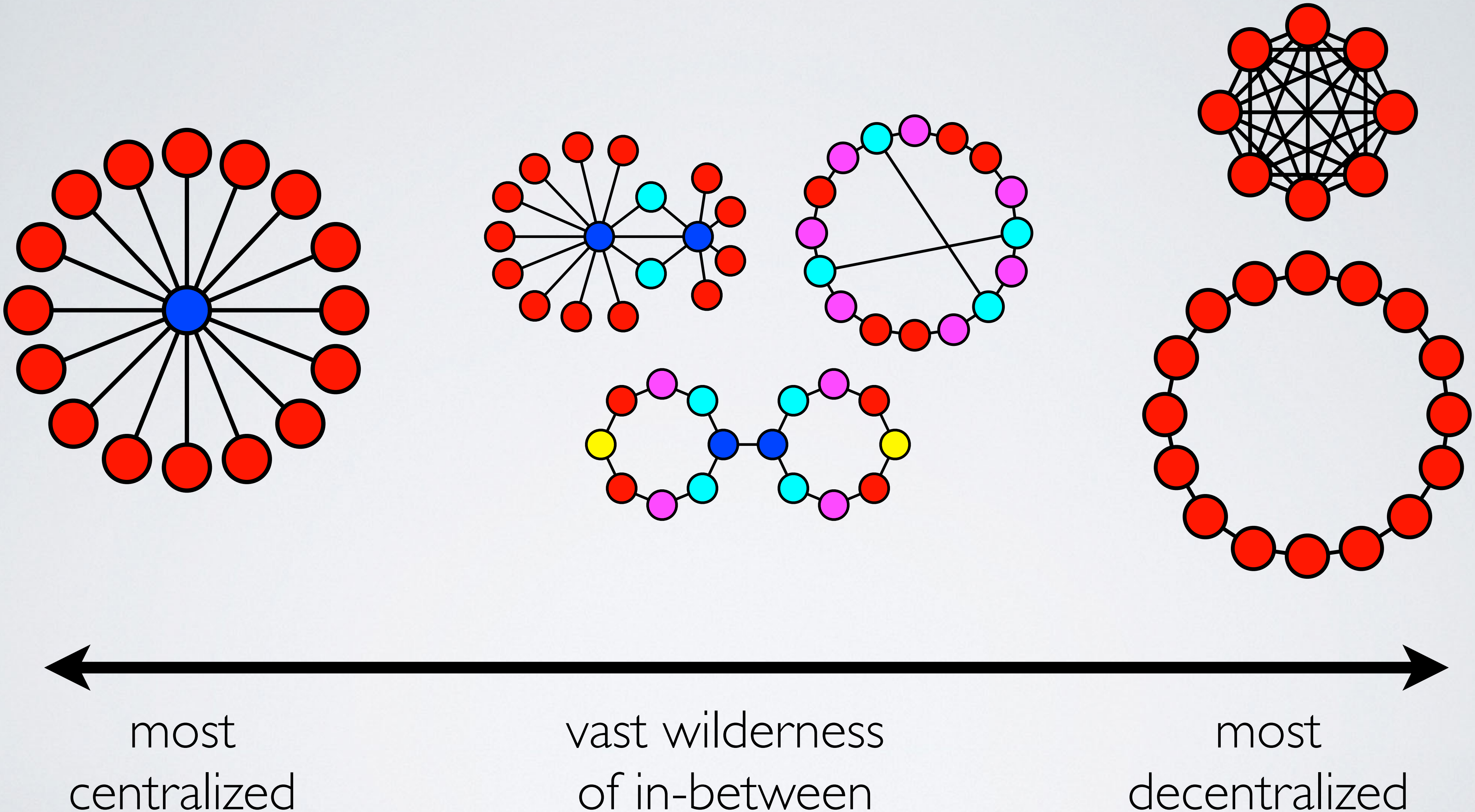


most
centralized

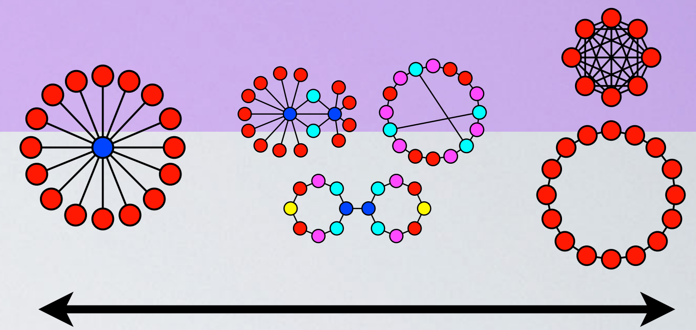
vast wilderness
of in-between

most
decentralized

network position

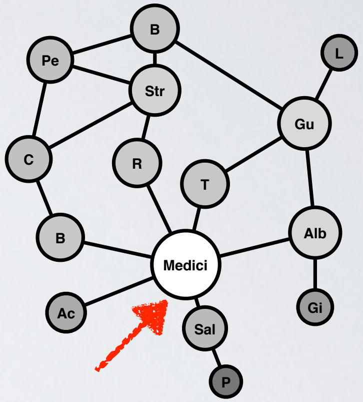


network position



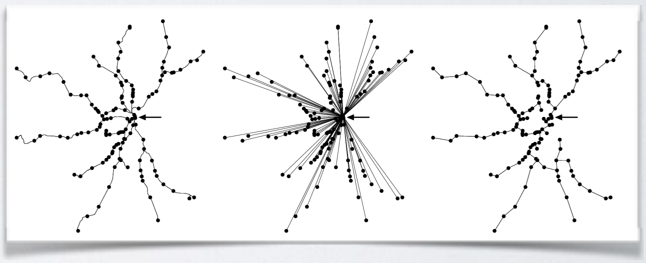
positions:

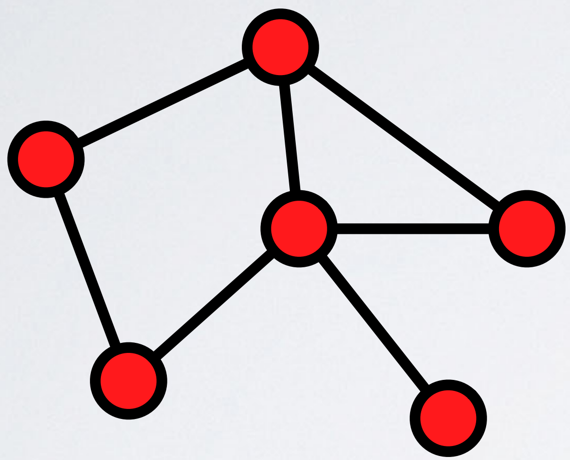
- *geometric* description of network structure
- core vs. periphery
- centrality = importance, influence



open questions:

- position and dynamics
- what does position predict?
- when does position *not* matter?

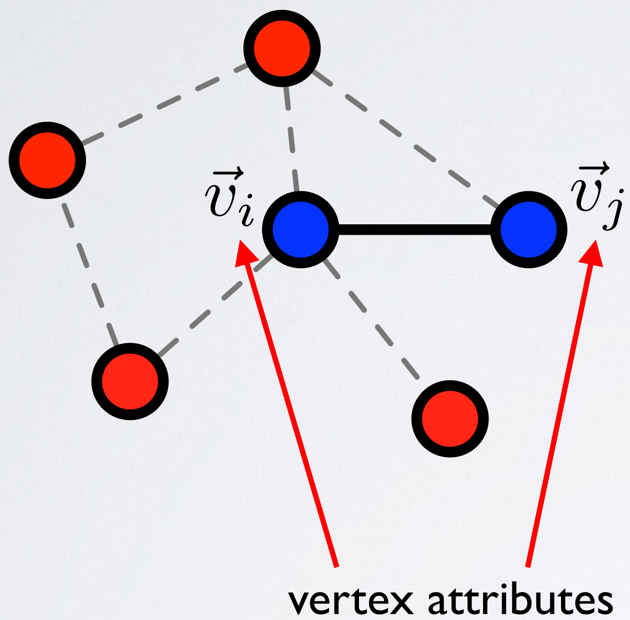




homophily and assortative mixing

like links with like

assortative mixing



homophily and assortative mixing

like links with like

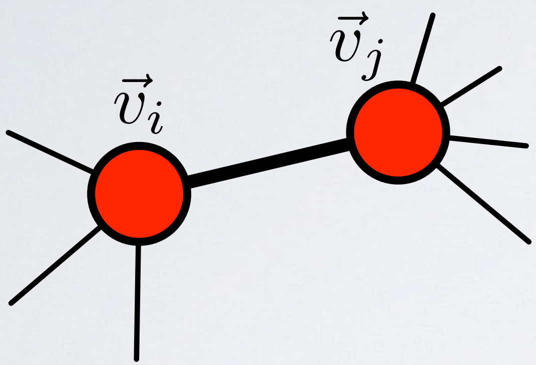
assortativity coefficient r quantifies homophily

three types:

- scalar attributes
- vertex degrees
- categorical variables

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

scalar attributes:

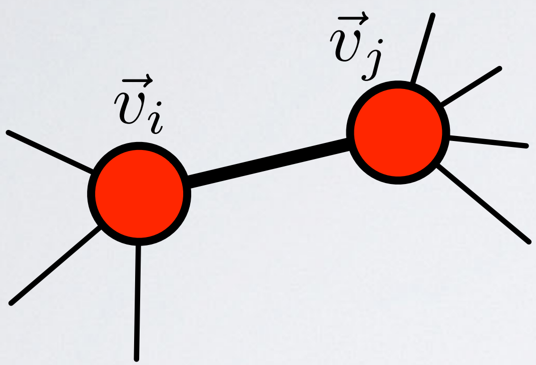
mean value across ties

$$\mu = \frac{1}{2m}\sum_i\sum_j A_{ij}v_i$$

$$\boxed{= \frac{1}{2m}\sum_i k_i v_i}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

scalar attributes:
covariance across ties

$$\mathrm{cov}(v_i, v_j) = \frac{\sum_{ij} A_{ij}(v_i - \mu)(v_j - \mu)}{\sum_{ij} A_{ij}}$$

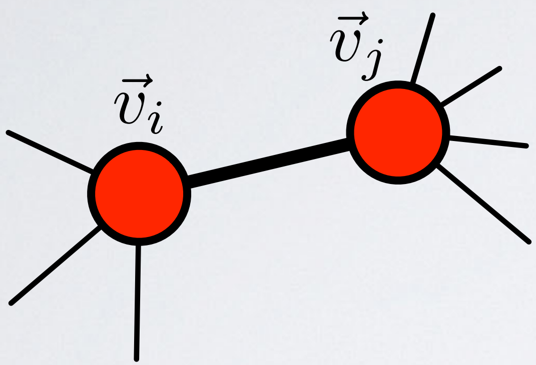
$$\left(\mu = \frac{1}{2m}\sum_i k_i v_i\right)$$

$$= \frac{1}{2m}\sum_{ij} A_{ij} v_i v_j - \mu^2$$

$$\boxed{= \frac{1}{2m}\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right) v_i v_j}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

assortativity coefficient (scalar)

$$r = \frac{\mathrm{cov}(v_i, v_j)}{\mathrm{var}(v_i, v_j)}$$

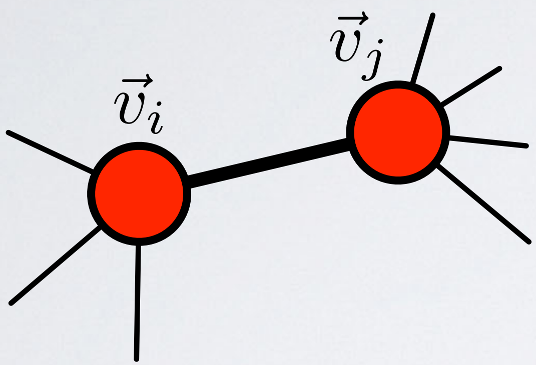
$$\boxed{= \frac{\sum_{ij} \left(A_{ij} - k_i k_j / 2m\right) v_i v_j}{\sum_{ij} k_i \delta_{ij} - k_i k_j / 2m}}$$

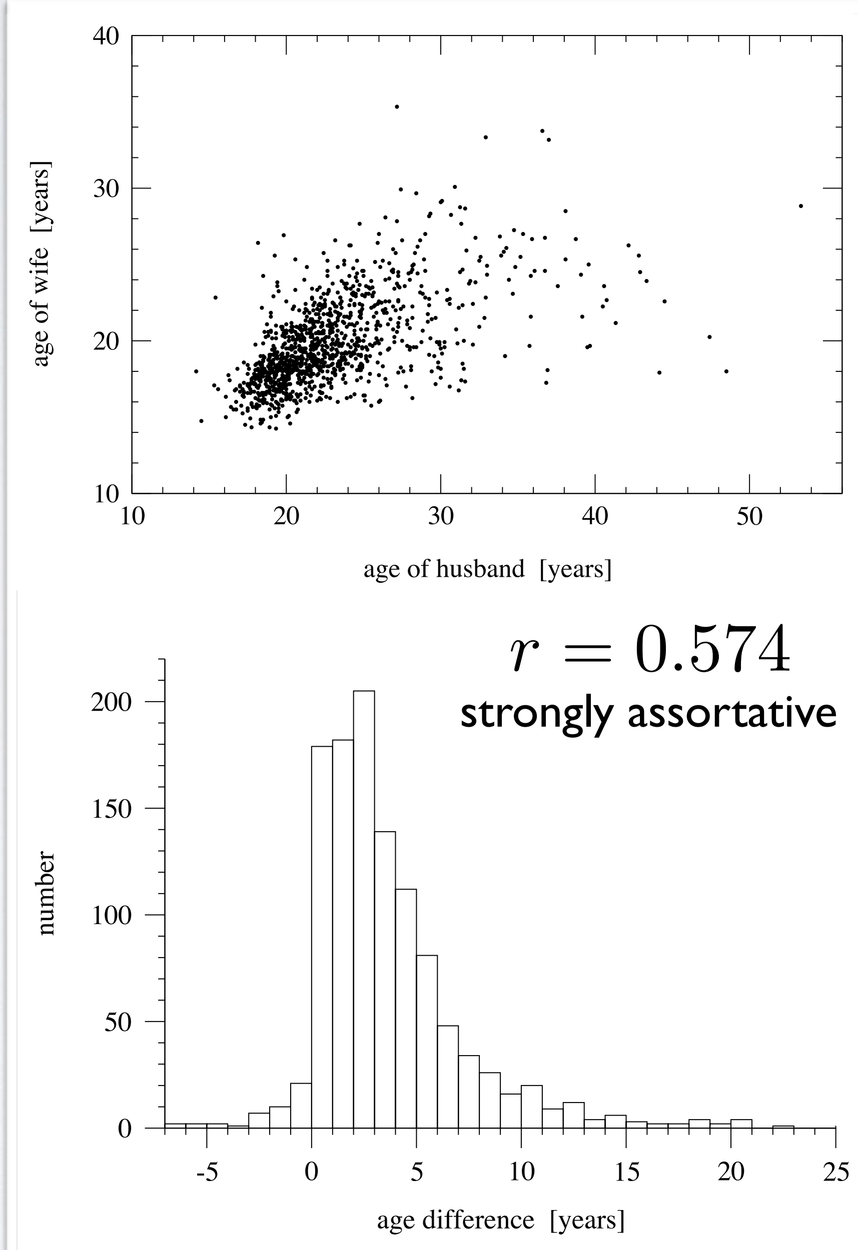
[this is just a Pearson correlation across edges]

$$-1 \leq r \leq 1$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



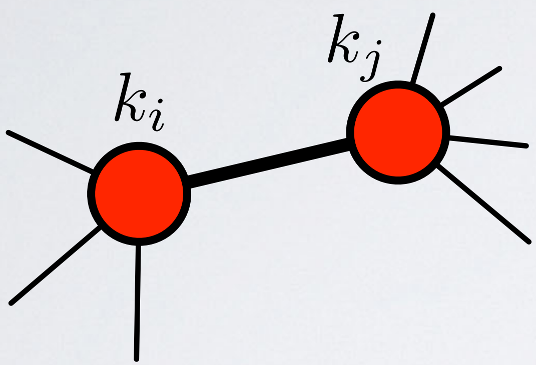


(top) scatter plot of ages of 1141 married couples at time of marriage [1995 US National Survey of Family Growth]

(bottom) histogram of age differences (M-F) for same data

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

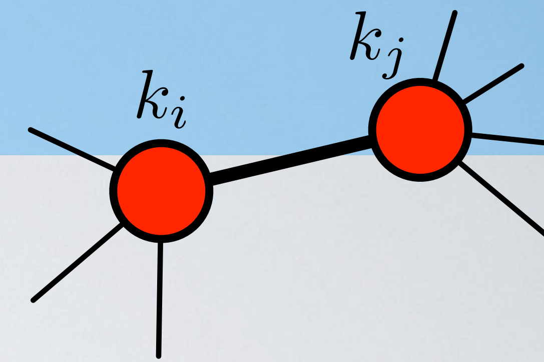
like links with like

degree:
just another scalar*

Newman, *Phys. Rev. E* **67**, 026126 (2003).

* the assortativity coefficient formula simplifies somewhat in this case. see the Ref in the left corner for more details

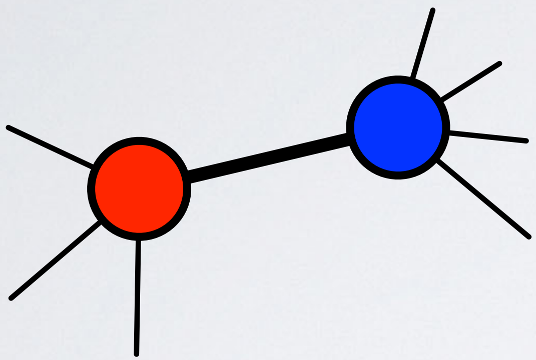
assortative mixing



	network	type	size n	degree assortativity r	error σ_r
social	physics coauthorship	undirected	52 909	0.363	0.002
	biology coauthorship	undirected	1 520 251	0.127	0.0004
	mathematics coauthorship	undirected	253 339	0.120	0.002
	film actor collaborations	undirected	449 913	0.208	0.0002
	company directors	undirected	7 673	0.276	0.004
	student relationships	undirected	573	−0.029	0.037
	email address books	directed	16 881	0.092	0.004
technological	power grid	undirected	4 941	−0.003	0.013
	Internet	undirected	10 697	−0.189	0.002
	World-Wide Web	directed	269 504	−0.067	0.0002
	software dependencies	directed	3 162	−0.016	0.020
biological	protein interactions	undirected	2 115	−0.156	0.010
	metabolic network	undirected	765	−0.240	0.007
	neural network	directed	307	−0.226	0.016
	marine food web	directed	134	−0.263	0.037
	freshwater food web	directed	92	−0.326	0.031

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

categorical variables:

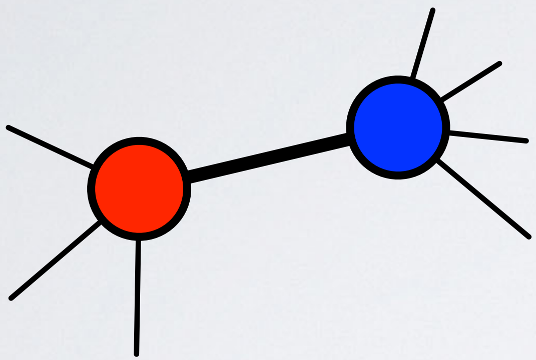
let e_{ij} be fraction of edges connecting vertices of type i to vertices of type j

matrix sum $$\sum_{ij} e_{ij} = 1$$

marginals $$\sum_{j} e_{ij} = a_i \qquad \sum_{i} e_{ij} = b_j$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

assortative mixing



homophily and assortative mixing

like links with like

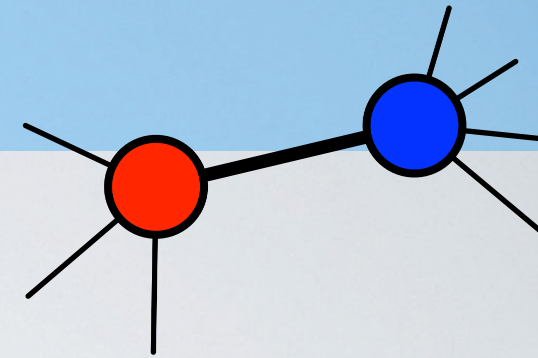
categorical variables:
assortativity coefficient*

$$r = \frac{\sum_i e_{ii} - \sum_i a_i b_i}{1 - \sum_i a_i b_i} = \frac{\mathrm{Tr}\,\mathbf{e} - ||\mathbf{e}^2||}{1 - ||\mathbf{e}^2||}$$

Newman, *Phys. Rev. E* **67**, 026126 (2003).

* this equation is equivalent to the popular *modularity* measure *Q* used to score the strength of community structure

assortative mixing



1992 study of heterosexual partnerships in San Francisco*
(bipartite network)

		women				
		black	hispanic	white	other	a_i
men	black	0.258	0.016	0.035	0.013	0.323
	hispanic	0.012	0.157	0.058	0.019	0.247
	white	0.013	0.023	0.306	0.035	0.377
	other	0.005	0.007	0.024	0.016	0.053
	b_i	0.289	0.204	0.423	0.084	

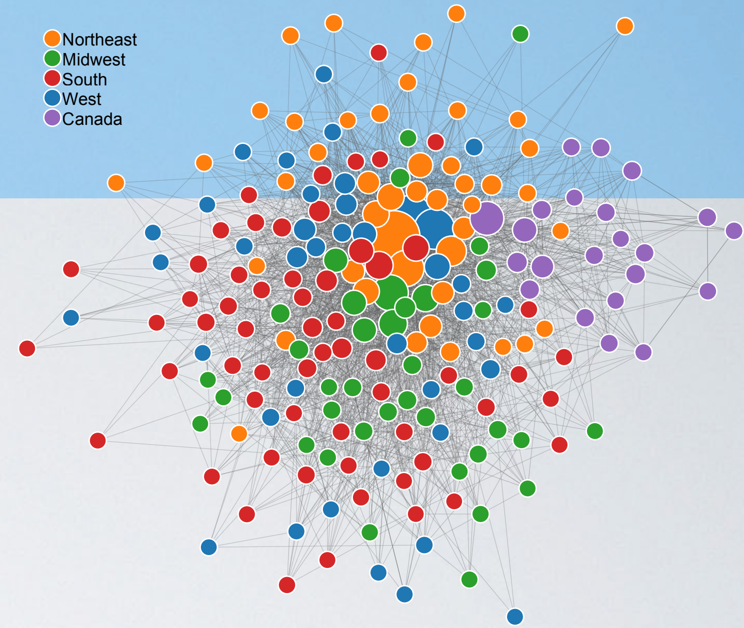
$$r = 0.621$$

strongly assortative

Newman, *Phys. Rev. E* **67**, 026126 (2003).

*Catania et al., *Am. J. Public Health* **82**, 284-287 (1992).

assortative mixing



4388 Computer Science faculty

vertices are PhD granting institutions in North America

edge (u, v) means PhD at u and now faculty at v

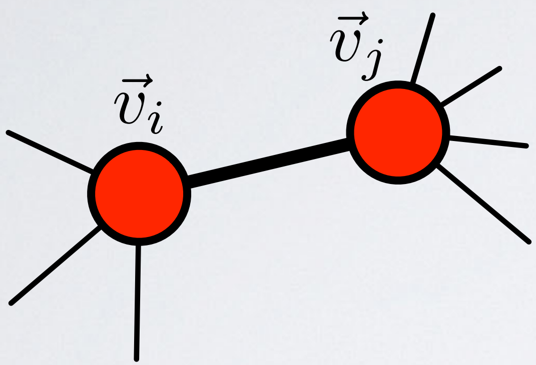
labels are US census regions + Canada

	Northeast	Midwest	South	West	Canada	a_i
Northeast	**0.119**	0.053	0.074	0.055	0.022	0.322
Midwest	0.031	**0.067**	0.061	0.026	0.011	0.196
South	0.025	0.027	**0.083**	0.024	0.006	0.166
West	0.049	0.033	0.043	**0.073**	0.011	0.209
Canada	0.006	0.005	0.005	0.005	**0.085**	0.107
b_i	0.229	0.185	0.267	0.184	0.135	

$$r = 0.264$$

moderately assortative

assortative mixing



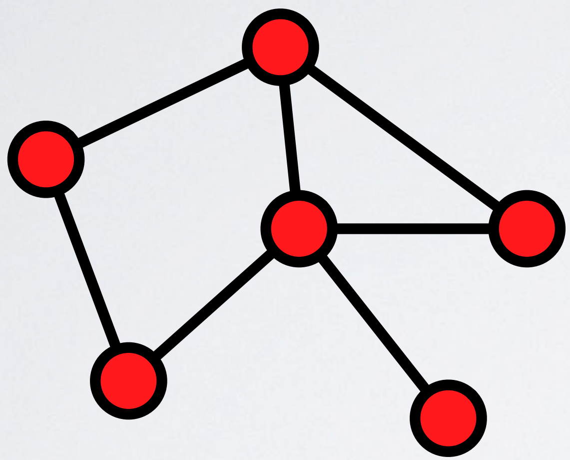
homophily and assortative mixing

like links with like

- random graphs tend to be disassortative $r \leq 0$ because the mixing is uniform
- social networks (apparently) highly assortative, in every way (attribute, degree, category)
- extremal values $r \approx \{-1, 1\}$ suggest underlying mechanism on that variable

Newman, *Phys. Rev. E* **67**, 026126 (2003).

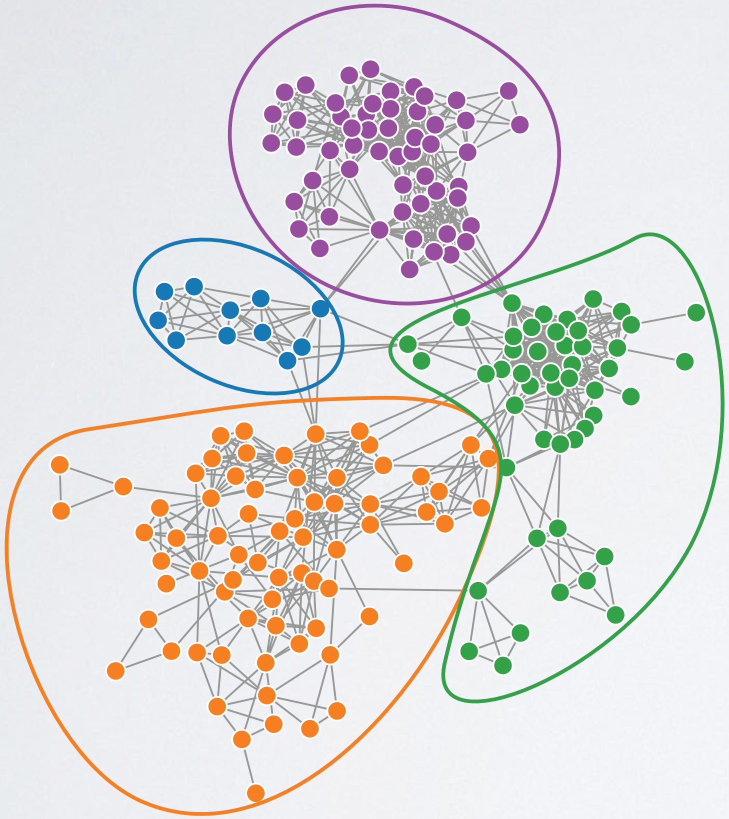
describing networks



community structure

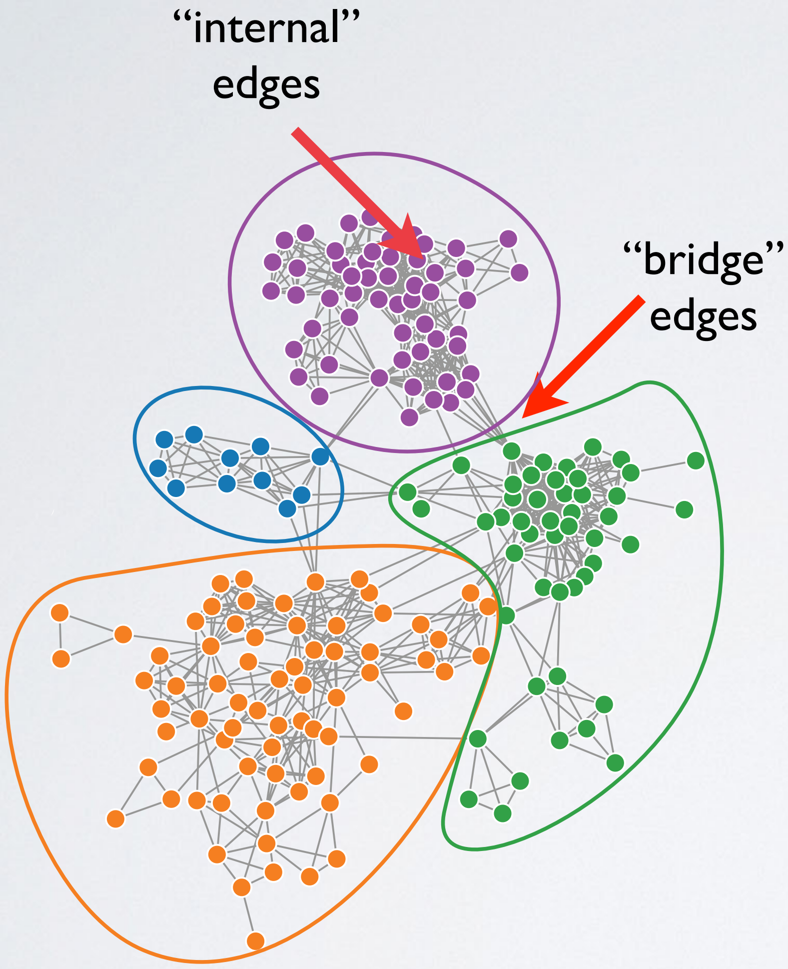
describing networks

community structure:
a *group of vertices* that connect to *other groups* in similar ways



assortative community structure
(edges inside the groups)

community structure



assortative community structure
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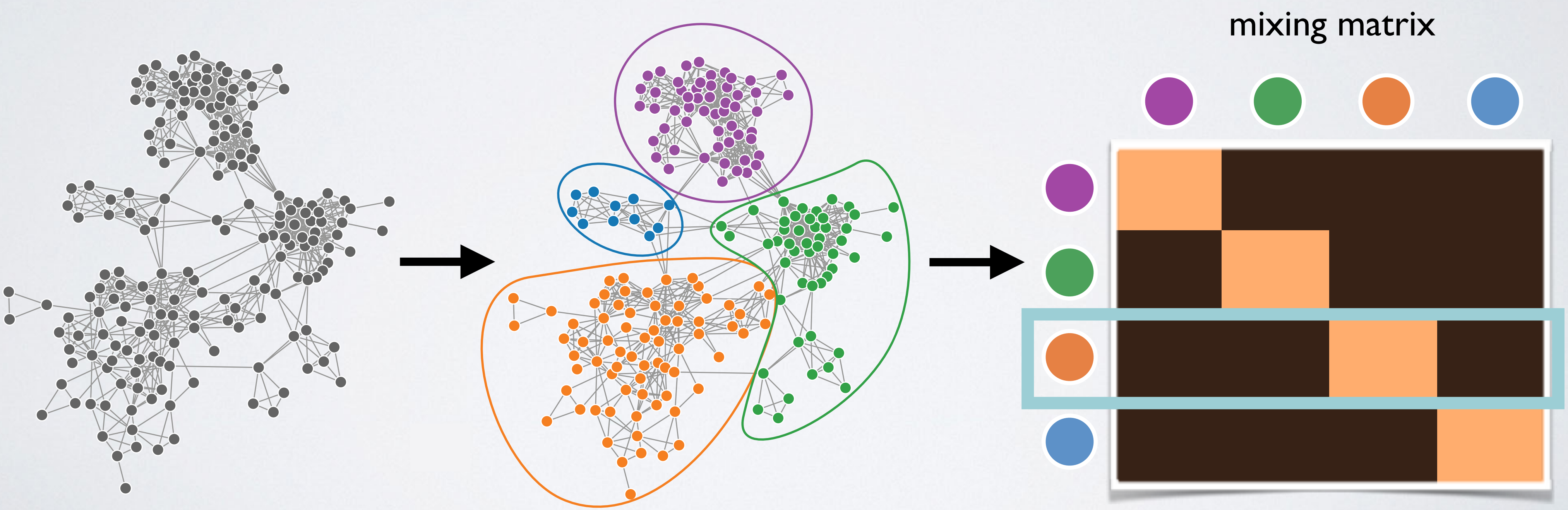
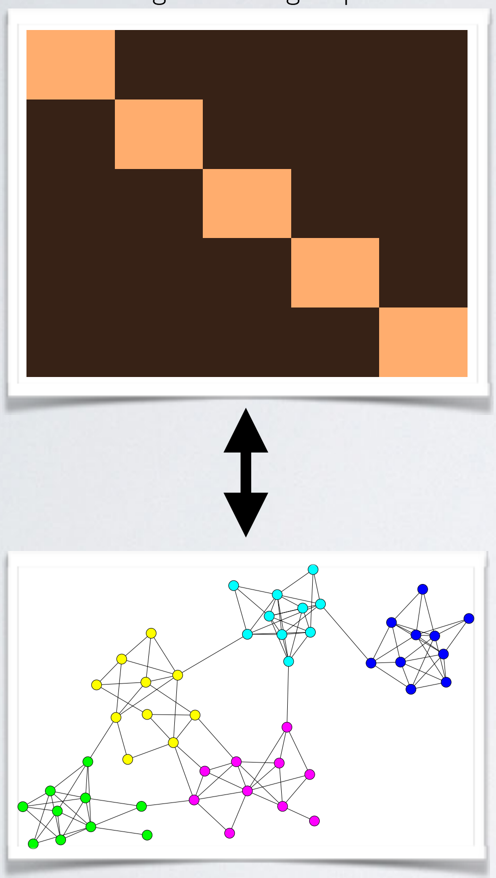
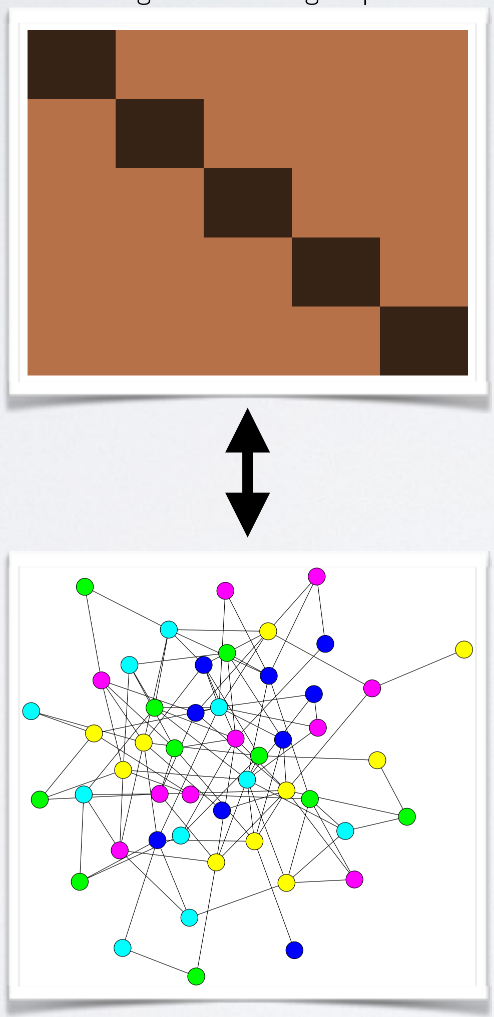


community structure

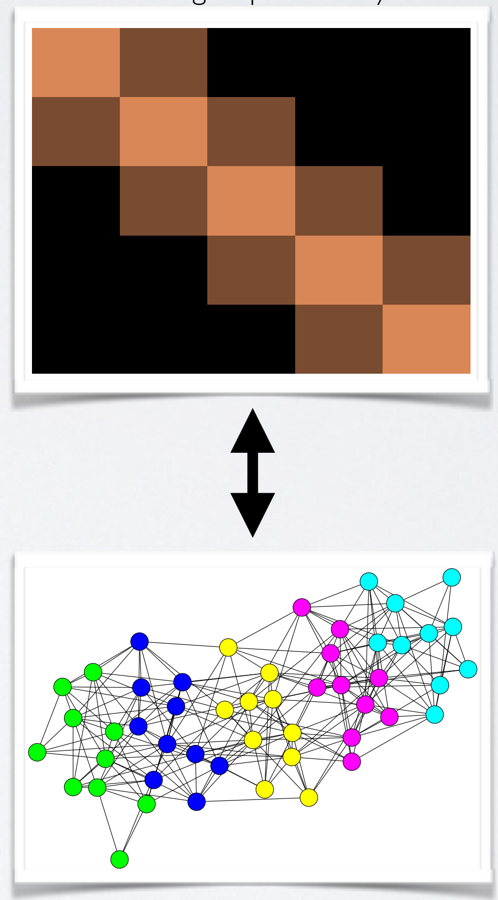
assortative
edges within groups



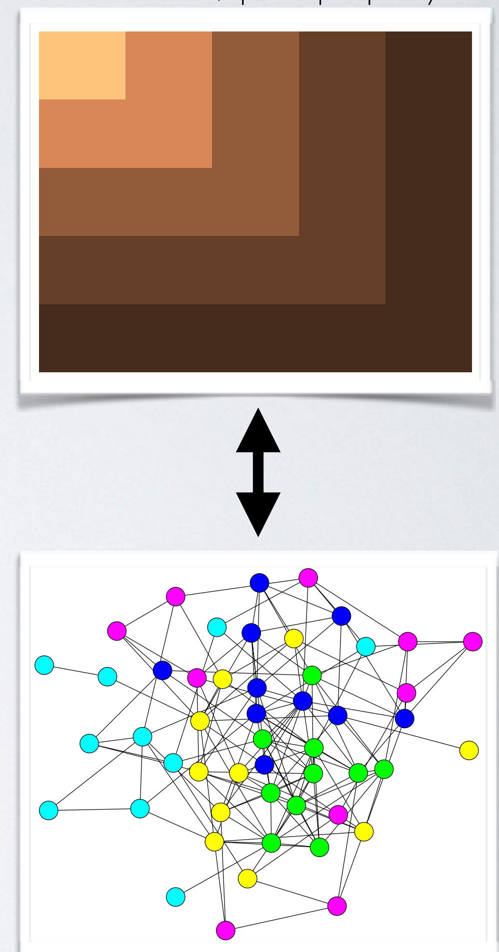
disassortative
edges between groups



ordered
linear group hierarchy



core-periphery
dense core, sparse periphery



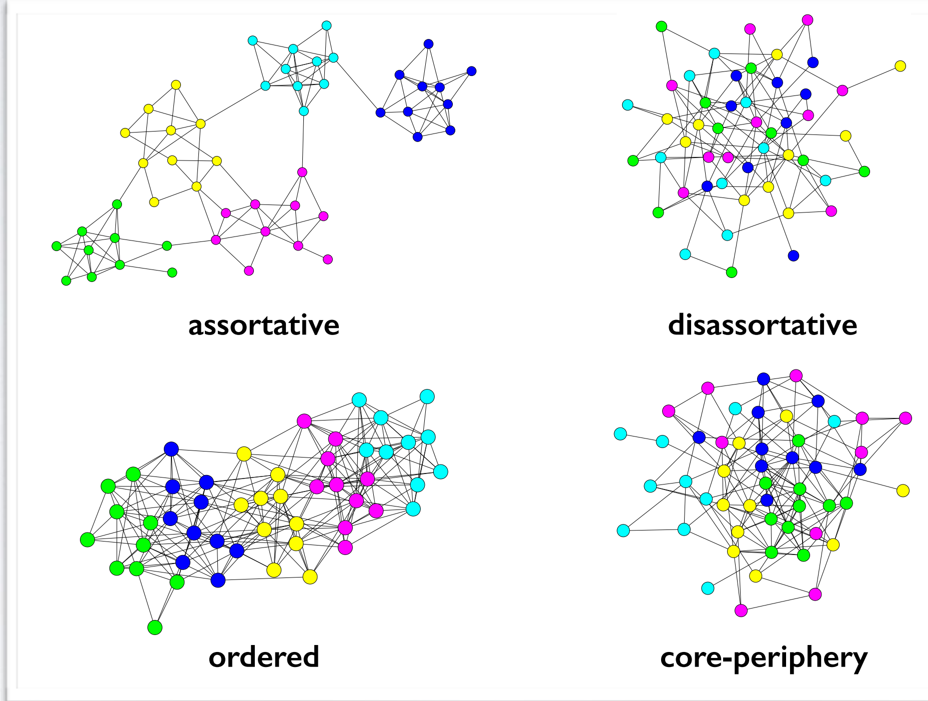
community structure

- enormous interest, especially since 2000
- dozens of algorithms for extracting various large-scale patterns
- hundreds of papers published
- spanning Physics, Computer Science, Statistics, Biology, Sociology, and more
- this was one of the first:

Community structure in social and biological networks

M. Girvan*†‡ and M. E. J. Newman*§ *PNAS* 2002

5700+ citations on Google Scholar

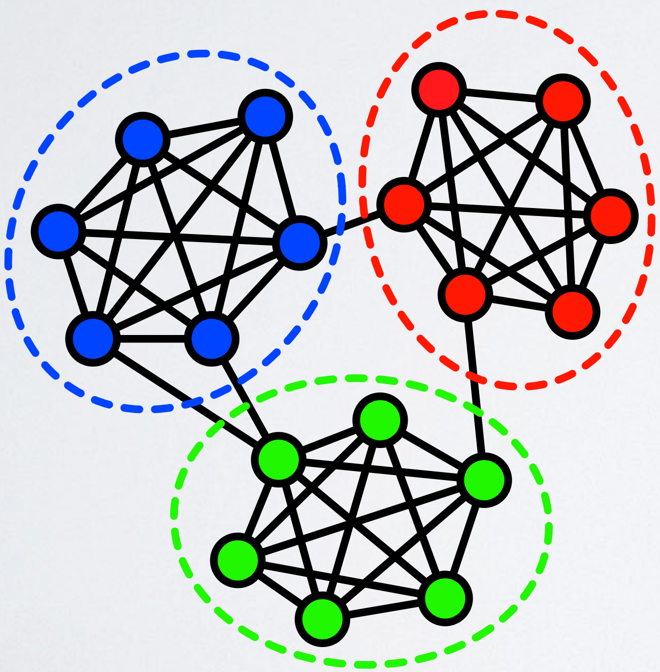


network communities

THE STRENGTH OF WEAK TIES:
A NETWORK THEORY REVISITED

1983

Mark Granovetter



most new job opportunities from "weak ties"

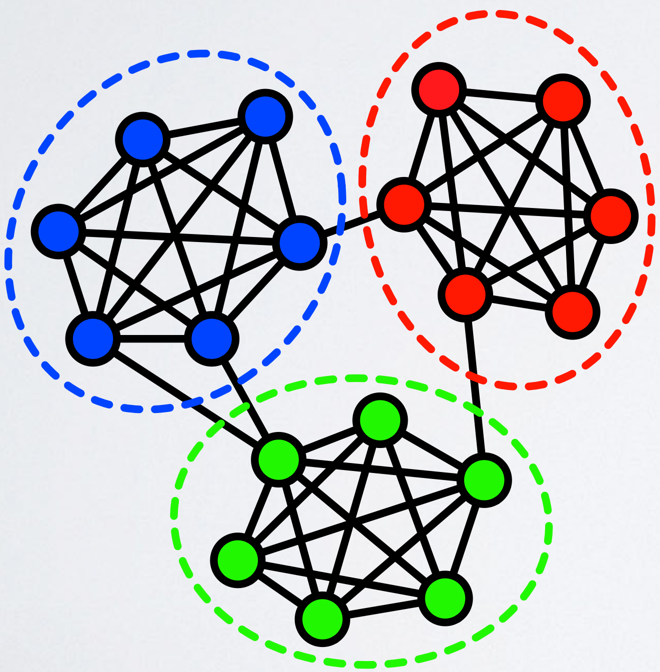
- within-community links = strong
- bridge links = weak

network communities

THE STRENGTH OF WEAK TIES:
A NETWORK THEORY REVISITED

Mark Granovetter

1983



most new job opportunities from "weak ties"

- within-community links = strong
- bridge links = weak

why?

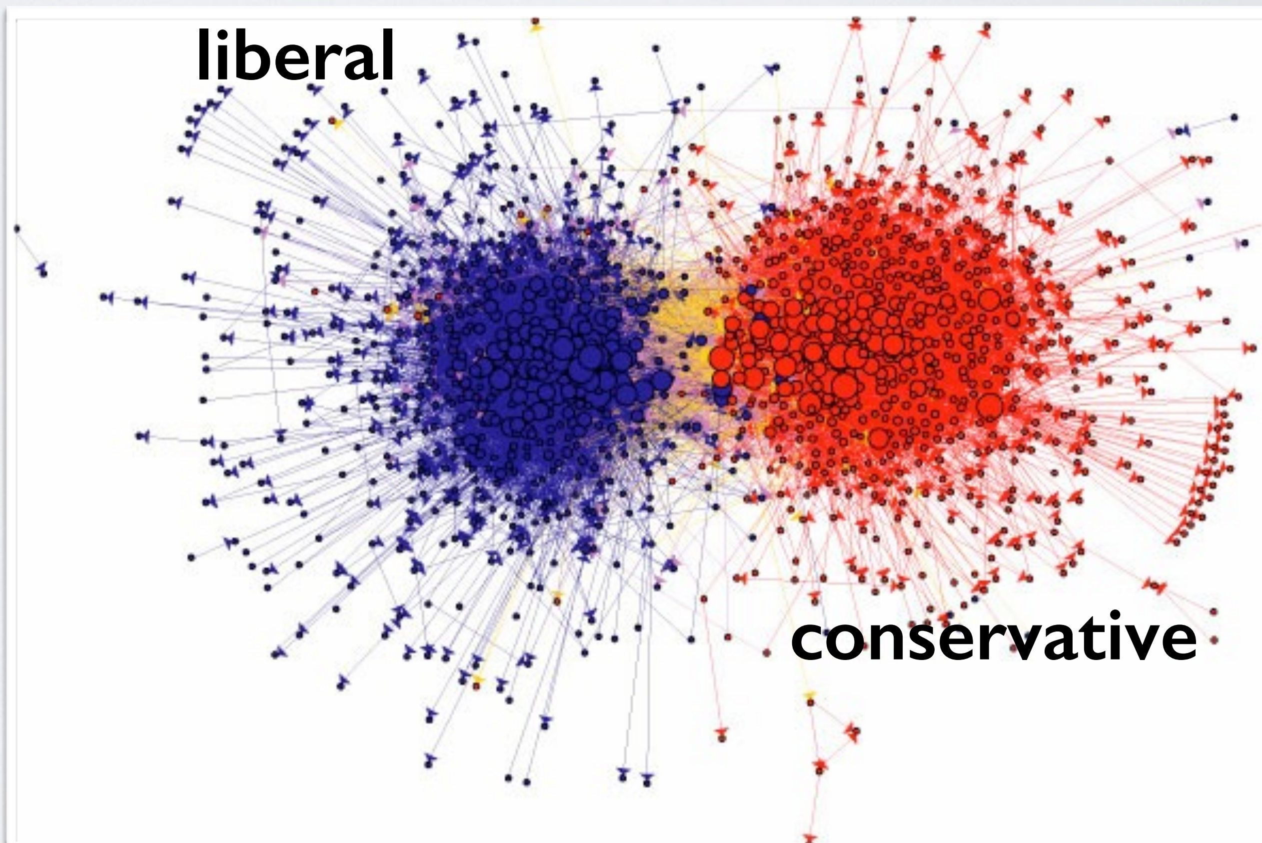
information propagates *quickly* *within* a community,
but *slowly between* communities

network communities

The Political Blogosphere and the 2004 U.S. Election:
Divided They Blog

Lada Adamic   Natalie Glance   2004

1494 blogs
759 liberal
735 conservative



network communities

Finding community structure in very large networks

Aaron Clauset, M. E. J. Newman, and Cristopher Moore 2004

amazon.com
co-purchasing network

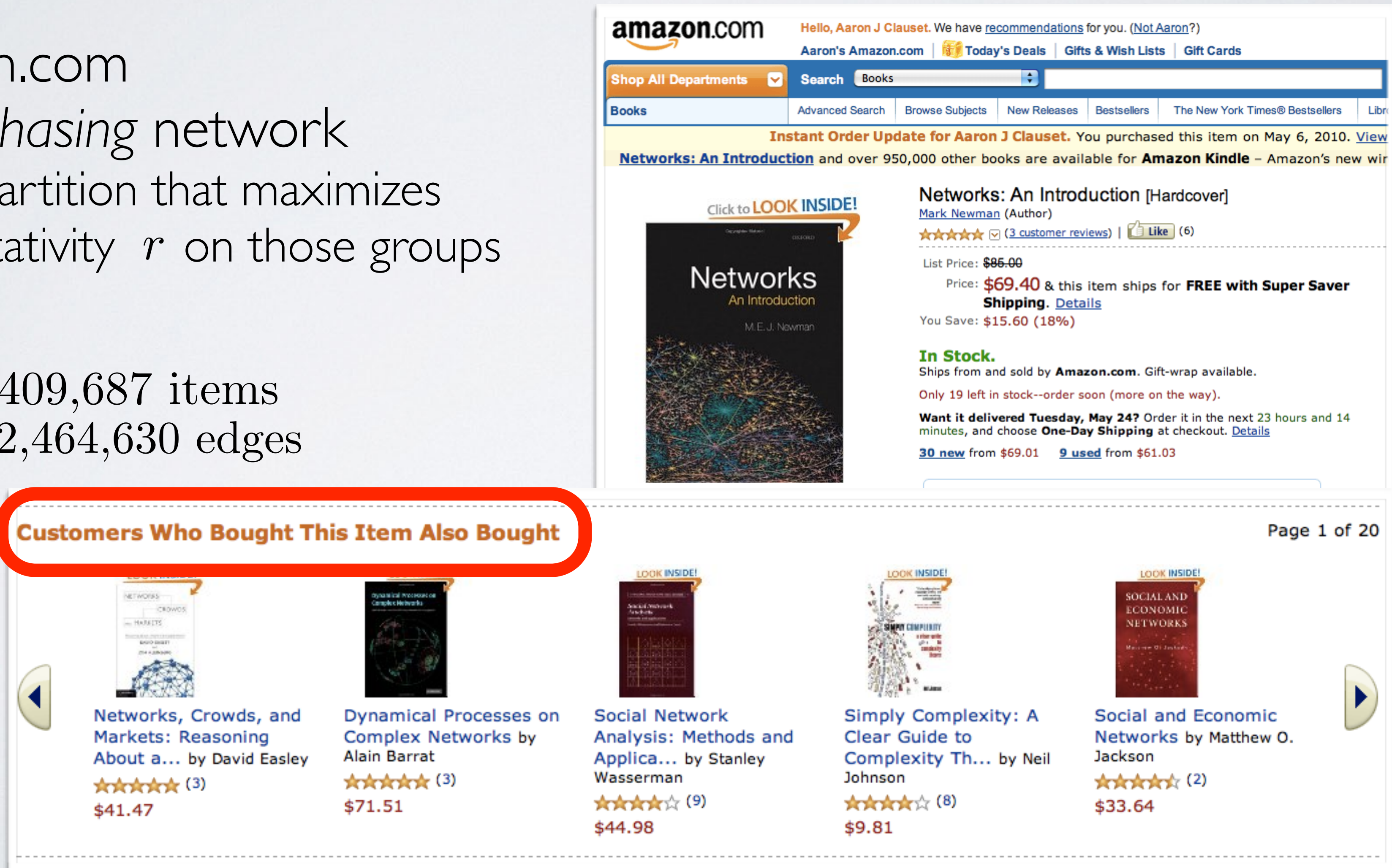
network communities

Finding community structure in very large networks

Aaron Clauset, M. E. J. Newman, and Cristopher Moore 2004

amazon.com
co-purchasing network
find partition that maximizes
assortativity r on those groups

$n = 409{,}687$ items
$m = 2{,}464{,}630$ edges

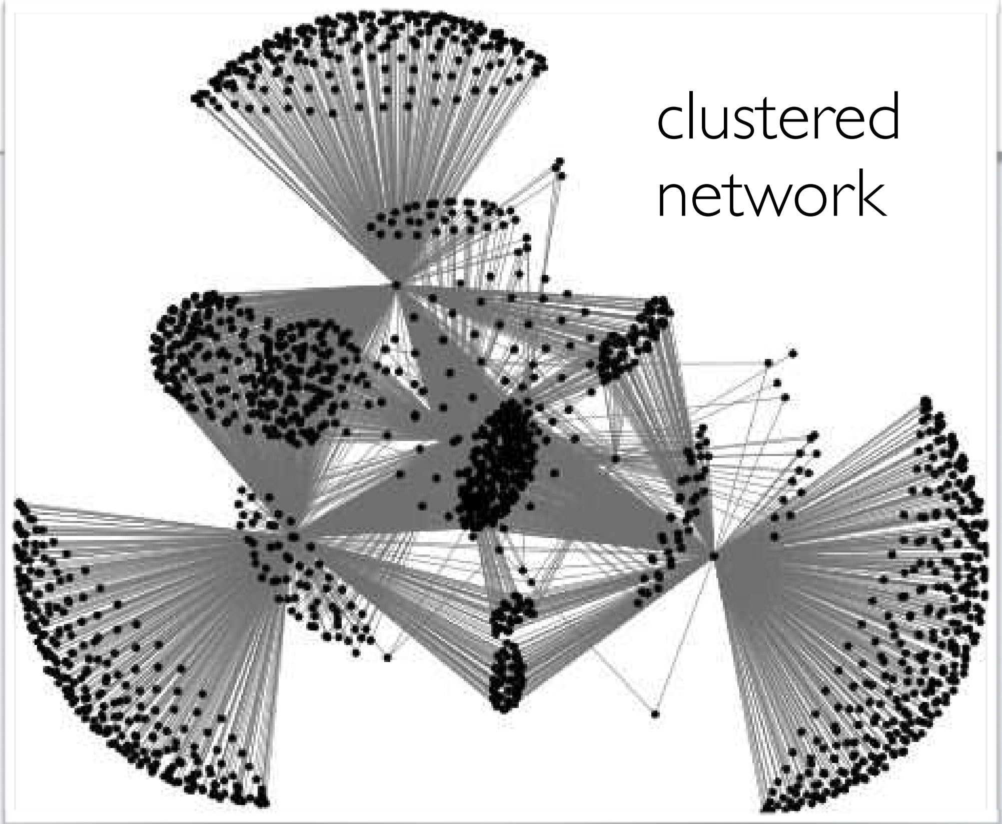


network communities

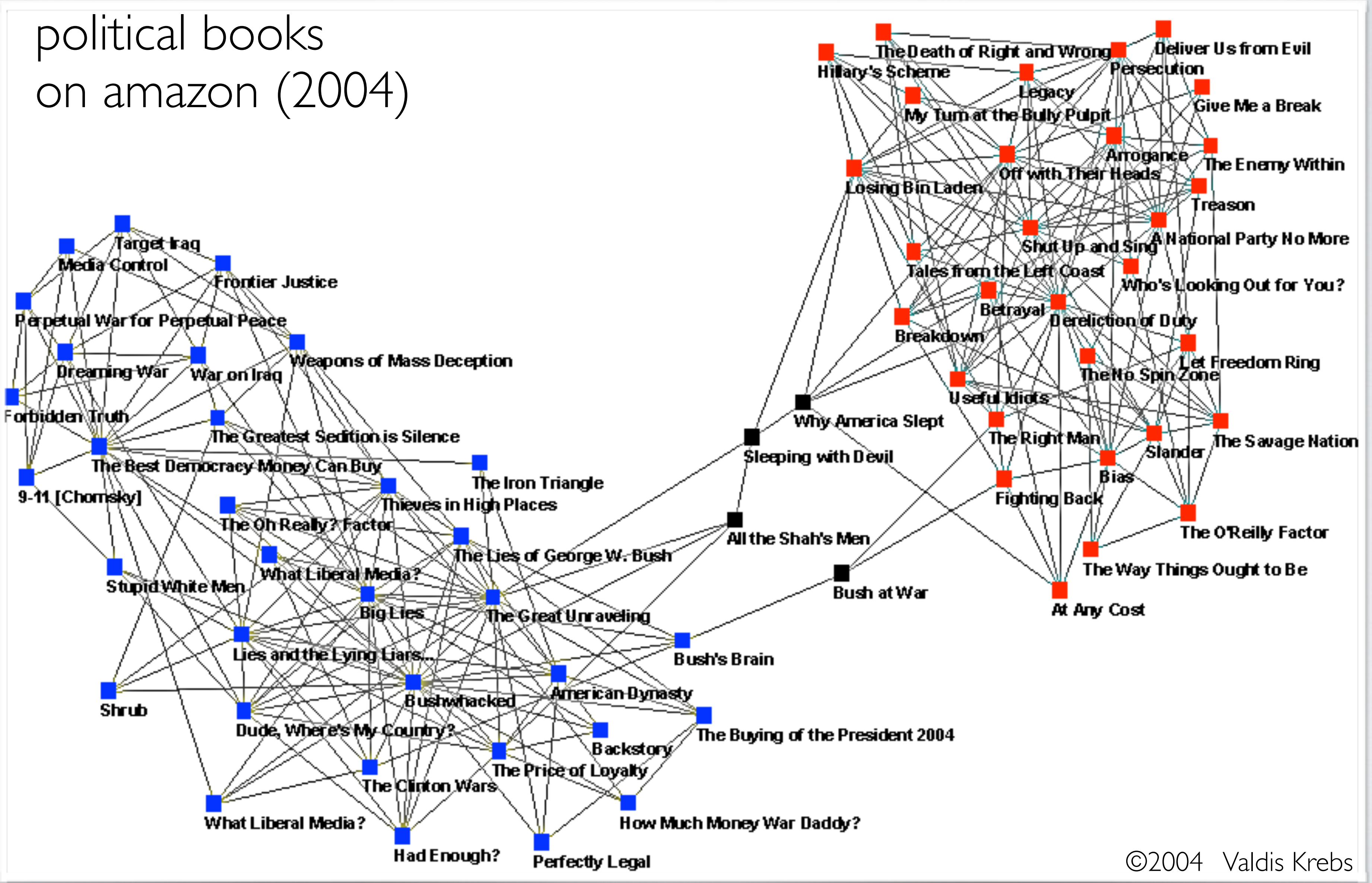
Rank	Size	Description
1	114538	General interest: politics; art/literature; general fiction; human nature; technical books; how things, people, computers, societies work, etc.
2	92276	The arts: videos, books, DVDs about the creative and performing arts
3	78661	Hobbies and interests I: self-help; self-education; popular science fiction, popular fantasy; leisure; etc.
4	54582	Hobbies and interests II: adventure books; video games/comics; some sports; some humor; some classic fiction; some western religious material; etc.
5	9872	classical music and related items
6	1904	children's videos, movies, music and books
7	1493	church/religious music; African-descent cultural books; homoerotic imagery
8	1101	pop horror; mystery/adventure fiction
9	1083	jazz; orchestral music; easy listening
10	947	engineering; practical fashion

purchases = interests

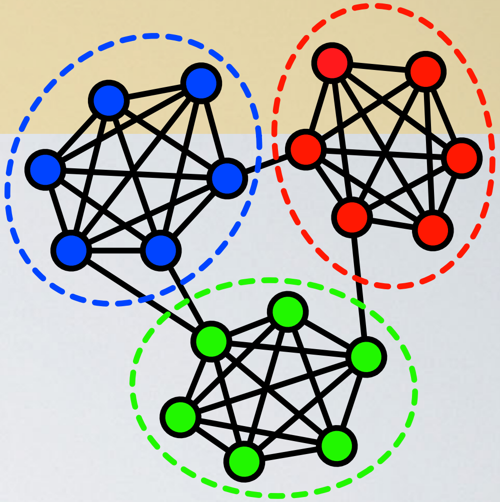
interests = clustered



network communities

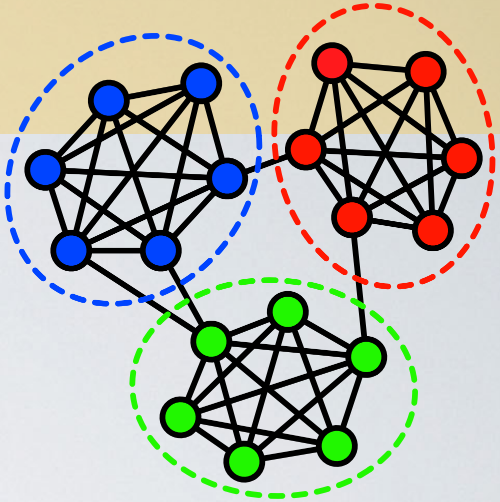


network communities



- *community = vertices with same pattern of inter-community connections*
- network macro-structure
- finding them like "network clustering"
- allow us to *coarse grain* system structure
  [decompose heterogeneous structure into homogeneous blocks]
- constrains network synchronization, information flows, diffusion, influence

network communities



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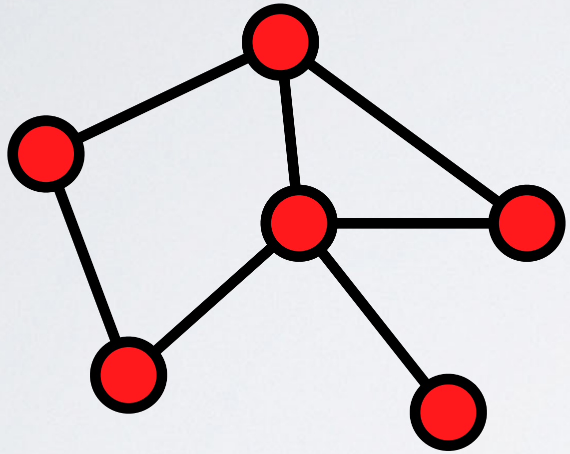
open questions:

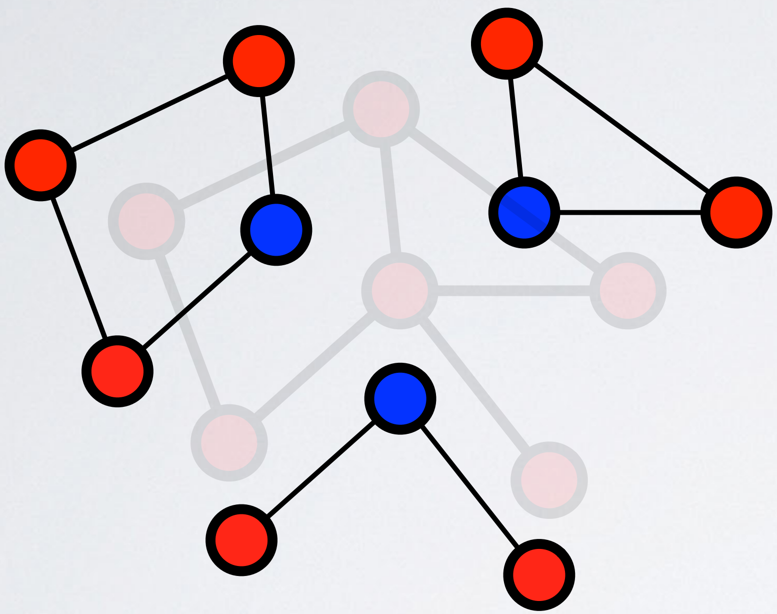
- what processes generate communities?
- what impact on dynamics? network function?

describing networks



motifs

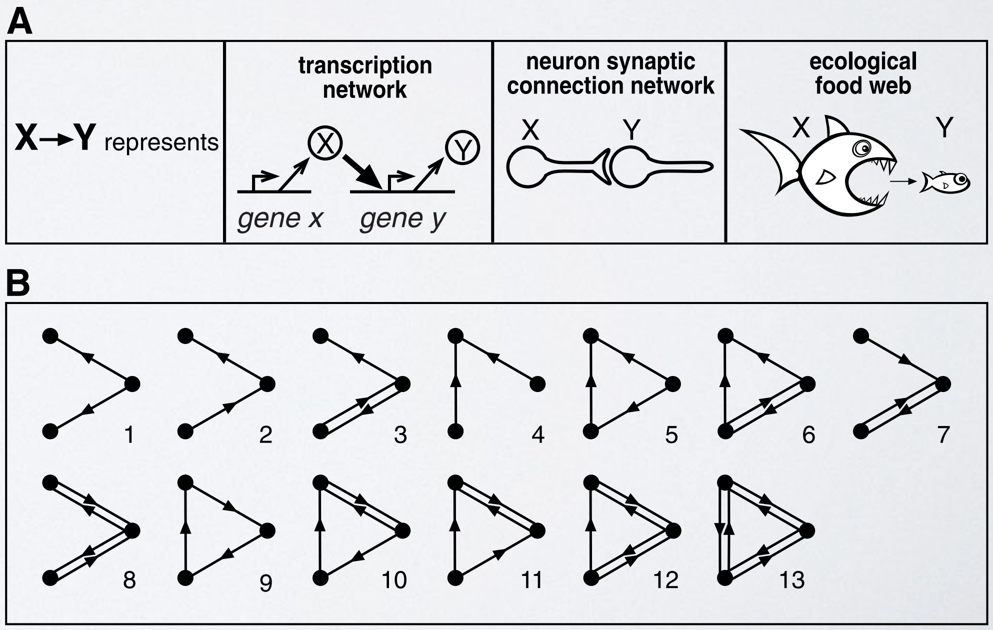
describing networks



motifs:

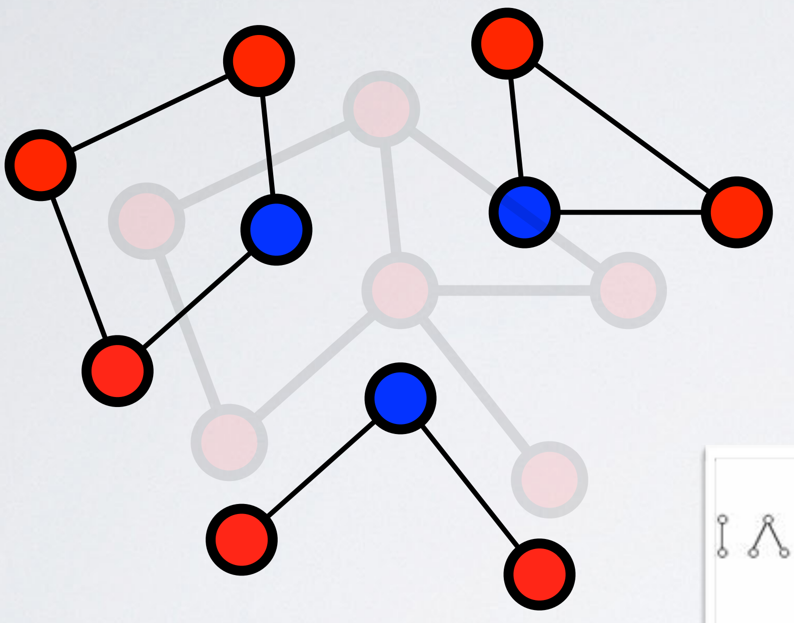
small subgraphs (of interest), which we then count

compare counts against null model (random graph model)



Milo et al., *Science* **298**, 824-827 (2002).

describing networks



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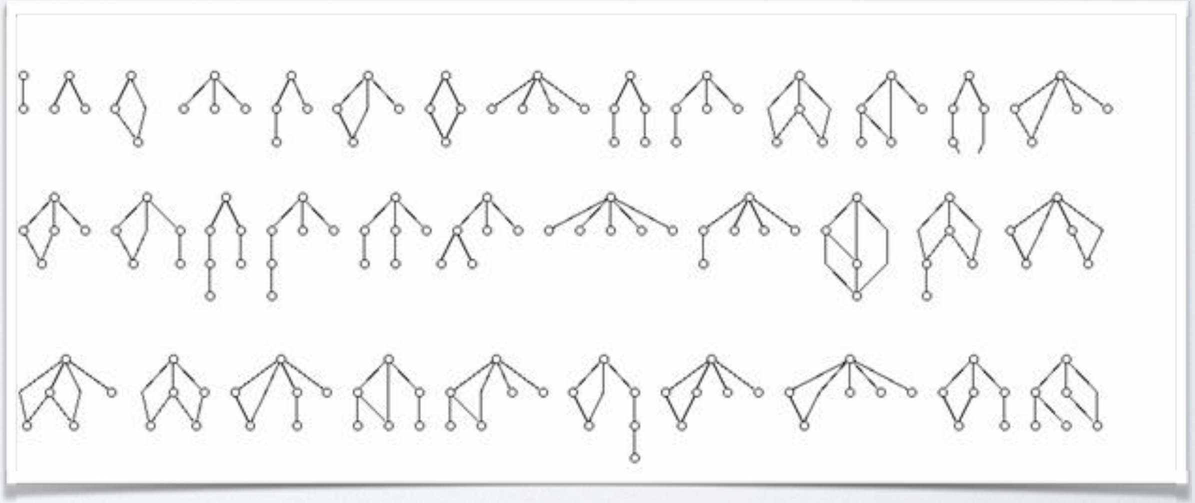

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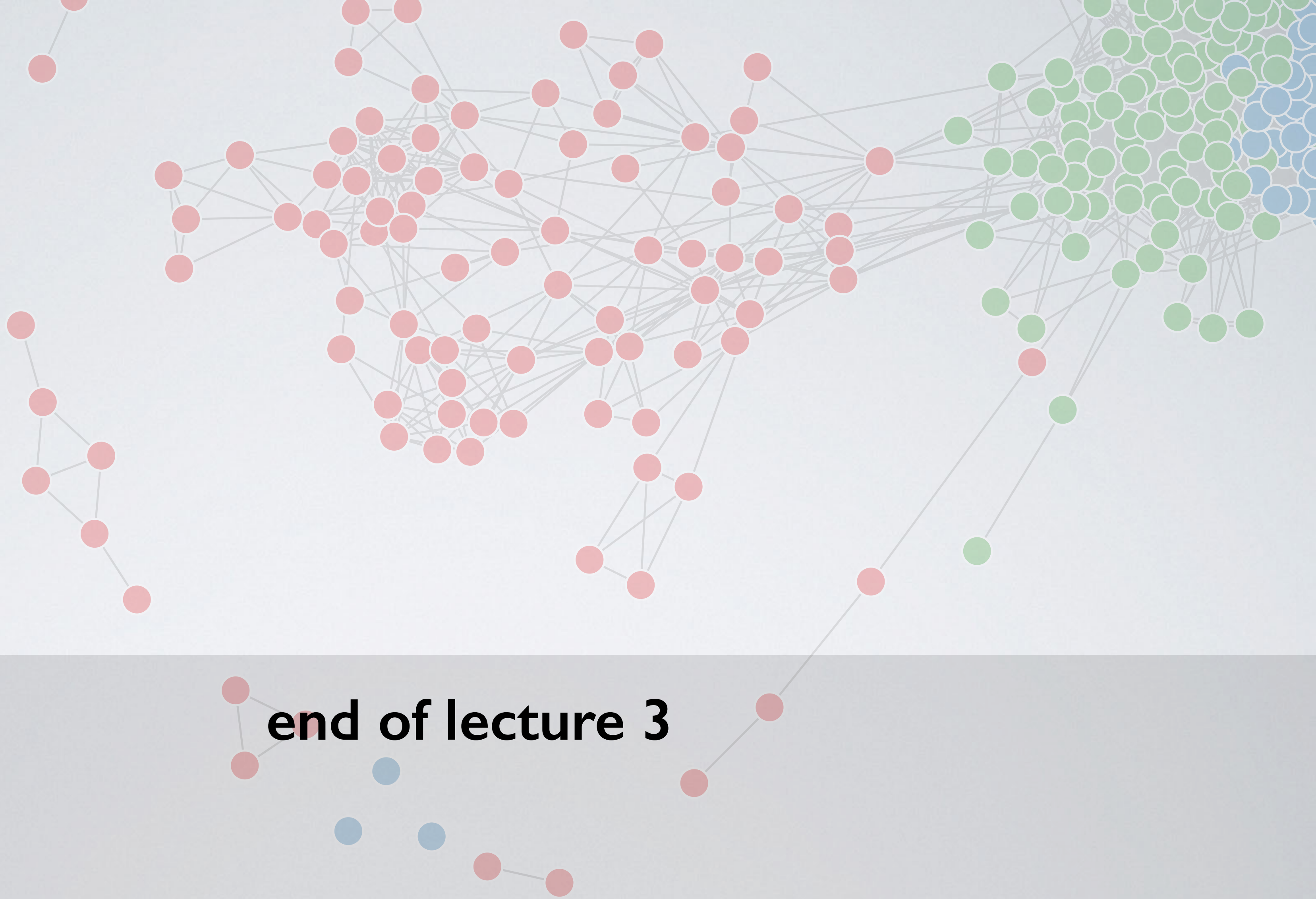

end of lecture 3

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