Structure vs. Strategy: The theory of market impact

Is there a physics of society?
October 25, 2007

J. Doyne Farmer
Santa Fe Institute

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The Virtues and Vices of Equilibrium, and the Future of Financial Economics

J. Doyne Farmer* and John Geanakoplos†

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Abstract

The use of equilibrium models in economics springs from the desire for parsimonious models of economic phenomena that take human reasoning into account. This approach has been the cornerstone of modern economic theory. We explain why this is so, extolling the virtues of equilibrium theory, then present a critique and describe why this approach is inherently limited, and why economics needs to move in new directions if it is to continue to make progress. We stress that this shouldn't be a question of dogma, but should be resolved empirically. There are situations where equilibrium models provide useful predictions and there are situations where they can never provide useful predictions. There are also many situations where the jury is still out, i.e., where so far they fail to provide a good description of the world, but where proper extensions might change this. Our goal is to convince the skeptics that equilibrium models can be useful, but also to make traditional economists more aware of the limitations of equilibrium models. We sketch some alternative approaches and discuss why they should play an important role in future research in economics.

Structure vs. strategy

- Strategy: Resolving consequences of strategic thinking by agents.
- Strategy: Things that don't depend on strategy, but may dictated purely by institutions, boundary conditions, emergent properties that come from interactions of agents with each other and with institutions, boundary conditions, etc.
  - e.g. crowd dynamics, traffic
- Ideal is to understand the interaction of both.

Theory of market impact
My collaborators

Austin Gerig
Santa Fe Institute

Fabrizio Lillo
University of Palermo

Demand elasticity of price

$$\frac{\Delta p/p}{\Delta q/q}$$

- What is the price change Δp for trade Δq ?
- Non dimensional
- Describes scale and shape* of demand - supply (excess demand), but not absolute level.
  - (if you let Δq be finite)

Market impact

- Average logarithmic return conditioned on signed trading volume.
- Roughly proportional to demand elasticity of price.

$$\text{market impact} = f(V) = E[R|V]$$

$$R = \log p(t+\tau)/p(t)$$

$$V = \Delta q$$

What is functional form of market impact?

- How does market impact depend on volume?
- Important for many reasons
  - Practical: Transaction cost (friction)
  - Allometry: Sets upper bound on size of funds
  - Interaction rule: Determines how much prices move, which determines what agents do, which determines how much prices move, ...
- Neoclassical method (utility maximization) fails to determine functional form.

Empirical studies

- Roughly 15-20 empirical studies
- All of them show concave impact, i.e. decreasing second derivative
- Why?
- Best fits to data have different functional forms
- Can we explain these diverse functional forms?

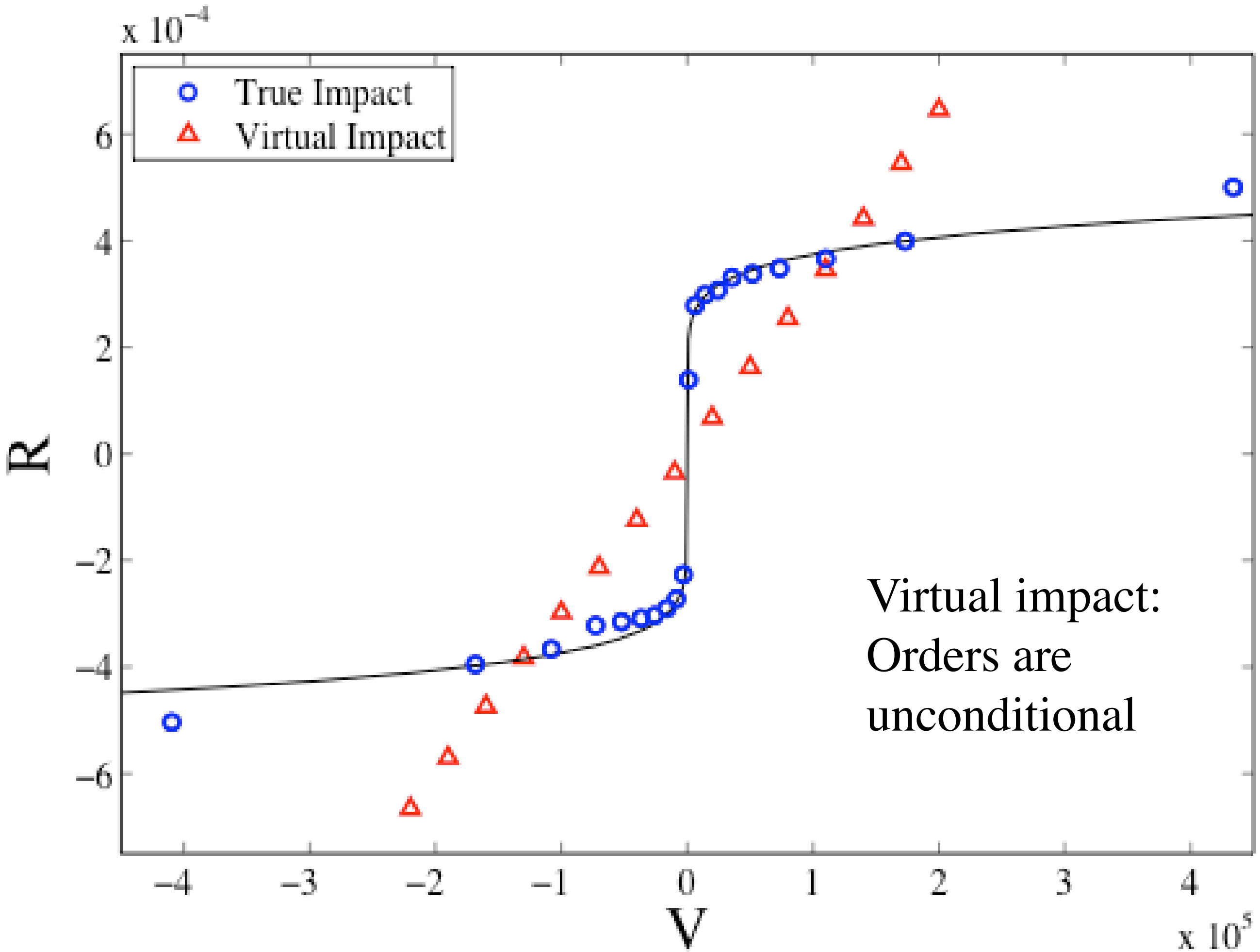
Different types of impact

Kind of impact	explanation	universal?
Single trade	conditioning order size on liquidity	no
Aggregate (sum of many trades)	Double synchronous random walks	yes
Hidden order sequence of linked trades	Asymmetric liquidity	yes?
Block trade	arbitrage with hidden orders	yes?

Why is single trade impact concave?

- Liquidity takers condition their orders on what is offered.
  - When offer is deep, they submit large orders
  - When offer is shallow, they submit small orders
  - Result is that observed impact grows slowly with size

Market impact f(V) for single trade



Farmer, Gillemot, Lillo, Mike, Sen (2004)
Weber and Rosenow (2006), Gerig (2007)

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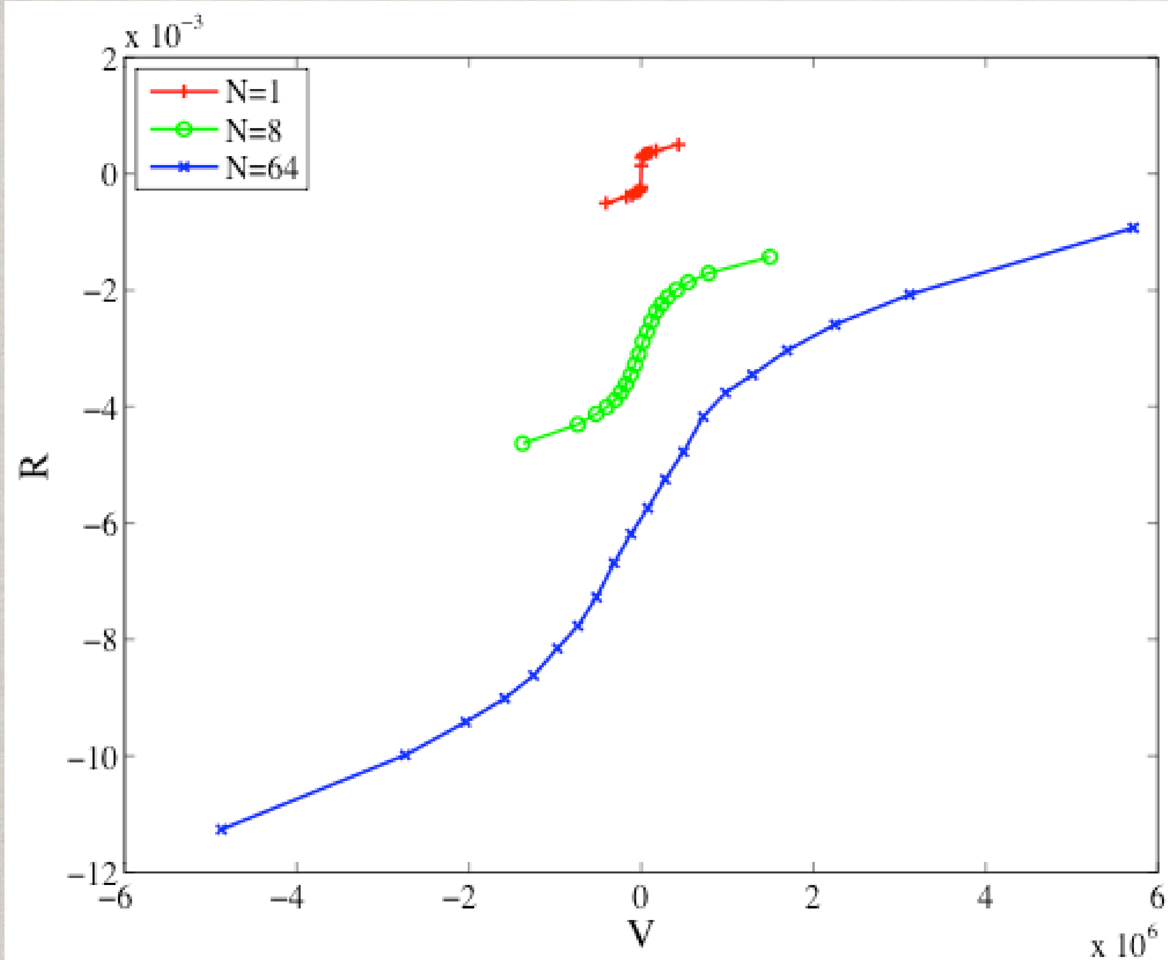
Aggregate Market Impact

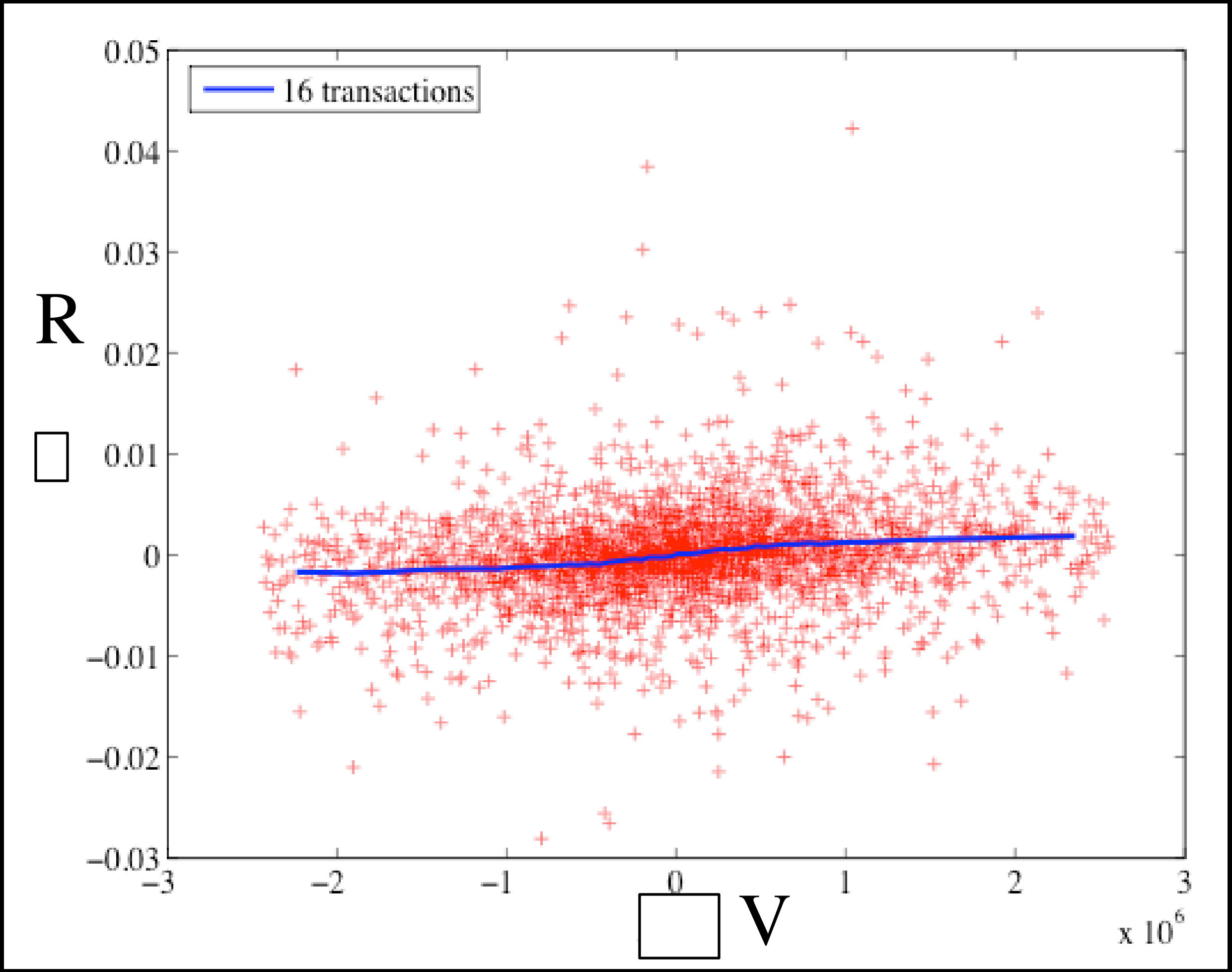


$$
\begin{aligned}
R &= E[R_t|V,N] \\
r_i &= \log(p_i/p_{t-1}) \\
R_t &= \sum_{i=t+1}^{t+N} r_i \\
V_t &= \sum_{i=t+1}^{t+N} v_i
\end{aligned}
$$

Return vs. signed volume for N successive transactions

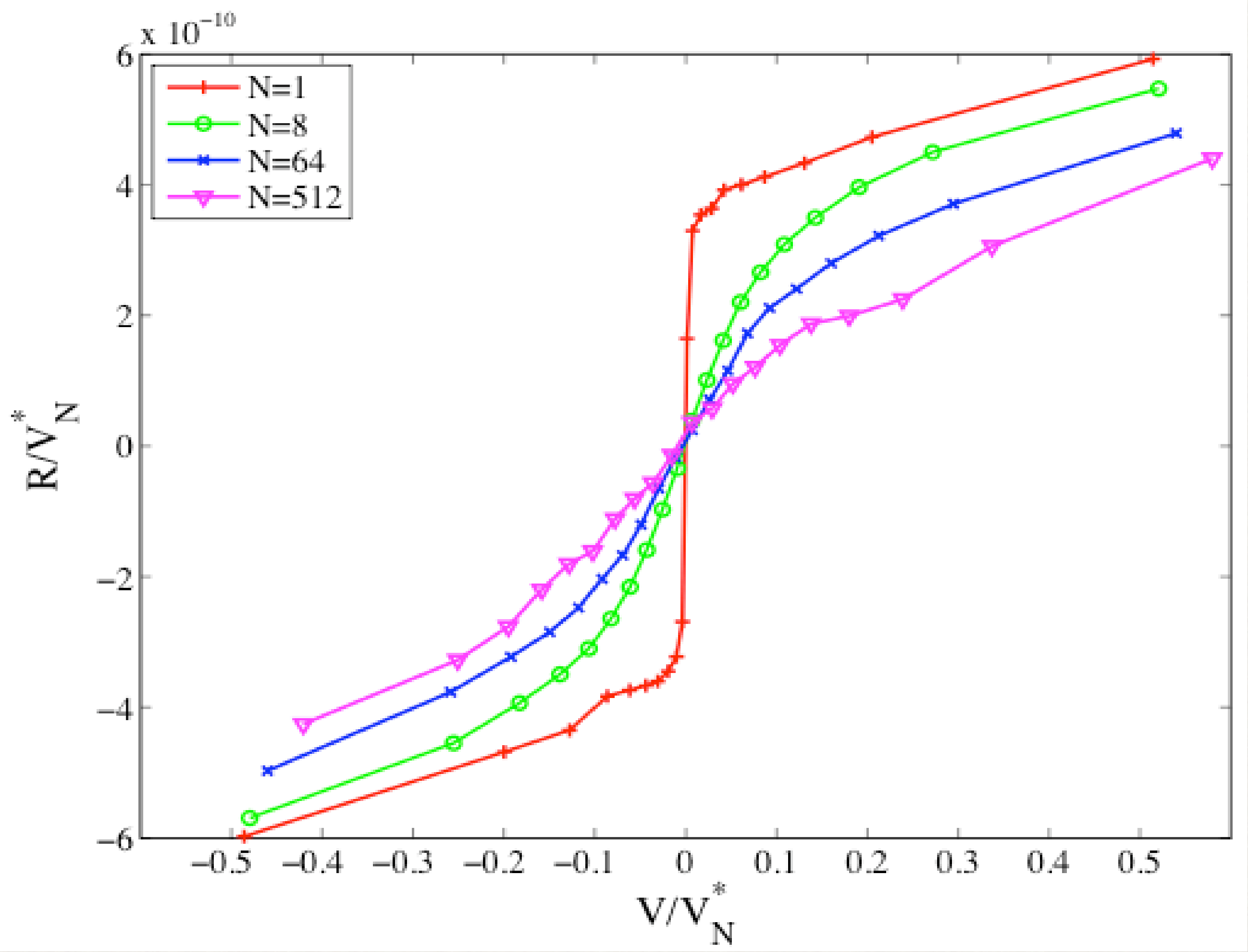
Vertical offset = -3 for N = 8 and -6 for N = 64

Price impacts of 16 trades in a row



- V: Net (signed) quantity traded in 16 transactions
- R: size of return in same 16 transactions

Renormalized market impact



Curves get straighter, slopes get flatter

$$V_N^* = 95\%tile\ of\ V - 5\%tile\ of\ V$$

Theory for aggregate impact

- Sequence of signs for volume are IID.
- Volumes sizes are heavy tailed.
- Returns are deterministic function of volume.

$$
\begin{aligned}
r_i = f(v_i) &\sim v_i^{\beta} \\
\pi(v_i) &\sim v_i^{-(1+\alpha)}
\end{aligned}
$$

- Generalize central limit theorem for synchronized dual random walks with the above relation between steps.

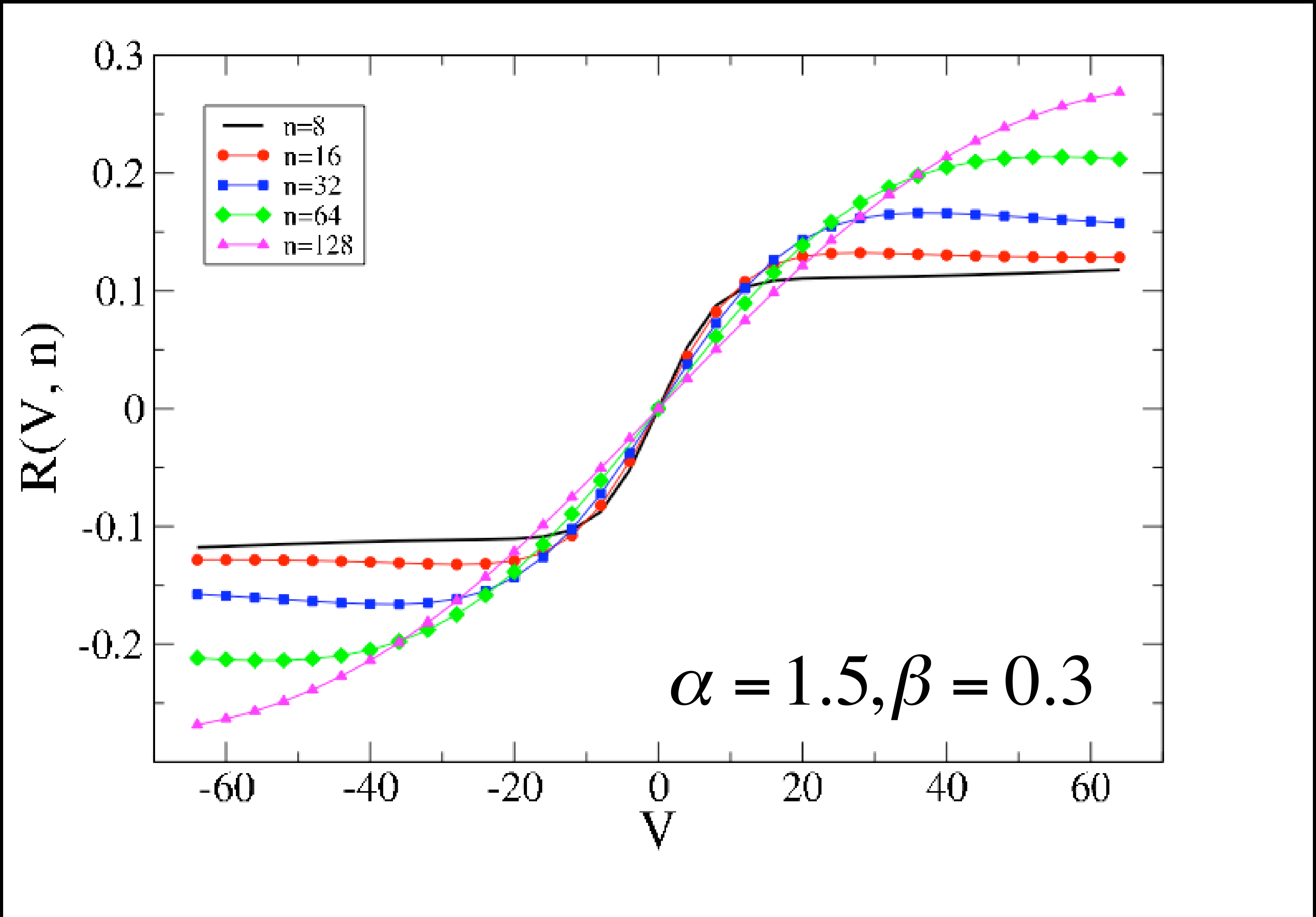
Result

For sufficiently small V, aggregate return R for N trades with signed volume V =

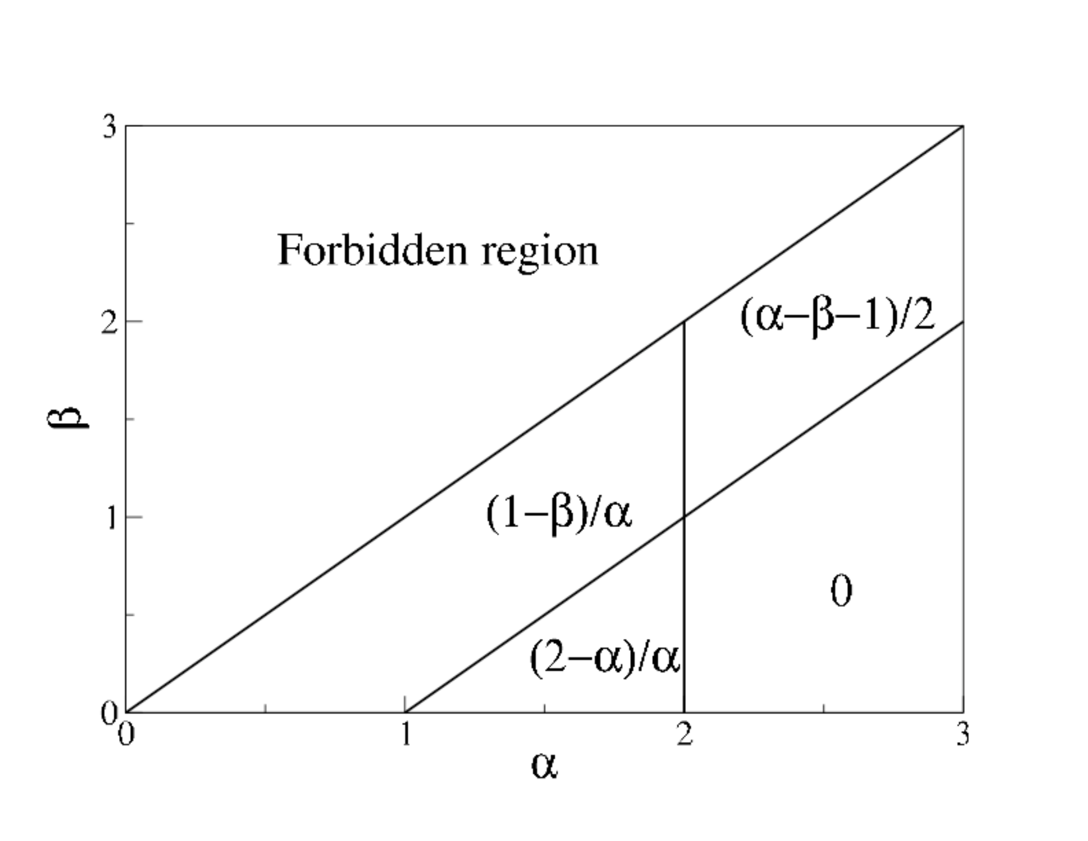
$$R(V, N+1) \sim V N^{-\kappa}$$

where κ is a constant.

Impact from theory

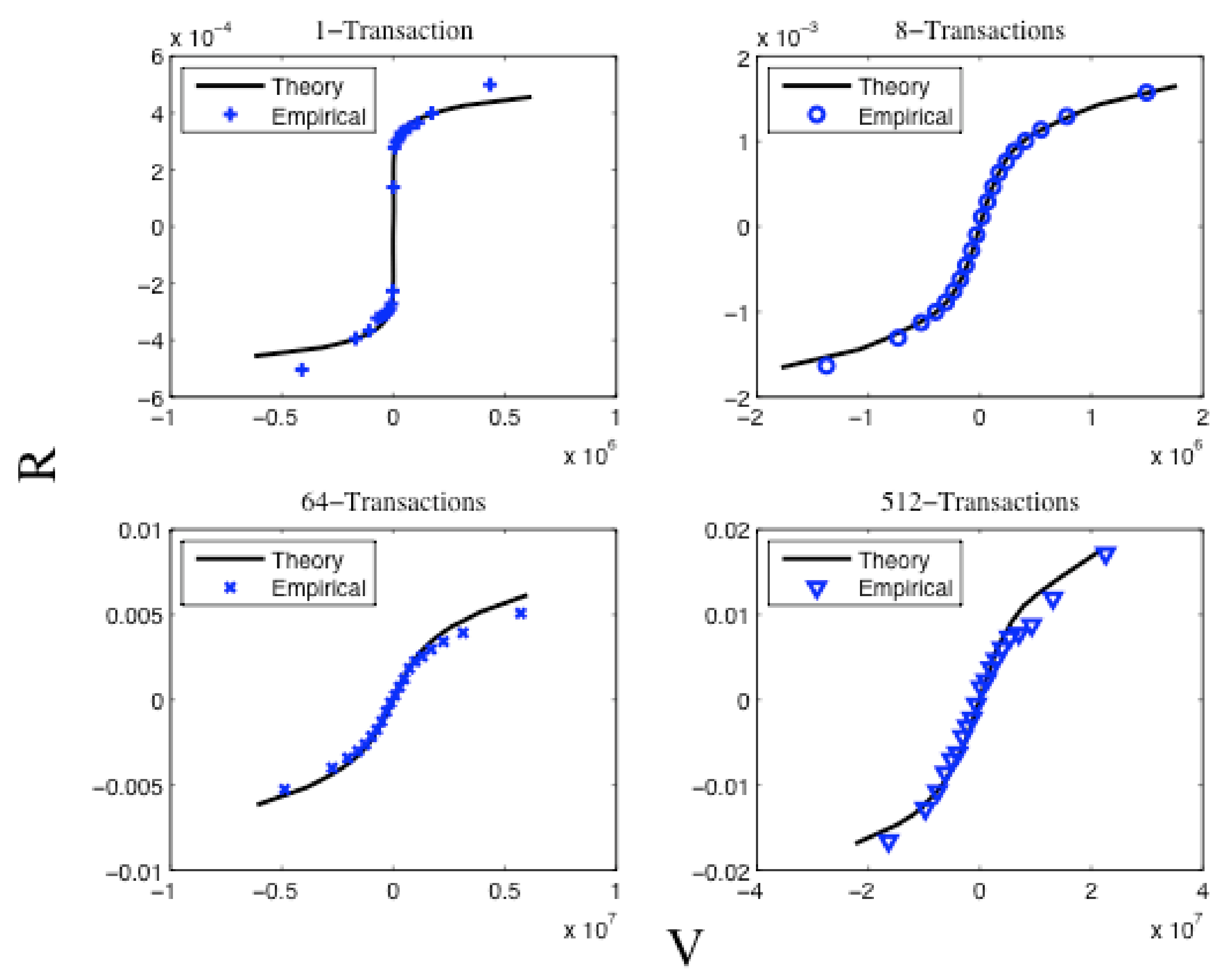


Kappa vs. alpha and beta



Lower right regions are independent of beta

i.e., they display universality.



1−Transaction
8−Transactions
64−Transactions
512−Transactions
Theory
Empirical
R
V
x 10^-4
x 10^-3
x 10^6
x 10^7

Different types of impact

Kind of impact	explanation	universal?
Single trade	conditioning order size on liquidity	no
Aggregate	Double synchronous random walks	yes
Hidden order sequence of linked trades	predictable order flow => asymmetric returns	yes?
Block trade	arbitrage with hidden orders	yes?

Gabaix, Gopikrishnan, Plerou, Stanley

- Neoclassical model of market impact
- Assume first order risk aversion

$$Utility \sim profit - (risk)^{\delta},\ \delta = 1$$

- Assume time to trade is proportional to trading size; risk then goes as $(size)^{1/2}$.
- Implies $market\ impact \sim (size)^{1/2}$

Market impact of large trading orders

- Large trading orders of size V are executed incrementally in small pieces.
- Strategic motivations were qualitatively understood by Kyle (1985).
- Our strategy mimics Black-Scholes. Key elements:
  - Hidden order distribution P(V)
  - Uniform execution.
  - Market efficiency (no easy profits, martingale)
  - New information is additive.
  - Information set of participants

Key elements of theory for impact of large trades

- Hidden order size distribution P(V)
- Uniform execution rate.
- Market efficiency (no easy excess profits)
- New information is additive.
- Information set of participants for detecting predictability of order flow

P(V)

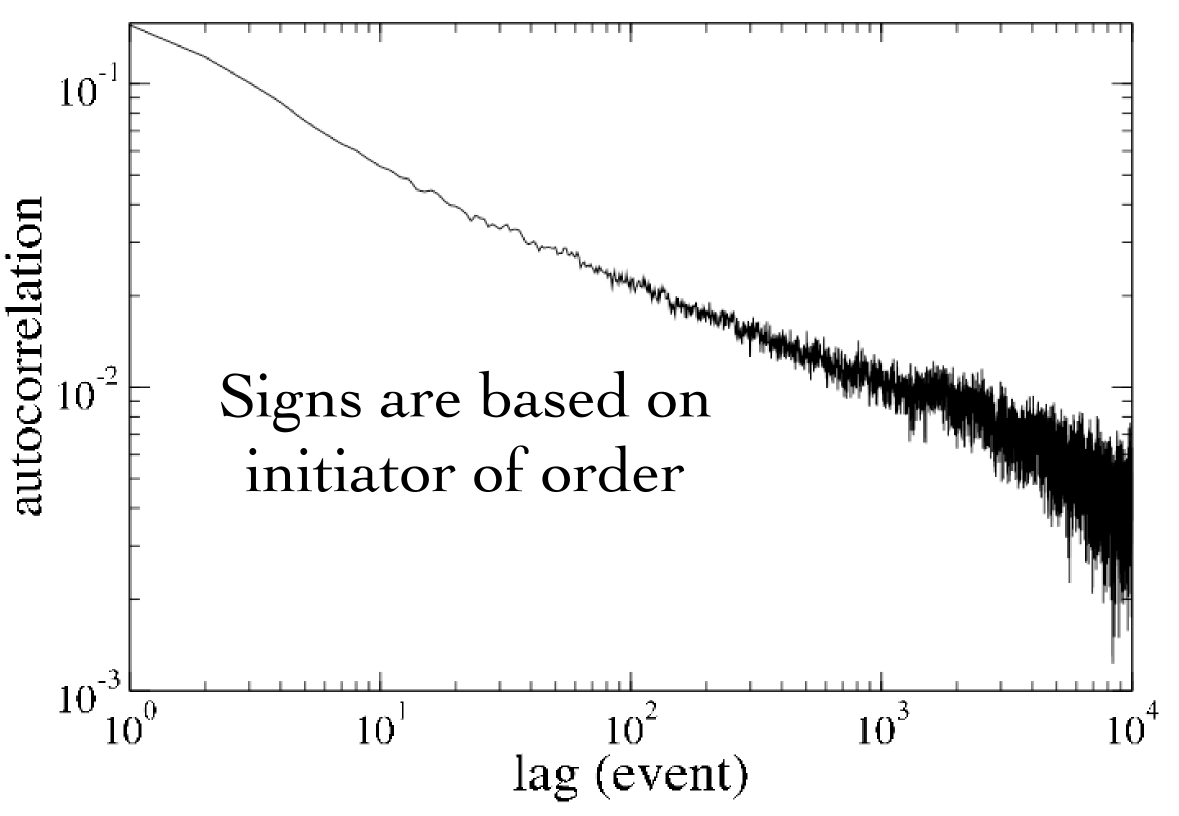
- *Order flow* is the sequence of transaction signs.
- Assume all transactions come from hidden orders of size V, which are split into uniform pieces and executed incrementally at a constant rate.
- The tail behavior of P(V) determines the persistence of order flow.
- E.g. $P(V > v) \sim v^{-\alpha}$ implies $C(\tau) \sim \tau^{-\gamma}$, with

$$\alpha = \gamma + 1$$

(Lillo, Mike and Farmer, 2005)

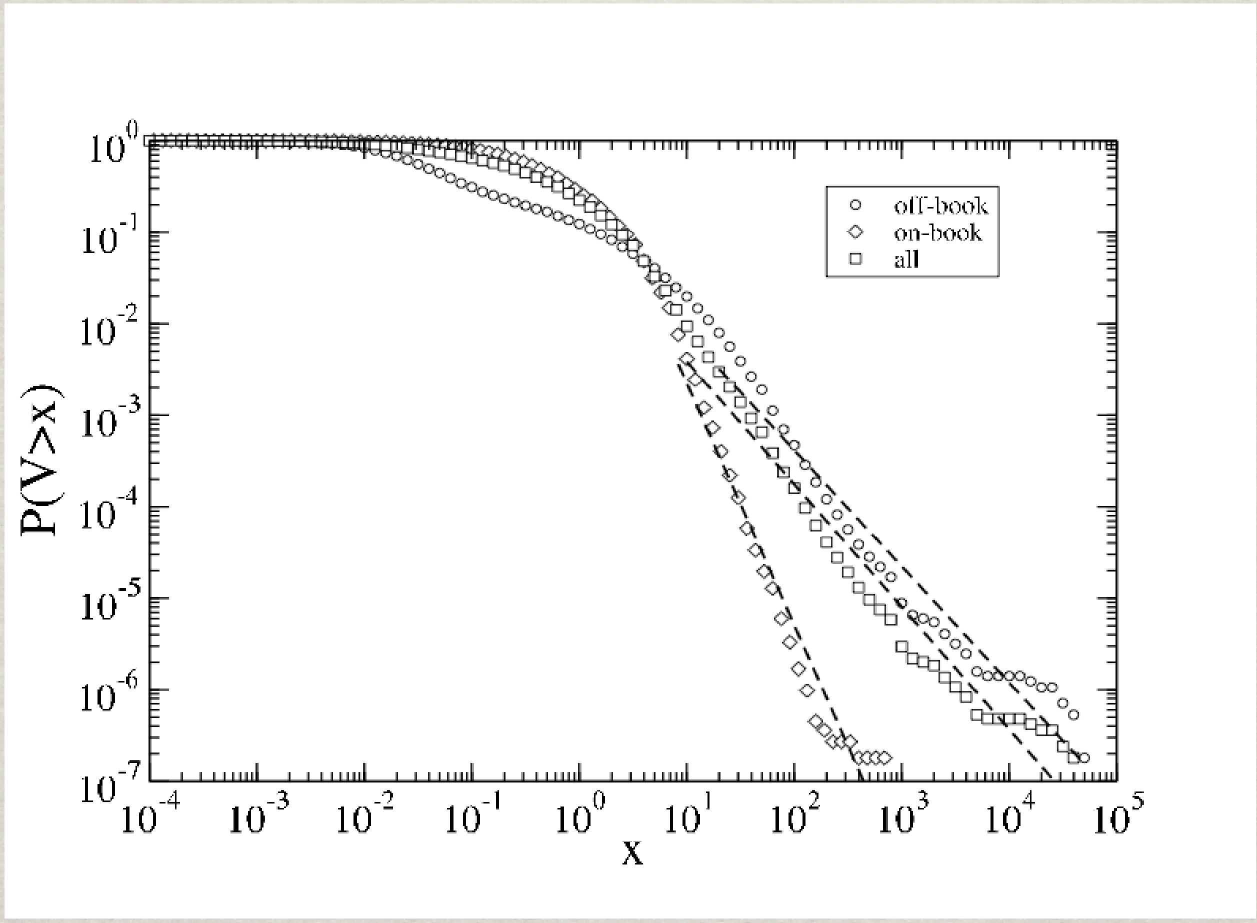
Autocorrelation of order flow (long memory of supply and demand)

Autocorrelation of trade signs



Bouchaud, Gefen, Potters, and Wyart (2004)
Lillo and Farmer (2004)

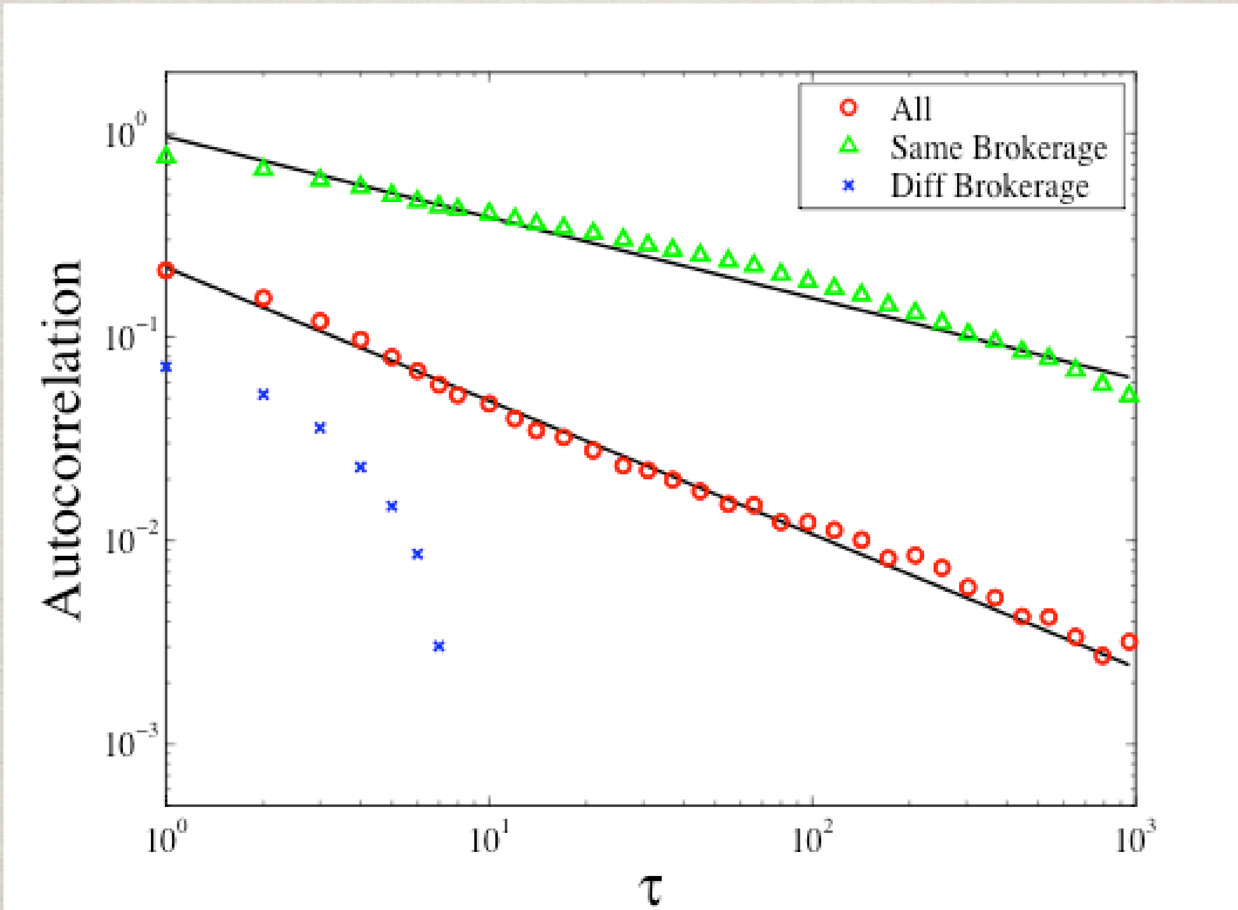
Cumulative distribution of trading volume for LSE Stock Astrazeneca



Gopikrishnan, Gabaix, Plerou and Stanley (2000)
Lillo, Mike, Farmer (2005)

Autocorrelation of trade signs (vs. time in #transactions)

Autocorrelation of trade signs
for same vs. different broker codes



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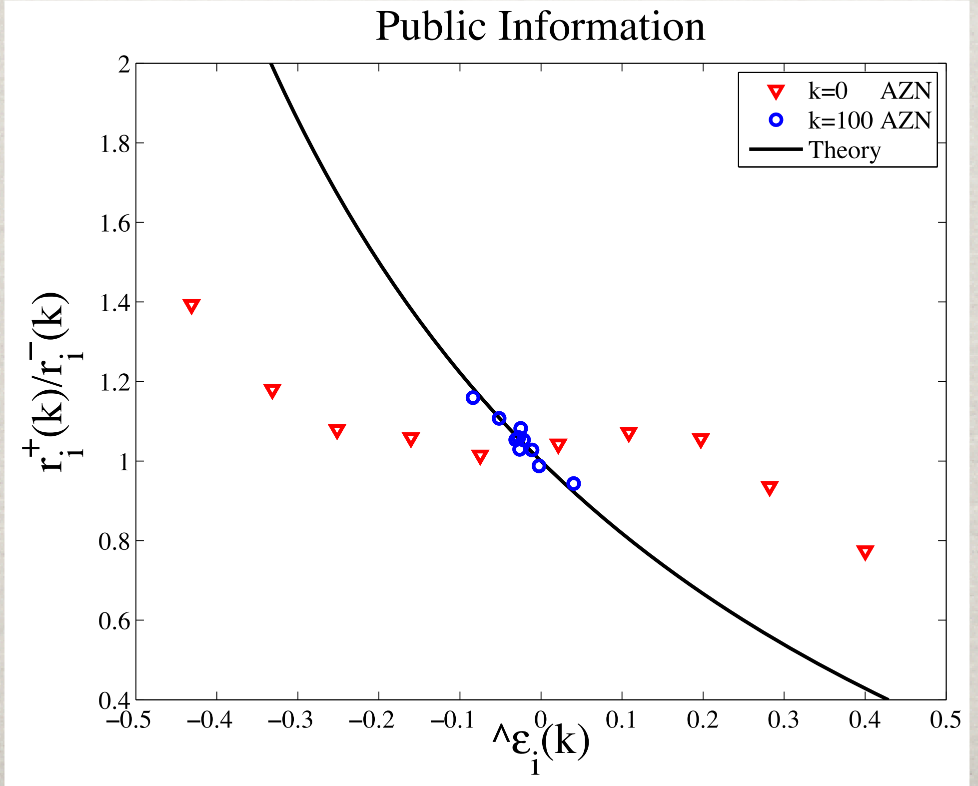
Market efficiency implies asymmetric price response

- To keep prices efficient (linearly unpredictable) when buy orders are really likely, responses to buy orders are proportionately smaller than responses to sell orders.



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Additivity of new information

$$
\begin{aligned}
s_t &= \text{order flow sign} \\
\hat{s}_t &= \text{order flow prediction}
\end{aligned}
$$

Assume that single trade (order flow) impact is of the form:

$$\phi(s_t, \hat{s}_t) = \phi(s_t - \hat{s}_t)$$

Efficiency implies that phi is linear and price response is symmetric, i.e. response to trades of same sign as hidden order is as much smaller as response to trades of opposite sign is bigger.

Why is market impact concave?

- As hidden orders develop the probability that they will continue increases and order flow becomes more predictable.
- Returns in the same direction get smaller*.
- This is what makes impact concave.
- (Assumption that information is additive can be used to set scale; can prove this to be compatible with market efficiency, this implies that single transaction impact has to be linear in deviation of predicted sign from actual sign)

Key elements of theory for impact of large trades

- Hidden order size distribution P(V)
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Two models for how market participants predict order flow

- Linear time series model, based on public information about order signs.
- Colored print model based on private information about status of hidden orders
  - Distribution of hidden orders
  - Position *n* within each hidden order
  - Do not know identity of agents or when a given hidden order will finish
  - Agents must detect end of hidden order

Market impact for linear times series model

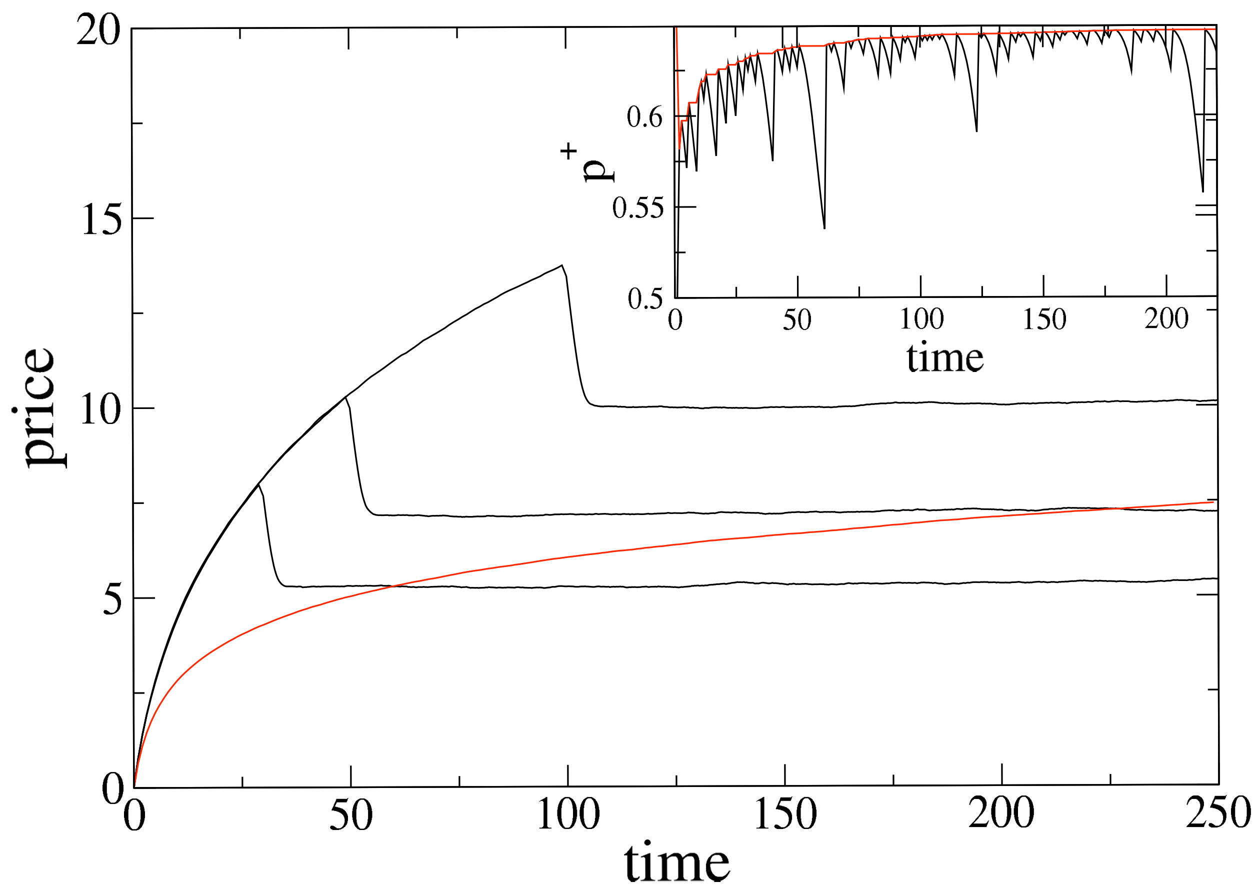
$$E[R|N] = r_0 \left(1 + \frac{2^{\phi-1}\theta^{-\phi}}{1-\phi}[(2N-1)^{1-\phi} - 1]\right) \sim N^{1-\phi}$$

$$\phi = 1 - \alpha/2 \qquad \theta = \text{execution rate}$$

Impact is completely temporary, but decaying slowly as a power law (Bouchaud et al., 2004)

Colored print model

Impact has a permanent component



Market force, ecology, evolution

- Key principle is market impact.

$$\Delta p_t = \sum_i f_I(\Delta x_i(p_t, p_{t-1}, \ldots, I')) + n_t$$

- Trading (Δx) changes the price.
- Agents observe price p, which causes trading.
- Trading affects price through interaction rule *f*.
- Agents are specialized and form a diverse, evolving ecology.

Closure: Long range wealth dynamics

- Short term price formation (modeled by impact) determines long-term profits.
  - Impact makes it possible to pairwise determine who benefits whom, and classify pairs of agents as competitive, predator-prey or mutualistic.
- Long-term profits determine influence of each agent on prices.
  - Get Lotka-Volterra style dynamics for capital.

Conclusions

- (Phenomenological) theory for all three types of market impact that explains functional forms.
- Uses a mixture of different approaches.
  - E.g. eq. approach useful for noneq. dynamics
- Are participants fully rational?
- Key fact is that volume is power law distributed, and orders are executed incrementally
- Why a power law with this exponent?
- What sets liquidity?
- Why is execution rate roughly constant?
- Closure to get a complete theory?