

# ***THE CONCEPTUAL FRAMEWORK OF (URBAN) SCALING***

***GEOFFREY WEST***

***SANTA FE INSTITUTE***

# GODZILLA

**MAY 16**

SEE IT IN REALD 3D AND IMAX 3D

**MENU** ×

**HOME**

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**STORY**

**PARTNERS**

**ULTIMATE FAN CONTEST**

**IMAX FAN ART CONTEST**

**MOTORESEARCH.NET**

**WORLDWIDE RELEASE DATES**

**GODZILLA SHOP**

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# ***WE LIVE IN AN EXPONENTIALLY EXPANDING SOCIO-ECONOMIC UNIVERSE!!***

***1800     < 4% THE US POPULATION WAS URBAN***

***2014     > 80% URBANISED***

***2006     > 50% WORLD'S POPULATION  
URBANISED***

***2050     > 75% URBANISED***

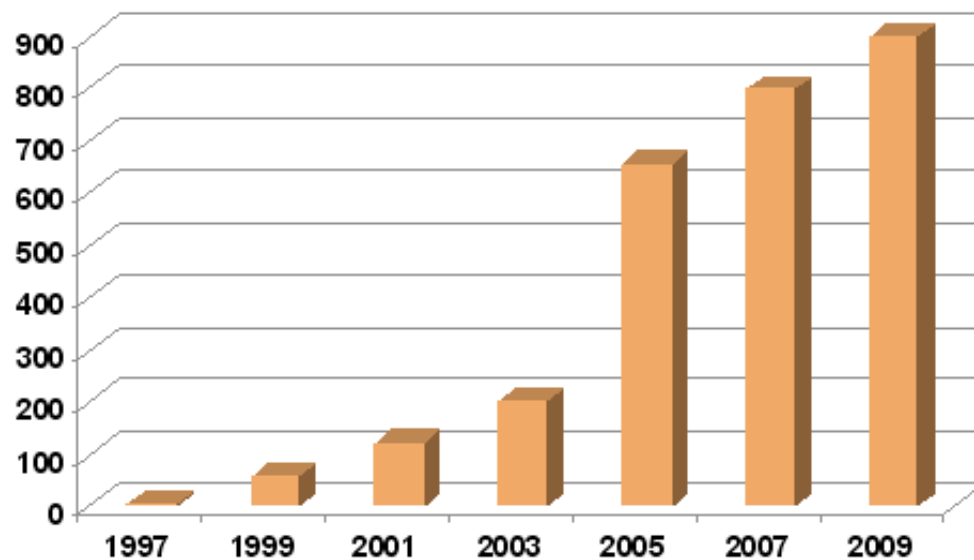
***EQUIVALENT TO URBANISING  
MORE THAN A MILLION PEOPLE  
EVERY WEEK FROM NOW TILL  
2050***

***OR.....TO ADDING A NEW YORK  
METROPOLITAN AREA EVERY TWO  
MONTHS FROM NOW TO 2050***





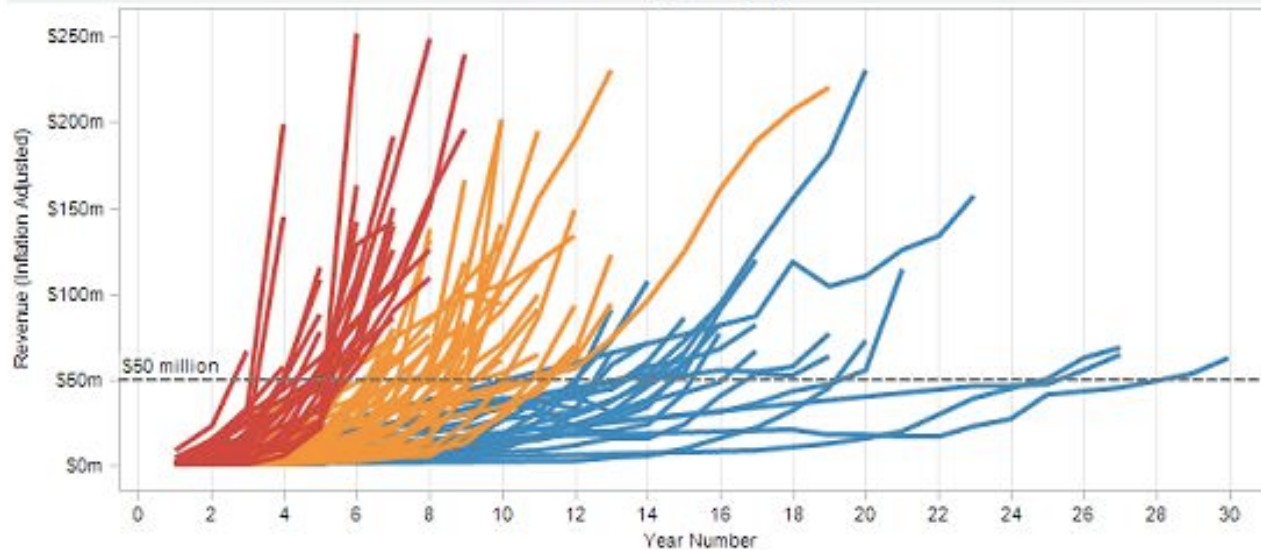




[Click to interact](#)

■ Rocket Ship
 ■ Hot Company
 ■ Slow Burner

Growth History by Company



Growth rates of 100 software companies from IPO Dashboard

## BRIDGE CAPITAL

Bridge funding, as its name implies, bridges the gap between your current financing and the next level of financing.



## MEZZANINE CAPITAL

Mezzanine capital is also known as expansion capital, and is funding to help your company grow to the next level, purchase bigger and better equipment, or move to a larger facility.

## STARTUP CAPITAL

Start-up, or working capital is the funding that will help you pay for equipment, rent, supplies, etc. for the first year or so of operation.

## SEED CAPITAL

Seed capital is the money you need to do your initial research and planning for your business.







**FATE OF THE PLANET**

**=**

**FATE OF OUR CITIES**













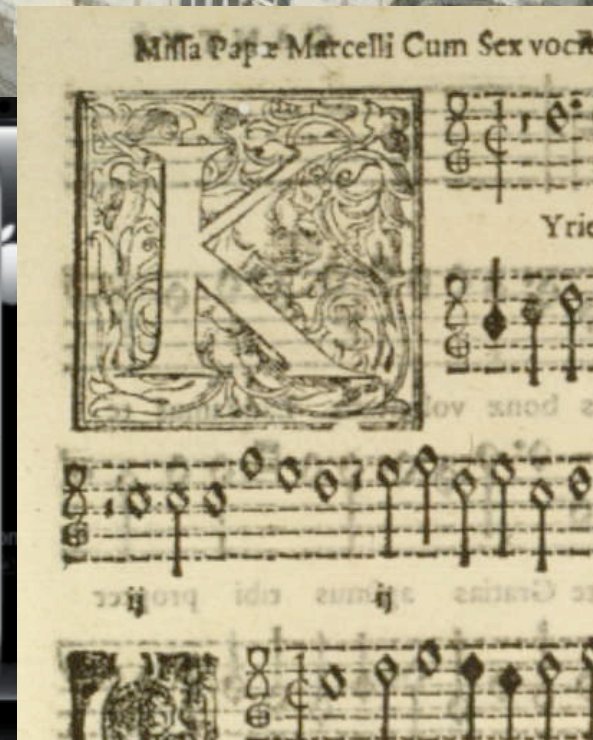








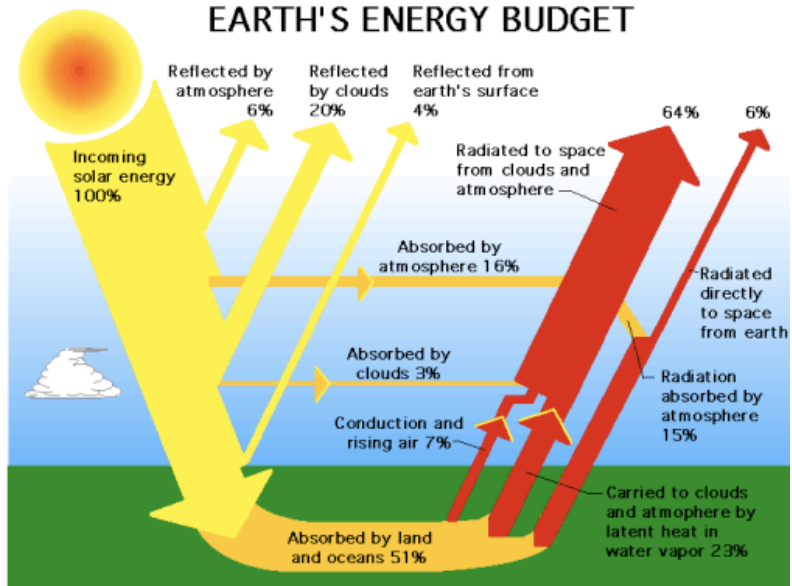








## EARTH'S ENERGY BUDGET



***SOCIO-ECONOMIC  
ENTROPY!!***













 **Evening  
Standard** 

**Classified** 79383

 **FRIDAY** 

**MASSIVE  
INCREASE  
IN KNIFE  
CRIME  
SURVEYS**

 **Evening  
Standard**

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# London After Climate Change?



**“What is the city but the  
people?”**

*William Shakespeare*













***CITIES ARE THE PROBLEM;***

***CITIES ARE THE SOLUTION!!***

# ***URGENTLY NEED A QUANTITATIVE, PREDICTIVE SCIENCE OF CITIES***

***RESILIENCE***

***EVOLVABILITY***

***GROWTH***

***SCALABILITY***



# ***NEED A SCIENCE OF CITIES***

**COMPLEMENT TO TRADITIONAL  
(QUALITATIVE) THEORIES AND MODELS**

***WHAT CAN WE LEARN FROM BIOLOGY AND  
PHYSICS?***

- **POPULATION, HEALTH, WELL-BEING,.....**
- **ENERGY, RESOURCES, FOOD,.....**  
***THERMODYNAMICS, METABOLICS,.....***
- **SOCIAL, POLITICAL, CULTURAL,.....**  
***ORGANISATION, STRUCTURE,.....***
- **ECONOMY, FINANCE, DEVELOPMENT,.....**  
***RISK, INFORMATION, INNOVATION, .....***
- **ECOLOGY, ENVIRONMENT, CLIMATE,.....**



***THESE ARE NOT INDEPENDENT***

***THEY ARE ALL HIGHLY  
COUPLED, INTER-RELATED,  
MULTI-SCALE COMPLEX  
ADAPTIVE SYSTEMS***

**ENERGY & RESOURCES  
(METABOLISM, INFRASTRUCTURE)**

**VS.**

**INFORMATION  
(GENOMICS, INNOVATION)**



***COARSE - GRAINED DESCRIPTION***

***WITH INCREASING RESOLUTION  
AND GRANULARITY***

***STATISTICAL/PROBABILISTIC***

***QUANTITATIVE, PREDICTIVE***

***WHY DO WE LIVE ~100 YEARS AND NOT 1000, OR 2-3 YEARS LIKE A MOUSE?***

***WHERE DOES A TIME-SCALE OF 100 YEARS COME FROM?***

***HOW IS IT GENERATED FROM FUNDAMENTAL MOLECULAR TIME-SCALES OF GENES AND RESPIRATORY ENZYMES?***



***WHY DO WE NEED TO SLEEP ABOUT EIGHT HOURS EACH NIGHT?***

***WHY DO MICE HAVE MANY MORE TUMOURS/GRAM OF TISSUE THAN WE DO AND WHALES HAVE ALMOST NONE?***

***WHAT'S THE DIFFERENCE BETWEEN GROWING BABIES IN YOUR BODY AND GROWING TUMORS (OR ORGANS)?***

***ARE CITIES AND COMPANIES JUST  
VERY LARGE ORGANISMS  
SATISFYING THE LAWS OF BIOLOGY?***

***WHY DO ALL COMPANIES DIE  
WHEREAS ALMOST ALL CITIES  
SURVIVE?***





***CAN THERE BE “NEWTON’S  
LAWS OF COMPLEX ADAPTIVE  
SYSTEMS”?***

# GEOMETRIC SCALING

ISOMETRIC – KEEP THE SAME SHAPE

i) **AREA ~ LENGTH x LENGTH**

$$A \sim L^2$$

ii) **VOLUME ~ LENGTH x LENGTH x LENGTH**

$$V \sim L^3$$

**VOLUME INCREASES MUCH FASTER THAN  
AREA**



i)  $V \sim A^{3/2}$  **SUPER-LINEAR SCALING**

ii)  $A \sim V^{2/3}$  **SUB-LINEAR SCALING**

**IF DENSITY IS FIXED, MASS  $\sim$  VOLUME:**

iii)  $M \sim V$  **LINEAR SCALING**

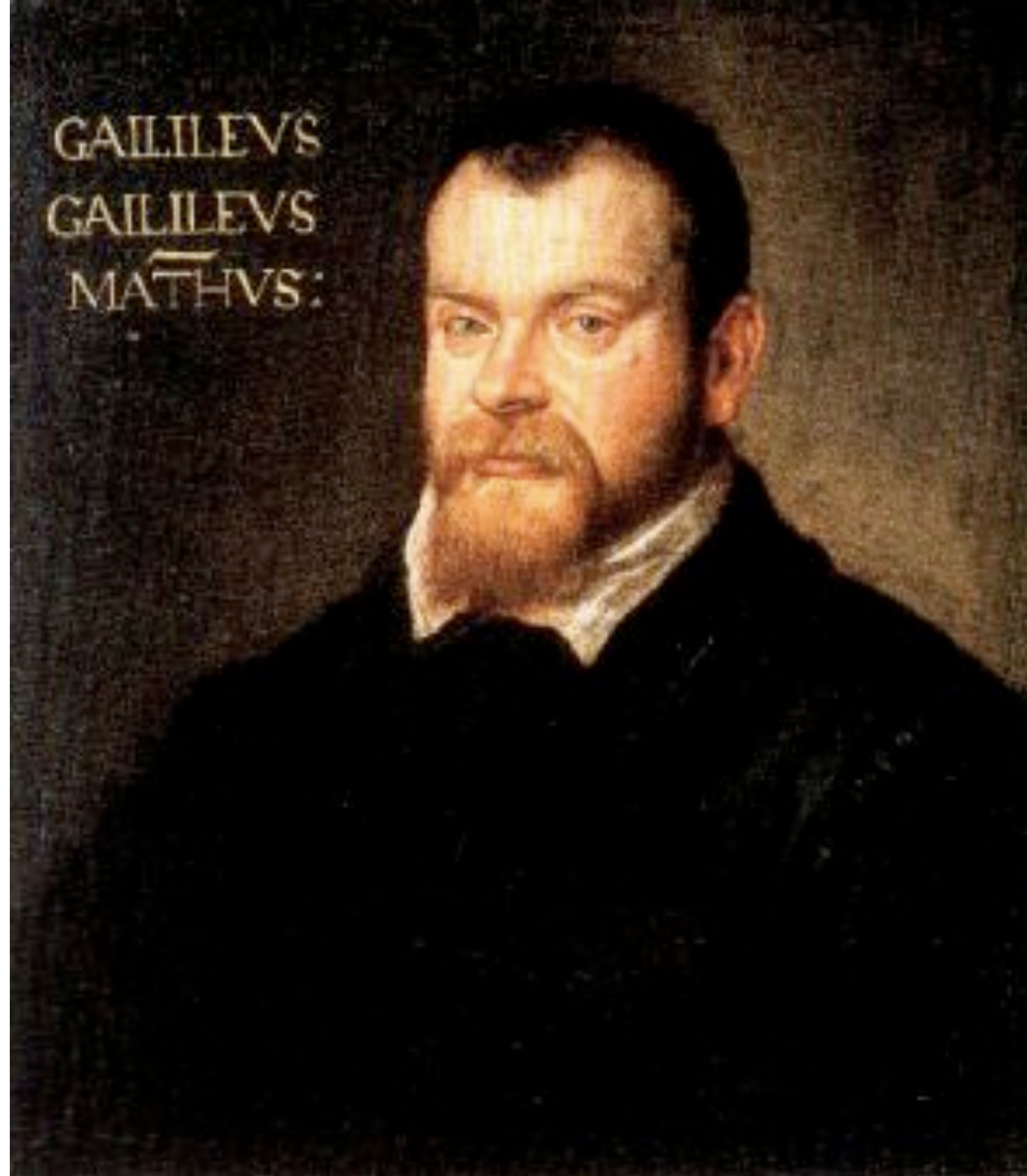
**SO  $A \sim M^{2/3}$**

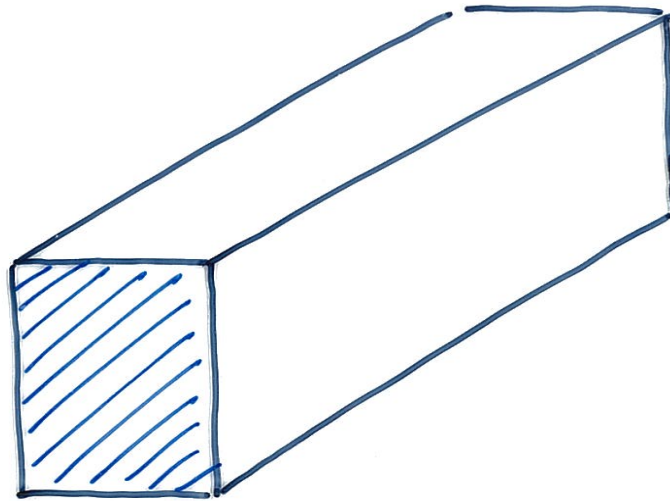
**NON-LINEAR**

**POWER LAWS**

**EXPONENTS**

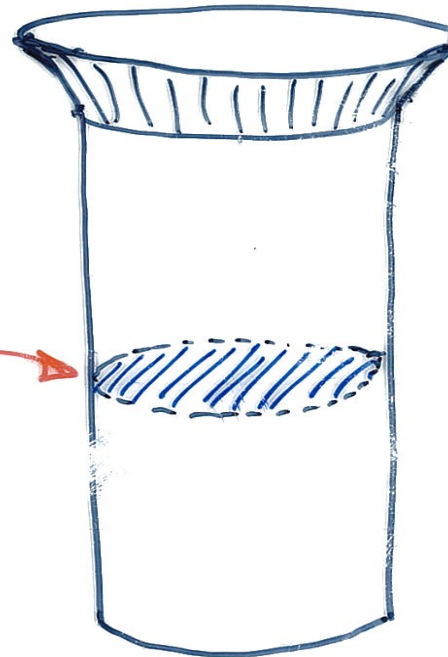
GAILLEVS  
GAILLEVS  
MATHVS:





STRENGTH

$\propto$  CROSS-SECTIONAL  
AREA





# **GALILEO**

**i) STRENGTH OF A LIMB OR BEAM**

**~ CROSS-SECTIONAL AREA ~  $L^2$**

**ii) WEIGHT SUPPORTED ~  $L^3$**

**AS SIZE INCREASES WEIGHT WILL  
EVENTUALLY EXCEED STRENGTH LEADING TO**

# **GALILEO**

**i) STRENGTH OF A LIMB OR BEAM**

**$\sim$  CROSS-SECTIONAL AREA  $\sim L^2$**

**ii) WEIGHT SUPPORTED  $\sim L^3$**

**AS SIZE INCREASES WEIGHT WILL  
EVENTUALLY EXCEED STRENGTH LEADING TO**

**COLLAPSE AND LIMITS TO GROWTH**

**TO AVOID NEED:**

**i)CHANGE DESIGN**

**ii)CHANGE MATERIALS**

**iii)OR BOTH**



**TO AVOID NEED:**

**i) CHANGE DESIGN**

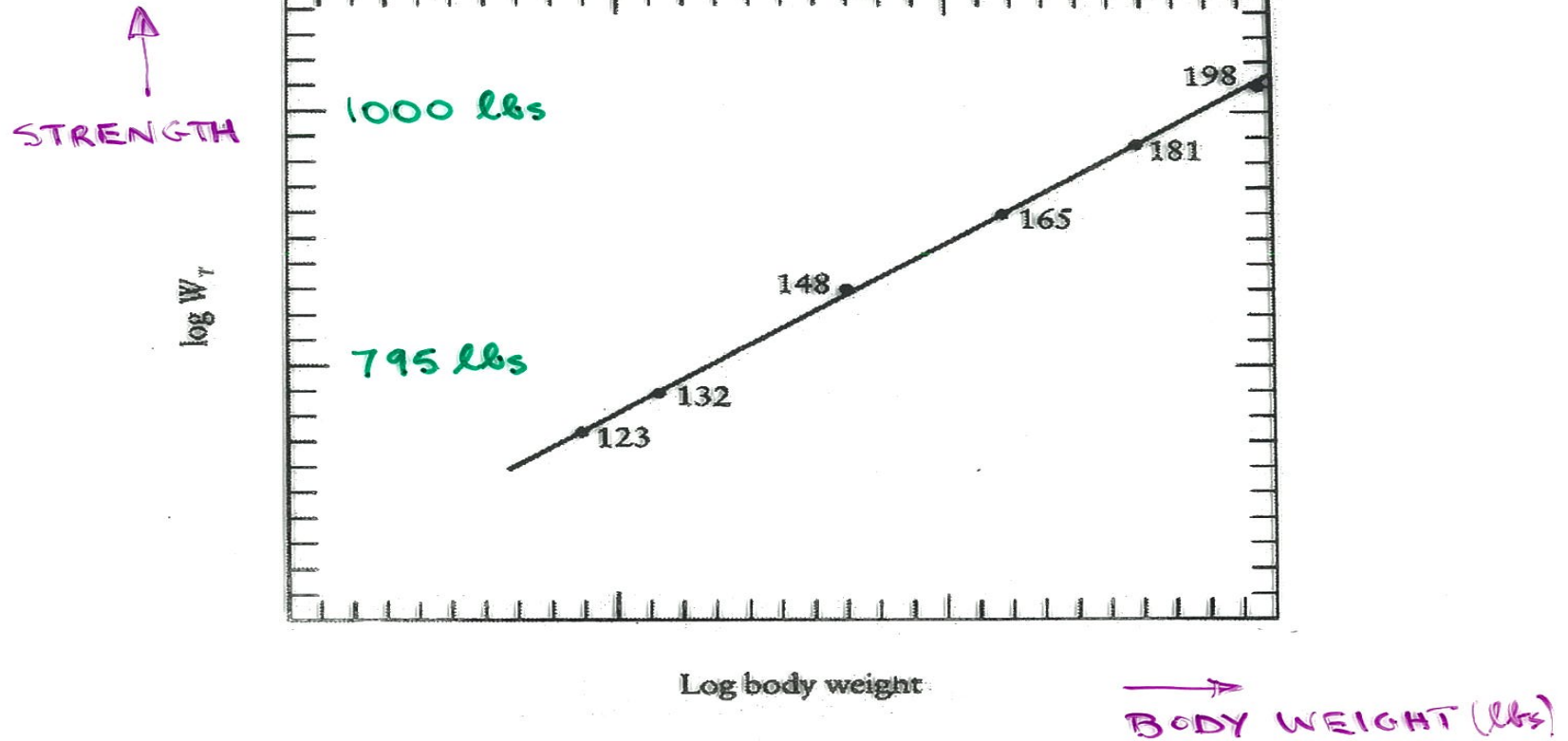
**ii) CHANGE MATERIALS**

**iii) OR BOTH**



**INNOVATION**

# WEIGHT-LIFTING ("PUMPING IRON") (PLOTTED LOGARITHMICALLY)



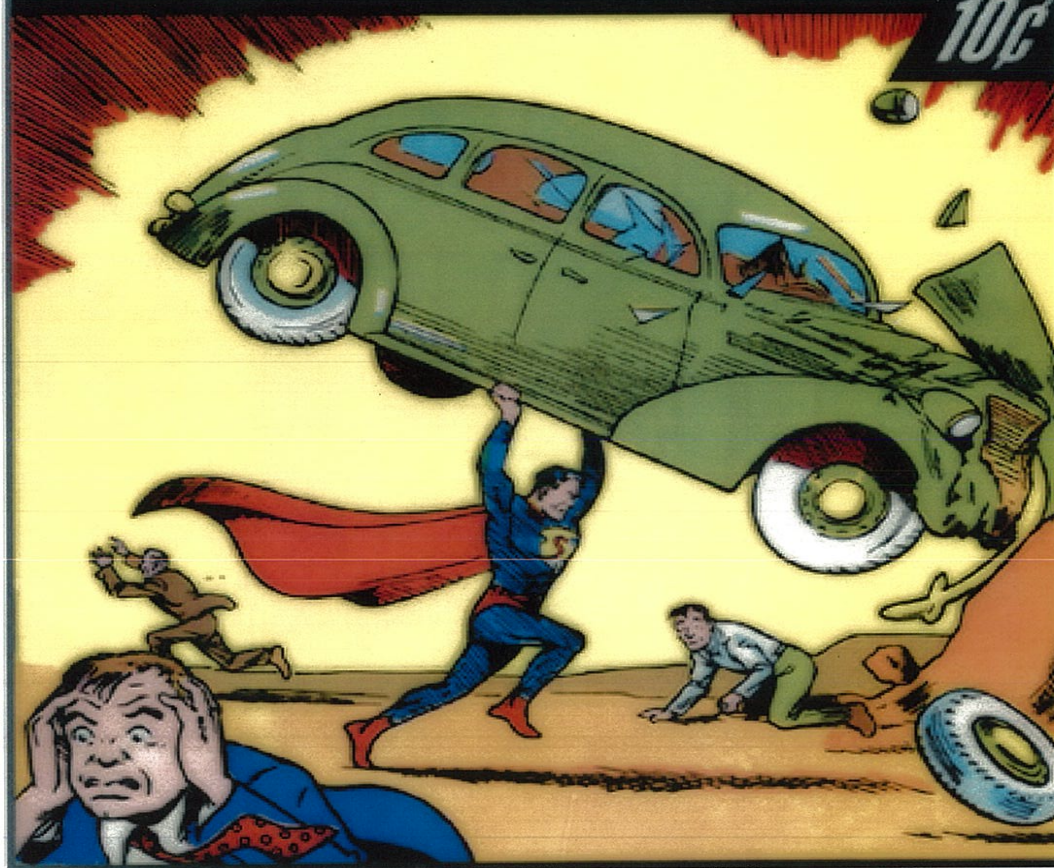
WHO IS THE STRONGEST AND WHO IS THE  
WEAKEST?

No. 1

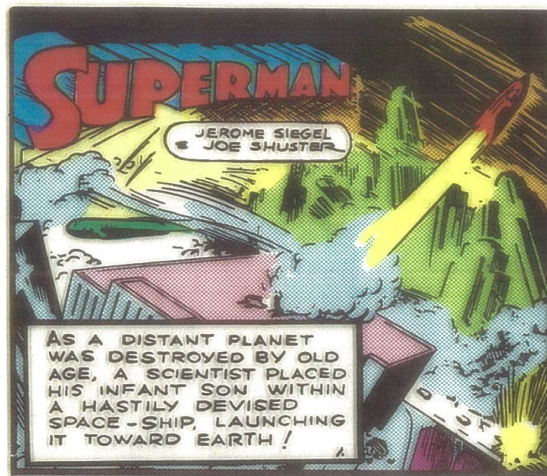
JUNE, 1938

# ACTION COMICS

10c



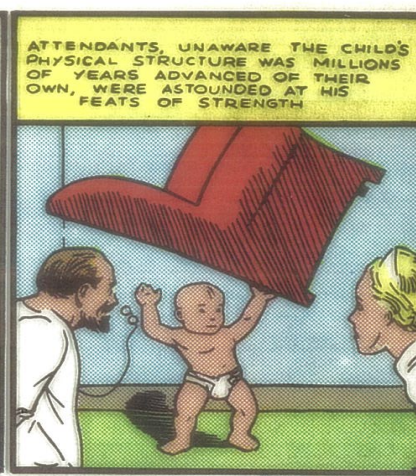




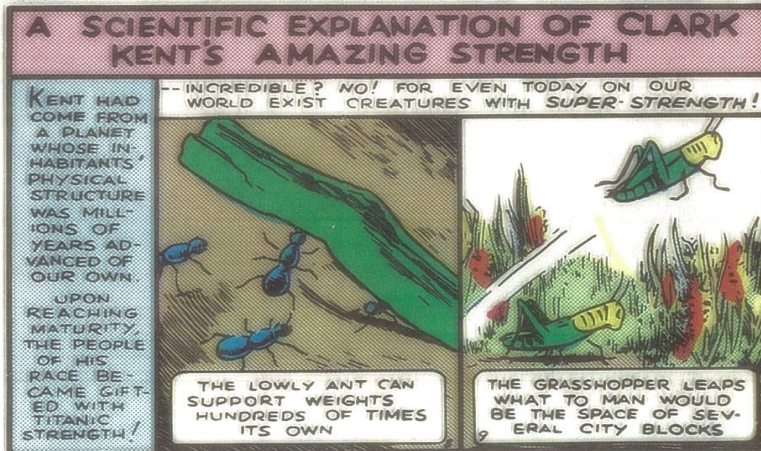
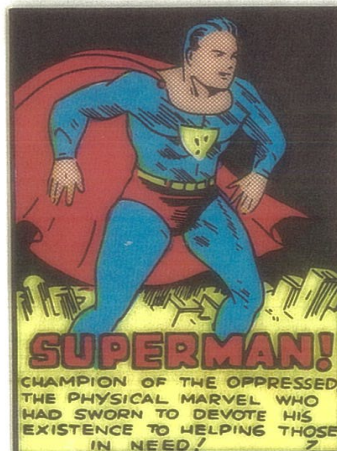
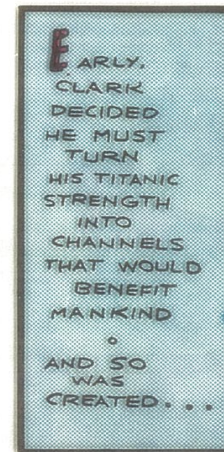
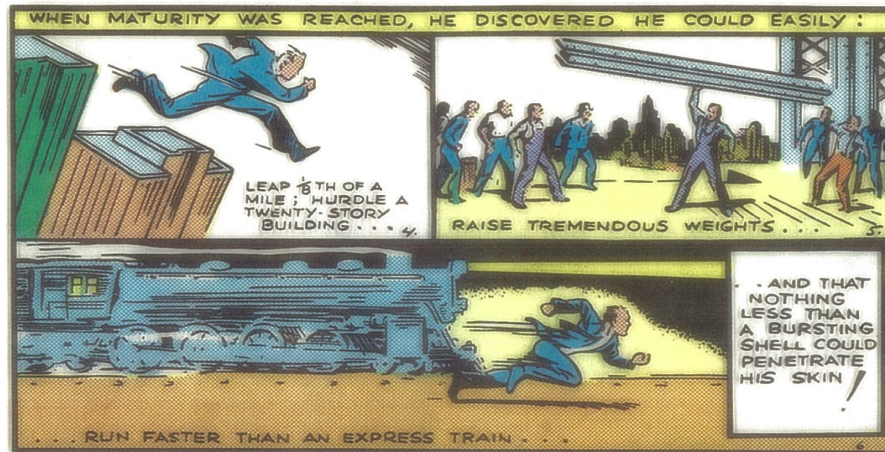
AS A DISTANT PLANET WAS DESTROYED BY OLD AGE, A SCIENTIST PLACED HIS INFANT SON WITHIN A HASTILY DEvised SPACE-SHIP, LAUNCHING IT TOWARD EARTH!



WHEN THE VEHICLE LANDED ON EARTH, A PASSING MOTORIST, DISCOVERING THE SLEEPING BABE WITHIN, TURNED THE CHILD OVER TO AN ORPHAN-AGE



ATTENDANTS, UNAWARE THE CHILD'S PHYSICAL STRUCTURE WAS MILLIONS OF YEARS ADVANCED OF THEIR OWN, WERE ASTOUNDED AT HIS FEATS OF STRENGTH





## A SCIENTIFIC EXPLANATION OF CLARK KENT'S AMAZING STRENGTH

KENT HAD COME FROM A PLANET WHOSE INHABITANTS' PHYSICAL STRUCTURE WAS MILLIONS OF YEARS ADVANCED OF OUR OWN.

UPON REACHING MATURITY, THE PEOPLE OF HIS RACE BECAME GIFTED WITH TITANIC STRENGTH!

--INCREDIBLE? NO! FOR EVEN TODAY ON OUR WORLD EXIST CREATURES WITH SUPER-STRENGTH!




THE LOWLY ANT CAN SUPPORT WEIGHTS HUNDREDS OF TIMES ITS OWN.



THE GRASSHOPPER LEAPS WHAT TO MAN WOULD BE THE SPACE OF SEVERAL CITY BLOCKS

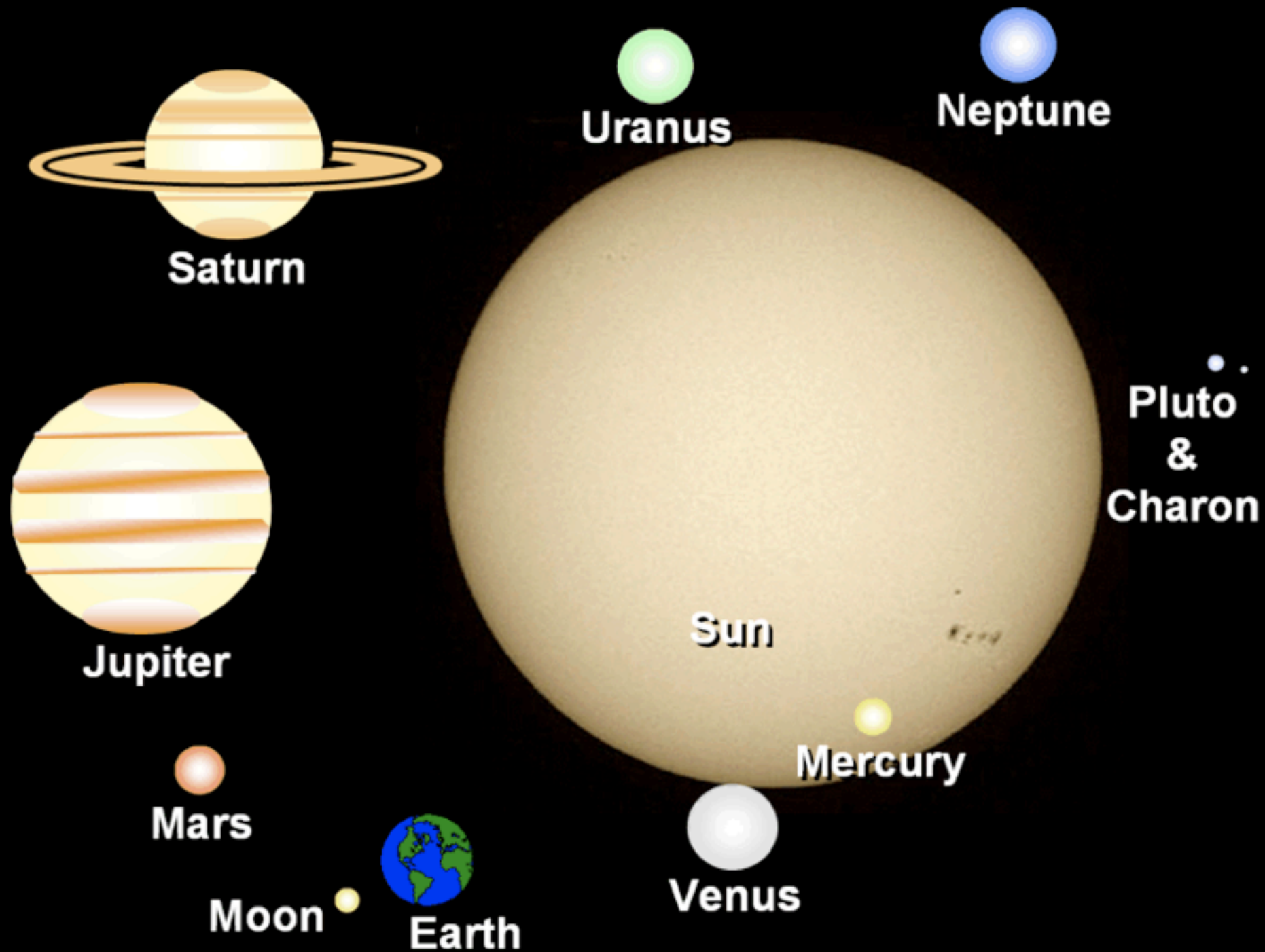
# Our Solar System

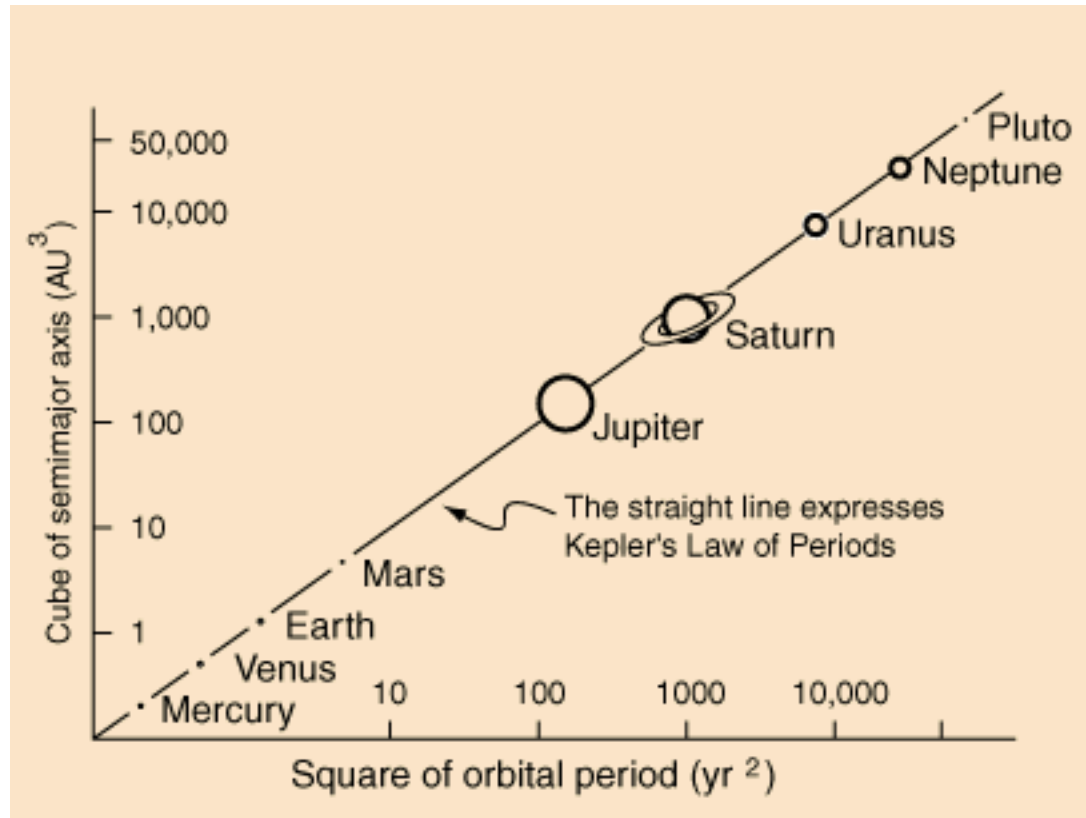
A detailed illustration of the Solar System. The Sun is on the left, a large glowing orange-yellow sphere. Eight planets are shown in their orbits, represented by concentric yellow lines. From left to right, the planets are: Mercury (small, grey), Venus (yellowish), Earth (blue and white), Mars (small, reddish), Jupiter (large, with brown and white bands), Saturn (yellowish with prominent rings), Uranus (light blue), and Neptune (darker blue). A comet with a long tail is shown in the lower right. The background is a dark blue space filled with stars and a dense field of grey asteroids in the inner solar system.

From our small world we have gazed upon the cosmic ocean for thousands of years. Ancient astronomers observed points of light that appeared to move among the stars.

**Select a Solar System body to learn more.**







$$T^2 \propto R^3$$

$$\frac{T^2}{R^3} = \text{CONSTANT} \quad \textbf{INVARIANT}$$

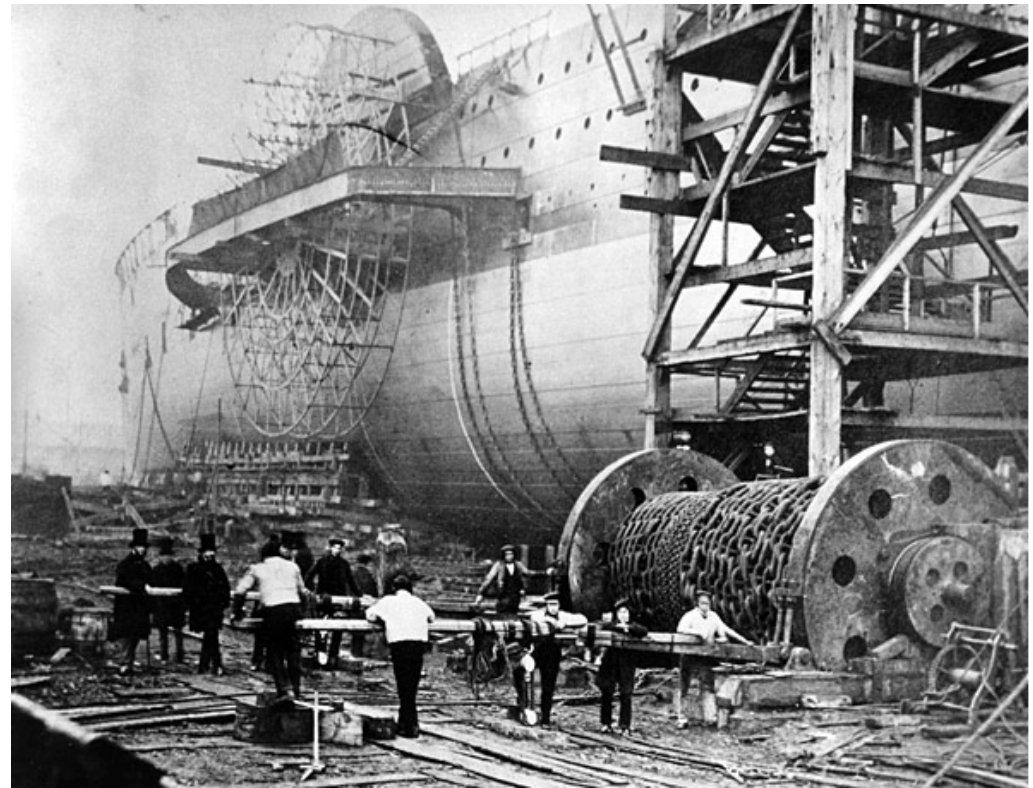
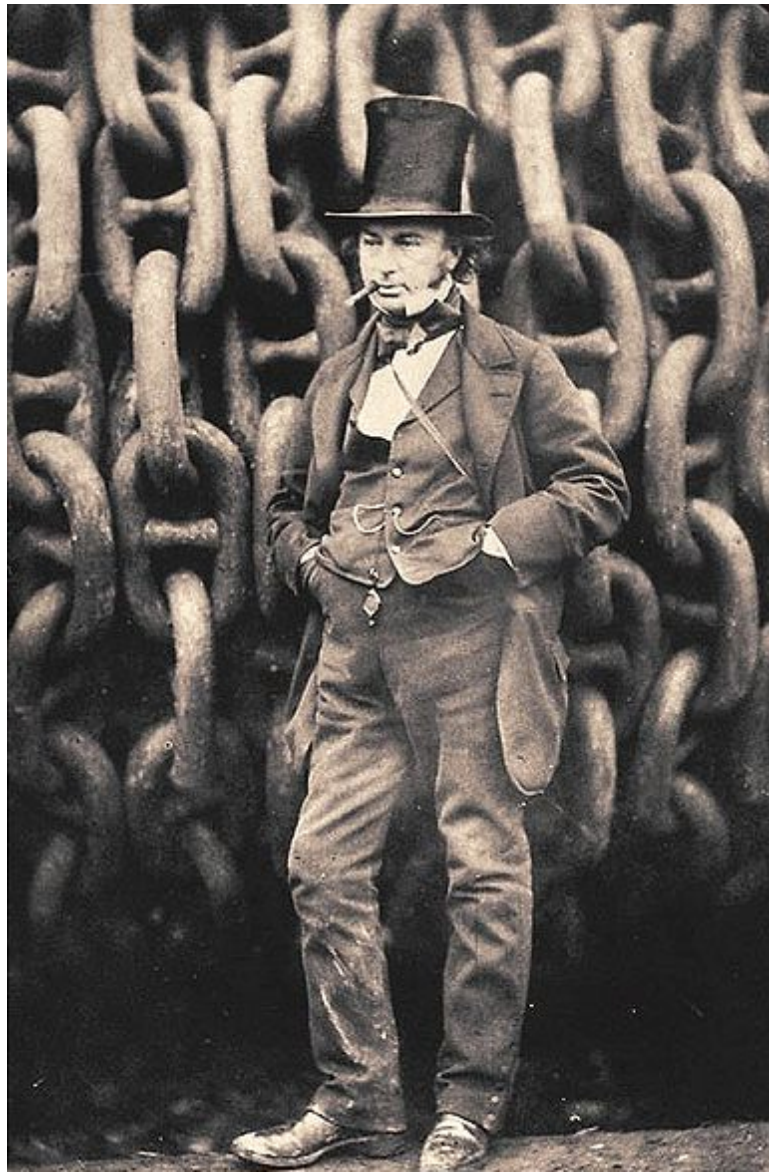
$$T \propto R^{3/2}$$

**POWER LAW WITH EXPONENT  $b$ :**

$$Y(X) = Y_0 X^b$$







$$Y_0 = Y(1)$$





***THE GREAT EASTERN (1858)***

Isambard Kingdom Brunel

1	Sir Winston Churchill	(1874–1965)		Politician
2	Isambard Kingdom Brunel	(1806–1859)		Engineer
3	Diana, Princess of Wales	(1961–1997)		Member of the British Royal family. Philanthropist.
4	Charles Darwin	(1809–1882)		Naturalist
5	William Shakespeare	(1564–1616)		Poet and playwright
6	Sir Isaac Newton	(1642–1727)		Physicist, mathematician, astronomer, natural philosopher and biblical scholar

William Froude

$$F \equiv \frac{v^2}{gL}$$



## MODELLING, SCALE-INVARIANCE

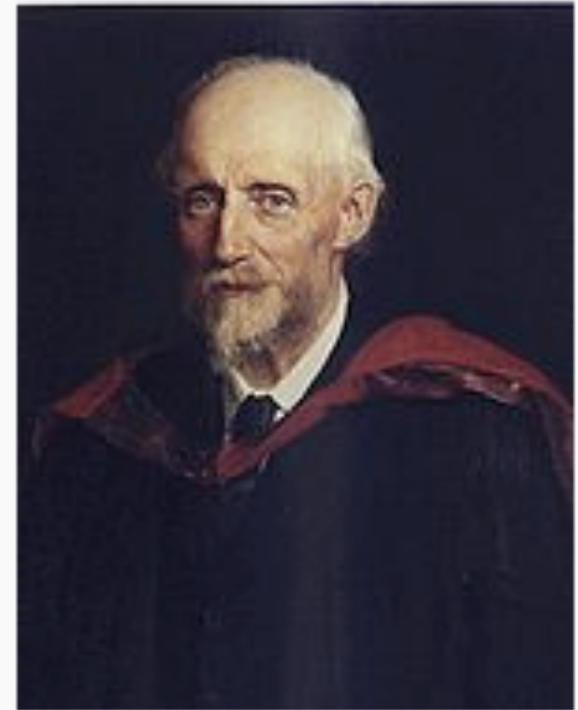
.....by which the results of small-scale tests could be used to predict the behaviour of full-sized hulls.





Lord Rayleigh (John Strutt)  
(1842-1919)

Osborne Reynolds



$$R = \frac{\rho v L}{\mu}$$

**MODELLING, SCALE-INVARIANCE**



## WE HAVE THEM!

\$150	Bicycle for	\$150
\$150	" "	\$130
\$150	" "	\$110
\$150	" "	\$85
\$135	" "	\$70

### BARGAINS IN CHEAP WHEELS.

Do not forget "Dunlap" and "Warwick" detachable tires and hollow rims! Most \$150 wheels do not have them, ours do. Come and see them.

## WRIGHT CYCLE EXCHANGE,

1015 WEST THIRD ST.,

Between Williams and Baxter.









## THE COMPANIES

### U.S.

Boeing  
Spirit  
Vought  
GE  
Goodrich

### CANADA

Boeing  
Messier-Dowty

### AUSTRALIA

Boeing

### JAPAN

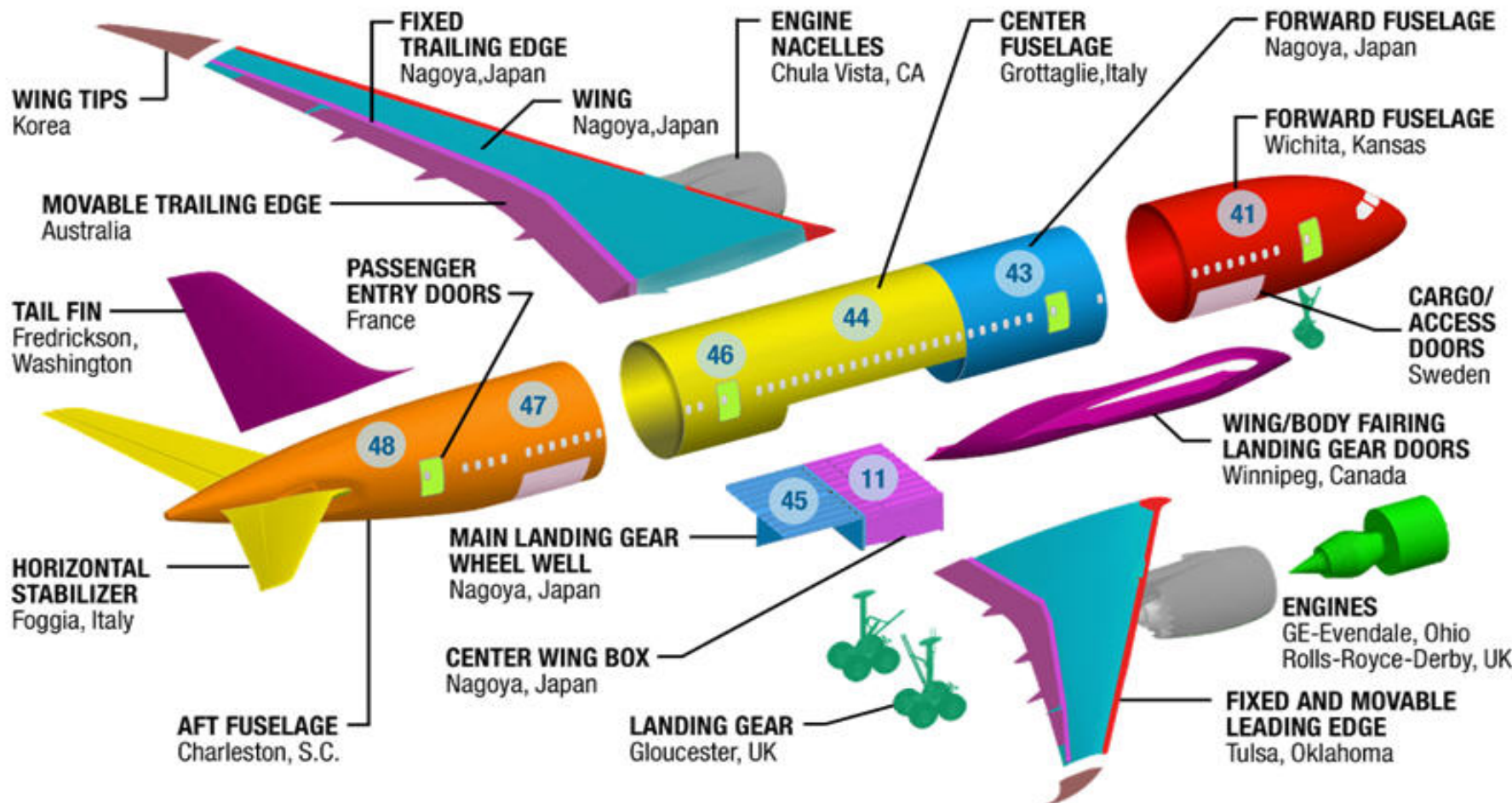
Kawasaki  
Mitsubishi  
Fuji

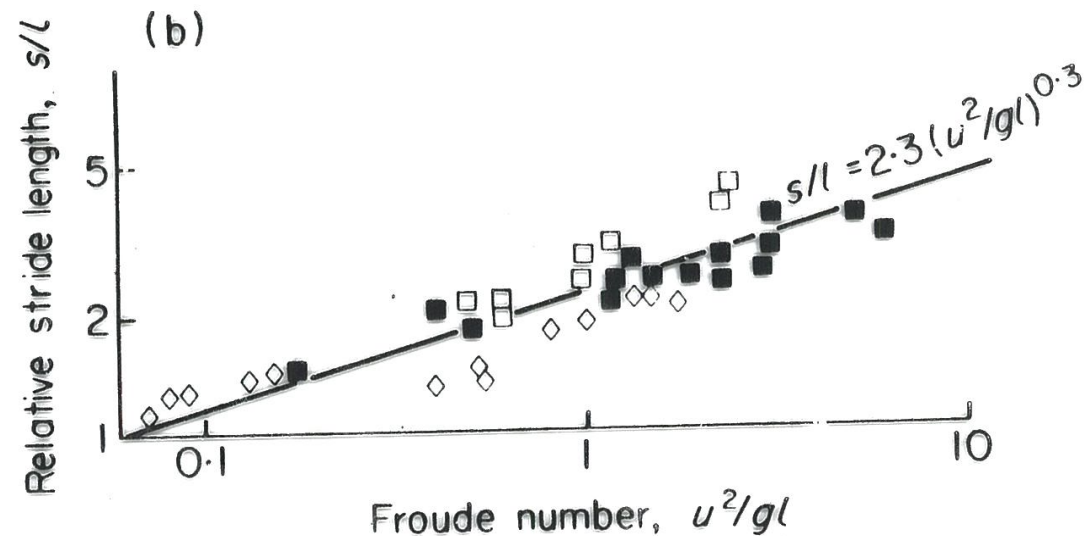
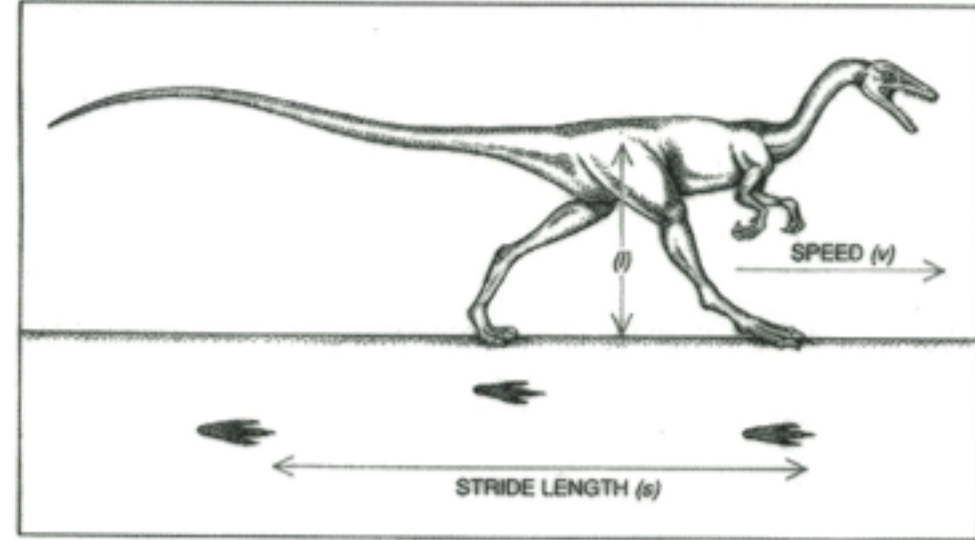
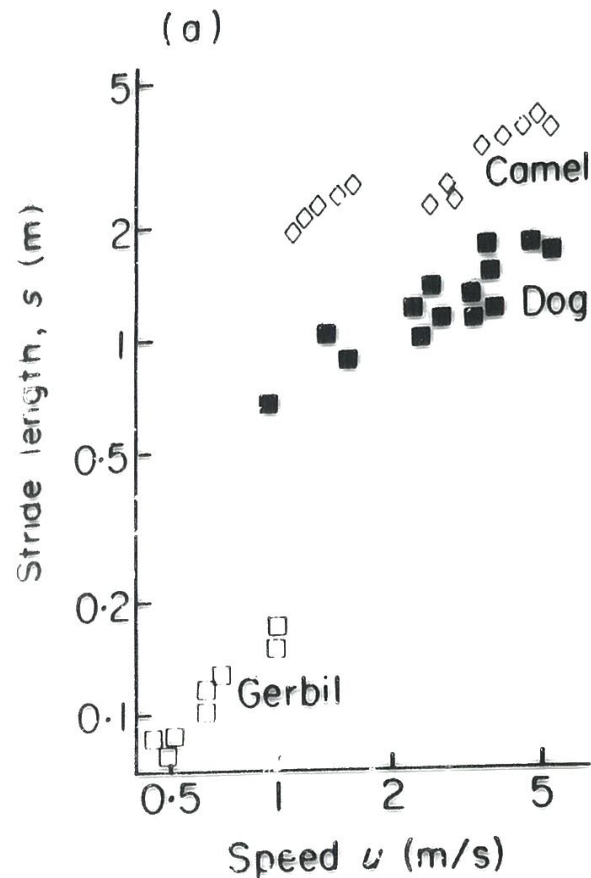
### KOREA

KAL-ASD

### EUROPE

Messier-Dowty  
Rolls-Royce  
Latecoere  
Alenia  
Saab





**Fig. 1.17**(a) Graphs of stride length ( $s$ ) against speed ( $u$ ) for several mammals. (b) Graphs of relative stride length ( $s/l$ ) against Froude number ( $u^2/gl$ ), based on the same data ( $l$  is leg length). The data are from films taken by the author and his associates.





**DINOSAUR TRACKS** provide a record of stride length and speed. A small, three-toed carnivore may have pursued a larger sauropod along this Texan trail. This pair of footprints was discovered by Ronald T. Bird at Paluxy Creek in 1944.



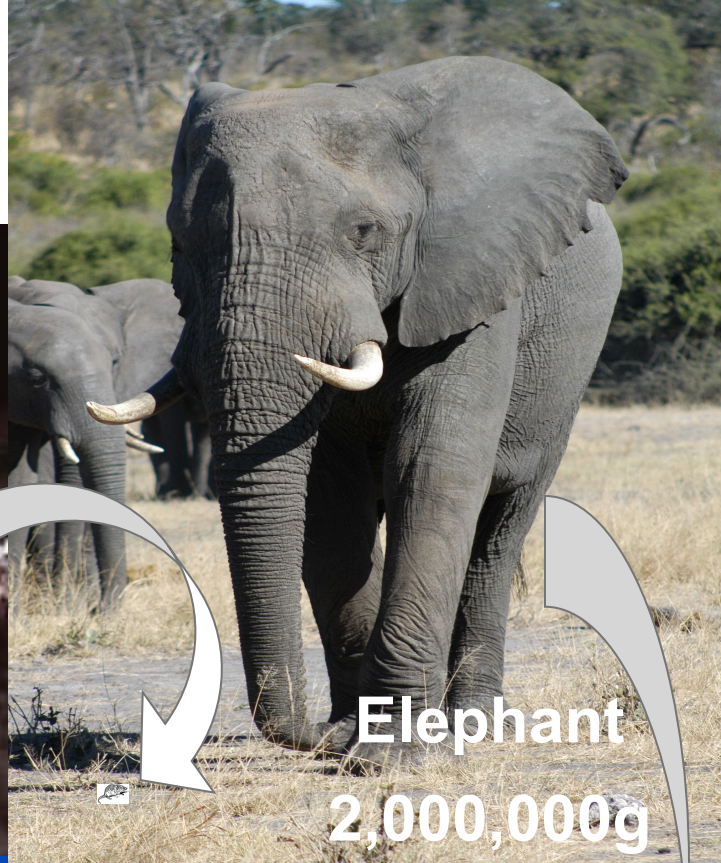
***SCALABILITY***

***RESILIENCE***

***EVOLVABILITY***

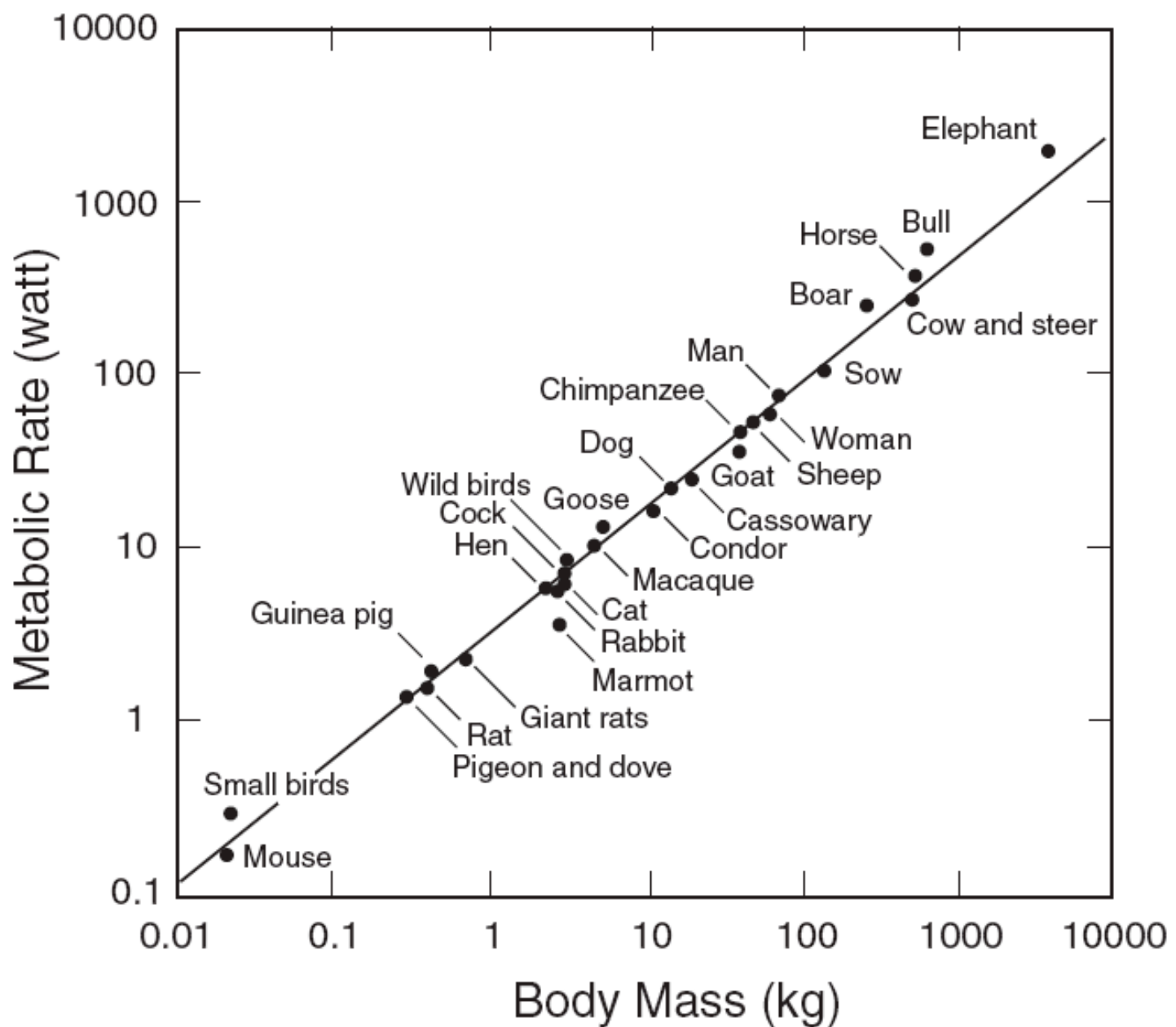
***GROWTH***

# Mammals vary in size by 8 orders of magnitude



**Blue Whale**  
200,000,000g

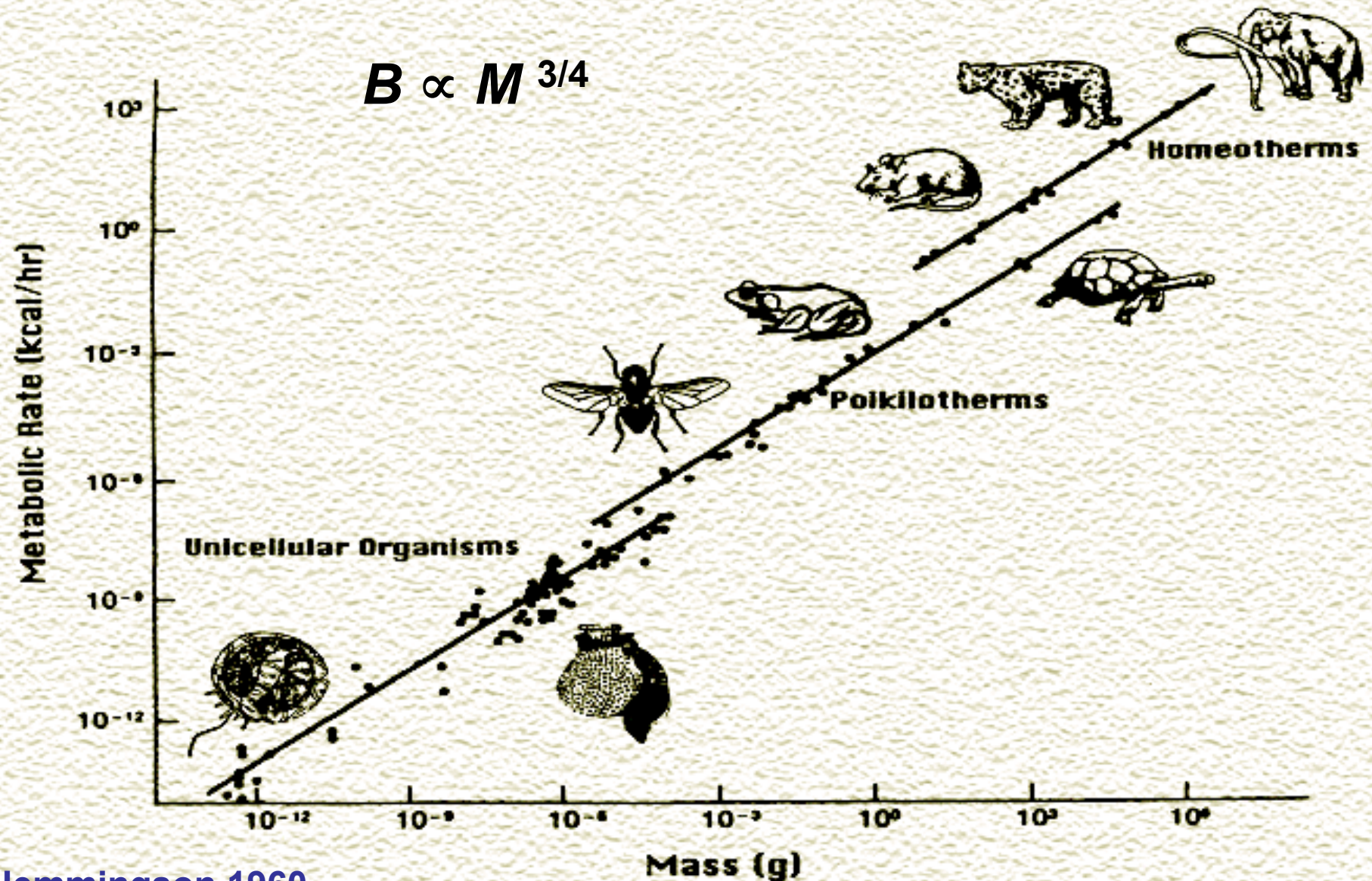


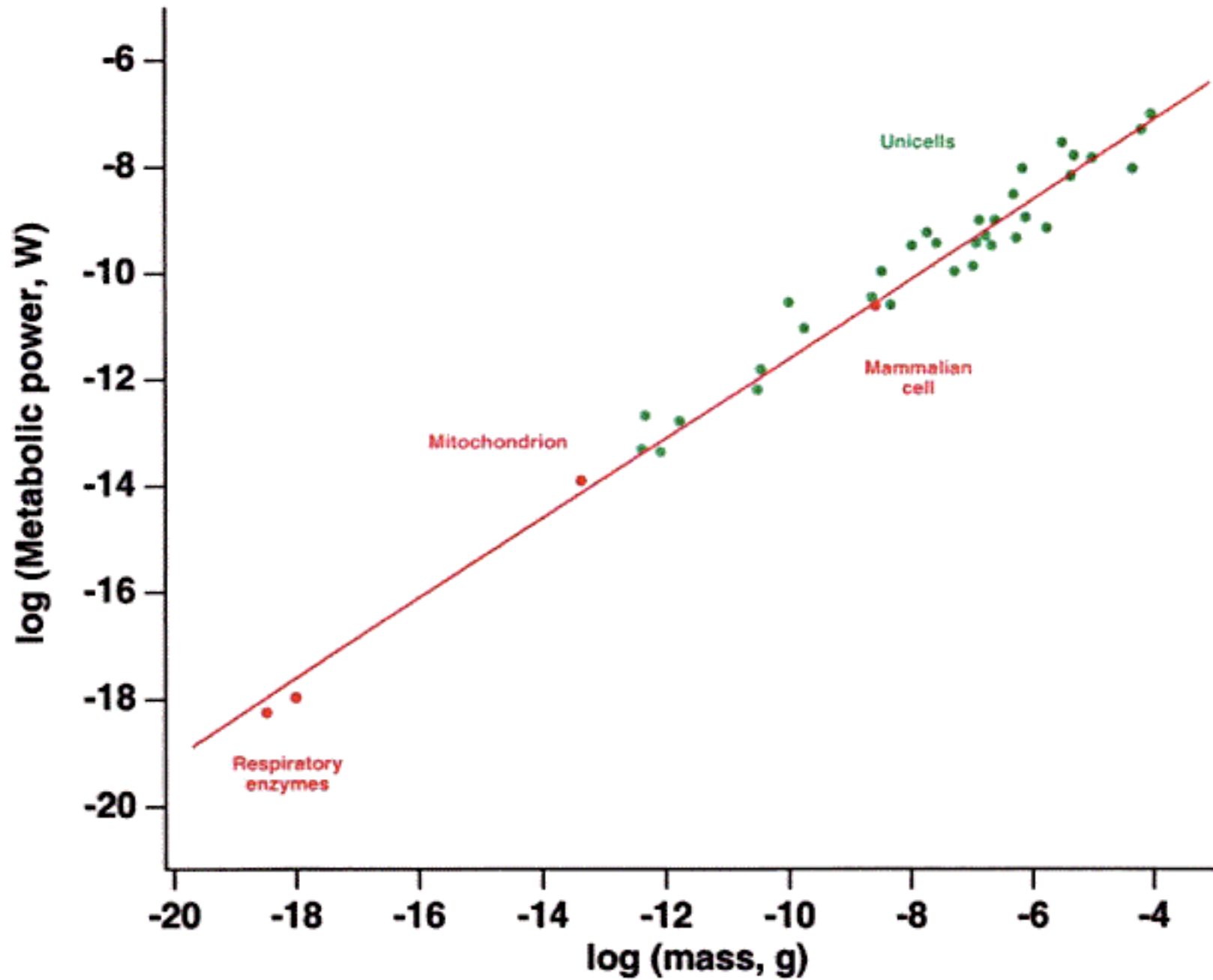


**SLOPE =  $\frac{3}{4} < 1$ ; SUB-LINEAR; ECONOMY OF SCALE**

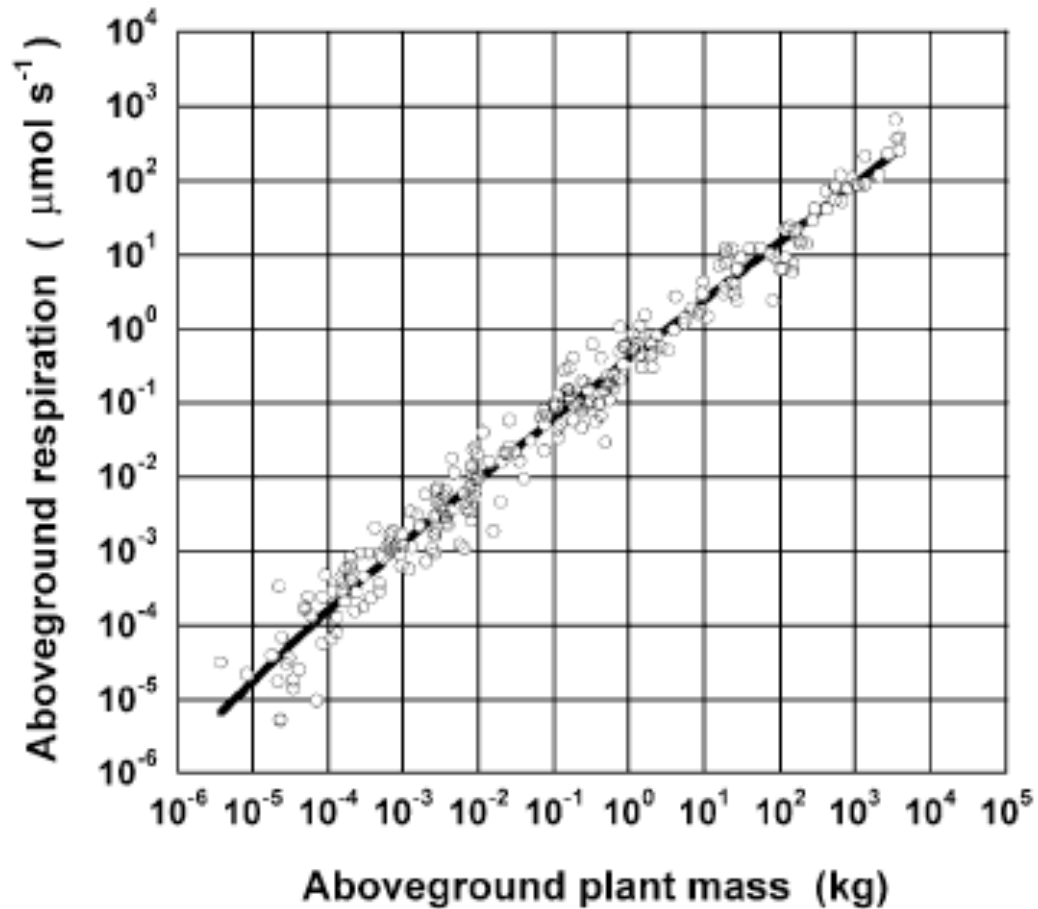


# Whole-organism metabolic rate ( $B$ ) scales as the $3/4$ power of body mass ( $M$ )





# PLANTS/TREES



$$B \propto M^{0.780 \pm 0.037}$$



**SINCE  $N_{\text{cells}} \sim M$  NAIVELY MIGHT EXPECT  $B \sim M$**

**HOWEVER,  $B \sim M^{3/4}$**

**OVER 27 ORDERS OF MAGNITUDE**

**SPECIFIC METABOLIC RATE (PER UNIT MASS)**

$$\frac{B}{M} \propto M^{-1/4}$$

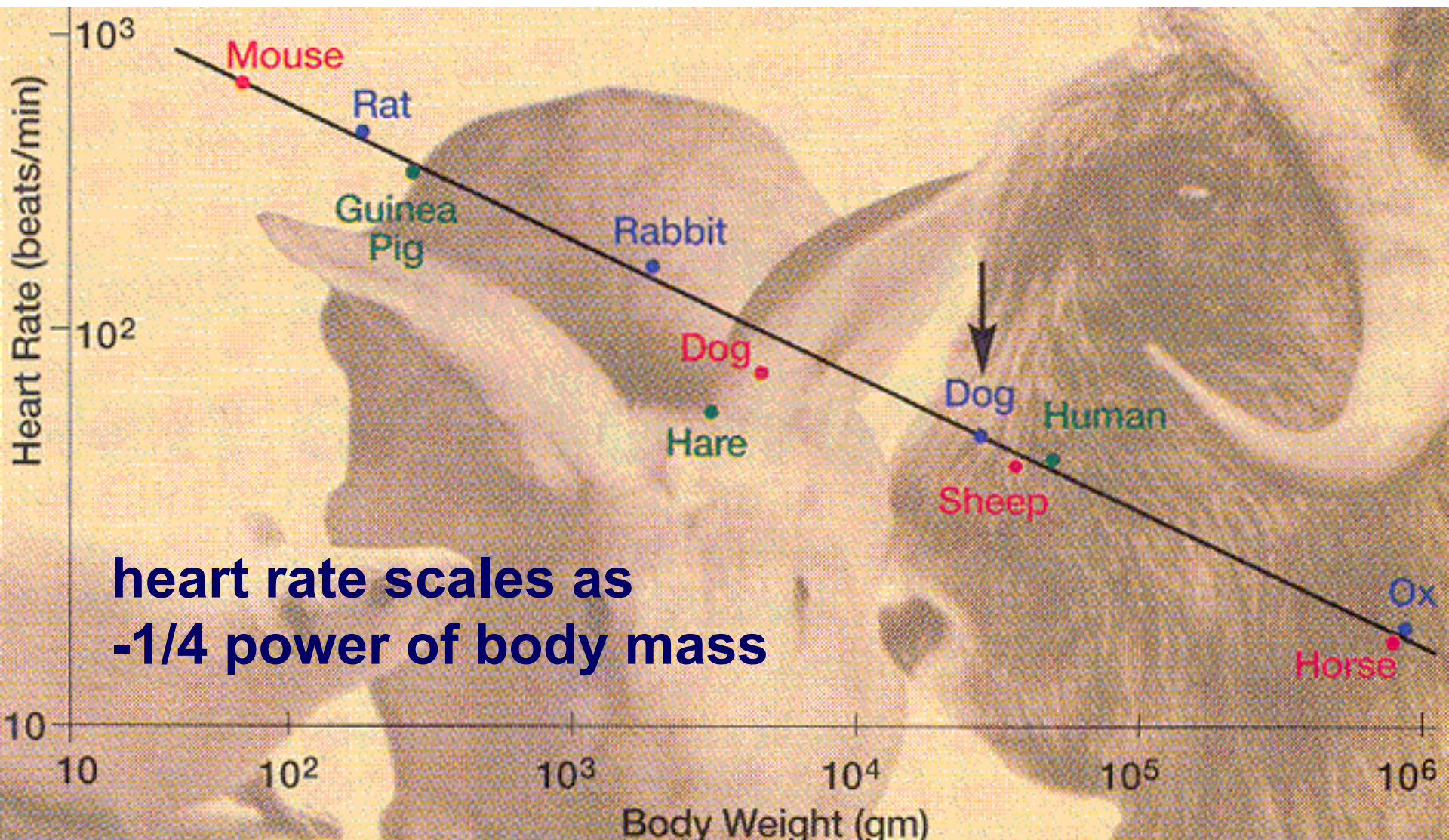
**SO METABOLIC RATE OF AVERAGE CELL**

$$B_{\text{cell}} \propto M^{-1/4}$$

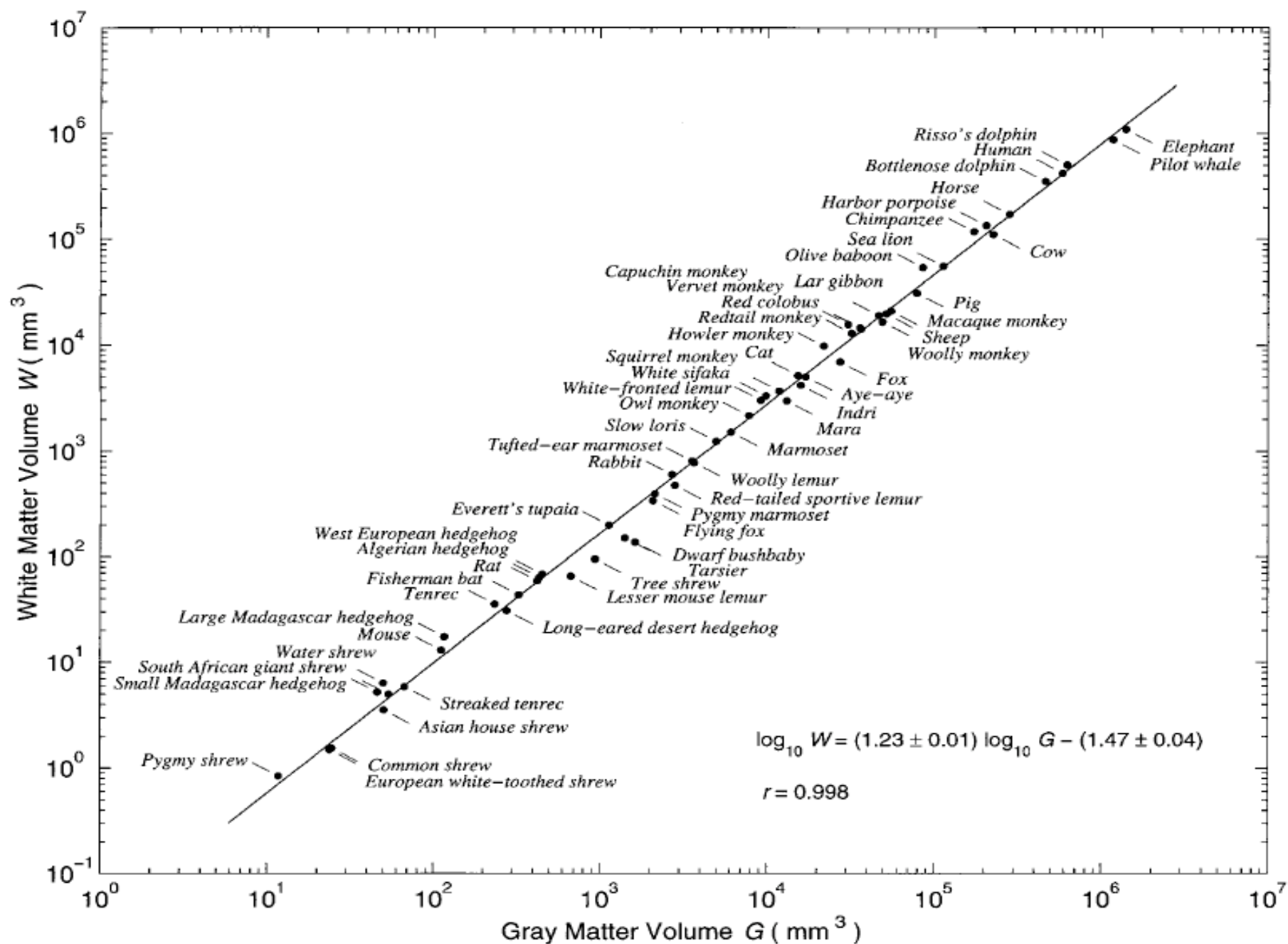
***EXTRAORDINARY SYSTEMATIC  
ECONOMY OF SCALE (THE BIGGER  
YOU ARE, THE LESS NEEDED PER  
“CAPITA”)***

***SIMILAR SCALING HOLDS TRUE  
FOR ALL PHYSIOLOGICAL  
PROCESSES AND LIFE HISTORY  
EVENTS OVER THE ENTIRE  
SPECTRUM OF LIFE***

# Metabolic rate sets the pace of life small animals live fast and die young



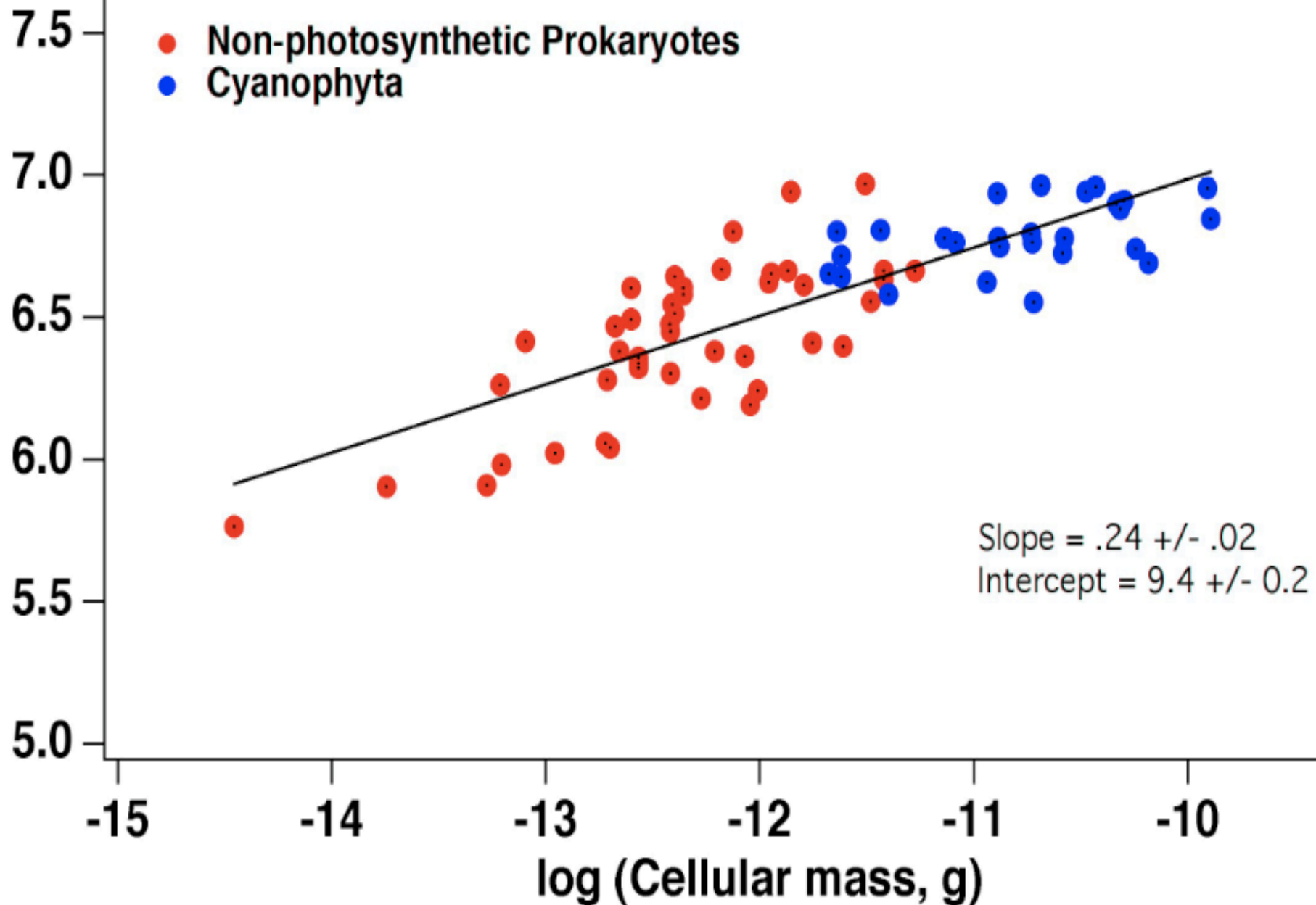




# Dependence of Prokaryotic Genome Length on Cellular Mass

log (genome length, bp)

- Non-photosynthetic Prokaryotes
- Cyanophyta



## LIFESPAN

$$T \sim M^{1/4}$$

IF HEART-RATE (NUMBER OF BEATS PER SEC.)

$$\sim M^{-1/4}$$

⇒ TOTAL NUMBER OF HEART-BEATS IN A

TYPICAL LIFE-TIME IS INDEPENDENT OF SIZE!

$$\approx 1.5 \times 10^9$$

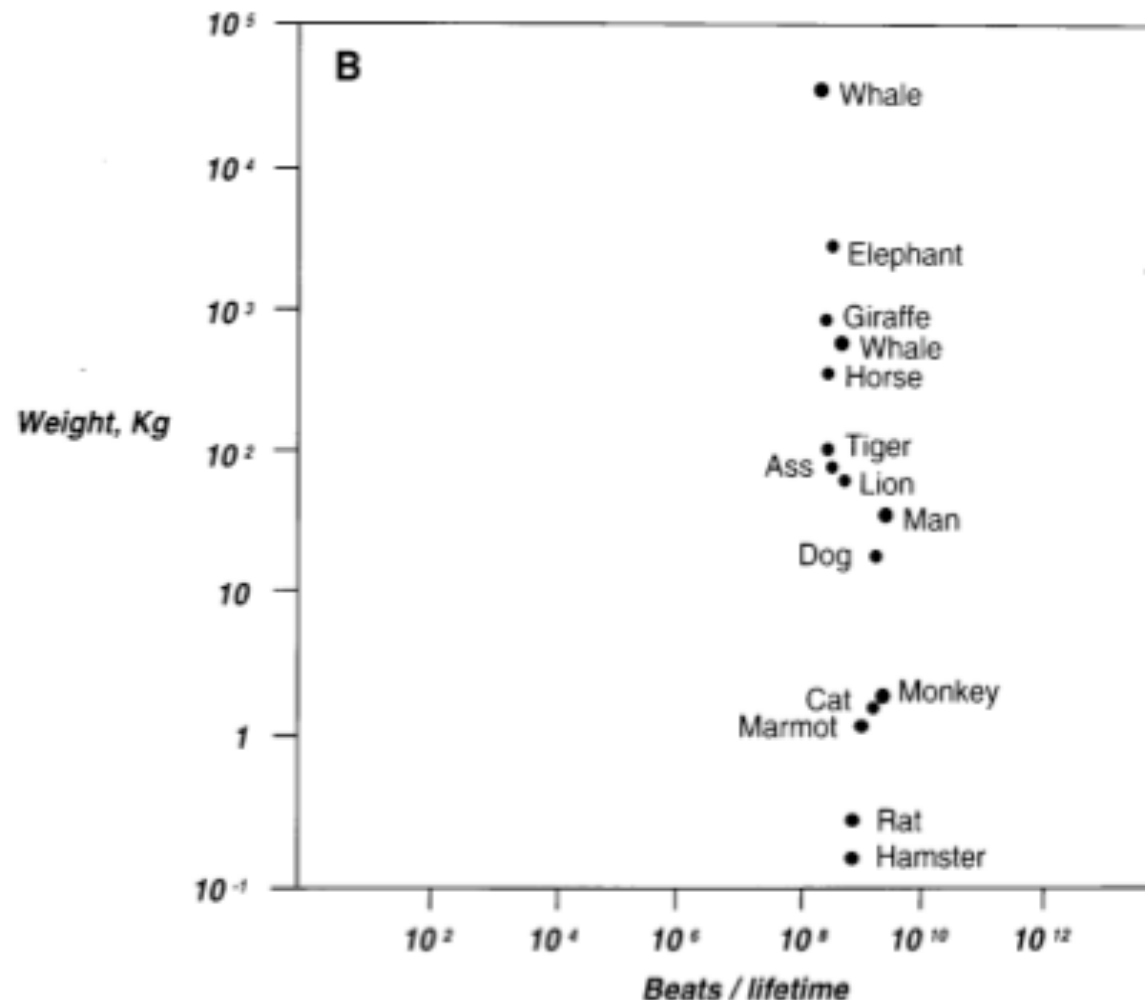
EACH ANIMAL SPECIES REGARDLESS OF SIZE

HAS APPROXIMATELY THE SAME NUMBER OF HEART-

BEATS IN ITS LIFE-TIME (ROUGHLY 1 BILLION)



**MORE FUNDAMENTALLY, ACROSS AEROBIC METABOLISM:  
THE NUMBER OF TURNOVERS IN A LIFETIME OF Cyto  
ENZYMES (RESPIRATORY COMPLEX) IS AN APPROXIMATE  
INVARIANT ( $\sim 10^{16}$ )**



LIFE IS THE MOST COMPLEX SYSTEM

SCALING LAWS ARE REMARKABLE BECAUSE

i) THEY EXIST

ii) THEY ARE VERY SIMPLE

iii) THEY ARE UNIVERSAL

} DOMINANCE OF  
 $1/4$  POWER

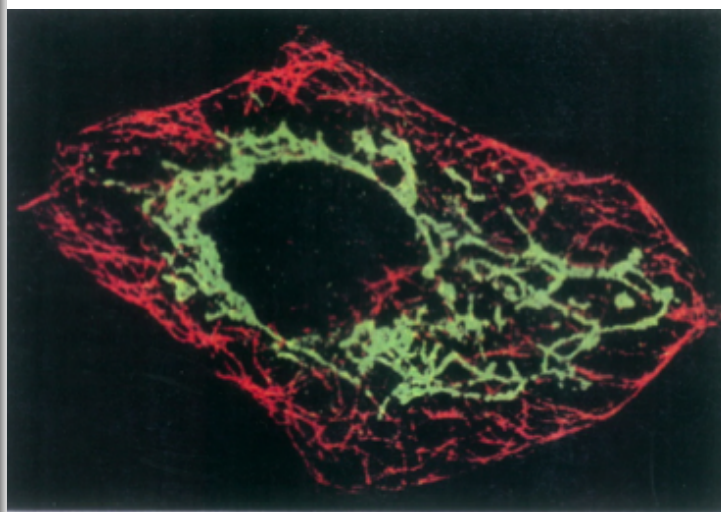
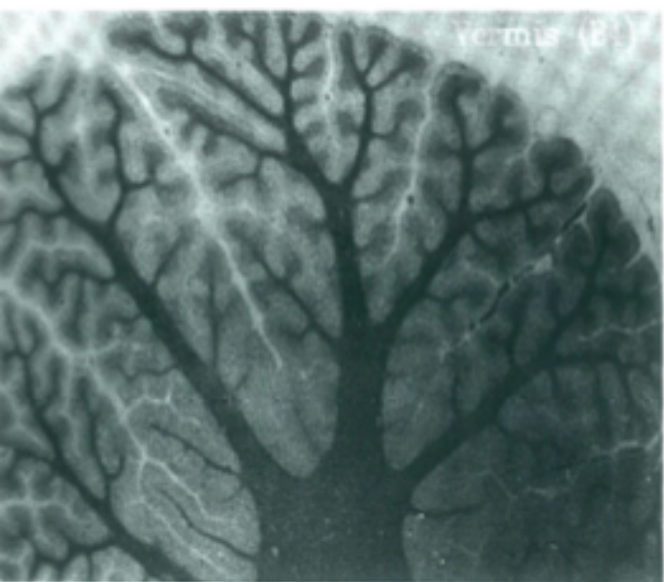
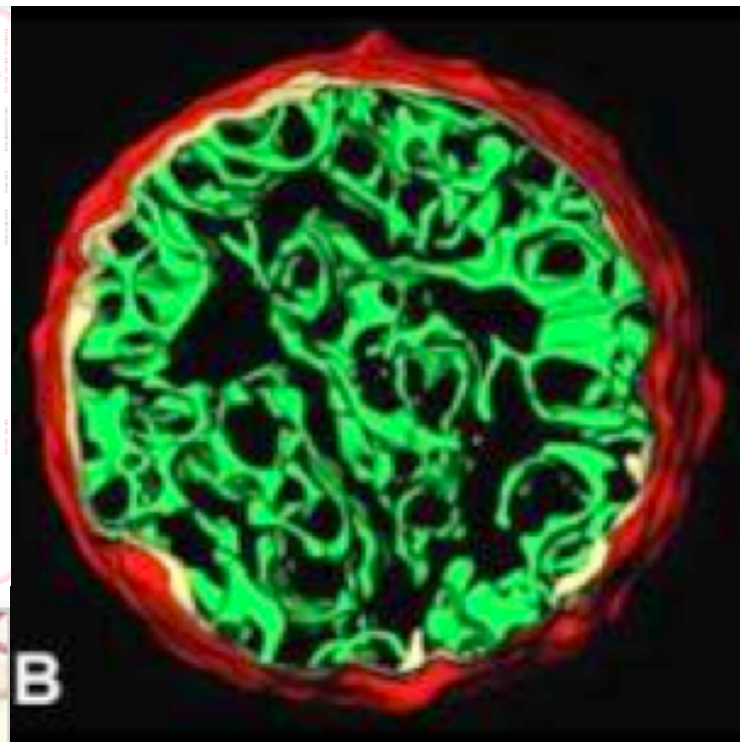
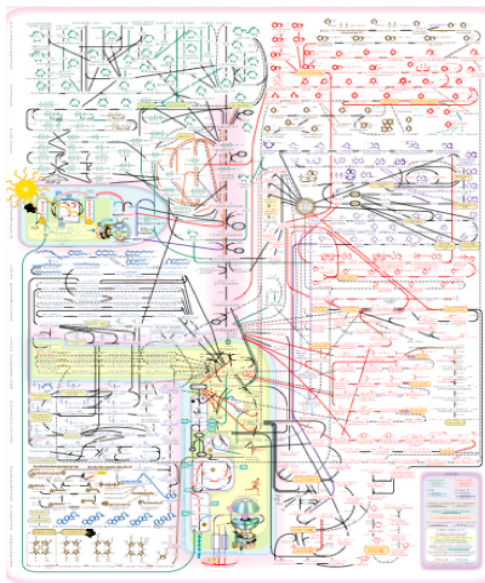
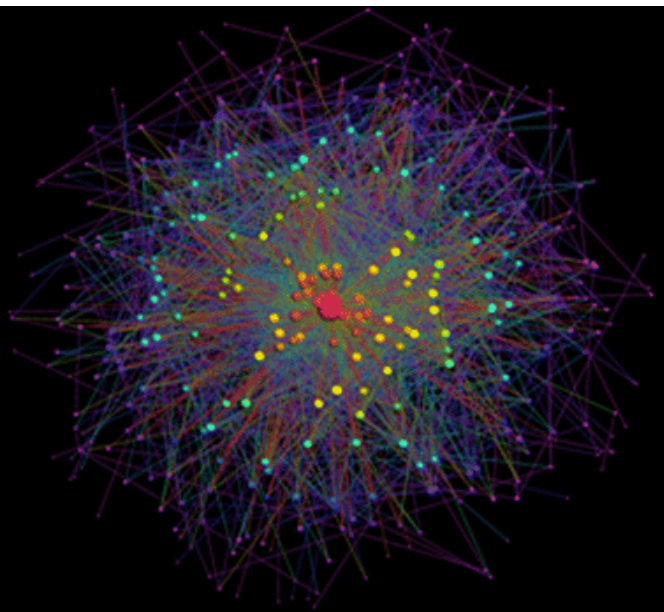
iv)  $\Rightarrow$  BIGGER IS MORE EFFICIENT

v) FEW QUANTITATIVE "LAWS" IN BIOLOGY

***NETWORKS!!!***

***(FRACTAL - LIKE)***





## FUNDAMENTAL PRINCIPLES

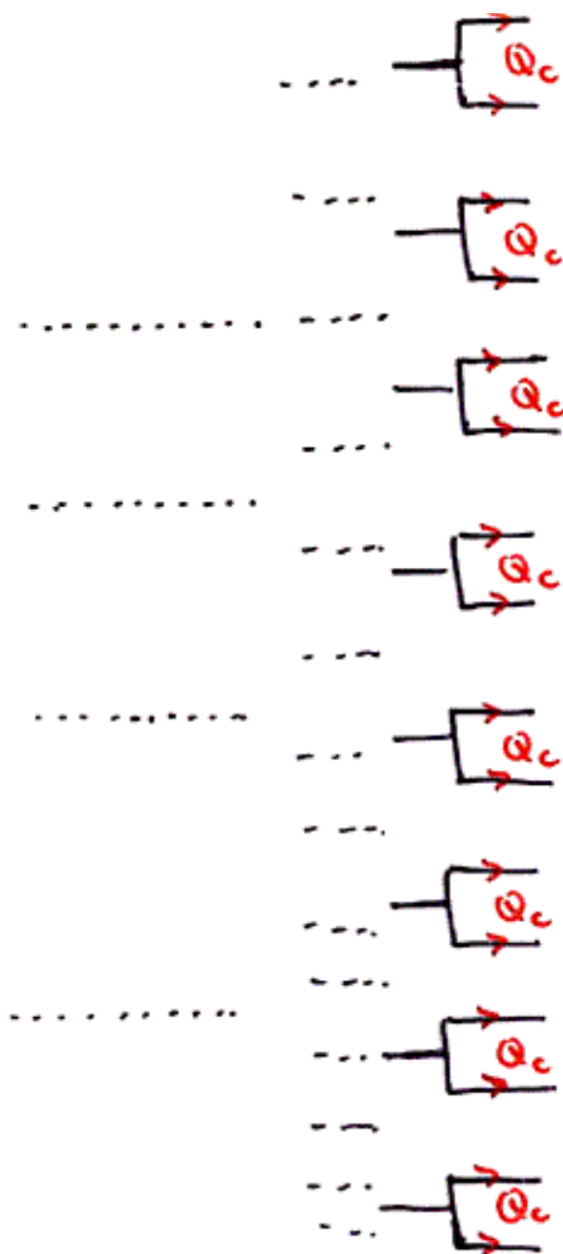
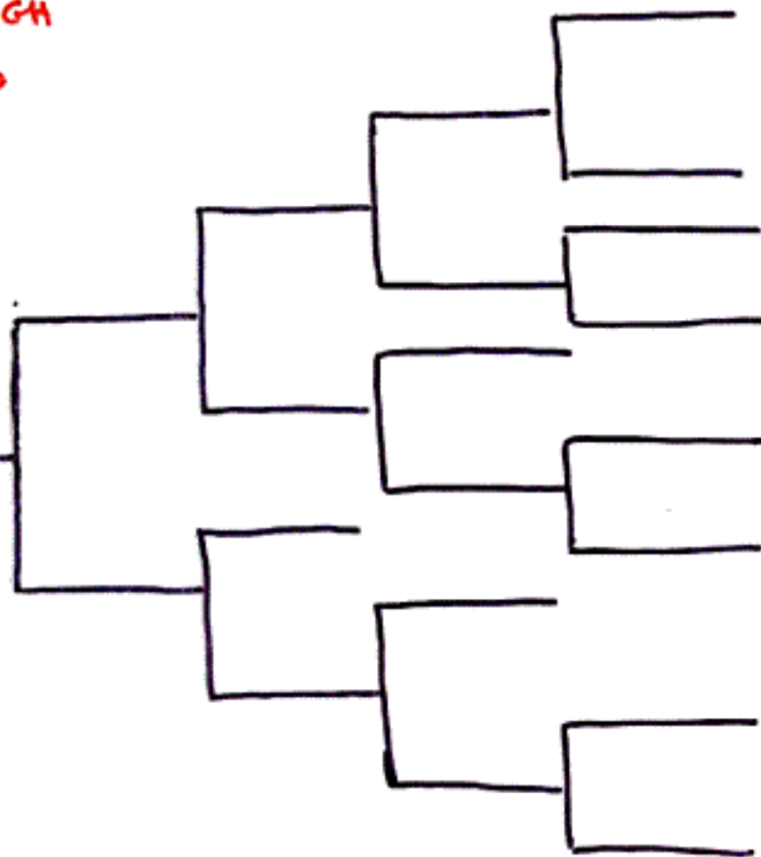
(NATURAL SELECTION)

- I. AT ALL SCALES ORGANISMS ARE SUSTAINED BY THE TRANSPORT OF ENERGY AND ESSENTIAL MATERIALS THROUGH HIERARCHICAL BRANCHING NETWORK SYSTEMS IN ORDER TO SUPPLY ALL LOCAL PARTS OF THE ORGANISM
- II. THESE NETWORKS ARE SPACE-FILLING
- III. THE TERMINAL BRANCHES OF THE NETWORK ARE INVARIANT UNITS
- IV. ORGANISMS HAVE EVOLVED BY NATURAL SELECTION SO AS TO
  - i) MINIMISE ENERGY DISSIPATED IN THE NETWORKS
  - AND/OR ii) MAXIMISE THE SCALING OF THEIR AREA OF INTERFACE WITH THEIR RESOURCE ENVIRONMENT

FLUID FLOW THROUGH  
AORTA  $Q_0 \propto B$



AORTA  
TRUNK  
CEO



CAPILLARIES  
PETIOLES  
MITOCHONDRIA



SINCE THE FLUID (BLOOD) TRANSPORTS OXYGEN,  
NUTRIENTS, ETC FROM THE AORTA TO THE  
CAPILLARIES

METABOLIC RATE  $\propto$  VOLUME FLOW RATE

$$B \propto Q_0$$

BUT THE CONSERVATION OF FLUID (BLOOD)

$$\Rightarrow Q_0 = N_c Q_c$$

TOTAL NUMBER OF CAPILLARIES      VOLUME FLOW RATE IN AVERAGE CAPILLARY

CAPILLARY IS AN INVARIANT UNIT

( $Q_c$  IS SAME FOR ALL MAMMALS)

$\Rightarrow$  NUMBER OF CAPILLARIES ( $N_c$ ) MUST SCALE IN SAME  
WAY AS THE METABOLIC RATE ( $B \propto Q_0$ )

SO, IF  $B \sim M^{3/4}$  THEN

$$N_c \sim M^{3/4} \quad (\text{NOT } N_c \sim M)$$

TOTAL NUMBER OF CELLS

$$N_{\text{cell}} \sim M \quad (\text{LINEAR})$$

TOTAL NUMBER OF CAPILLARIES

$$N_c \sim M^{3/4}$$

MISMATCH !

⇒ NUMBER OF CELLS FED BY A SINGLE  
CAPILLARY INCREASES AS  $M^{1/4}$

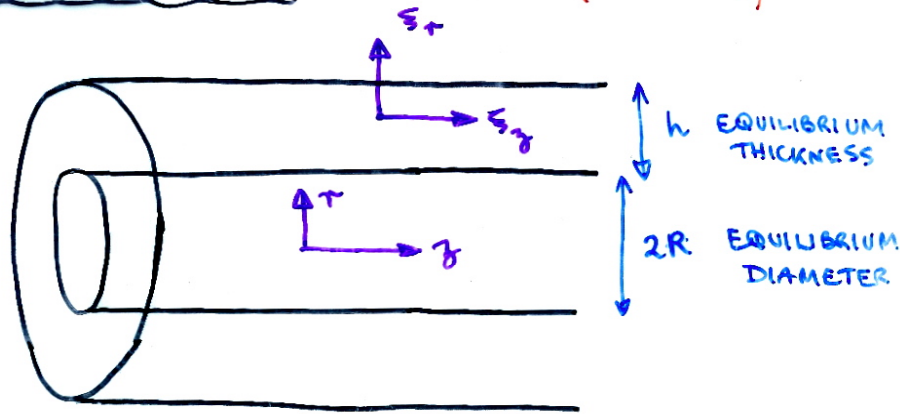
(ANOTHER MANIFESTATION THAT EFFICIENCY  
INCREASES WITH SIZE)

IMPORTANT IMPLICATIONS FOR GROWTH AND DEATH !

# 1) PULSATILE TREATMENT

(WOMERSLEY)

(19)



## a) FLUID :

STRESS TENSOR:  $\theta_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - p \delta_{ij}$

(NEWTONIAN)

COEFFICIENTS OF VISCOSITY (pointing to  $\lambda$  and  $\mu$ )

PRESSURE (pointing to  $p$ )

STRAIN TENSOR:  $e_{ij} = \frac{1}{2} (\partial_i v_j + \partial_j v_i)$

VELOCITY (pointing to  $v_i$  and  $v_j$ )

EQ<sup>n</sup>. OF MOTION:  $\rho \frac{Dv_i}{Dt} = \partial_j \theta_{ij}$

NAVIER-STOKES EQ<sup>n</sup>.

COVARIANT DERIVATIVE  $\frac{D}{Dt} = \frac{\partial}{\partial t} + v_i \partial_i$

EQ<sup>n</sup>. OF CONTINUITY:  $\frac{\partial \rho}{\partial t} + \partial_i (\rho v_i) = 0$



b) WALLS :

$$\sigma_{ij}^w = \lambda e_{kk}^w \delta_{ij} + 2B e_{ij}^w - p \delta_{ij} \quad \text{HOOKE'S LAW}$$

ELASTIC MODULI

$$e_{ij}^w = \frac{1}{2} (\partial_i \xi_j + \partial_j \xi_i)$$

$$\rho_w \frac{Du_i}{Dt} = \partial_j \sigma_{ij}^w \quad ; \quad u_i = \frac{\partial \xi_i}{\partial t} \quad \text{NAVIER EQ'NS.}$$

$$\partial_i \xi_i = 0$$

NEGLECT NON-LINEARITIES :

$$\rho_w \frac{\partial^2 \xi_i}{\partial t^2} = B \partial^2 \xi_i - \partial_i p$$

SOLVE USING FOURIER AS WITH FLUID, WALLS AND

FLUID COUPLED VIA BOUNDARY CONDITIONS : CONTINUITY

OF VELOCITY AND FORCE :  $u_r = \frac{\partial \xi_r}{\partial t}$

AND  $\int_{\text{SURFACE}} \sigma_{ij} dS_j$  CONTINUOUS

CAN BE SOLVED : BIG MESS!

SIMPLIFY USING THIN WALL APPROXIMATION

$$\text{i.e. } \frac{h}{R} \ll 1$$

IMPEDANCE :  $\bar{Z}(\omega) = \frac{\rho c_0^2}{\pi R^2 c} \frac{1}{J_0(ikR)}$

WHERE  $c = \frac{\omega}{k}$  PHASE VELOCITY

AND  $c_0 = \left( \frac{Bh}{2\rho R} \right)^{1/2}$

$B = \frac{E}{1 - \sigma^2}$

YOUNG'S MODULUS  
POISSON RATIO

[ KORTEWEG - MOENS VELOCITY,  
FIRST DERIVED BY YOUNG ]

TYPICALLY  $kR = \frac{2\pi R}{\lambda} \ll 1$  so

$\bar{Z}(\omega) \approx \frac{\rho c_0^2}{\pi R^2 c}$

WITH  $\left( \frac{c}{c_0} \right)^2 = - \frac{J_2(i^{3/2}\alpha)}{J_0(i^{3/2}\alpha)}$  DISPERSION RELATION

WHERE  $\alpha \equiv \left( \frac{\omega \rho}{\mu} \right)^{1/2} R$  DIMENSIONLESS WOMERSLEY NUMBER

$\Rightarrow$  ATTENUATION AND DISPERSION

ii) OPTIMISATION (ENERGY MINIMISATION)

(17)

EX: FOR GIVEN VOLUME OF BLOOD MINIMISE ENERGY

OUTPUT (THEREFORE MINIMISE RESISTANCE) SUBJECT

TO SPACE FILLING GEOMETRY, USE LAGRANGE

MULTIPLIERS AND CONSIDER:

$$F(r_k, l_k, n_k) = Z(r_k, l_k, n_k) + \lambda V_b(r_k, l_k, n_k) + \sum_{k=0}^N \lambda_k n_k^k l_k^3$$

DEMAND  $\frac{\partial F}{\partial r_k} = \frac{\partial F}{\partial l_k} = \frac{\partial F}{\partial n_k} = 0$

LAGRANGE EQ<sup>ns</sup>

USE:  $Z(r_k, l_k, n_k) = \sum_{k=0}^N \frac{8\mu l_k}{\pi r_k^4 n_k}$

$$V_b(r_k, l_k, n_k) = \sum_{k=0}^N \pi r_k^2 l_k n_k^k$$

e.g.  $\frac{\partial F}{\partial r_k} = 0 \Rightarrow r_k^6 n_k^{2k} = \text{CONSTANT}$

i.e.  $\beta_k = \frac{r_{k+1}}{r_k} = \frac{1}{n^{1/3}}$  IND<sup>p</sup> OF  $k$  NOT  $\frac{1}{n^{1/2}}$  !



NOW VARY MASS : MINIMISE ENERGY LOSS ( $\dot{Q}_0$  CHANGES)

$$F(r_k, l_k, n, M) = \underset{\downarrow M^a}{\dot{E}(M)} + \lambda \left[ \sum_{k=0}^N \pi r_k^2 l_k n^k - \underset{\downarrow M^b}{V_b(M)} \right] \\ + \sum_{k=0}^N \lambda_k [n^k l_k^3 - \underset{\downarrow N_0 l_0^3 \sim M^a}{V_n(M)}] + \lambda_m M$$

FOR FIXED  $M$ , AS ABOVE ;

$$\frac{\partial F}{\partial M} \sim a M^{a-1} + \lambda b M^{b-1} + (\sum \lambda_k) a M^{a-1} + \lambda_m$$

$$= 0 \quad \Rightarrow \quad \underline{b=1}$$

$$\text{i.e.} \quad \underline{V_b \sim M}$$

IN  $d$  DIMENSIONS

$$B \propto M^{\frac{d}{d+1}}$$

WE LIVE IN 3 SPATIAL DIMENSIONS SO  $B \propto M^{3/4}$

⇒ "3" REPRESENTS DIMENSIONALITY OF SPACE

"4" INCREASE IN DIMENSIONALITY DUE TO  
FRACTAL-LIKE SPACE FILLING

LIFE HAS TAKEN ADVANTAGE OF THE POSSIBILITY OF  
USING SPACE-FILLING FRACTAL-LIKE SURFACES  
(WHERE ENERGY AND RESOURCES ARE EXCHANGED)

TO MAXIMISE ENERGY TRANSFER FROM THE  
ENVIRONMENT

NON-FRACTAL :

AREA

$M^{2/3}$  ← DIMENSIONALITY OF SPACE (VOLUME)

BIOLOGICAL (FRACTAL)

$$M^{3/4}$$

BY ANALOGY : LIFE EFFECTIVELY OPERATES IN  
FOUR SPATIAL DIMENSIONS

[FIVE IF TIME IS INCLUDED]

# Cardiovascular

Variable	Exponent	
	Predicted	Observed
Aorta radius $r_o$	$3/8 = 0.375$	0.36
Aorta pressure $\Delta p_o$	$0 = 0.00$	0.032
Aorta blood velocity $u_o$	$0 = 0.00$	0.07
Blood volume $V_b$	$1 = 1.00$	1.00
Circulation time	$1/4 = 0.25$	0.25
Circulation distance $l$	$1/4 = 0.25$	ND
Cardiac stroke volume	$1 = 1.00$	1.03
Cardiac frequency $\omega$	$-1/4 = -0.25$	-0.25
Cardiac output $\dot{E}$	$3/4 = 0.75$	0.74
Number of capillaries $N_c$	$3/4 = 0.75$	ND
Service volume radius	$1/12 = 0.083$	ND
Womersley number $\alpha$	$1/4 = 0.25$	0.25
Density of capillaries	$-1/12 = -0.083$	-0.095
O <sub>2</sub> affinity of blood $P_{50}$	$-1/12 = -0.083$	-0.089
Total resistance $Z$	$-3/4 = -0.75$	-0.76
Metabolic rate $B$	$3/4 = 0.75$	0.75



## Respiratory

Variable	Exponent	
	Predicted	Observed
Tracheal radius	$3/8 = 0.375$	0.39
Interpleural pressure	$0 = 0.00$	0.004
Air velocity in trachea	$0 = 0.00$	0.02
Lung volume	$1 = 1.00$	1.05
Volume flow to lung	$3/4 = 0.75$	0.80
Volume of alveolus $V_A$	$1/4 = 0.25$	ND
Tidal volume	$1 = 1.00$	1.041
Respiratory frequency	$-1/4 = -0.25$	-0.26
Power dissipated	$3/4 = 0.75$	0.78
Number of alveoli $N_A$	$3/4 = 0.75$	ND
Radius of alveolus $r_A$	$1/12 = 0.083$	0.13
Area of alveolus $A_A$	$1/6 = 0.083$	ND
Area of lung $A_L$	$11/12 = 0.92$	0.95
O <sub>2</sub> diffusing capacity	$1 = 1.00$	0.99
Total resistance	$-3/4 = -0.75$	-0.70
O <sub>2</sub> consumption rate	$3/4 = 0.75$	0.76

**Table 1 Predicted values of scaling exponents for physiological and anatomical variables of plant vascular systems.**


Variable	Plant mass		Branch radius		
	Exponent predicted	Symbol	Symbol	Exponent	
				Predicted	Observed
Number of leaves	$\frac{3}{4}$ (0.75)	$n_0^L$	$n_k^L$	2 (2.00)	2.007 (ref. 12)
Number of branches	$\frac{3}{4}$ (0.75)	$N_0$	$N_k$	-2 (-2.00)	-2.00 (ref. 6)
Number of tubes	$\frac{3}{4}$ (0.75)	$n_0$	$n_k$	2 (2.00)	n.d.
Branch length	$\frac{1}{4}$ (0.25)	$l_0$	$l_k$	$\frac{2}{3}$ (0.67)	0.652 (ref. 6)
Branch radius	$\frac{3}{8}$ (0.375)	$r_0$			
Area of conductive tissue	$\frac{7}{8}$ (0.875)	$A_0^{CT}$	$A_k^{CT}$	$\frac{7}{3}$ (2.33)	2.13 (ref. 8)
Tube radius	$\frac{1}{16}$ (0.0625)	$a_0$	$a_k$	$\frac{1}{8}$ (0.167)	n.d.
Conductivity	1 (1.00)	$K_0$	$K_k$	$\frac{8}{3}$ (2.67)	2.63 (ref. 12)
Leaf-specific conductivity	$\frac{1}{4}$ (0.25)	$L_0$	$L_k$	$\frac{2}{3}$ (0.67)	0.727 (ref. 17)
Fluid flow rate			$\dot{Q}_k$	2 (2.00)	n.d.
Metabolic rate	$\frac{3}{4}$ (0.75)	$\dot{Q}_0$			
Pressure gradient	$-\frac{1}{4}$ (-0.25)	$\Delta P_0/l_0$	$\Delta P_k/l_k$	$-\frac{2}{3}$ (-0.67)	n.d.
Fluid velocity	$-\frac{1}{8}$ (-0.125)	$u_0$	$u_k$	$-\frac{1}{3}$ (-0.33)	n.d.
Branch resistance	$-\frac{3}{4}$ (-0.75)	$Z_0$	$Z_k$	$-\frac{1}{3}$ (-0.33)	n.d.
Tree height	$\frac{1}{4}$ (0.25)	$h$			
Reproductive biomass	$\frac{3}{4}$ (0.75)				
Total fluid volume	$\frac{26}{24}$ (1.0415)				

## PLANTS

**VERY DIFFERENT  
EVOLVED  
ENGINEERING  
DESIGN (NON-  
PULSATILE FIBRE  
BUNDLES) BUT  
SAME NETWORK  
PRINCIPLES**

Table 1. Similarity of predicted scaling relations for branches within a tree [quantities denoted by uppercase symbols and subscripts  $i$  (20)], and for trees within a forest (denoted by lowercase symbols and subscripts  $k$ )\*

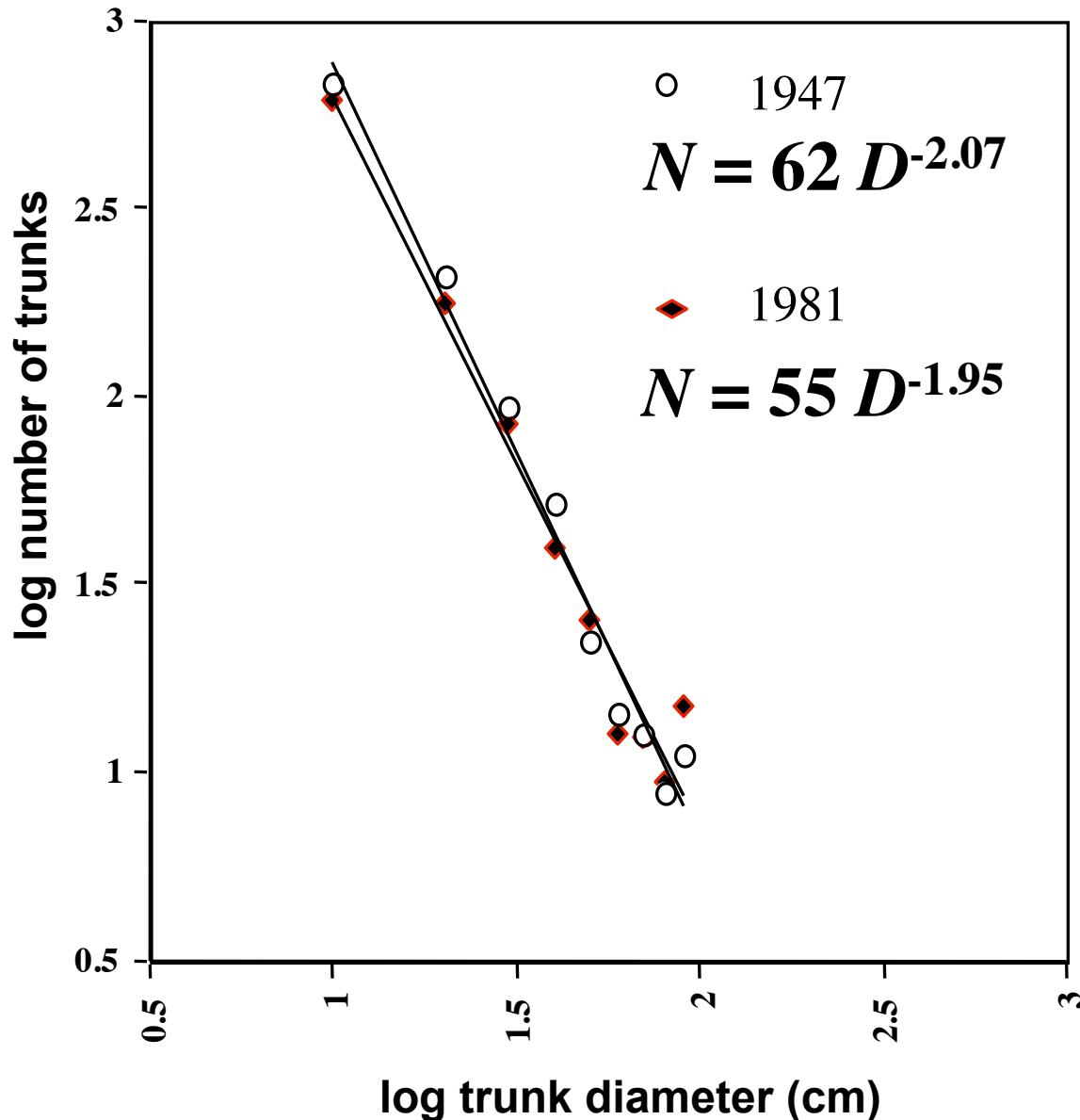
Scaling quantity	Individual tree	Entire forest
Area preserving	$\frac{R_{i+1}}{R_i} = \frac{1}{n^{1/2}}$	$\frac{r_{k+1}}{r_k} = \frac{1}{\lambda^{1/2}}$
Space filling	$\frac{L_{i+1}}{L_i} = \frac{1}{n^{1/3}}$	$\frac{l_{k+1}}{l_k} = \frac{1}{\lambda^{1/3}}$
Biomechanics	$R_i^2 = L_i^3$	$r_k^2 = l_k^3$
Size distribution*	$\Delta N_i \propto R_i^{-2} \propto M_i^{-3/4}$	$\Delta n_k \propto r_k^{-2} \propto m_k^{-3/4}$
Energy and material flux*	$B_i \propto R_i^2 \propto N_i^L \propto M_i^{3/4}$	$B_k \propto r_k^2 \propto n_k^L \propto m_k^{3/4}$

Stand property	Predicted stem radius,  based scaling function
Size class neighbor separation	$d_k \propto r_k$
Canopy scaling	$r_k^{\text{can}} \propto r_k^{2/3}$
Canopy spacing	$d_k^{\text{can}} = c_1 r_k \left[ 1 - \left( \frac{r_k}{r_{\bar{k}}} \right)^{1/3} \right]$
Energy Equivalence	$\Delta n_k B_k \propto r_k^0$
Total forest resource use	$B_{\text{Tot}} \propto \sum \Delta n_k r_k^2 \leq \dot{R}$
Mortality rate	$\mu_k \approx A r_k^{-2/3}$
Size distribution	$N_k \approx \frac{\dot{R}}{(K+1)b_0} r_k^{-2}$

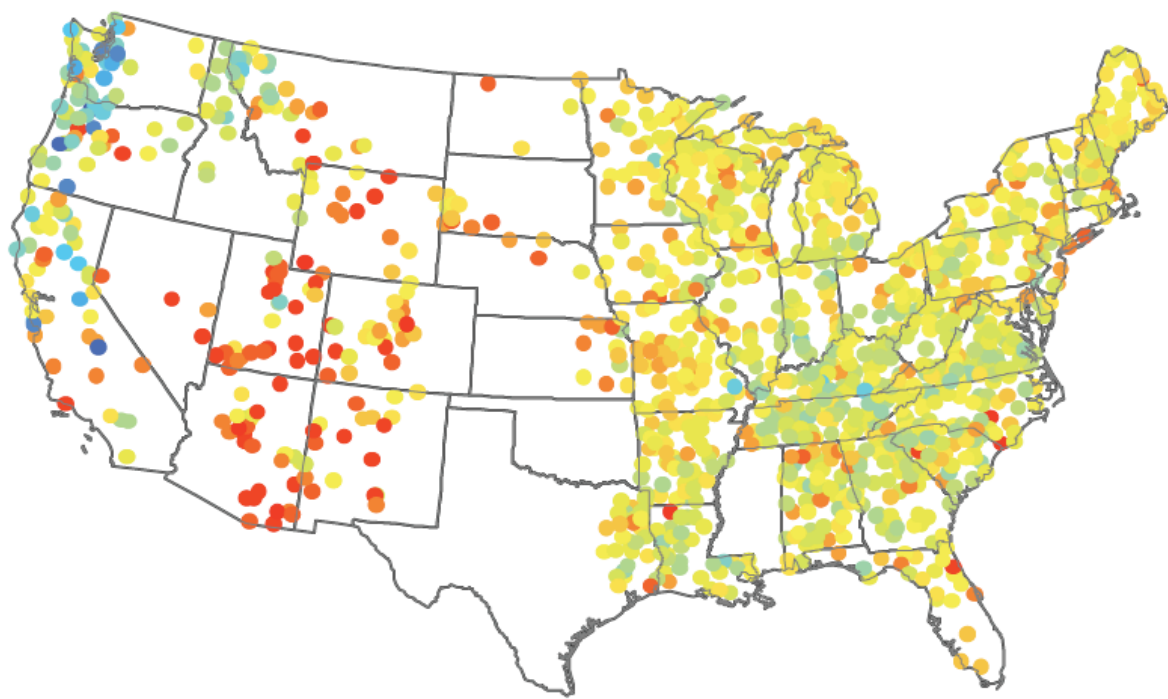
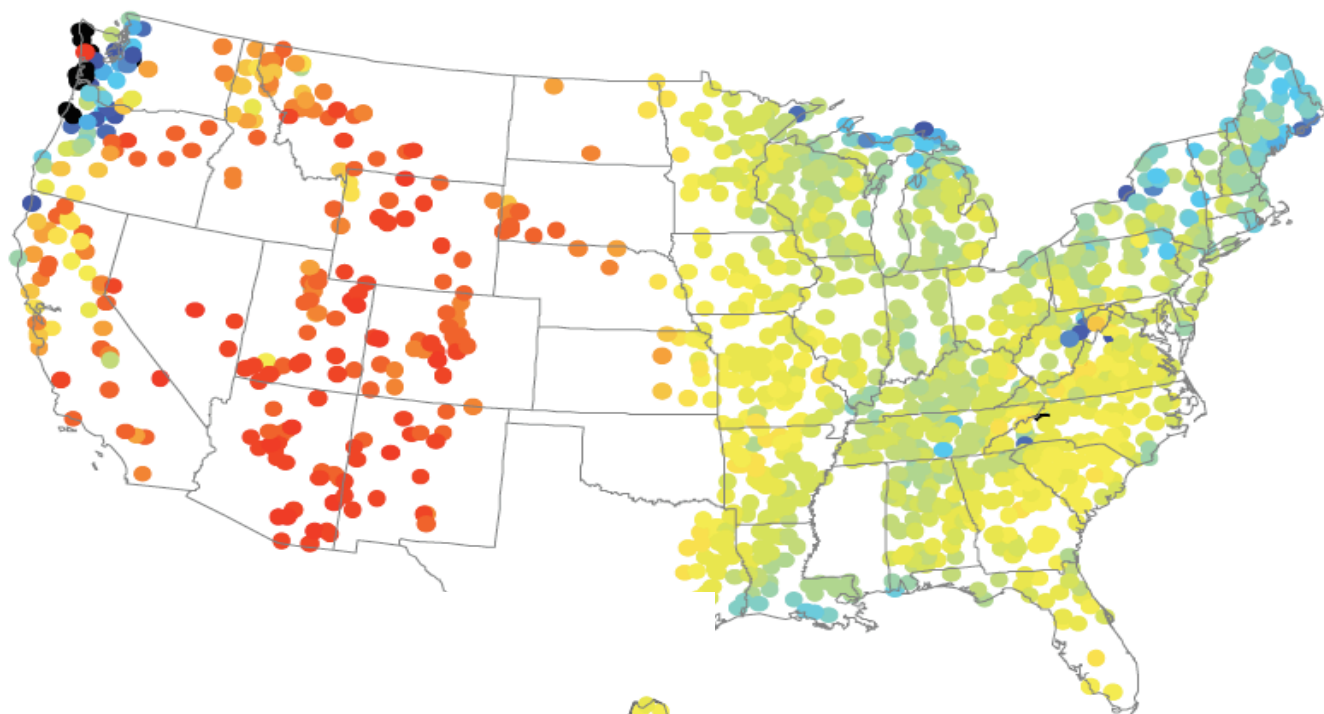


# INTERSPECIFIC SIZE DISTRIBUTION

## All species in a Malaysian Rainforest



Manokaran and  
Kochummen (1987)



## HYDRODYNAMIC RESISTANCE OF THE NETWORK

$$\sim \frac{1}{M^{3/4}}$$

TOTAL RESISTANCE DECREASES WITH SIZE !!

SMALL MAY BE BEAUTIFUL BUT LARGE IS  
MORE EFFICIENT !!

BLOOD PRESSURE  $\sim M^0$

AORTA BLOOD VELOCITY  $\sim M^0$

} INVARIANT !

RADIUS OF A WHALE'S AORTA  $\sim 30$  cm

RADIUS OF A SHREW'S AORTA  $\sim \frac{1}{10}$  mm



THIS DECREASE OF  $B_c$  WITH SIZE IS DRIVEN

BY THE HEGEMONY OF THE NETWORK

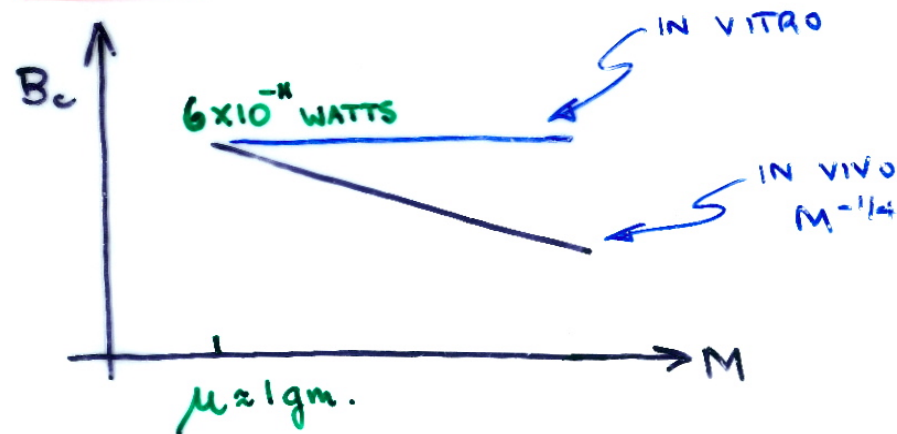
(CONTROLS FUNDAMENTAL BIOCHEMICAL RATES)

⇒ IF THE NETWORK WERE REMOVED SO CELLS

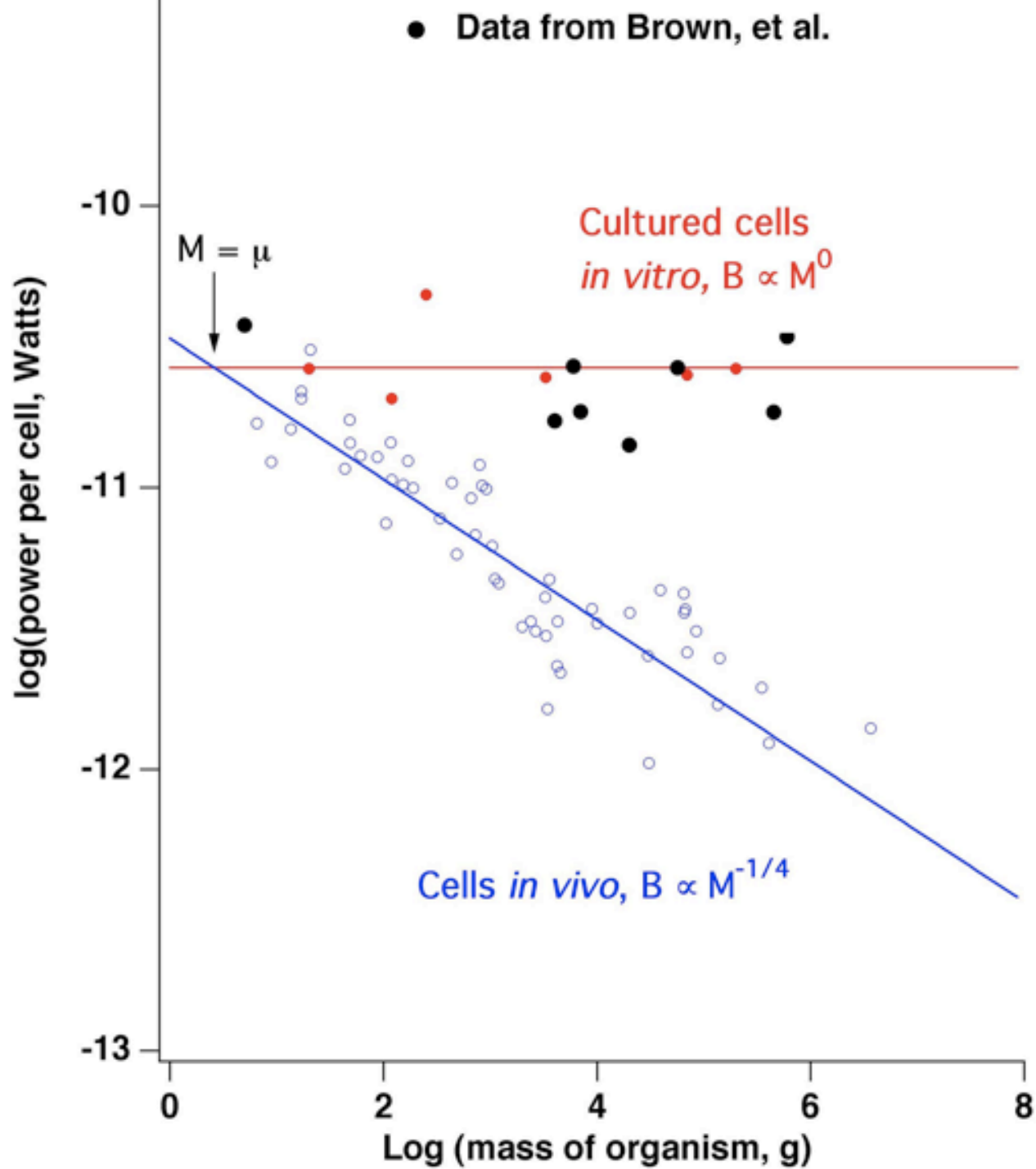
BECOME FREE (IN VITRO)  $B_c$  SHOULD BECOME

INDEPENDENT OF WHAT MAMMAL THEY ORIGINATED

IN: PREDICT



- **Data from Brown, et al.**

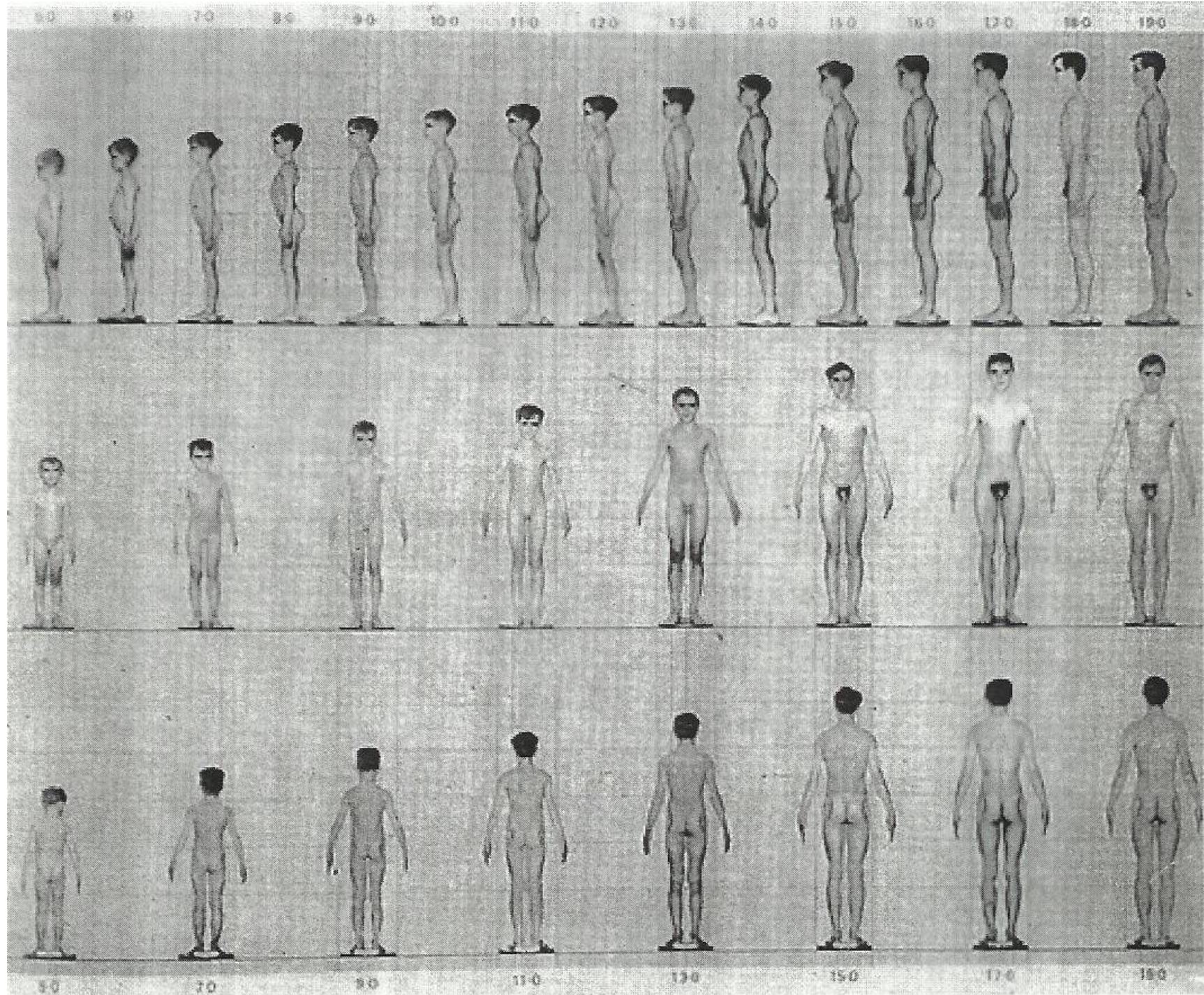


***NETWORK GEOMETRY AND DYNAMICS  
CONTROLS THE PACE OF LIFE AT ALL  
SCALES LEADING TO AN EMERGENT  
“UNIVERSAL” TIME SCALE***

$$B_{cell} \propto \frac{B}{M} = B_0 M^{-1/4}$$

***THE PACE OF LIFE SYSTEMATICALLY  
SLOWS WITH INCREASING SIZE***





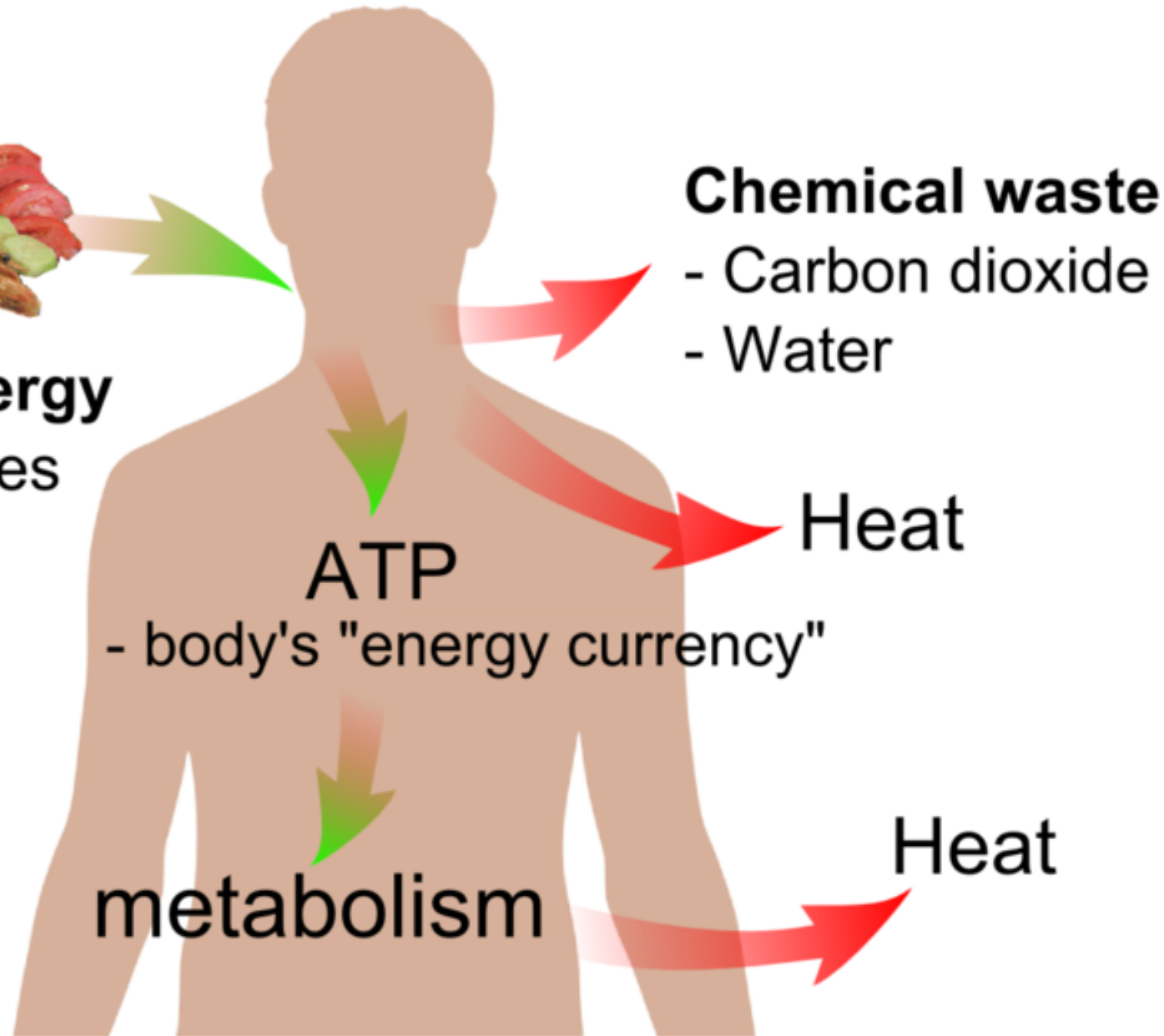


# Energy and human life



## **Chemical energy**

- Carbohydrates
- Fats
- Others



**INCOMING METABOLISED ENERGY**



***MAINTENANCE***  
***(of existing cells)***

**+**

***GROWTH***  
***(of new cells)***

$$B = N_{cells} B_{cell} + E_{cell} \frac{dN_{cell}}{dt}$$

**IN TERMS OF MASS AT AGE  $t$**

$$\Rightarrow \frac{dm}{dt} = am^{3/4} - bm$$

where

$$a \equiv \frac{B_0 m_c}{E_c}$$

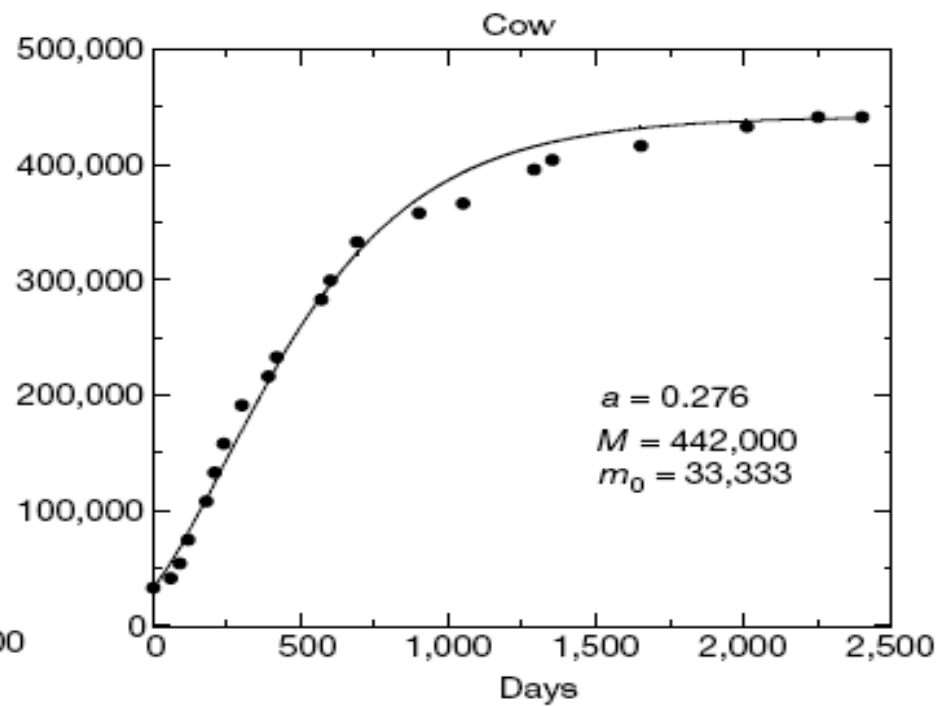
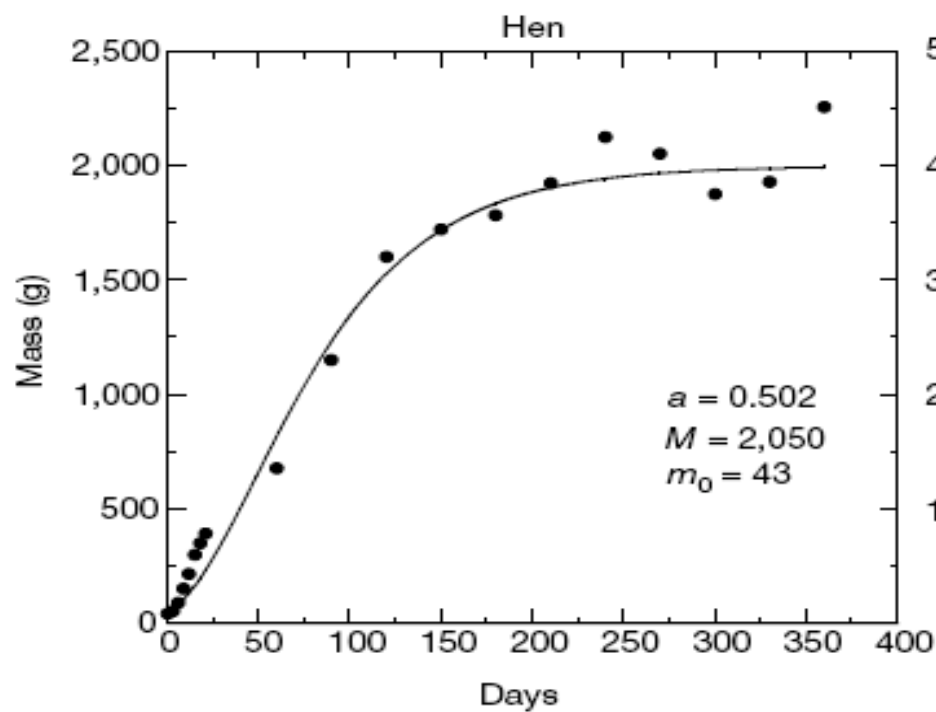
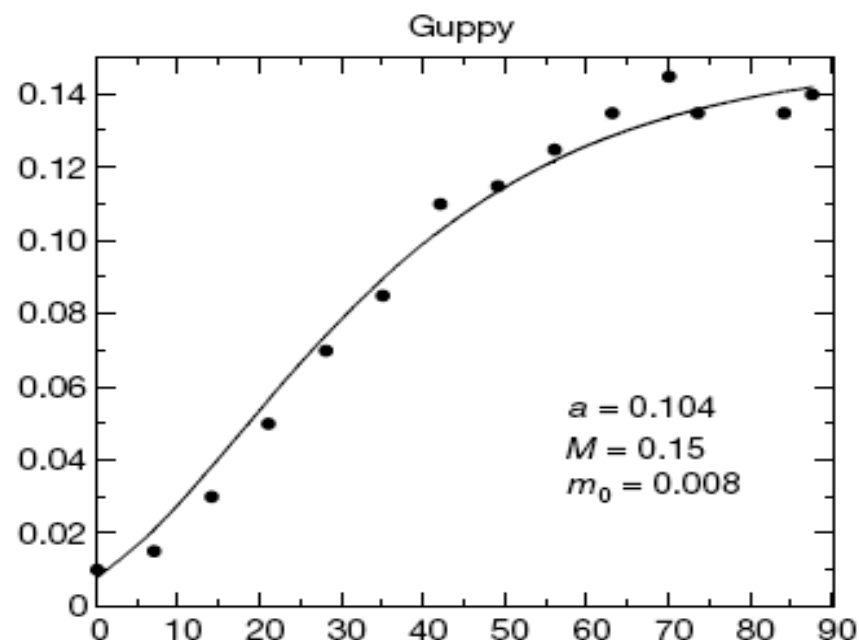
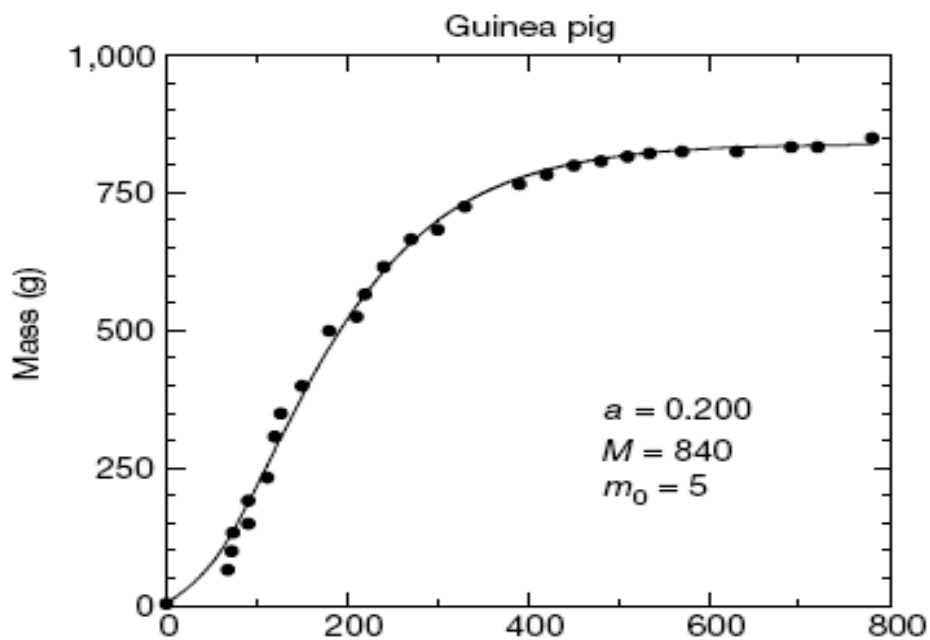
$$b \equiv \frac{B_c}{E_c}$$

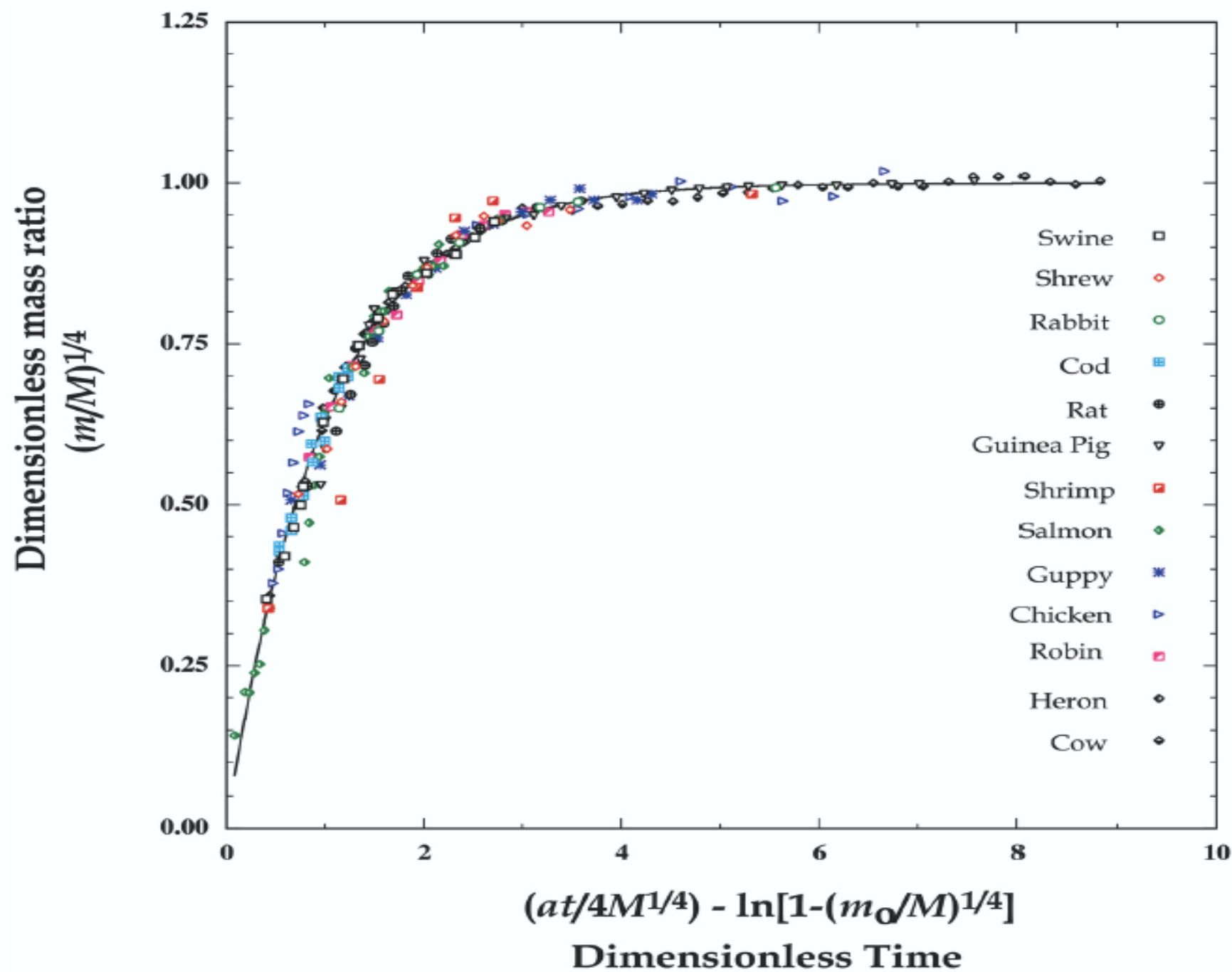
SOLUTION:

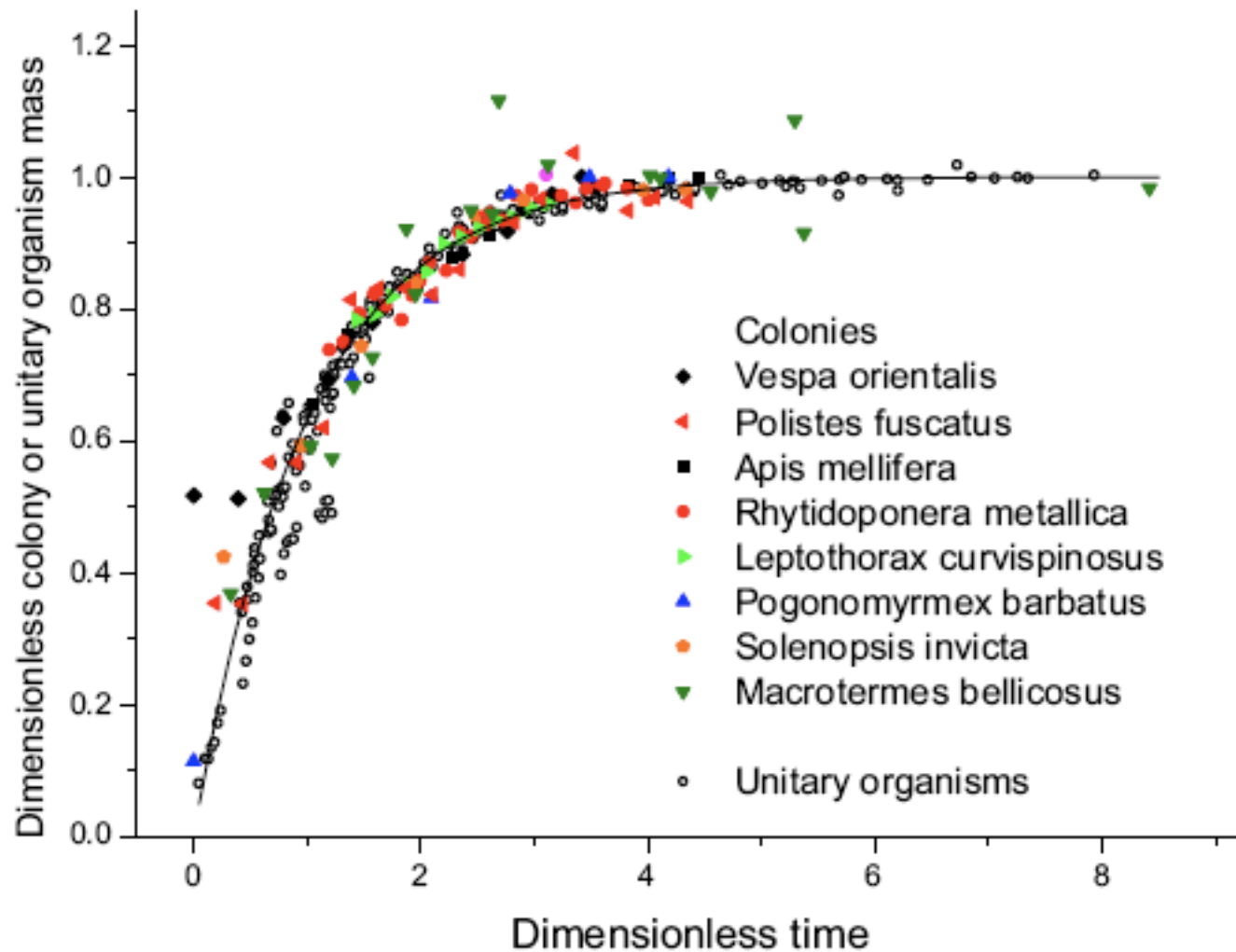
$$\left(\frac{m}{M}\right)^{1/4} = 1 - \left[1 - \left(\frac{M_0}{M}\right)^{1/4}\right] e^{-at/4M^{1/4}}$$

WHERE  $M_0$  = MASS AT BIRTH ( $m = M_0$  WHEN  $t = 0$ )









C. Hou, M.Kaspari, M.H. vander Zanden, J.F. Giloolly, PNAS (2010)



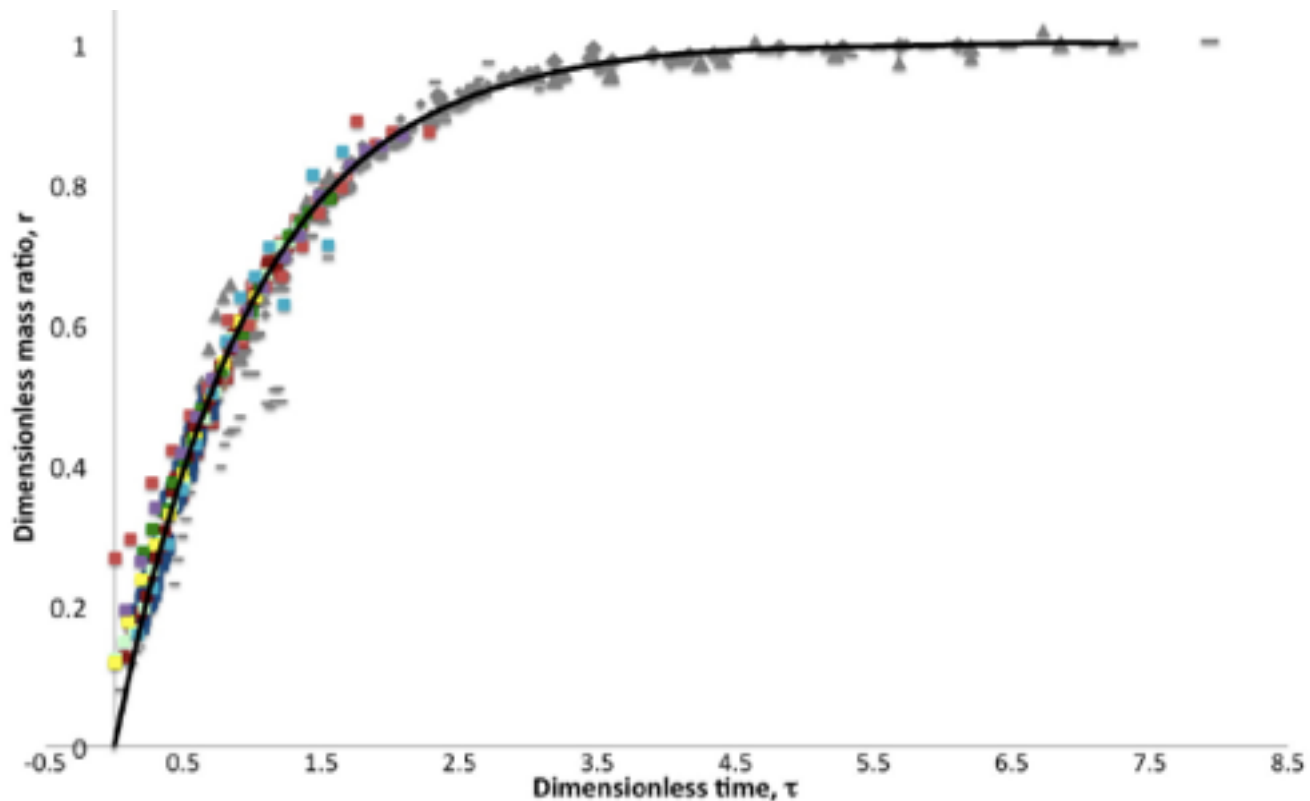
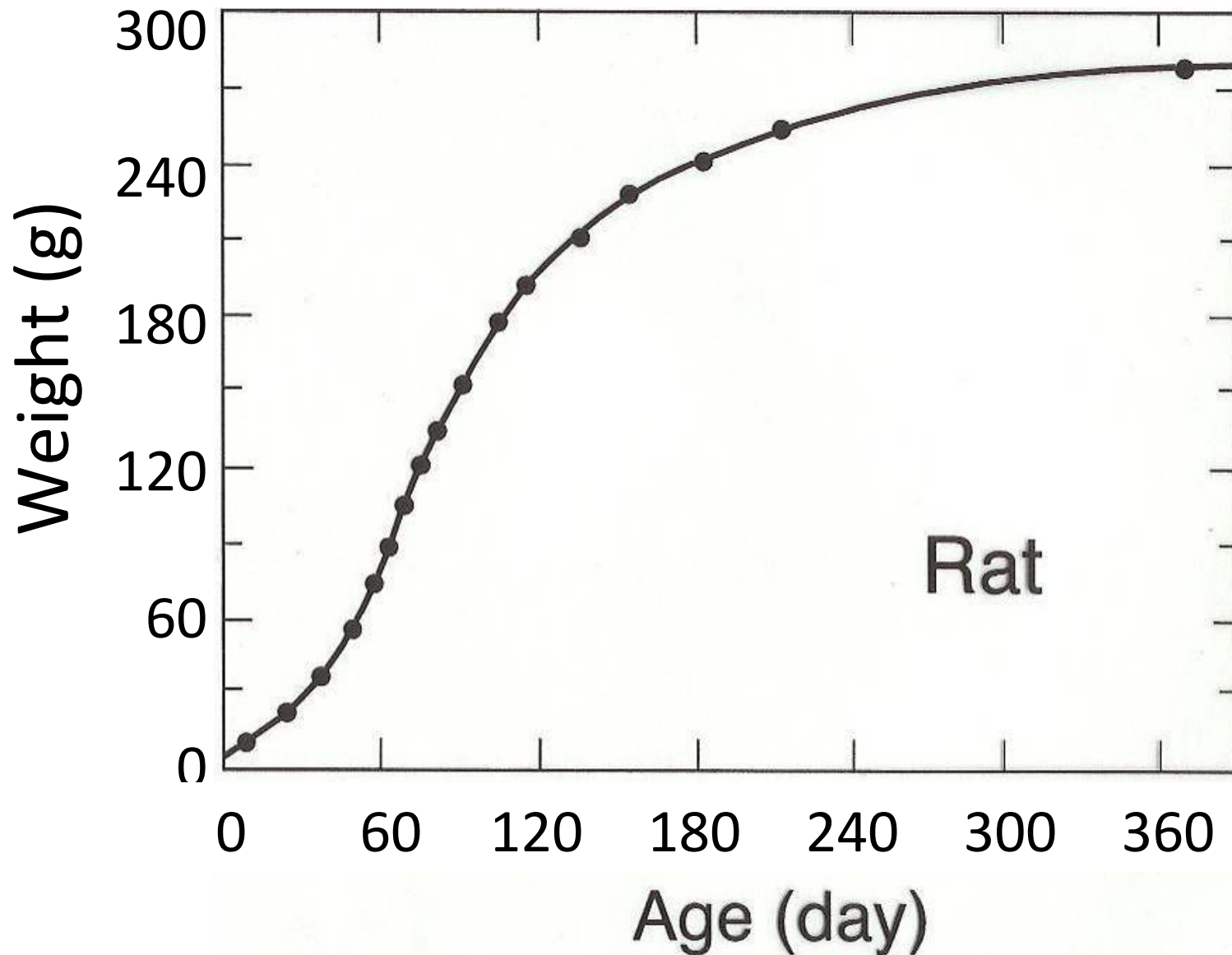


FIGURE 5. Plots of dimensionless ratio versus dimensionless time as defined by Eqs. (29)-(30). Data in grey are for ontogenetic growth from 13 species of animals (see [90] for original data sources), ranging from guppy to cod to guinea pig, and data in color with square symbols are for tumor growth trajectories for C3H mammary carcinoma (dark blue), EMT6 mammary carcinoma (dark red), KHJJ mammary carcinoma (light green), NCTC (dark green), Flank (yellow), Primary fibroadenoma (red), Primary Osteosarcoma (light blue), and Walker Carcinoma (purple) tumor



**SUB-LINEAR SCALING (SLOPE < 1)  
LEADS TO BOUNDED GROWTH**

# GODZILLA

**MAY 16**

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**LENGTH**      350 ft

**WEIGHT**       $1.7 \times 10^7$  Kg =  $1.7 \times 10^4$  tons

**BASAL METABOLIC RATE**     $2 \times 10^7$  calories a day = 1 megawatt

**WEIGHT OF HEART**               $10^5$  Kg = 100 tons

**RADIUS OF HEART**              30 ft

**HEART RATE**              2.5 times a minute

**VOLUME OF BLOOD**               $2 \times 10^6$  litres

**DIAMETER OF AORTA**              10 ft

**SLEEP**    < 1 hour a day

**LIFESPAN**    2000 years



**JAMES BROWN (UNM/SFI)**  
**BRIAN ENQUIST (U. ARIZONA)**  
**WOODY WOODRUFF (LANL)**  
**VAN SAVAGE (HARVARD)**  
**JAMIE GILOOLLY (U. FLORIDA)**  
**DREW ALLEN (UCSB)**  
**MICHELLE GIRVAN (U. MARYLAND)**  
**ALEX HERMAN (UCSF)**  
**CHRIS KEMPES (MIT)**

# GENERALISED SCALING

**i) SUPPOSE THE POPULATION SIZE CHANGES BY A FACTOR  $\lambda$ :**

$$N \rightarrow \lambda N$$

**ii) THIS INDUCES A CHANGE IN SOME METRIC FROM  $Y(N)$  TO  $Y(\lambda N)$ :**

$$Y(N) \rightarrow Y(\lambda N) = Z(\lambda, N)Y(N)$$

# GENERALISED SCALING

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$$Y(N) \rightarrow Y(\lambda N) = Z(\lambda, N)Y(N)$$

**RENORMALISATION GROUP**

*M. Gell-Mann & F. E. Low (1954) Physical Review 95 (5): 1300–1312*

iii) FOR **ARBITRARY**  $Z(\lambda, N)$  THIS CAN BE SOLVED TO GIVE THE GENERAL SOLUTION:

$$Y(N) = Y_0 N^{b(N)}$$

WHERE THE GENERALISED EXPONENT,  $b(N)$ , DEPENDS ON  $N$  AND IS GIVEN BY:

$$b(N) = \frac{\int_0^{\ln N} \gamma(N) d \ln N}{\ln N}$$

WITH

$$\gamma(N) \equiv \frac{\partial Z(1, N)}{\partial \lambda}$$



$$Y(N) = Y_0 N^{b(N)}$$

iv) THE “NATURAL” VARIABLE IS  $\ln N$

v) WHEN DO WE GET SIMPLE POWER LAWS  
WITH EXPONENTS  $b(N)$  INDEPENDENT OF  $N$ ?

ANSWER: WHEN  $\gamma(N)$  IS INDEPENDENT OF  $N$

→ WHEN  $Z(\lambda, N)$  IS INDEPENDENT OF  $N$ :

$$Y(\lambda N) = Z(\lambda) Y(N)$$

**SELF-SIMILAR (FRACTALITY)**

# GENERALISE TO “DYNAMICAL” REPRESENTATION

$$Y(N) \rightarrow Y[N, g(N)]$$

$g(N)$  “STRENGTH OF INTERACTION”  
THEN RG SOLUTION IS

$$Y(N) \equiv Y[N, g(N)] = Y(N_0) e^{\int_{\beta(g)}^g \frac{\gamma(g)}{\beta(g)} dg} F\left[N e^{\int_{\beta(g)}^g \frac{dg}{\beta(g)}}\right]$$

WHERE

$$\beta(g) \equiv \frac{\partial g(N)}{\partial N}$$

(FIXED POINTS)

**LUIS BETTENCOURT (SFI - PHYSICS)**  
**JOSE LOBO (ASU - URBAN ECONOMICS)**  
**DEBORAH STRUMSKY (UNC - ECONOMICS)**  
**HYEJIN YOUN (OXFORD - PHYSICS)**  
**MARCUS HAMILTON (SFI/UNM - ANTHROPOLOGY)**  
**NATHANIEL RODRIGUEZ (SFI – COMPUTER SCIENCE)**  
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***ROCKEFELLER FOUNDATION***

***McDONNELL FOUNDATION***

***TEMPLETON FOUNDATION***

# ***WE LIVE IN AN EXPONENTIALLY EXPANDING SOCIO-ECONOMIC UNIVERSE!!***

***1800 < 4% OF THE US POPULATION WAS URBAN***

***2011 > 80%***

***2006 > 50% WORLD'S POPULATION URBANISED***

***2050 > 75%***

***EVERY WEEK FROM NOW TILL 2050  
OVER ONE MILLION PEOPLE ARE BEING  
ADDED TO OUR CITIES***

# ***URGENTLY NEED A QUANTITATIVE, PREDICTIVE SCIENCE OF CITIES***

***RESILIENCE***

***EVOLVABILITY***

***GROWTH***

***SCALABILITY***

# ***NEED A SCIENCE OF CITIES***

**COMPLEMENT TO TRADITIONAL  
(QUALITATIVE) THEORIES AND MODELS**

***WHAT CAN WE LEARN FROM BIOLOGY AND  
PHYSICS?***

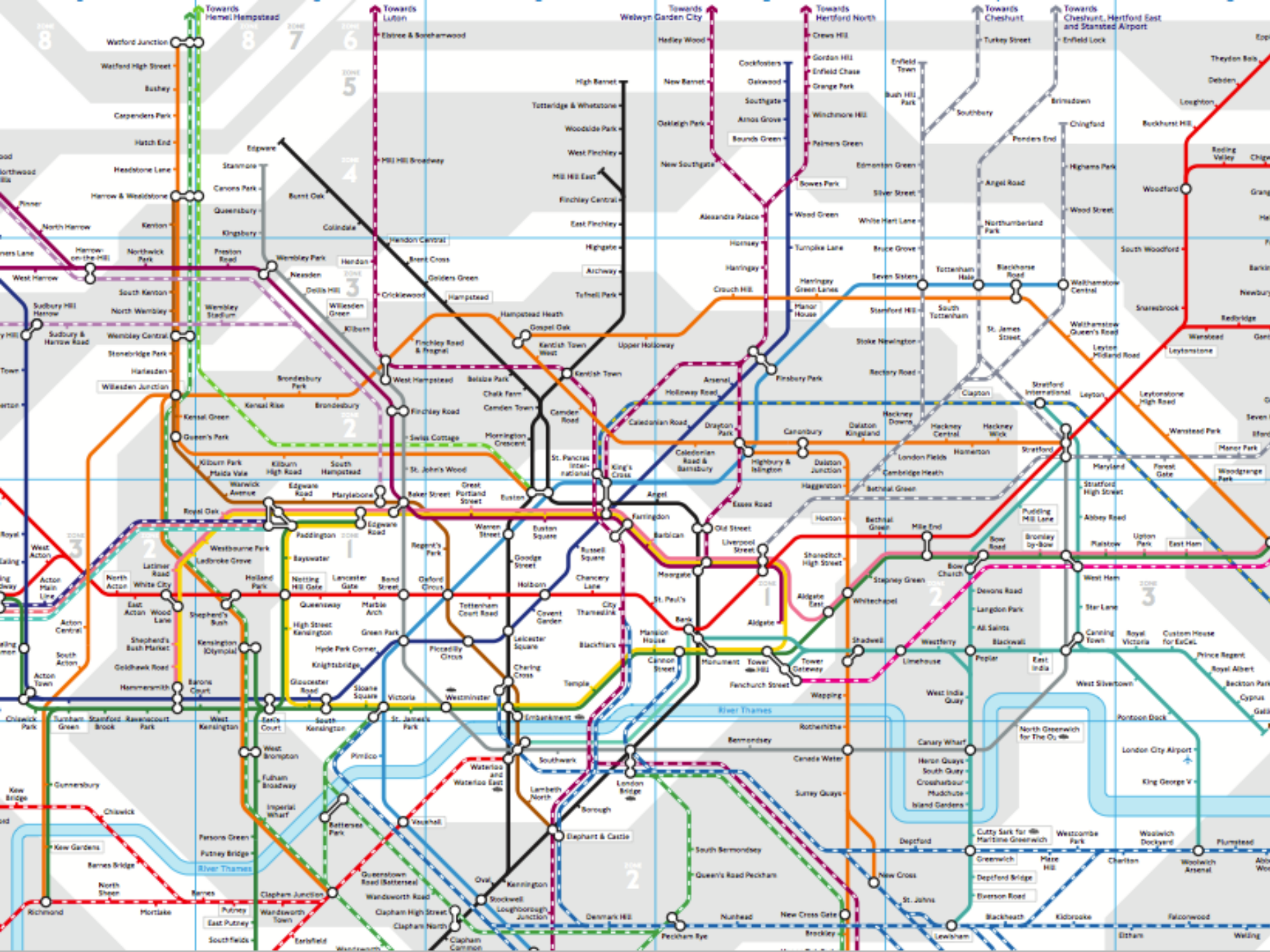


**ARE CITIES (AND COMPANIES)  
SCALED VERSIONS OF EACH  
OTHER?**

**DO THEY MANIFEST  
“UNIVERSALITY”?**







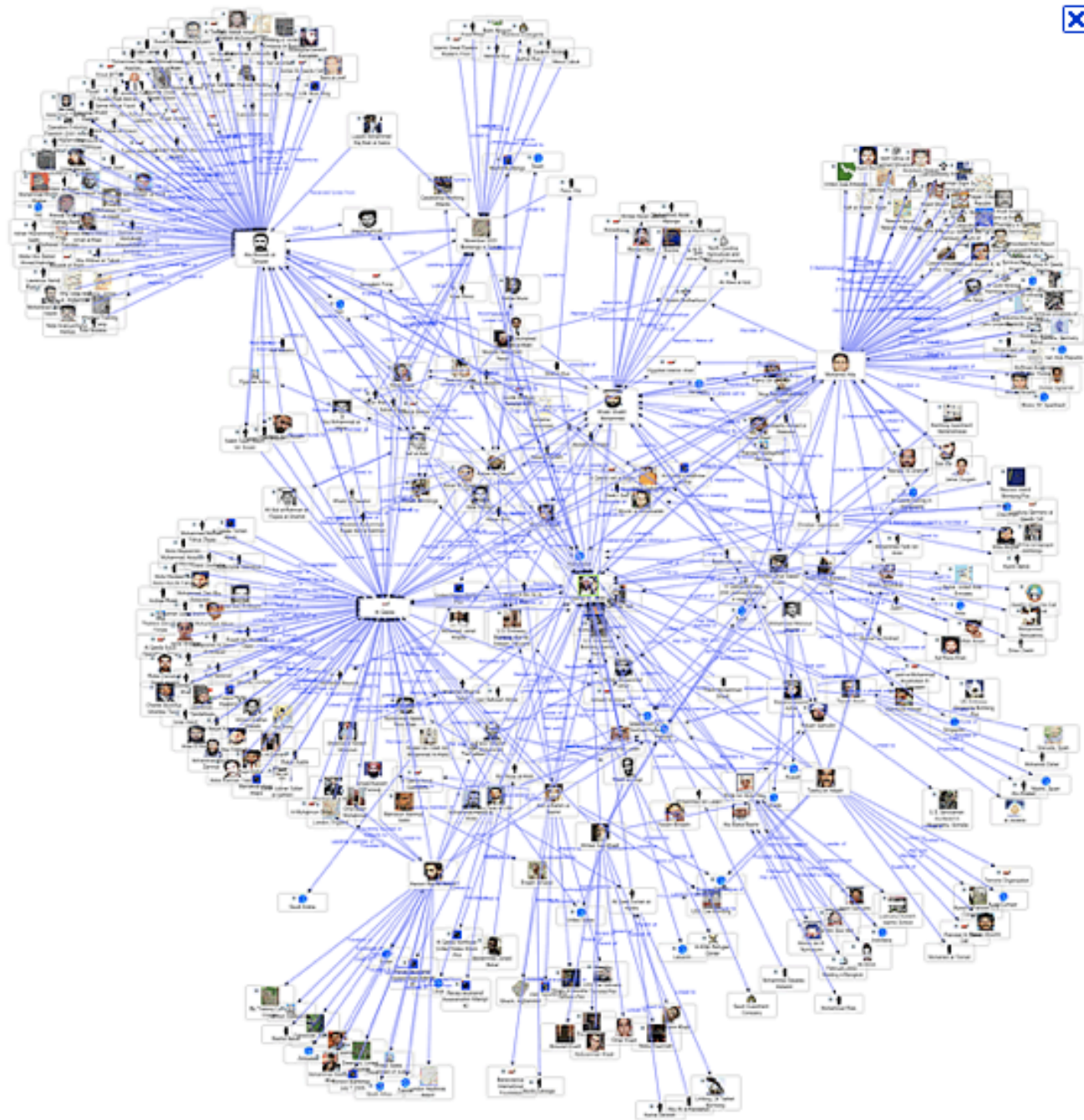




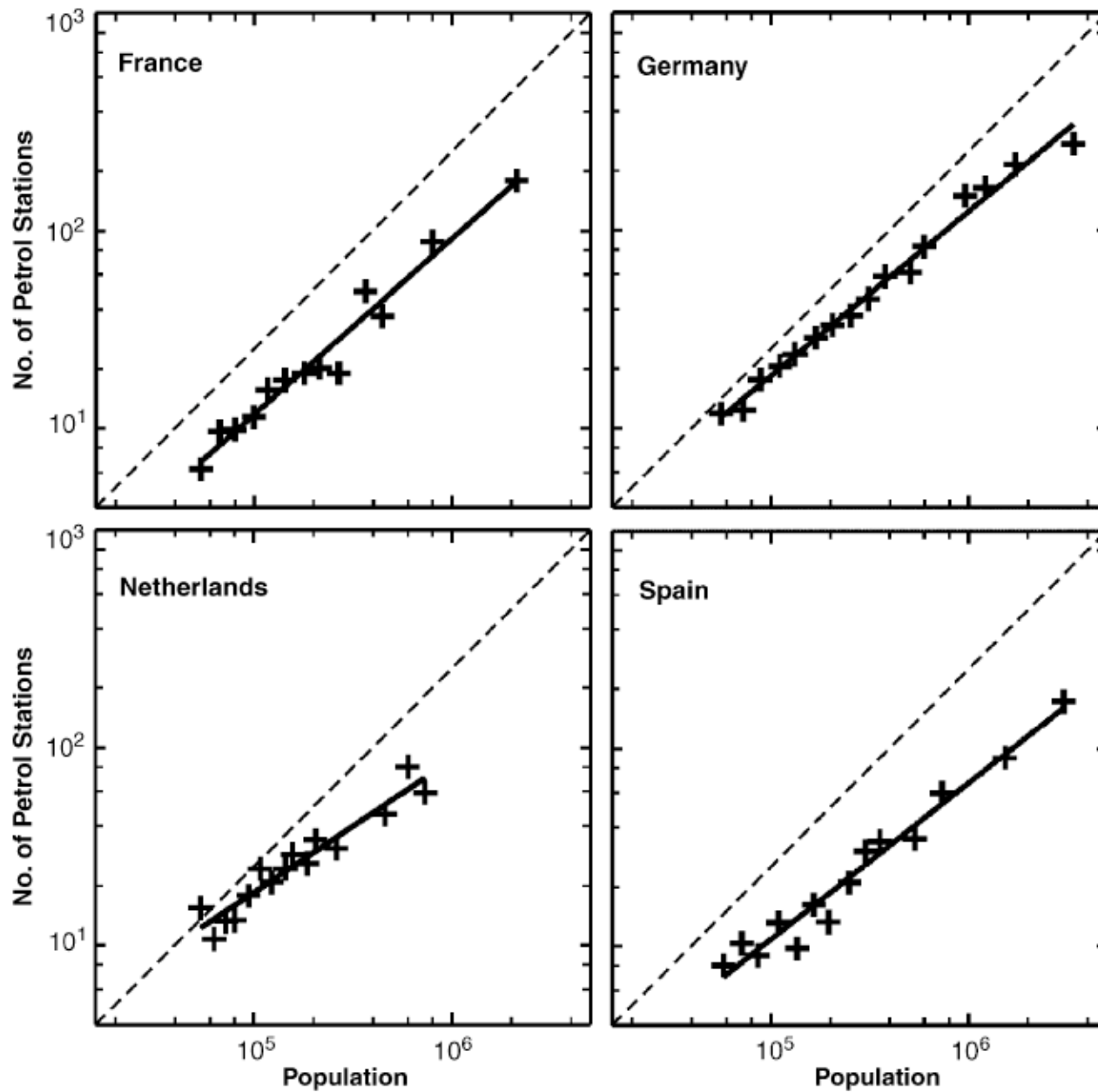
**“What is the city but the  
people?”**

*William Shakespeare*

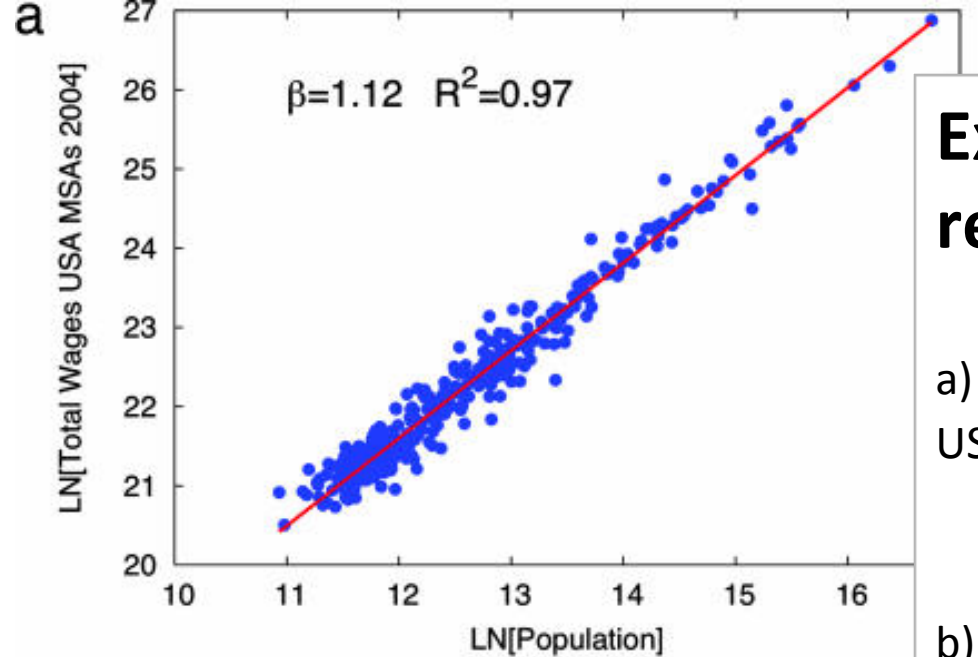








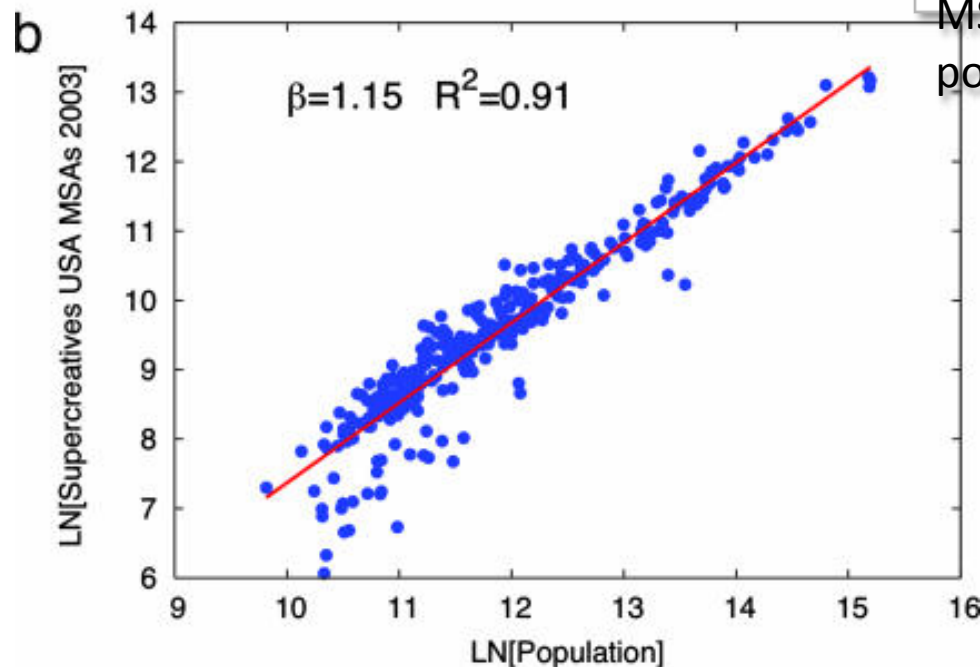




## Example of scaling relationships

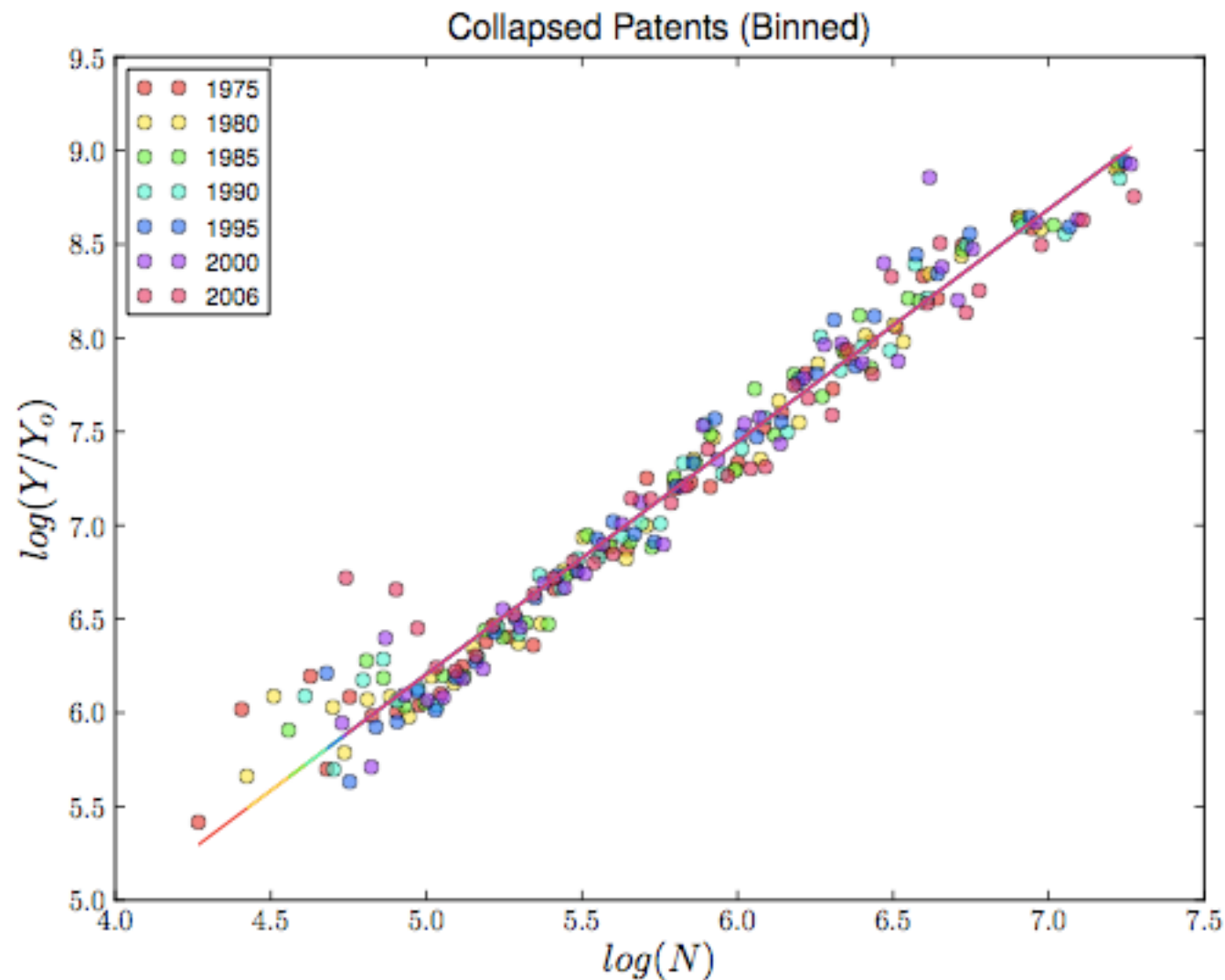
a) Total **WAGES** per MSA in 2004 for the USA vs. metropolitan population.

b) **SUPERCREATIVE** employment per MSA in 2003, for the USA vs. metropolitan population.

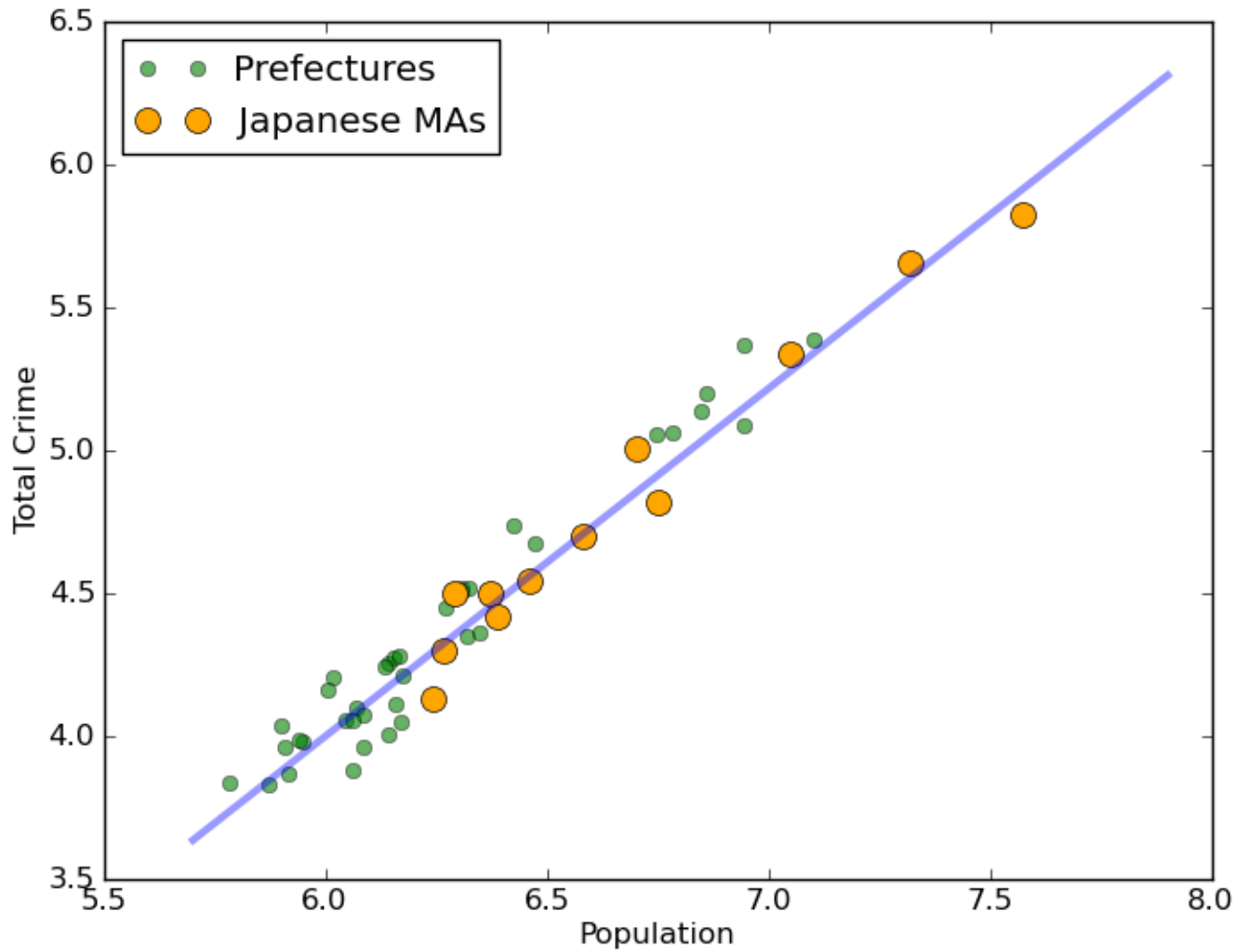


***SUPER-LINEAR  
SCALING***

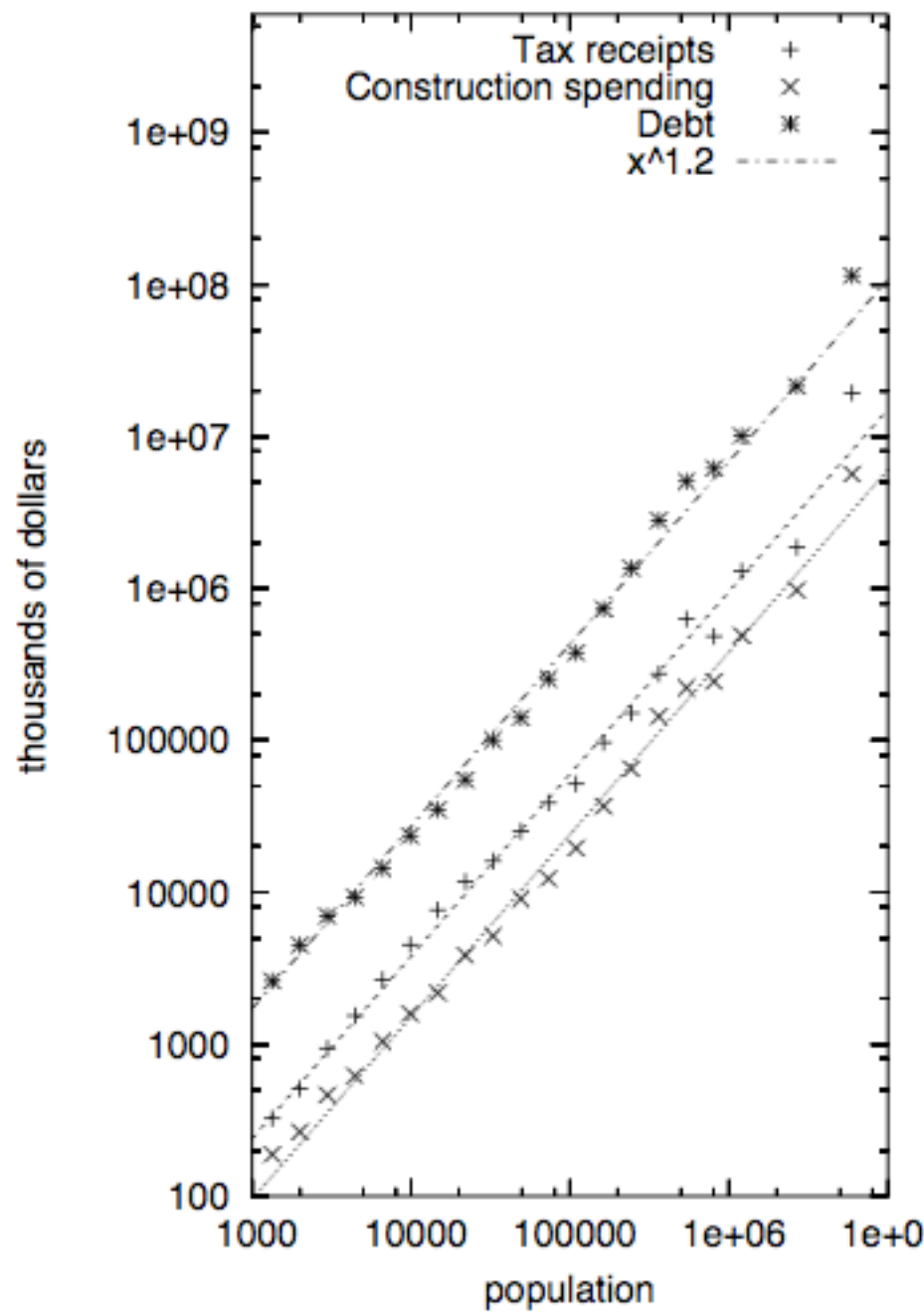
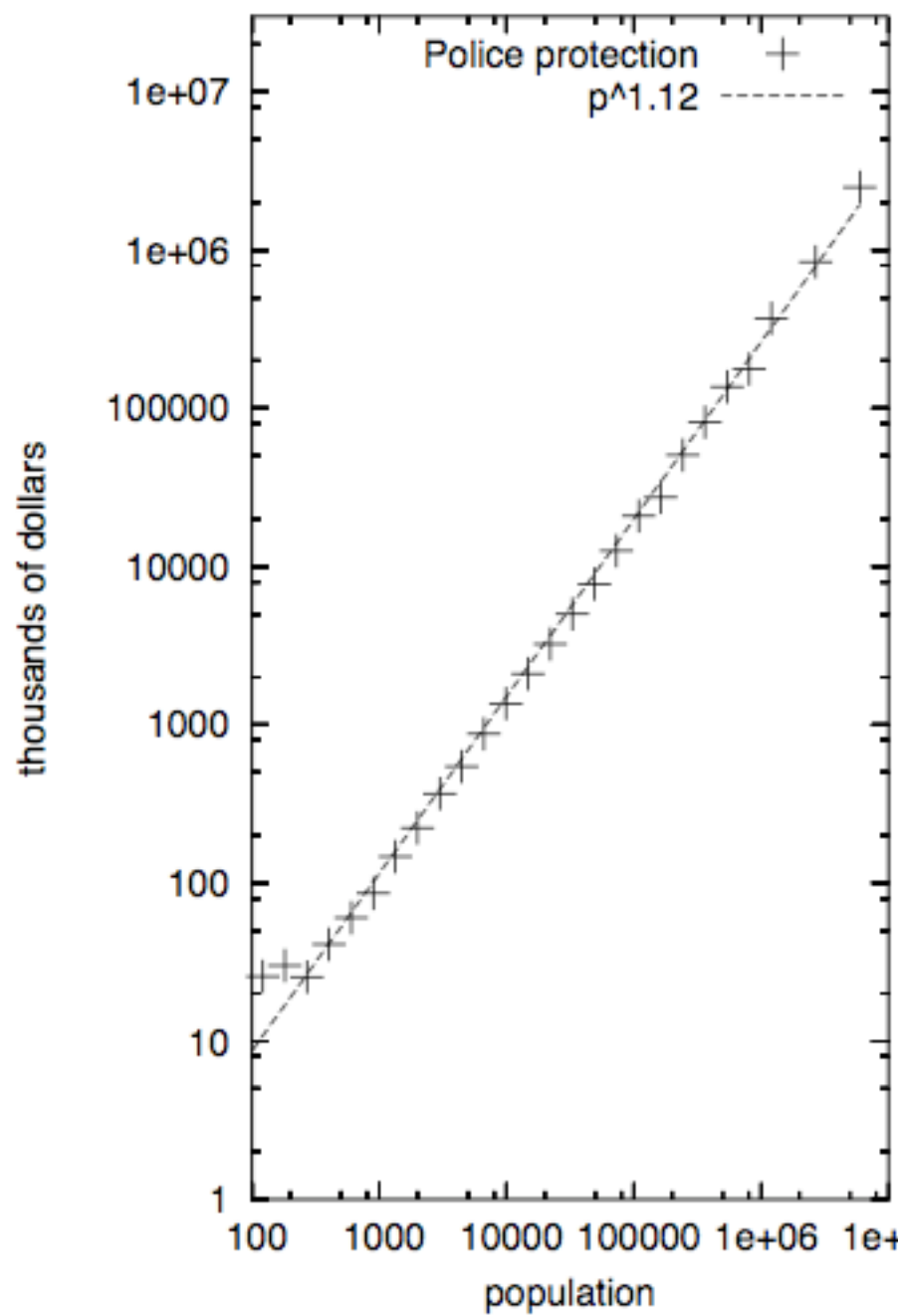
# Innovation measured by Patents



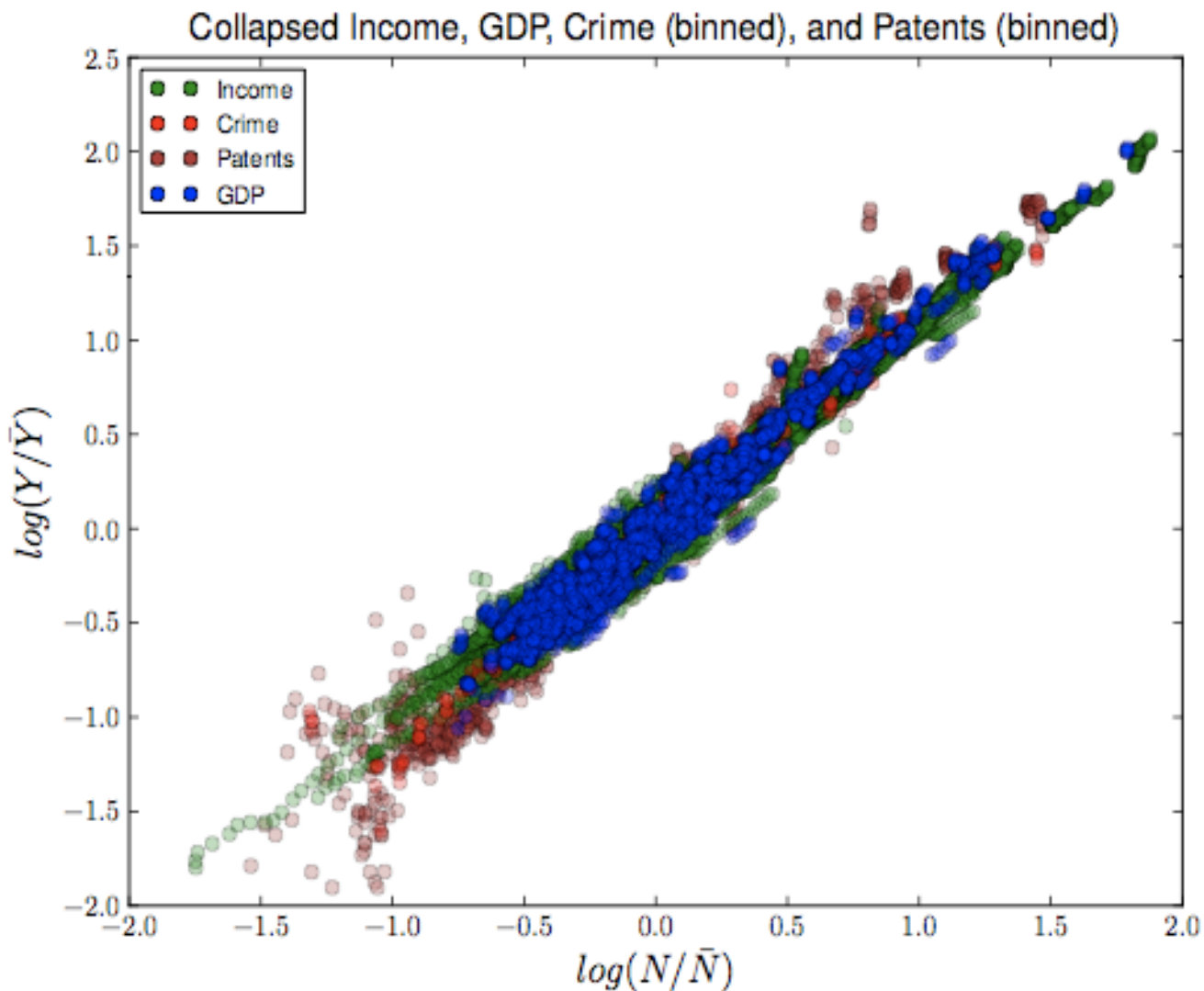
# TOTAL CRIME (JAPAN)



Slope = 1.21      [1.08, 1.35]







***UNIVERSALITY***

# THE GOOD, THE BAD & THE UGLY

**DOUBLING THE SIZE OF A CITY  
ON AVERAGE SYSTEMATICALLY  
INCREASES**

**INCOME, WEALTH, PATENTS,  
COLLEGES, CREATIVE PEOPLE,  
POLICE, AIDS & FLU, CRIME, SOCIAL  
INTERACTIONS,.....**

**ALL BY APPROXIMATELY 15%  
REGARDLESS OF CITY**

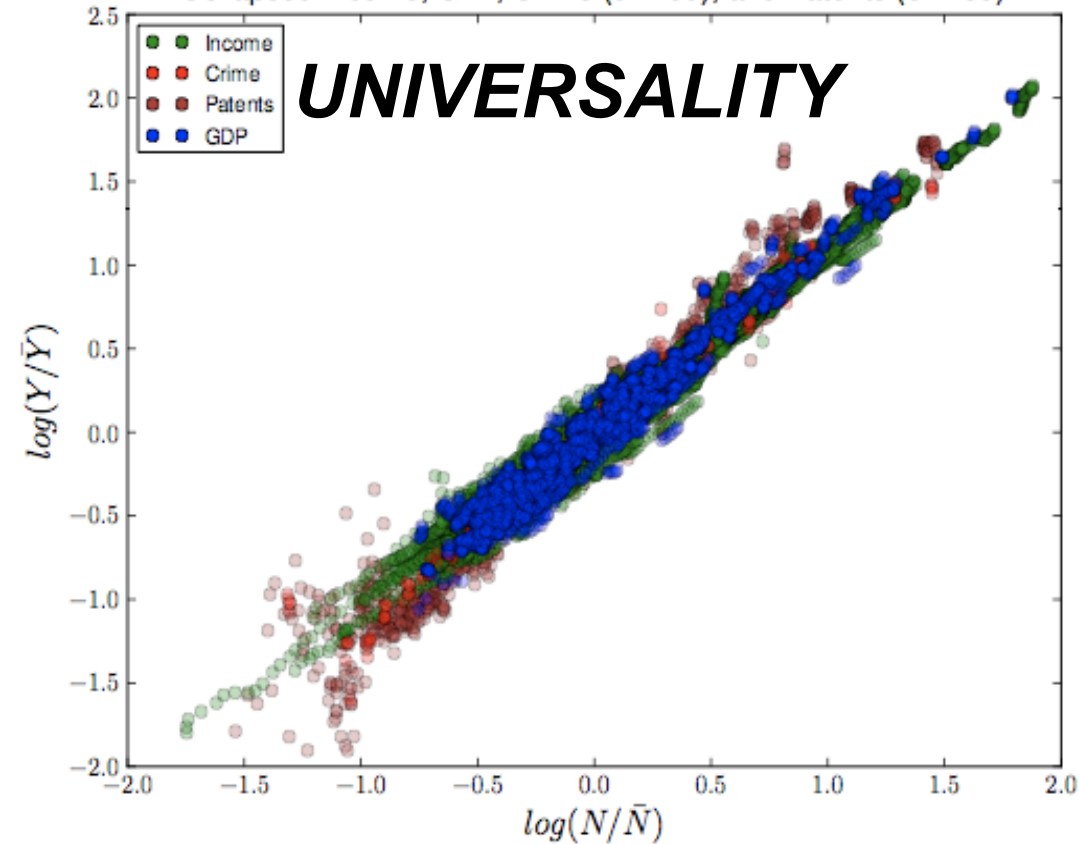
***AND.....***

***SAVES APPROXIMATELY 15%  
ON ALL INFRASTRUCTURE  
(ROADS, ELECTRICAL LINES,  
GAS STATIONS,.....)***

# ***UNIVERSALITY OF SOCIAL NETWORKS (CLUSTERING HIERARCHIES)***

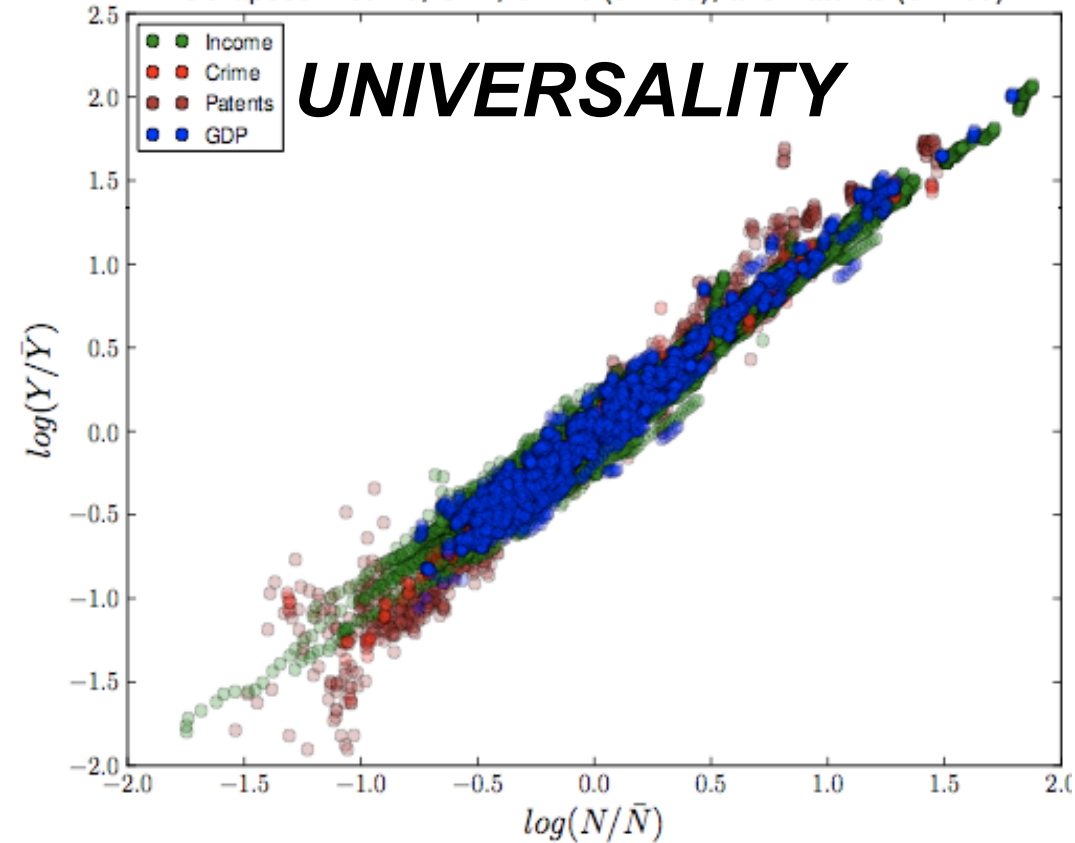


Collapsed Income, GDP, Crime (binned), and Patents (binned)

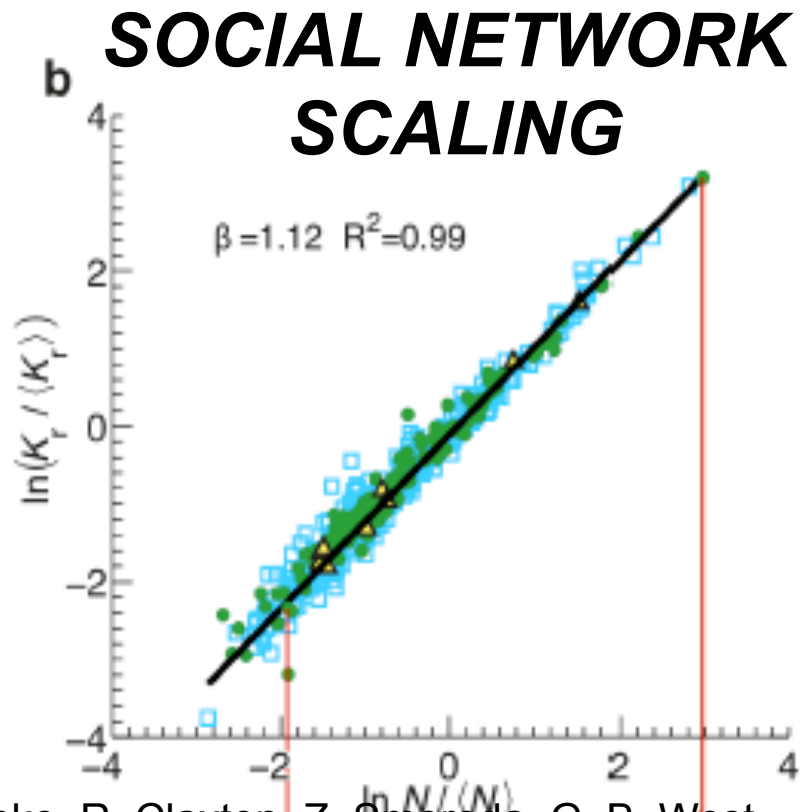


**SUPER-LINEAR  
SCALING  
SLOPE  $\sim 1.15 > 1$**

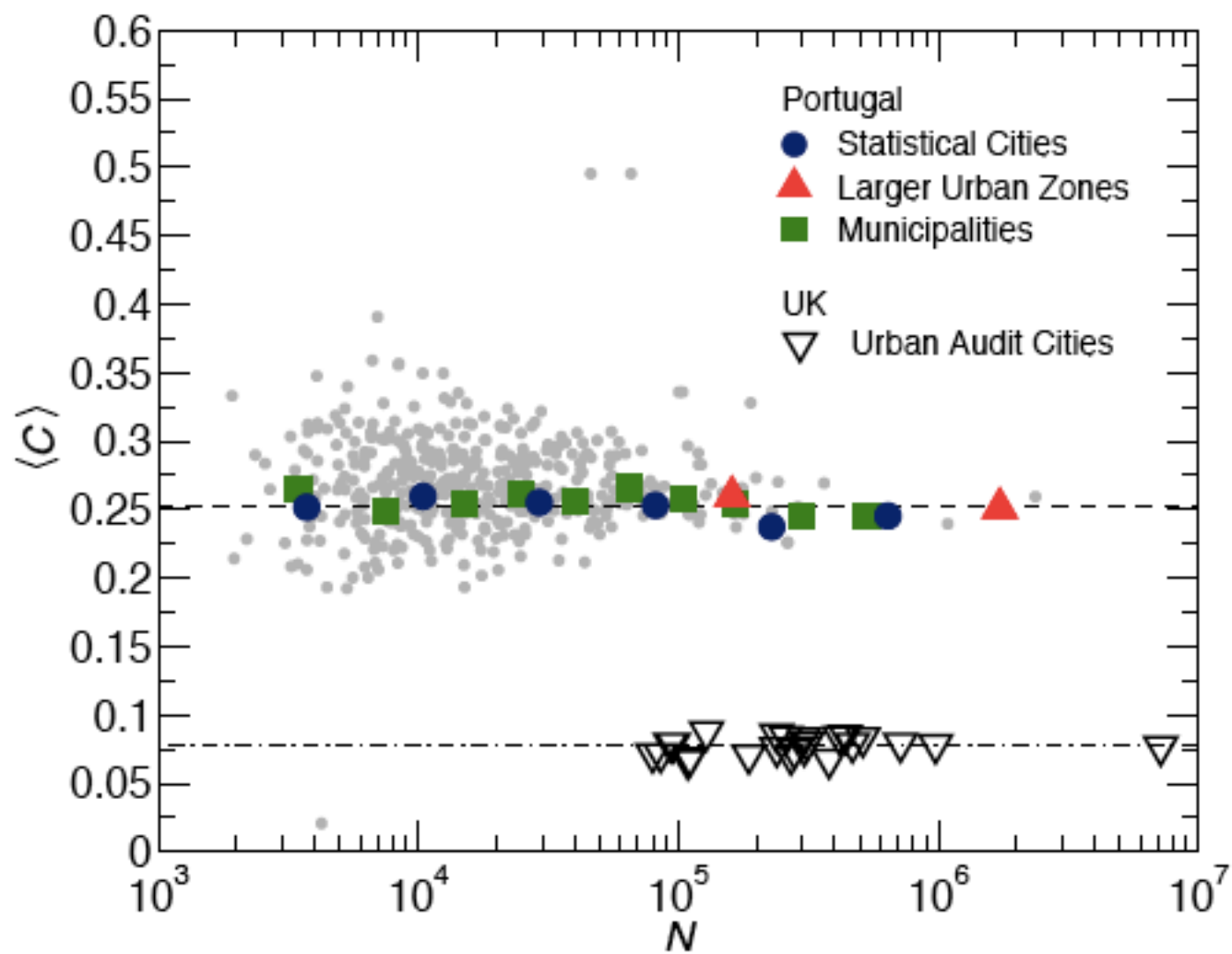
Collapsed Income, GDP, Crime (binned), and Patents (binned)



**SUPER-LINEAR  
SCALING**  
SLOPE  $\sim 1.15 > 1$



M. Schläpfer, L. M. A. Bettencourt, S. Grauwin, M. Raschke, R. Claxton, Z. Smoreda, G. B. West and C. Ratti. P. Roy. Soc (tbp)



***FINANCIAL MARKETS,  
ECONOMIES,  
GLOBAL WARMING,  
ENVIRONMENT,  
URBANISATION,  
HEALTH,  
CRIME,  
POLLUTION,.....***

***ARE NOT INDEPENDENT***

***THEY ARE ALL HIGHLY COUPLED, INTER-  
RELATED COMPLEX ADAPTIVE SYSTEMS***



# ***NETWORK DYNAMICS DETERMINES THE PACE OF LIFE***

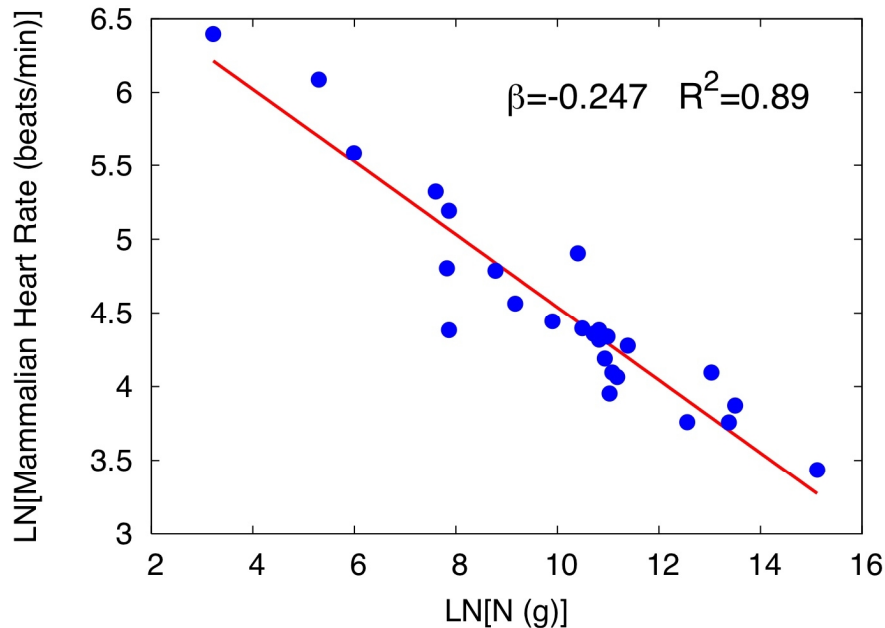
***IF THE SLOPE IS  $< 1$***

***PACE OF LIFE  
SLOWS DOWN***

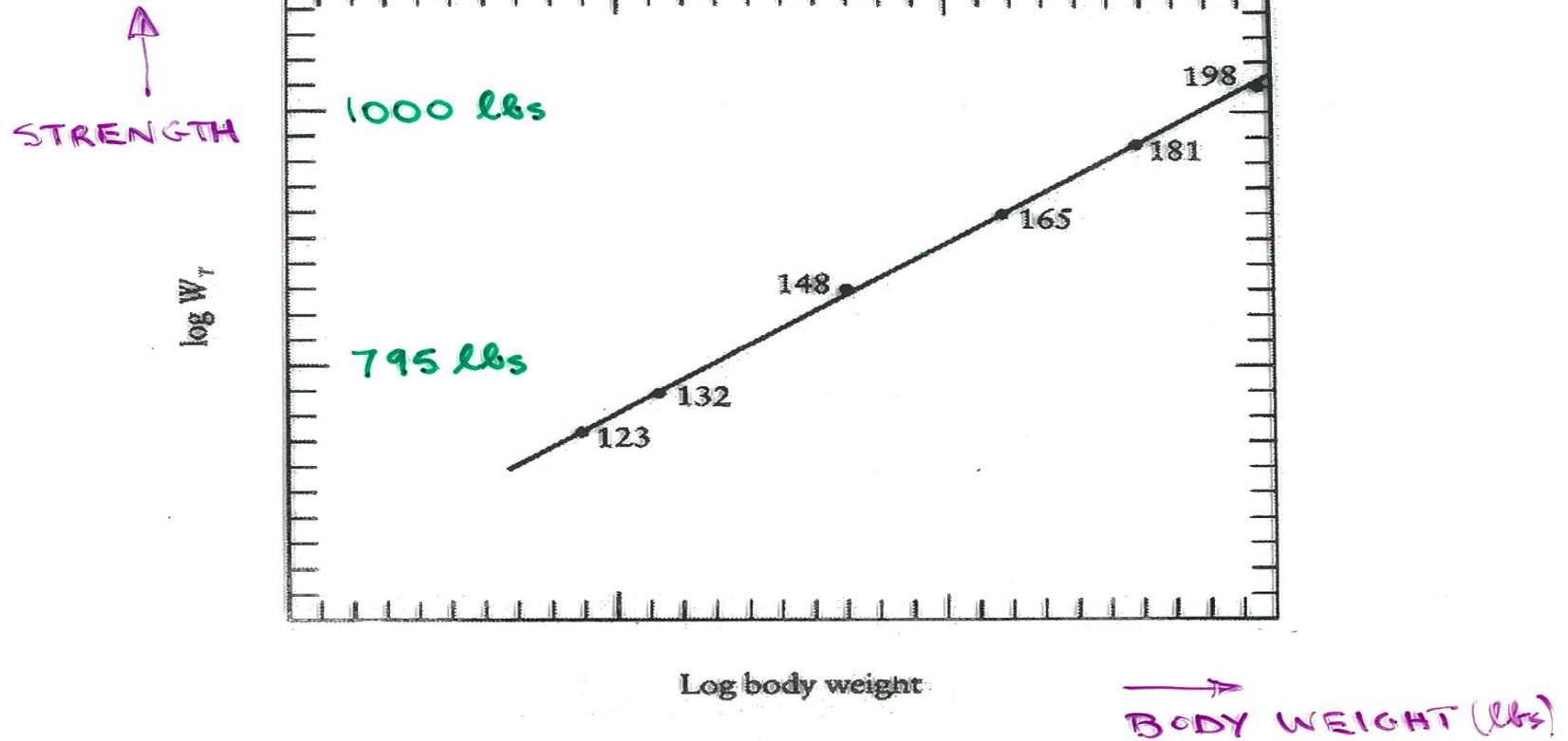
***IF THE SLOPE IS  $> 1$***

***PACE OF LIFE  
SPEEDS UP***

# Pace of biological life vs. Pace of social life



# WEIGHT-LIFTING ("PUMPING IRON") (PLOTTED LOGARITHMICALLY)



WHO IS THE STRONGEST AND WHO IS THE  
WEAKEST?

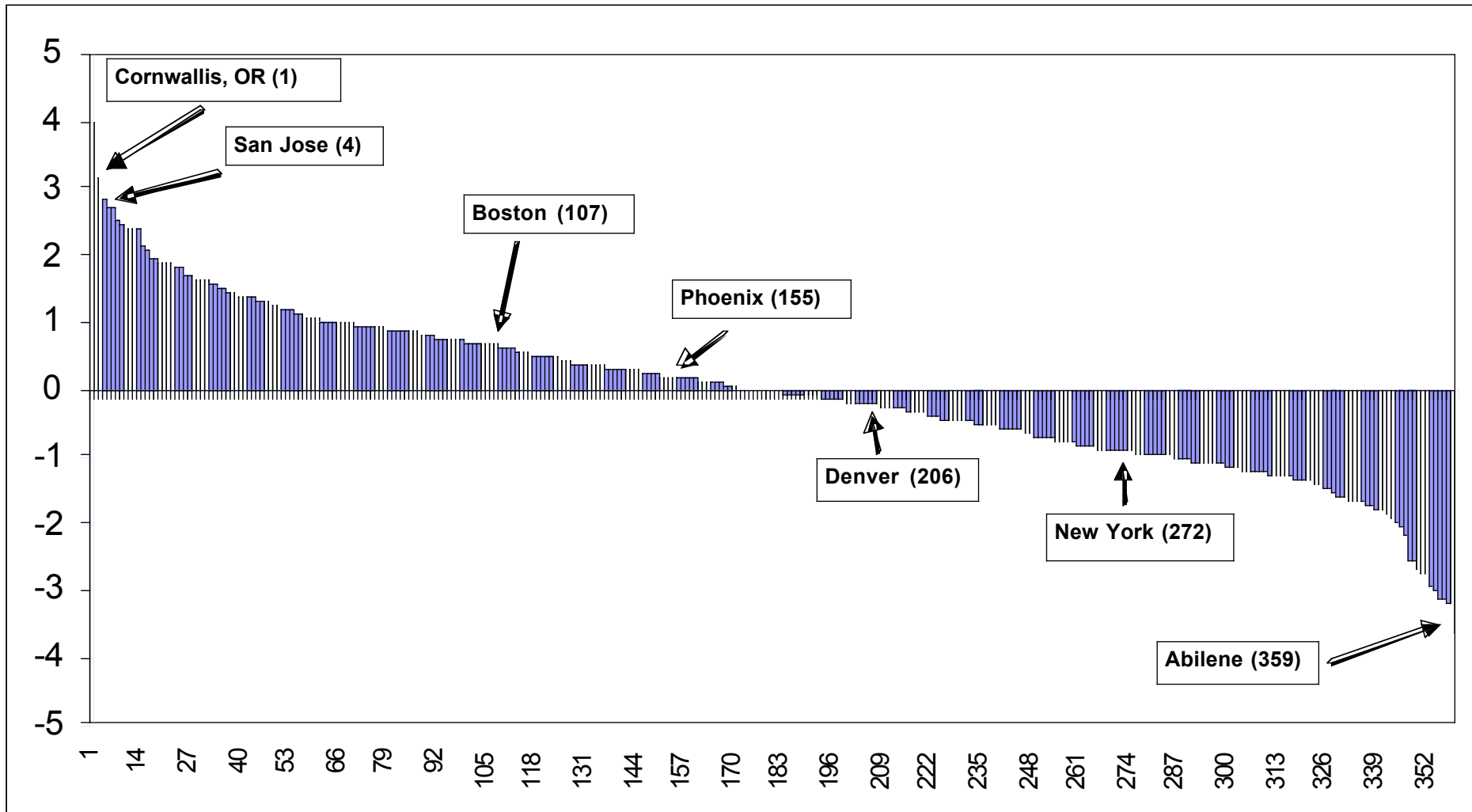
**Average “idealised, universal”  
characteristics of cities and companies of a  
given size (constrained by underlying  
principles and dynamics of network  
structures) as manifested in scaling laws**

**vs.**

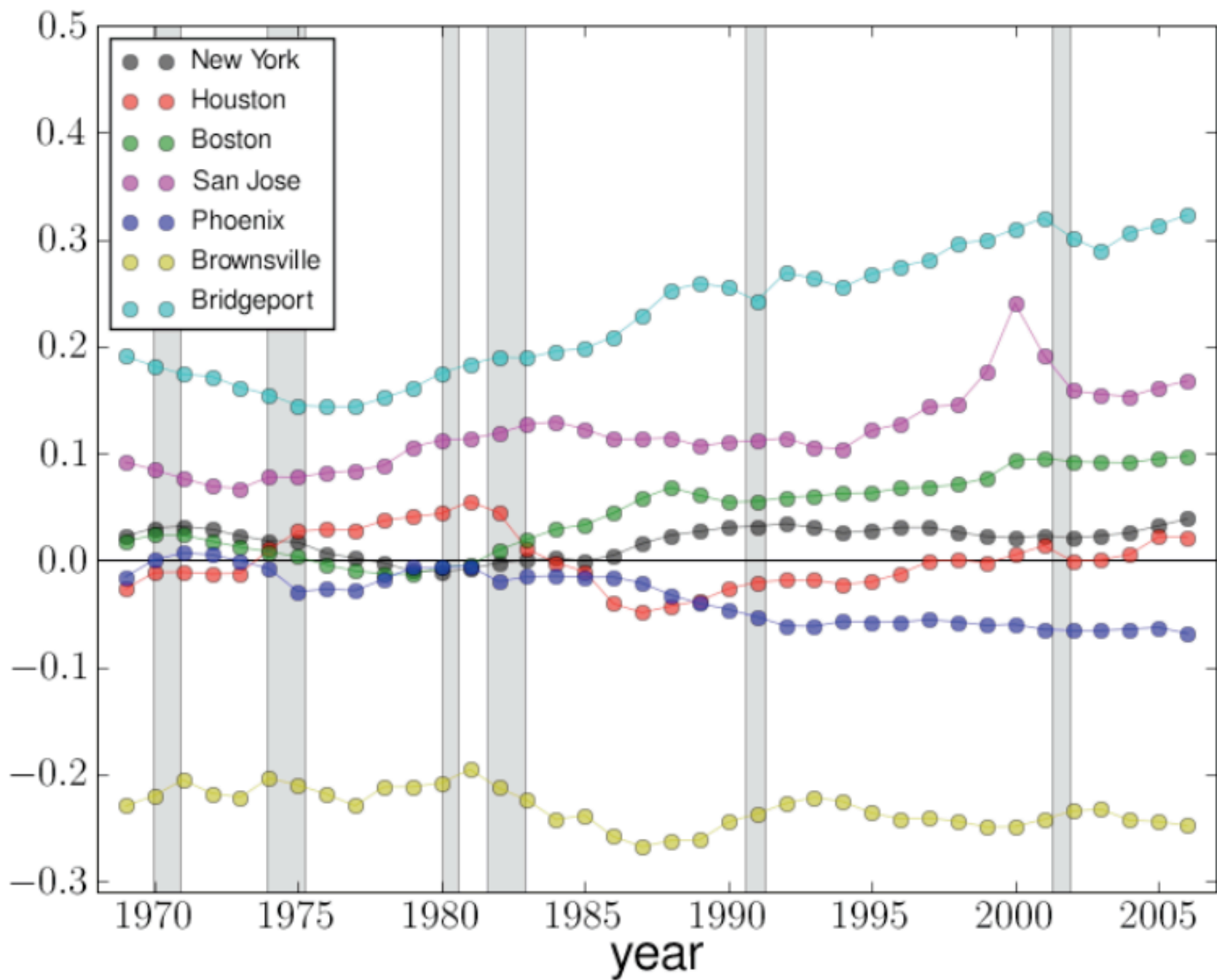
**Characteristics of specific cities and  
companies as measured by their deviations  
from scaling laws representing their  
individuality and local environment and  
conditions**



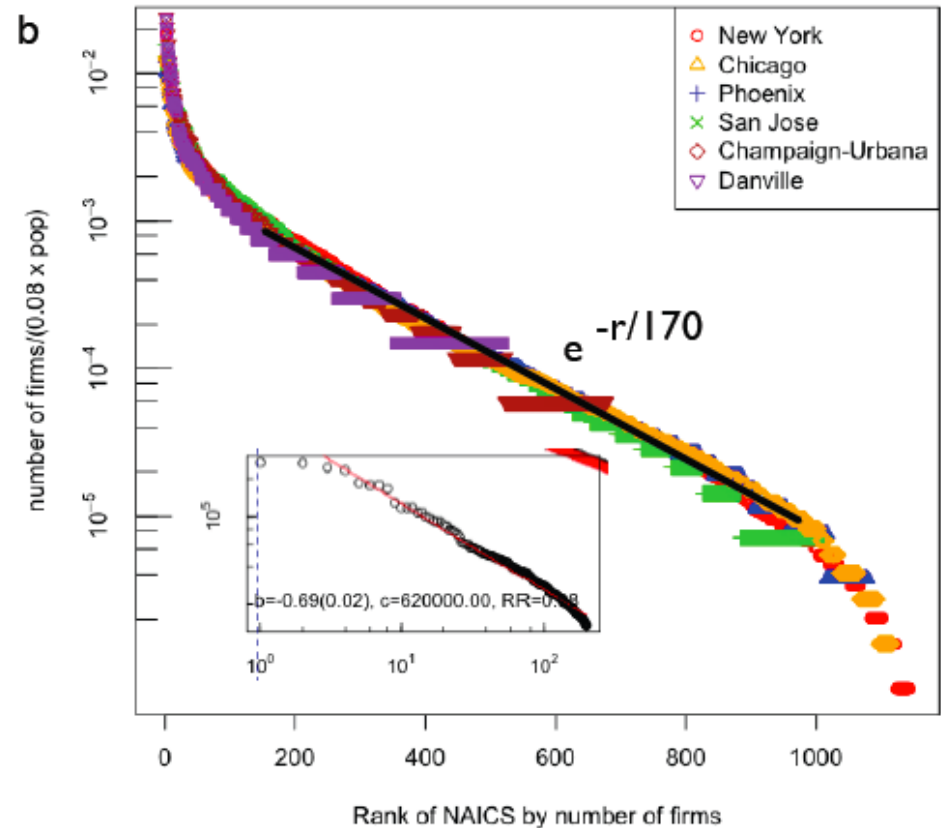
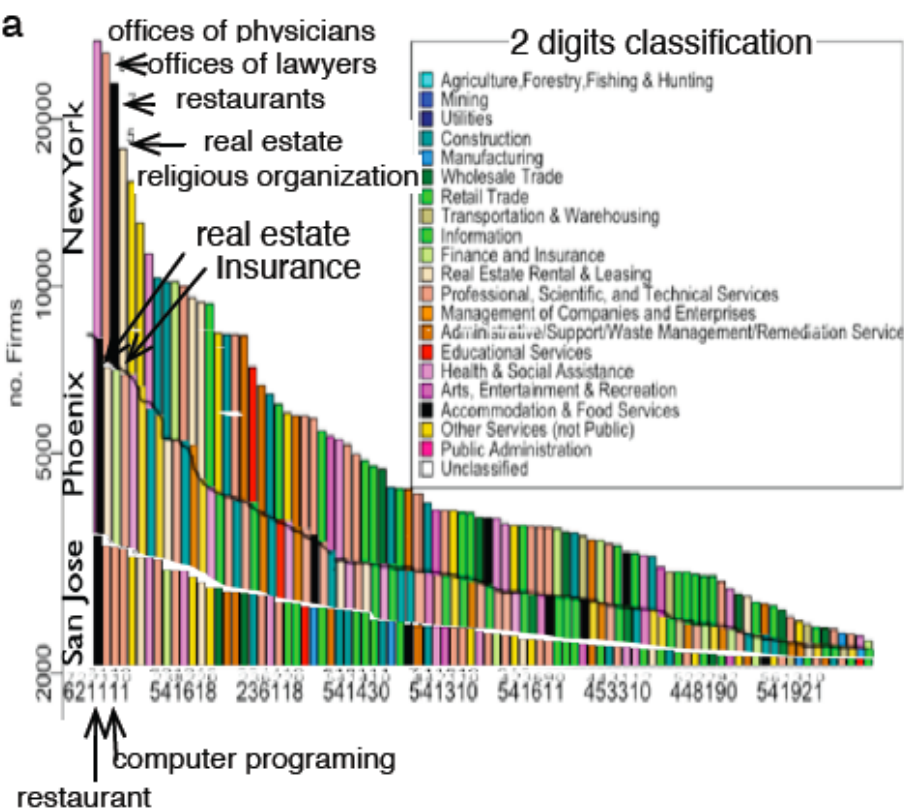
# 2003 Patenting Rankings



Scale Adjusted Urban Indicators



# DIVERSITY OF FIRMS AND OCCUPATIONS



**IF NUMBER OF ESTABLISHMENTS OF TYPE  $j$   
SCALES AS**

$$n_j \propto N^{\beta_j}$$

**THEN ITS RANKING SCALES AS**

$$x_j \propto N^{(1-\beta_j)/\gamma} \quad \mathbf{x_j < x_0}$$

$$x_j \propto (1 - \beta) \ln N \quad \mathbf{x_j > x_0}$$

**SO BUSINESS TYPES WHOSE ABUNDANCES SCALE  
SUPER-LINEARLY (PROFESSIONAL, SERVICE, e.g.  
LAWYERS, DOCTORS,....) INCREASE IN RANK WITH  
INCREASING CITY SIZE WHEREAS THOSE THAT SCALE  
SUB-LINEARLY (e.g. AGRICULTURE, MINING, FISHING,  
.....) DECREASE IN RANK**



***SOCIO-ECONOMIC QUANTITIES DEPEND ON  
“TWO-BODY” INTERACTIONS (INFORMATION  
EXCHANGE) AND THEREFORE NUMBER AND  
DENSITY OF SOCIAL INTERACTIONS:***

$$Y(N) \propto N_{\text{int}}$$

***[UNLIKE BIOLOGY WHERE  $Y(N) \sim N$ ]***

***IF EVERYONE INTERACTED WITH  
EVERYONE ELSE, THEN***

$$Y(N) \propto N_{\text{int}} \sim N^2$$

***EFFECTIVE INTERACTION SPATIAL AREA  
FOR AVERAGE INDIVIDUAL =  $\varepsilon^2$***

**EACH INDIVIDUAL INTERACTS WITH  $\Delta N$  OTHERS:**

$$\Delta N \approx \rho \varepsilon^2$$

**TOTAL NUMBER OF INTERACTIONS  $\approx N\Delta N \approx N\rho \varepsilon^2$**

**SOCIO-ECONOMIC METRICS**

$$Y(N) \propto (N\Delta N)Y_0 \sim (N\rho\varepsilon^2)Y_0 \sim \frac{N^2}{A}(\varepsilon^2 Y_0)$$

$$Y(N) \sim \left(\frac{\varepsilon^2}{A}\right)N^2 Y_0$$

**IF ROADS, CABLES, ETC ARE SPACE-FILLING  
(THEY SERVICE EVERYONE) WITH TOTAL LENGTH  
 $L$ , THEN**

**AREA**  $A \sim L\varepsilon$

$$\Rightarrow \left( \frac{L}{N} \right) \left( \frac{Y}{N} \right) \approx \varepsilon Y_0 \quad \text{INVARIANT!}$$

**IF**

$$L \approx L_0 N^{\beta_I}$$

$$R \approx R_0 N^{\beta_{SE}}$$

**WITH**

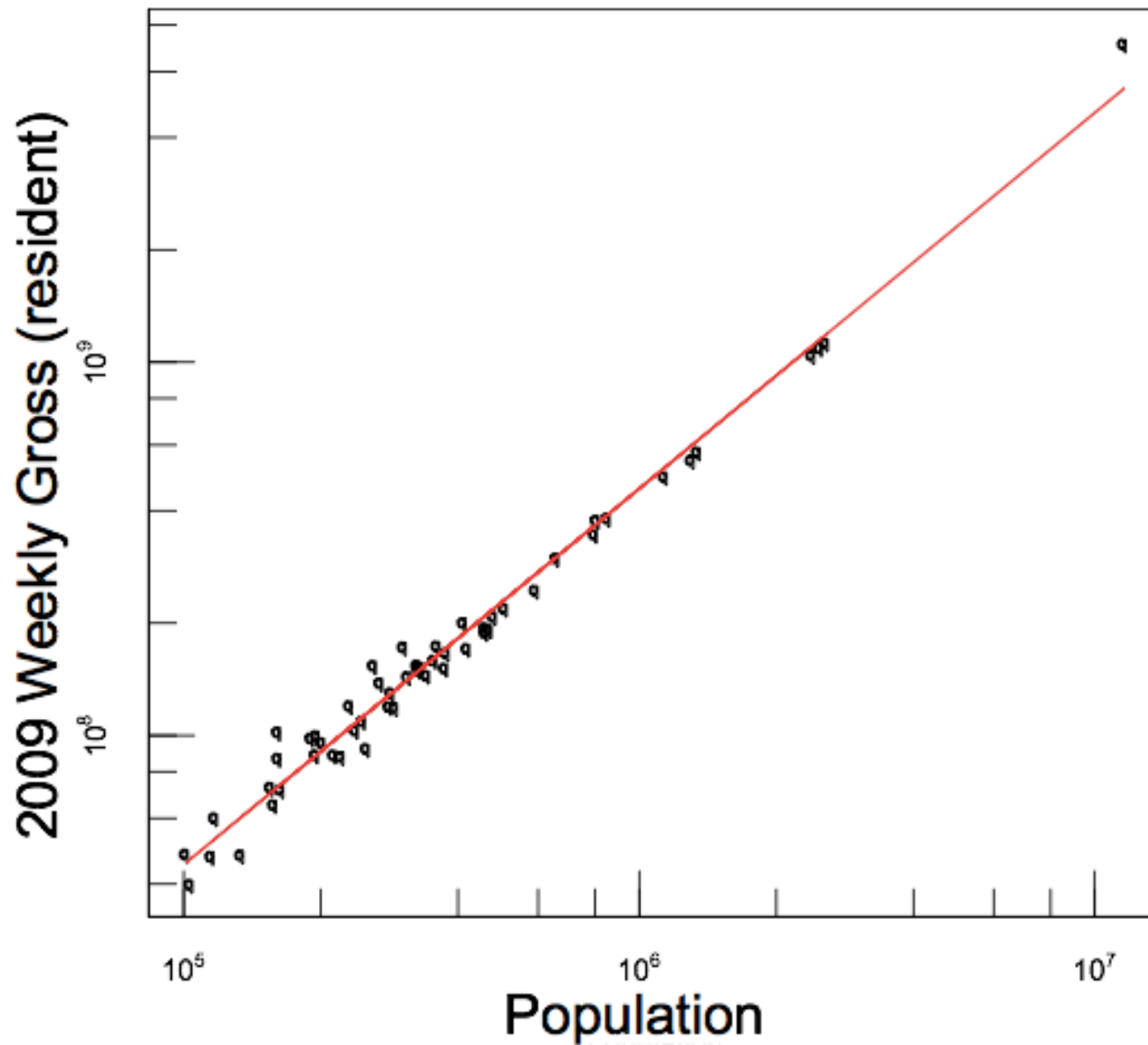
$$\beta_I = 1 + \varepsilon_I$$

$$\beta_{SE} = 1 + \varepsilon_{SE}$$

**THEN**

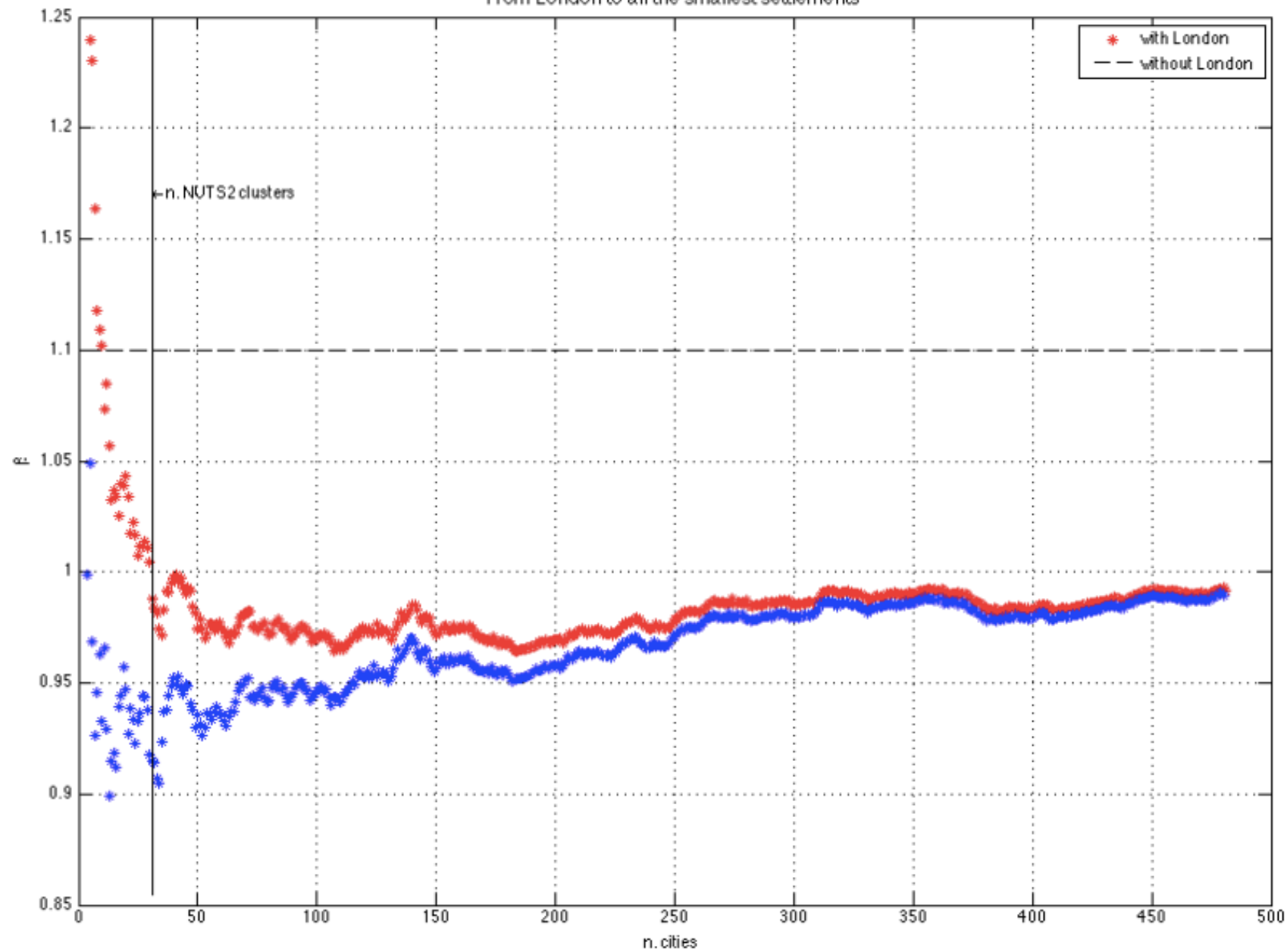
$$\varepsilon_I = \varepsilon_{SE} \quad (\sim 0.15)$$

**→ CAN DETERMINE THE SOCIAL  
INFORMATIONAL NETWORK SCALING FROM  
THE “METABOLIC” NETWORK SCALING**





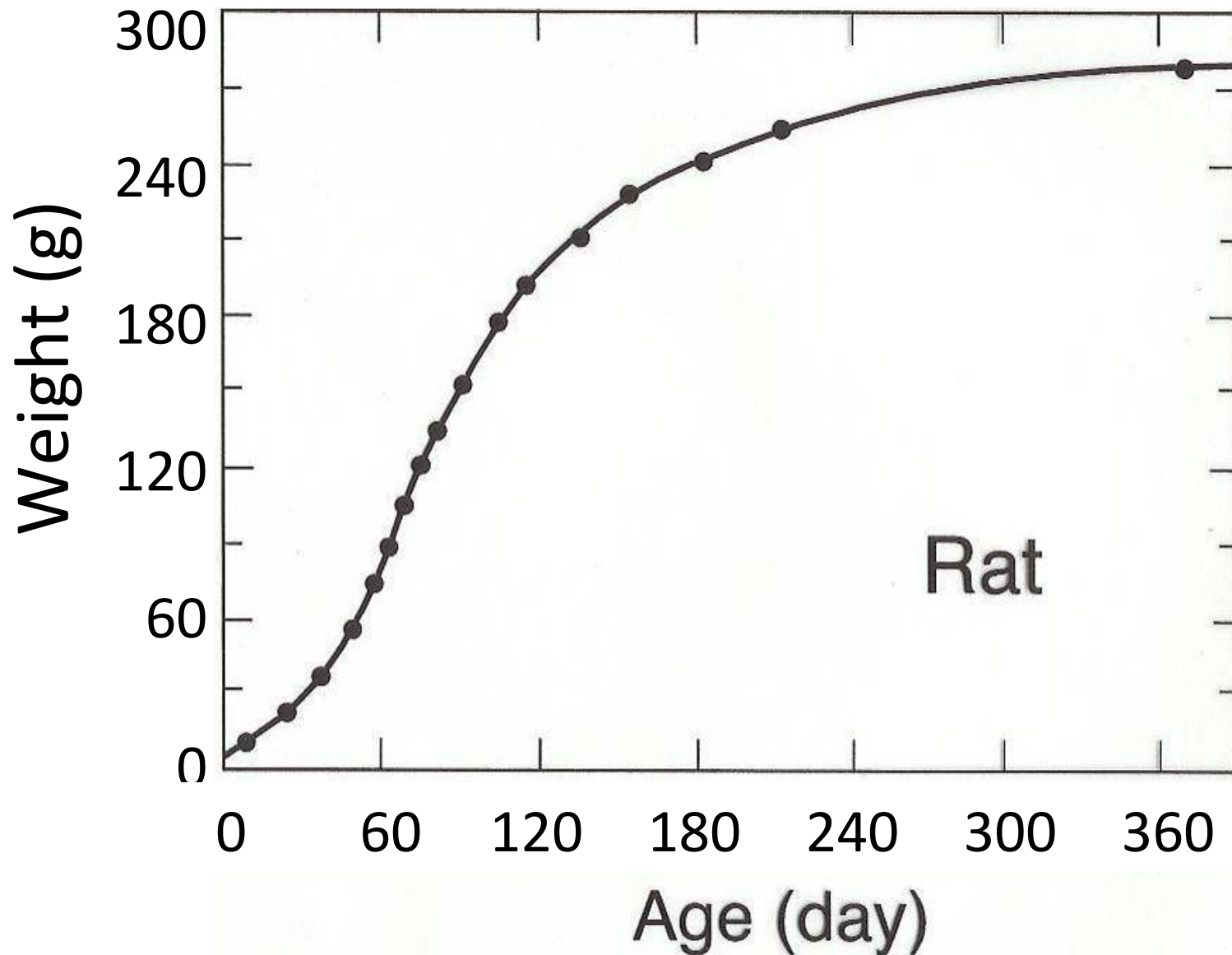
From London to all the smallest settlements



# The Editors: Is London's success causing the UK a problem?

The BBC's Economics Editor, Stephanie Flanders, visits London, Birmingham and Manchester and discovers wide discrepancies between the capital's economy and the rest of the country.

She says London's ebullient economy is subsidising other parts of the country but there is a lot of resentment in other big provincial cities.



**SUB-LINEAR SCALING ( $b < 1$ )  
LEADS TO BOUNDED GROWTH**

# Growth Equation

**Total incoming rate** (Resources, Products, Patents,... .. “Energy” or “Dollar” equivalent)

$\approx$  **Maintenance** (Repair, Replacement, Sustenance, ...) +

**Growth**



$$R = \sum_{i=1}^n Y_i(N) = \sum_{j=1}^N r_j + \frac{d}{dt} \sum_{j=1}^N c_j$$

**$n$  = NUMBER OF “DRIVERS”  $Y_i$  CONTRIBUTING TO THE CITY “METABOLISM”**

**$r_j$  = RATE AT WHICH THESE RESOURCES ARE USED BY THE  $j^{\text{th}}$  INDIVIDUAL (MAINTAIN HIS/HER/ITS LIFE-STYLE, ETC)**

**$c_j$  = COST OF ADDING A NEW INDIVIDUAL TO THE CITY POPULATION**

**SCALING LAWS TELL US THAT EACH  $i$  SCALES AS**  $Y_i(N) = Y_i(1)N^{\beta_i}$

**WITH  $\beta_i \approx \beta \approx 1.15$**   
**APPROXIMATELY THE SAME FOR ALL  $i$ ,**  
**SO**

$$R(N) = R(1)N^{\beta}$$

**INTRODUCE AVERAGE COSTS:**

$$R_0 \equiv \frac{1}{N} \sum_{j=1}^N r_j$$

$$E_0 \equiv \frac{1}{N} \sum_{j=1}^N c_j$$

$$R \approx NR_o + E_o \frac{dN}{dt}$$

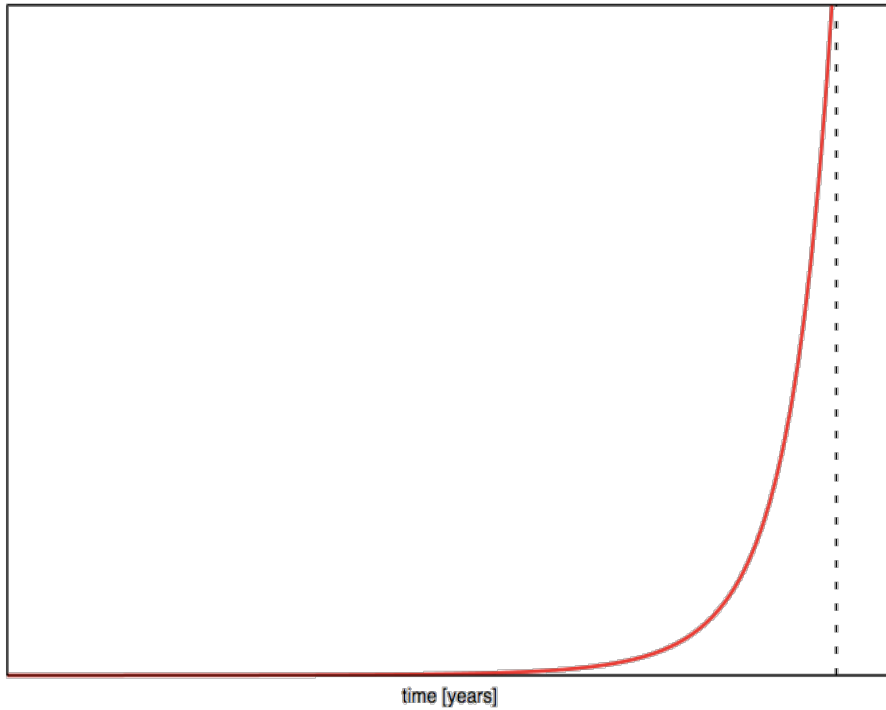
$$\frac{dN}{dt} = \left( \frac{R_1}{E_0} \right) \left[ N^\beta - \left( \frac{R_0}{R_1} \right) N \right]$$

**SOLUTION:**

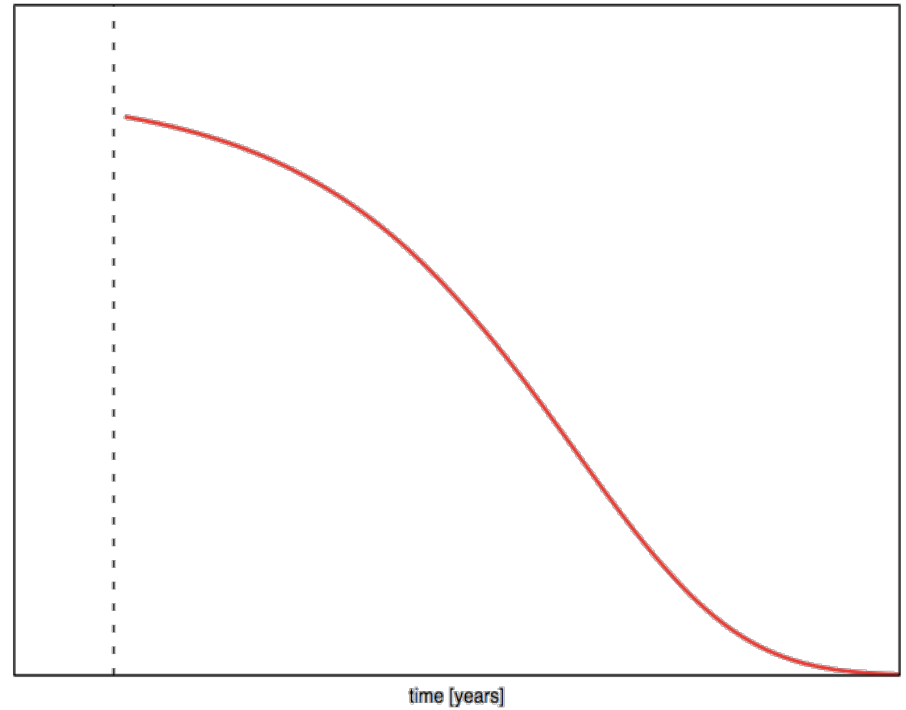
$$N^{1-\beta} = \frac{R_1}{R_0} + \left[ N^{1-\beta}(0) - \frac{R_1}{R_0} \right] e^{-\frac{R_0}{E_0}(1-\beta)t}$$

**CHARACTER OF SOLUTION SENSITIVE TO  $\beta >, =, < 1$**

**$b > 1$  (SUPER-LINEAR)**



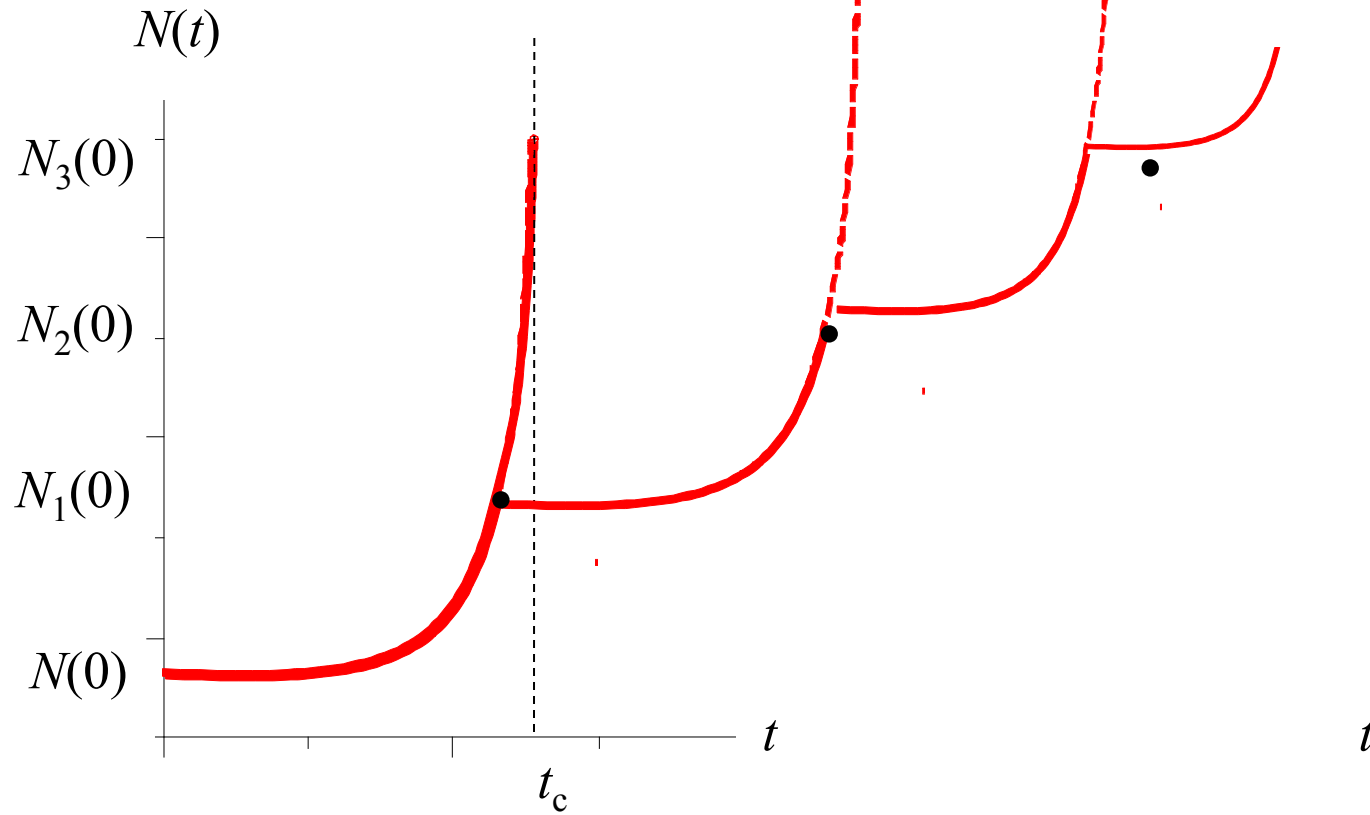
**SUPER-EXPONENTIAL  
UNBOUNDED GROWTH**



**COLLAPSE**



# UNBOUNDED GROWTH REQUIRES **ACCELERATING** CYCLES OF INNOVATION TO AVOID COLLAPSE



**UNBOUNDED GROWTH LEADING TO  
“FINITE-TIME SINGULARITY” & COLLAPSE**

**UNLESS INNOVATIONS (SYSTEMATICALLY)  
OCCUR FASTER AND FASTER**

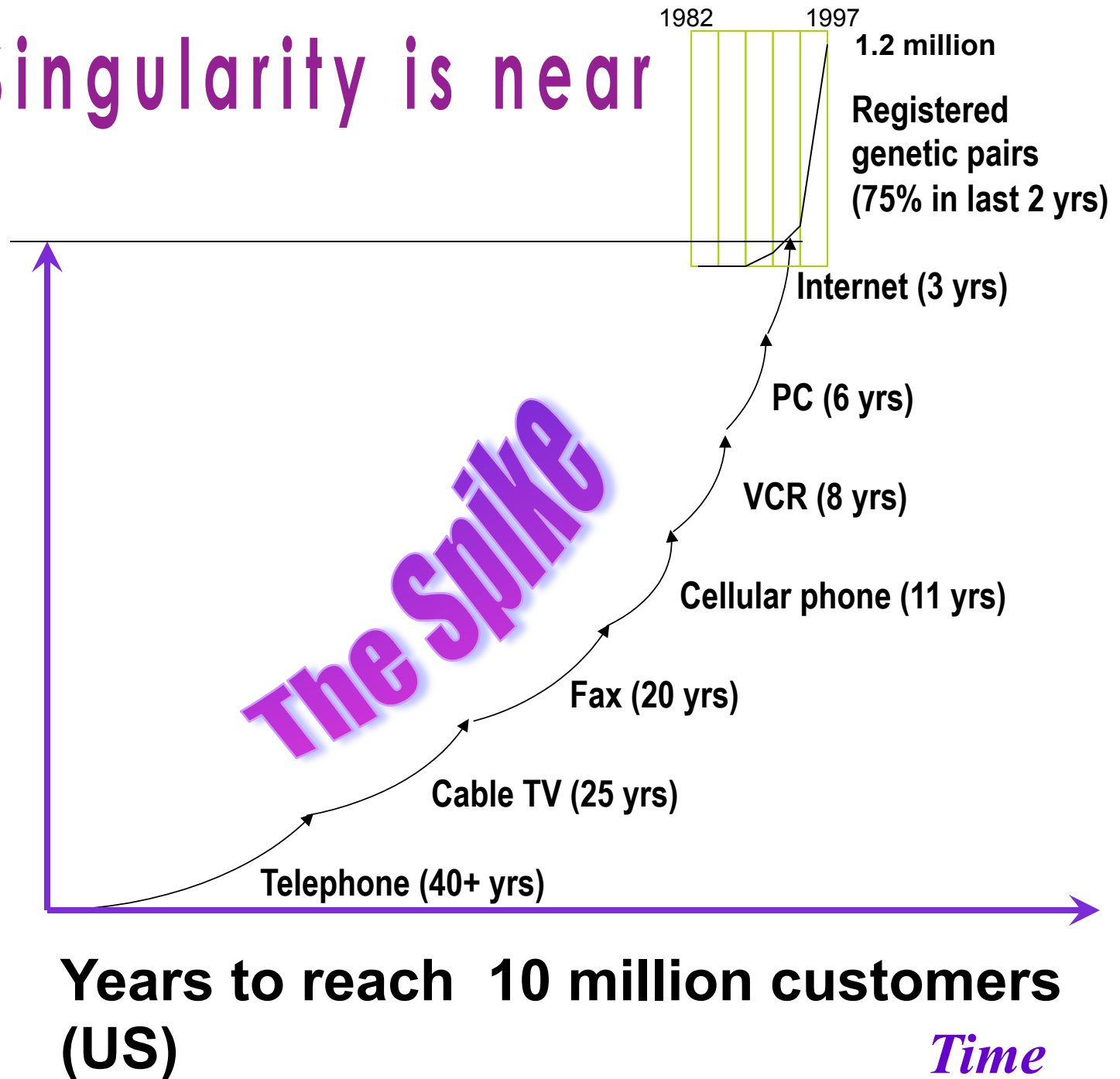
**CONTINUOUS TENSION BETWEEN:**

**INNOVATION & WEALTH CREATION vs  
ECONOMIES OF SCALE**



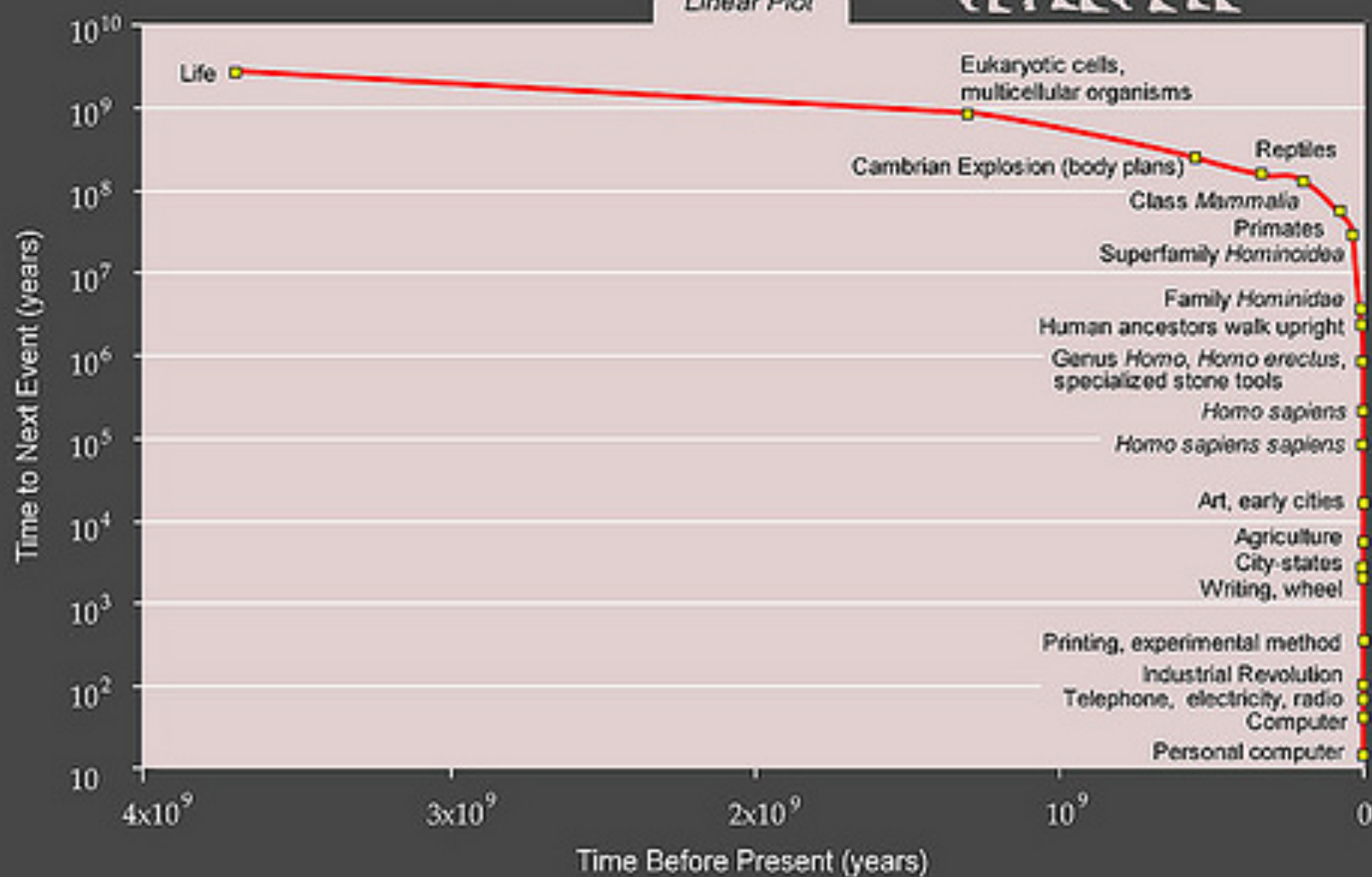
Damien  
Broderick

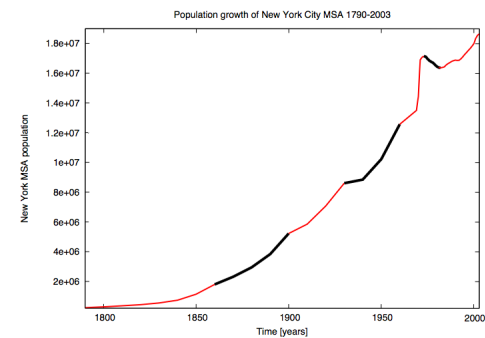
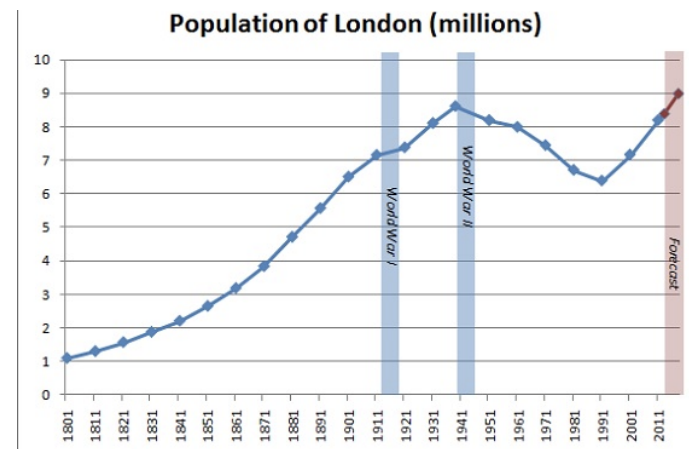
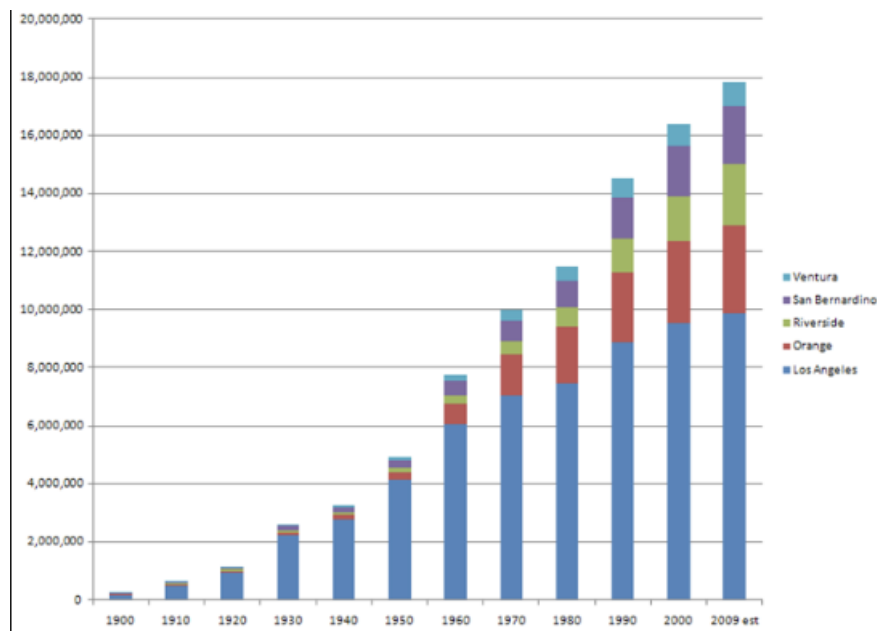
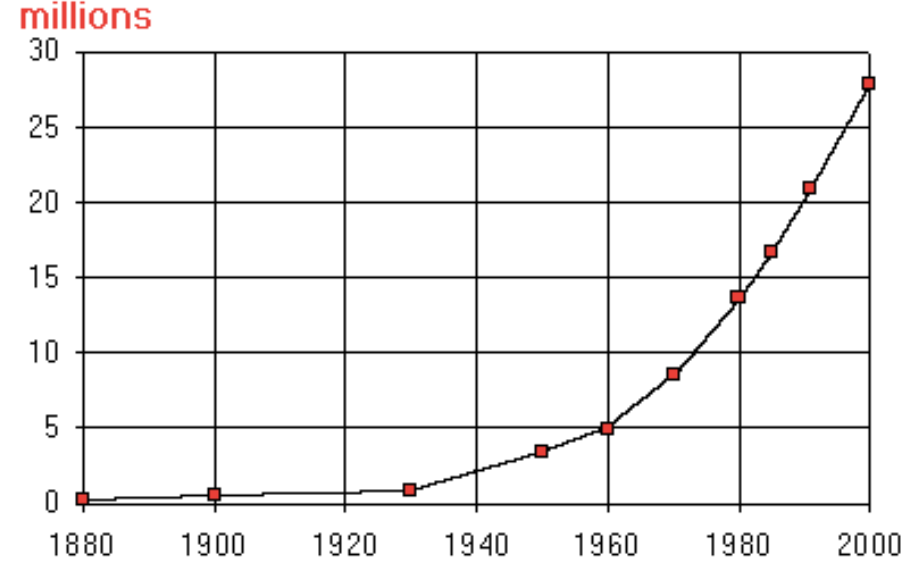
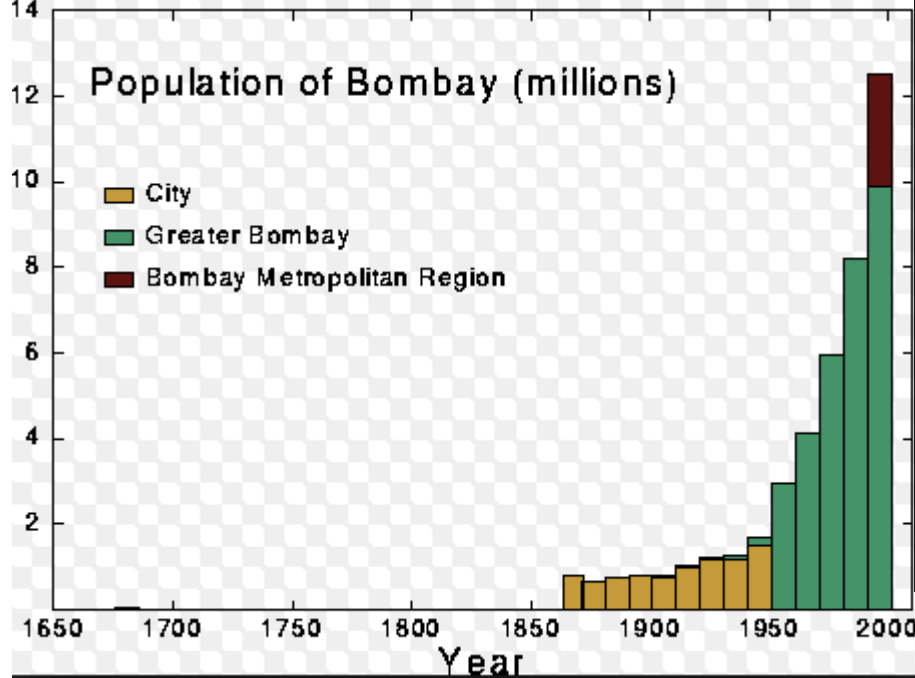
# Singularity is near



## Countdown to Singularity

Linear Plot







***NEED A NEW PARADIGM, A NEW  
INTEGRATED CONCEPTUAL  
FRAMEWORK:***

***SYSTEMIC, HOLISTIC, QUANTITATIVE,  
MECHANISTIC, COMPUTATIONAL,  
PREDICTIVE***

***COUPLED WITH, INSPIRED BY,  
MOTIVATED BY, INSPIRING AND  
MOTIVATING,  
“BIG DATA”***

**BUT MINDLESS BIG DATA IS  
(PROBABLY) BAD AND EVEN  
DANGEROUS**

**WITHOUT SOME CONCEPTUAL  
FRAMEWORK**

**HOW MUCH, WHERE, WHEN, WHAT, WHY?**

**.....AND THERE IS NO VIRGIN  
DATA**



# Big Data Needs a Big Theory to Go with It

Just as the industrial age produced the laws of thermodynamics, we need universal laws of complexity to solve our seemingly intractable problems

By Geoffrey West

As the world becomes increasingly complex and interconnected, some of our biggest challenges have begun to seem intractable. What should we do about uncertainty in the financial markets? How can we predict energy supply and demand? How will climate change play out? How do we cope with rapid urbanization? Our traditional approaches to these problems are often qualitative and disjointed and lead to unintended consequences. To bring scientific rigor to the challenges of our time, we need to develop a deeper understanding of complexity itself.



[Pin it](#)

Image: Eva Vazquez

**OUR “NATURAL” METABOLIC RATE ~ 90 watts**

**OUR SOCIAL METABOLIC RATE ~ 11,000 watts !!!**

**WE ARE EQUIVALENT TO A 30,000 Kg GORILLA !!!**

**REPRODUCTION RATE OF ~ ONE OFFSPRING PER  
15 years**





