Spatial predator-prey ecological system: The influence of local interactions

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Introduction

Predator-Prey (P-P) systems have been widely described using classical P-P Lolka-Voltera models (Bjornstad *et al.* 1999, Holmes *et al*. 1994). These models studied the temporal or spatio-temporal dynamics of these systems.
Previous spatial P-P models argued that the formation of spatial patterns is possible only under strong constraints and by introducing particular forms of the interactions (Homes *et al.* 1994). In this study we explore the possibility of formation of spatial patterns throughout the inclusion of a simple local interaction in a standard P-P spatial Lolka-Voltera model.

The model

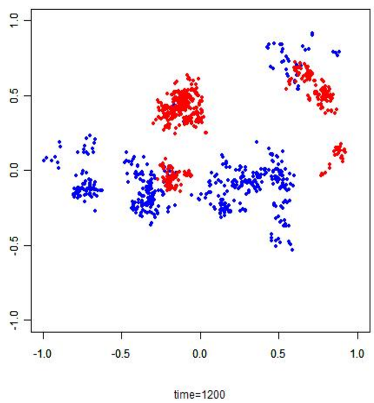
$$\frac{\partial N(x,t)}{\partial t} = D_N \nabla^2 N(x,t) + rN(x,t) - \alpha N(x,t) \int_{|x-l|<L_1} P(l,t)dl$$
$$\frac{\partial P(x,t)}{\partial t} = D_P \nabla^2 P(x,t) - mP(x,t) + \beta P(x,t) \int_{|x-l|<L_2} N(l,t)dl,$$

where D_N and D_P are the diffusion coefficients, r is the prey growth rate, m is the predator growth rate and α and β are the predation interaction coefficients.

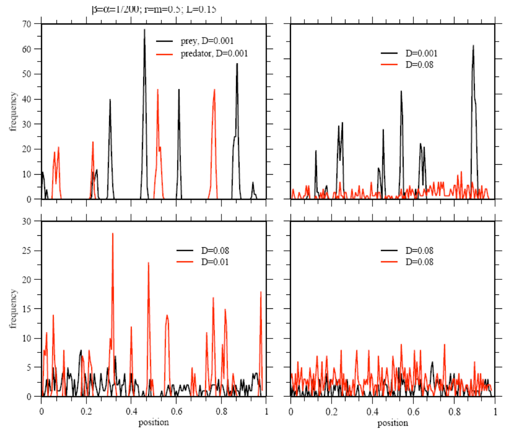
Results

Individual-based simulations

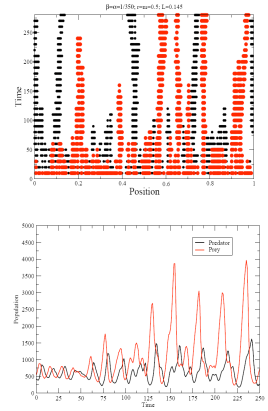
P-P in a 2-D space



P-P in a 1-D space



P-P in time



Analytical results

We calculate the stationary non trivial solutions: $\bar{N} = \frac{m}{2\beta L_2}$ and $\bar{P} = \frac{r}{2\alpha L_1}$ and we perturb it:

$$N = \bar{N} + A_N exp[\lambda t + ikx]$$
$$P = \bar{P} + A_P exp[\lambda t + ikx]$$

The equations (1-2) reduce to:

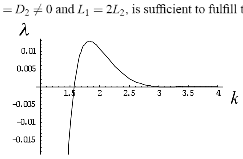
$$(\lambda + k^2 D_N)A_N + \frac{\alpha m}{\beta}\frac{Sin(kL_1)}{kL_1}A_P = 0$$
$$(\lambda + k^2 D_P)A_P - \frac{\beta r}{\alpha}\frac{Sin(kL_2)}{kL_2}A_N = 0.$$

satisfied by the following $\lambda = \lambda(K)$ condition:

$$\lambda = \frac{1}{2}\left[-(D_N+D_P)k^2 \pm \left[(D_N+D_P)^2k^4 - 4(D_N D_P k^4 + rm\frac{sin(kL_1)sin(kL_2)}{k^2 L_1 L_2}\right]^{1/2}\right]$$

The condition for the existence of a solution with spatial patterns stable in time is: $\lambda(K) > 0$.

This constrain is satisfied in a number of cases. For example, for $D_1 = D_2 \neq 0$ and $L_1 = 2L_2$, is sufficient to fulfill the relation: $\frac{2D^2}{rm} < 0.012726$.
On the other side, for $L_1 = L_2$ no stable solutions can be found.



Growth rate (λ) vs. wave number (K)

Conclusion

-Simulations show the existence of spatial patterns stable in time.

-The analytical results show that this patterns can exists in some regions of the parameter space.

Perspectives

-Explore the possible differences existing between results obtained via individual-based simulations and the prediction of the analytical results (effects of demographic fluctuations).

-Build up a direct comparison with real data.