The Science of Cities as Complex Systems I

Luís M. A. Bettencourt
Santa Fe Institute

bettencourt@santafe.edu @BettencourtLuis

Our Group:

Clio Andris (SFI, PennState)
Christa Brelsford (SFI/ASU)
Andres Gomez-Lievano (ASU)
Marcus Hamilton (SFI)
Joe Hand (SFI)
José Lobo (ASU)
Nathaniel Rodriguez (UI, Bloomington)
Horacio Samaniego (LANL, Valdivia)
Markus Schläpfer (SFI/MIT)
Deborah Strumsky (UNC Charlotte)
Geoffrey West (SFI)
HyeJin Youn (SFI, Oxford)

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http://www.santafe.edu/research/cities-scaling-and-sustainability/

Masdar City (Abu Dhabi, UAE)
credit: Foster + Partners

Lagos
credit: African Arguments

credit: Eddie Moore/ ABQ Journal

credit: lostinthestreet Street Photography

By its nature, the metropolis provides what otherwise could be given only by traveling; namely, the **strange**.

-Jane Jacobs
The Death and Life of Great American Cities, 1961.

The city according to different disciplines

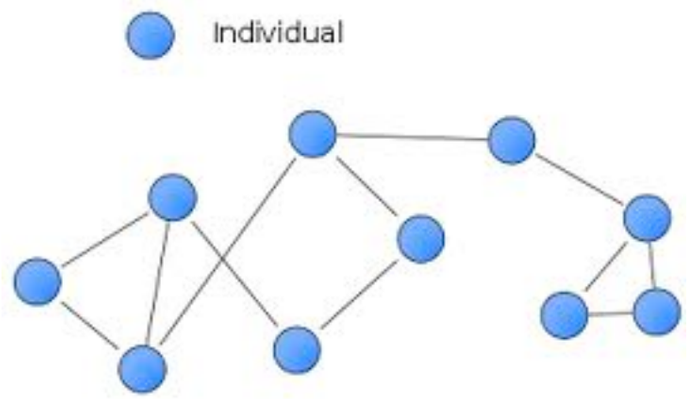
Economics

production, competitive advantage
economies in transportation and production
knowledge spillovers

$$Y=A\ L^{\alpha}\ K^{1-\alpha}$$

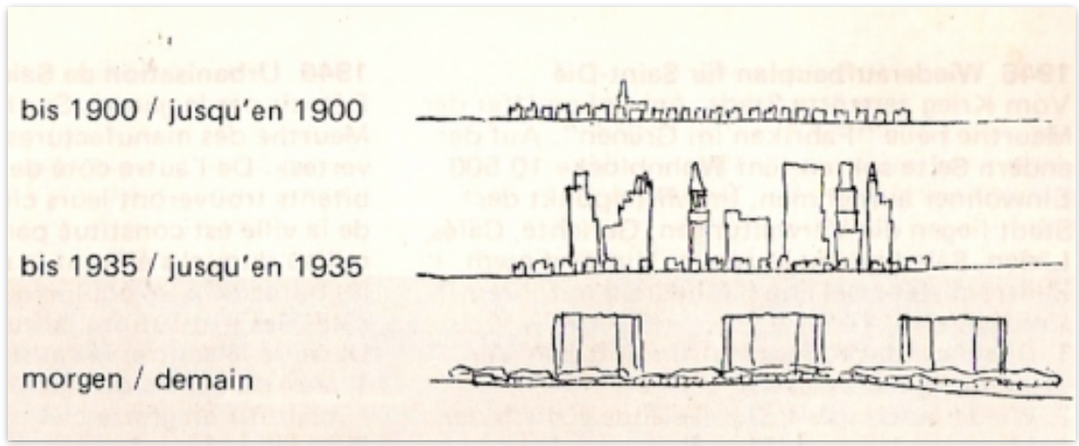
Sociology & Social Psychology

Social interactions & networks
change, exclusion, segregation



Urban Planning

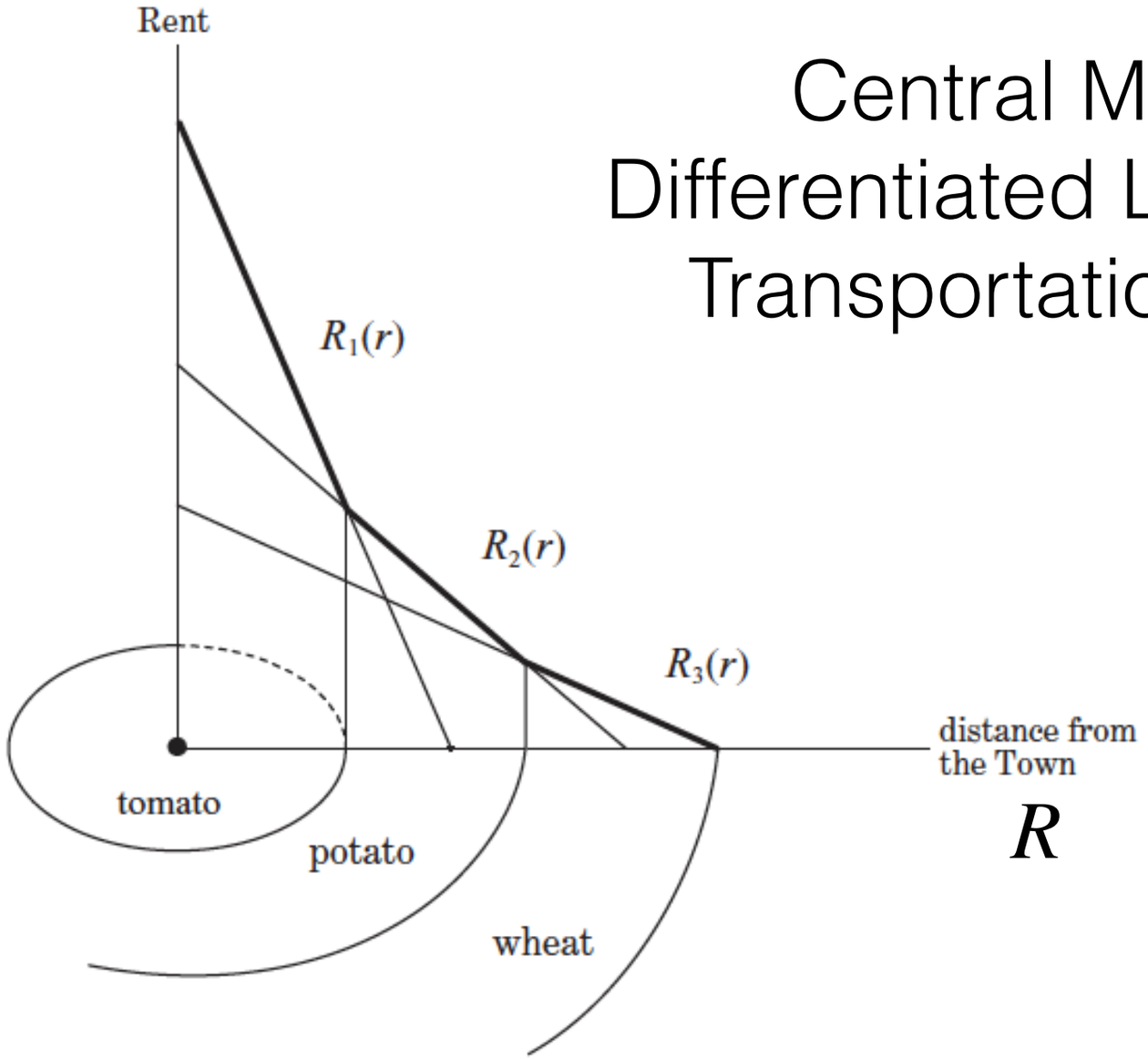
Design of space and infrastructure
Locations, Flows, Zoning
“Form determines Function”



Le Corbusier, Ville Radieuse, 1935

Spatial Equilibria in the Isolated State

von Thünen Model [1826, beginnings Central Place & Locational Theory]



Central Market
Differentiated Land Uses
Transportation costs

net profit

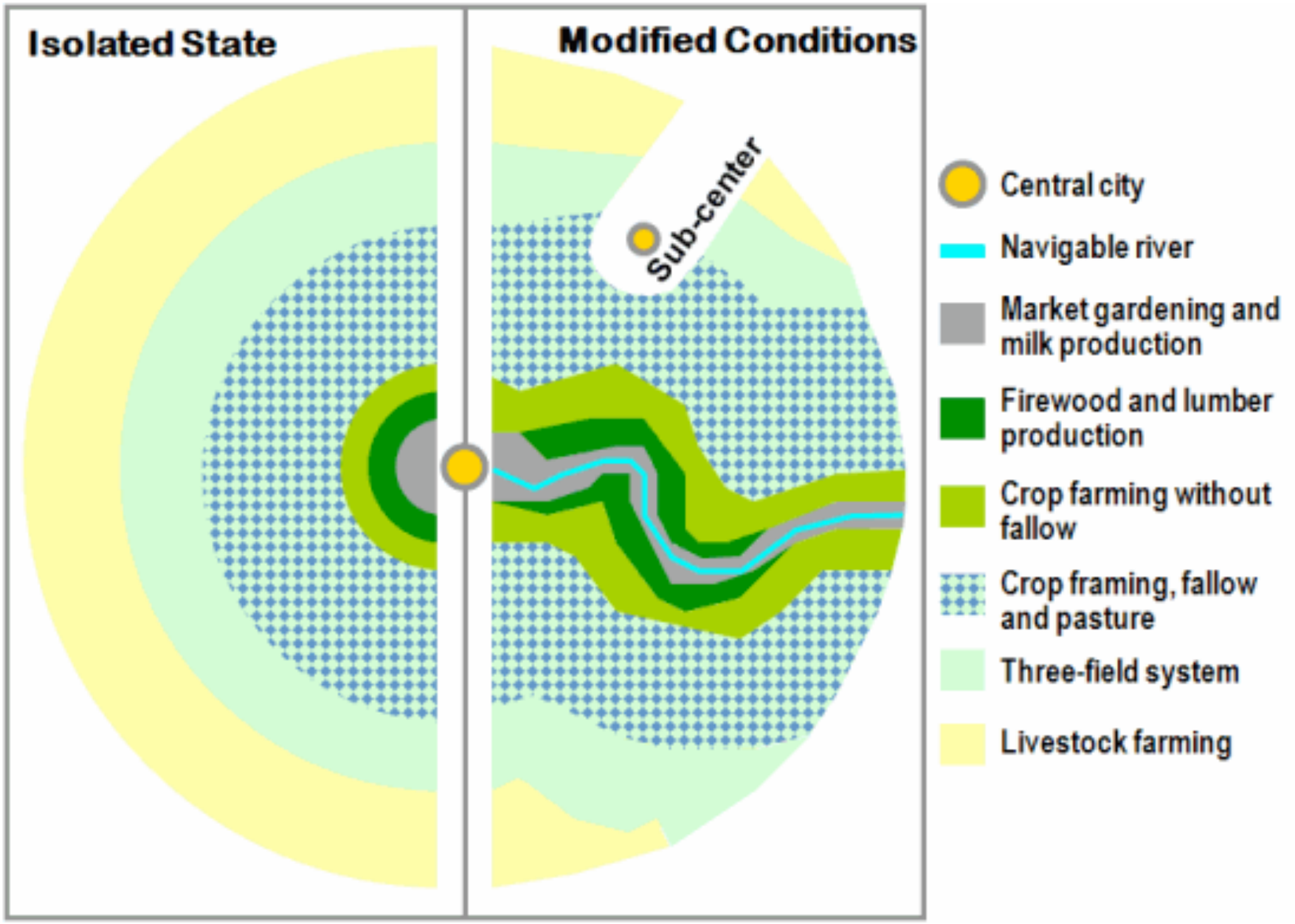
$$Rent_i = Y_i\left[(P_i - C_i) - T_i \times R\right].$$

crop yield

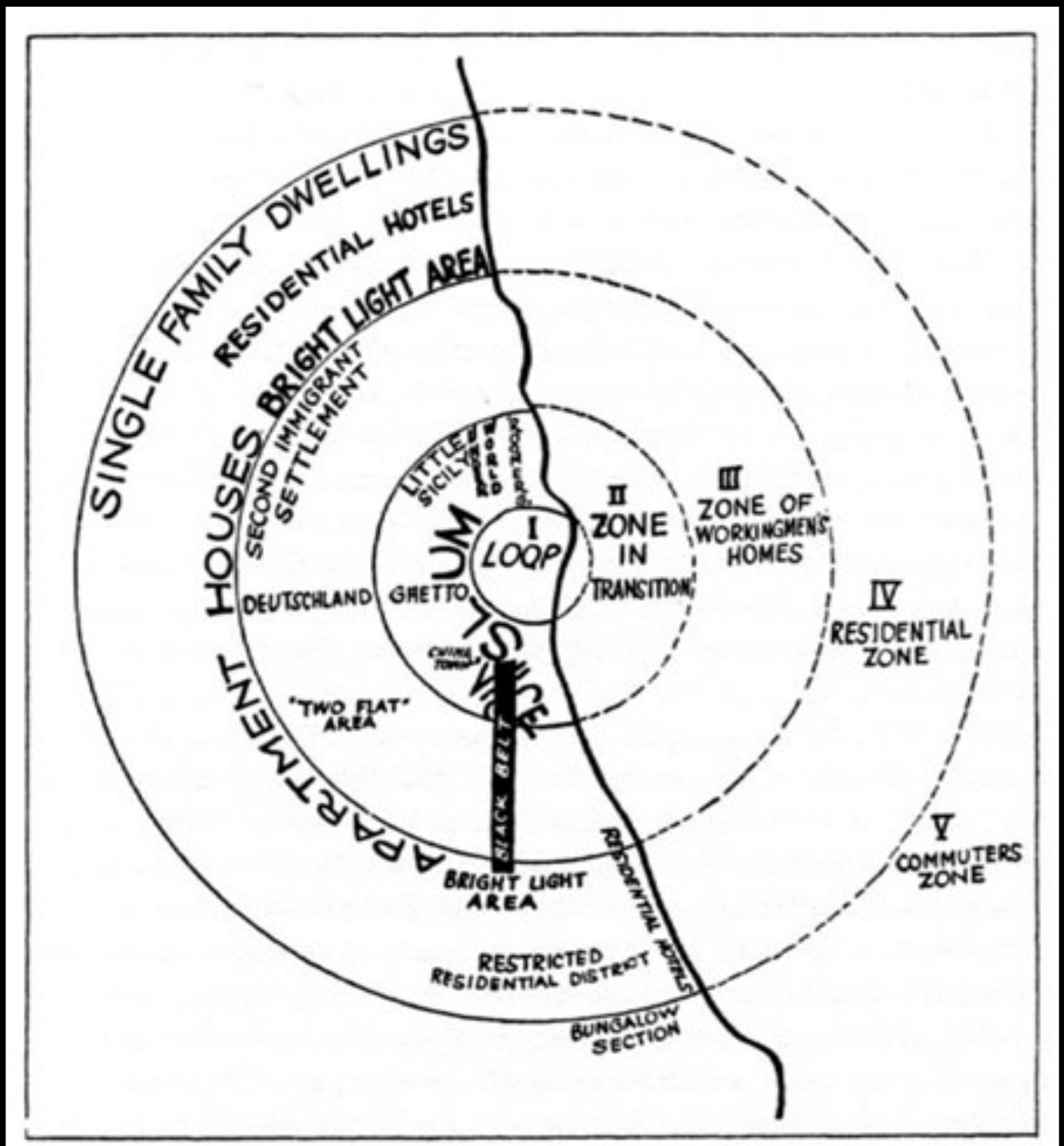
transportation cost

Fujita 2010 The Evolution of Spatial Economics

The “Isolated State” model can include geography



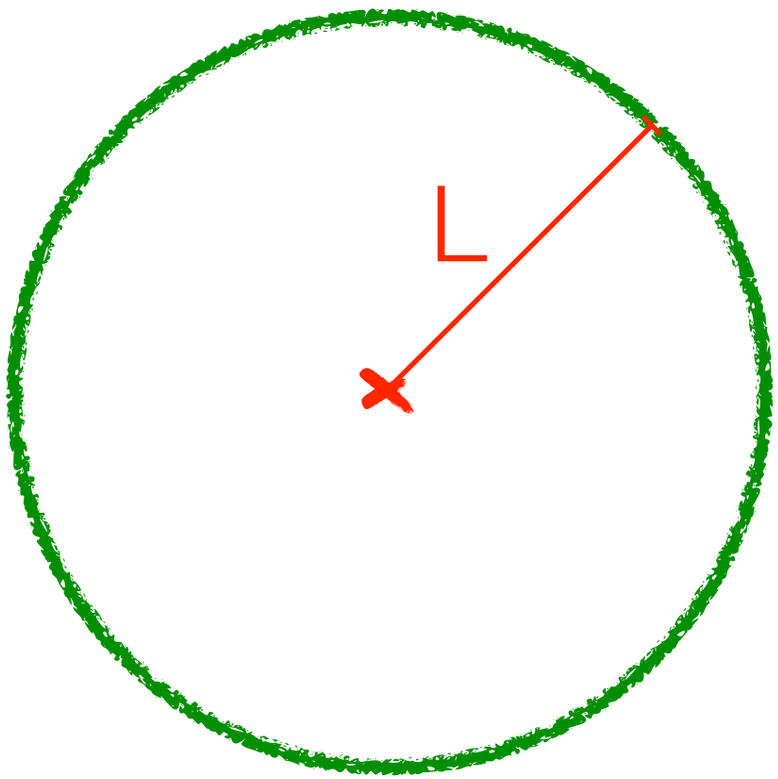
credit: http://lewishistoricalsociety.com



E. Burgess1925

Urban Morphology

Alonso 1960 (Müth-Mills,...,Beckmann,...) Model of the Monocentric City.



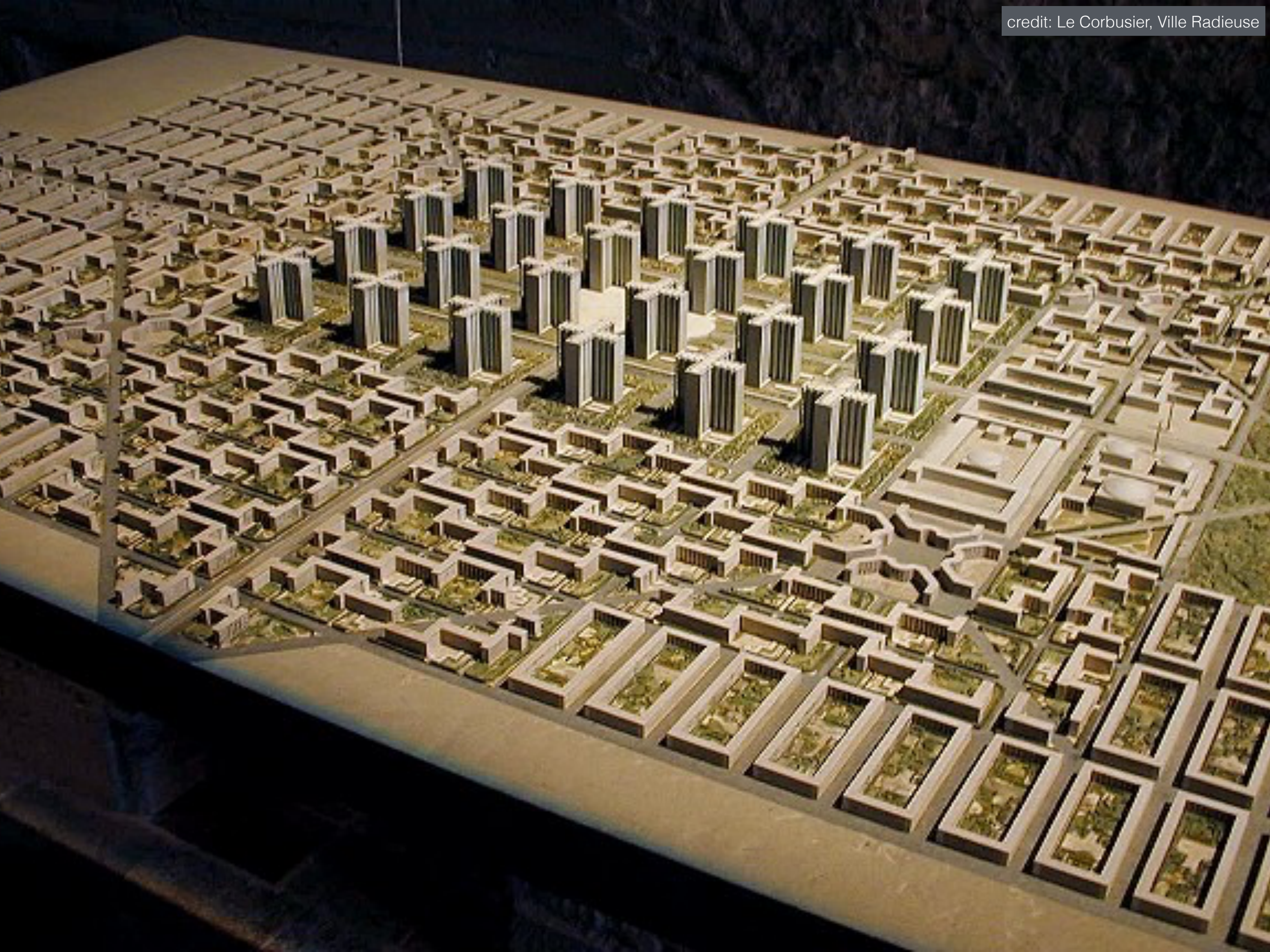
Land Rents = R(l)
Commuting Costs = C(l)

y = R(l)-C(l)

C(l=0)=0; R(l=L) = r

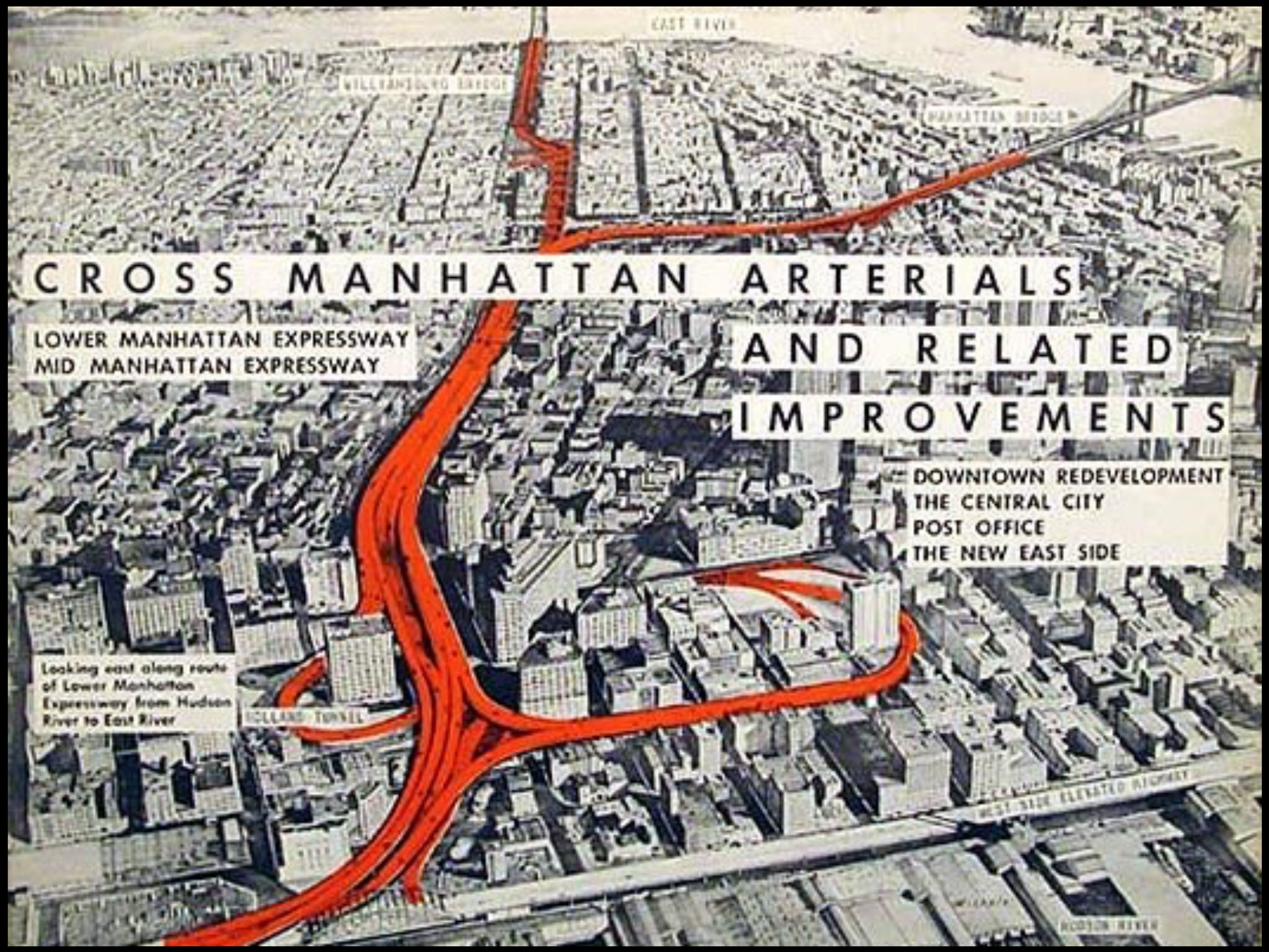
y = the “budget” per capita
set via a production function.

This is the basis of New Economic Geography Fujita, Krugman & Venables,1999



credit: Le Corbusier, Ville Radieuse

Tianjin
credit: Zhang Lei/World Press Photo 2016

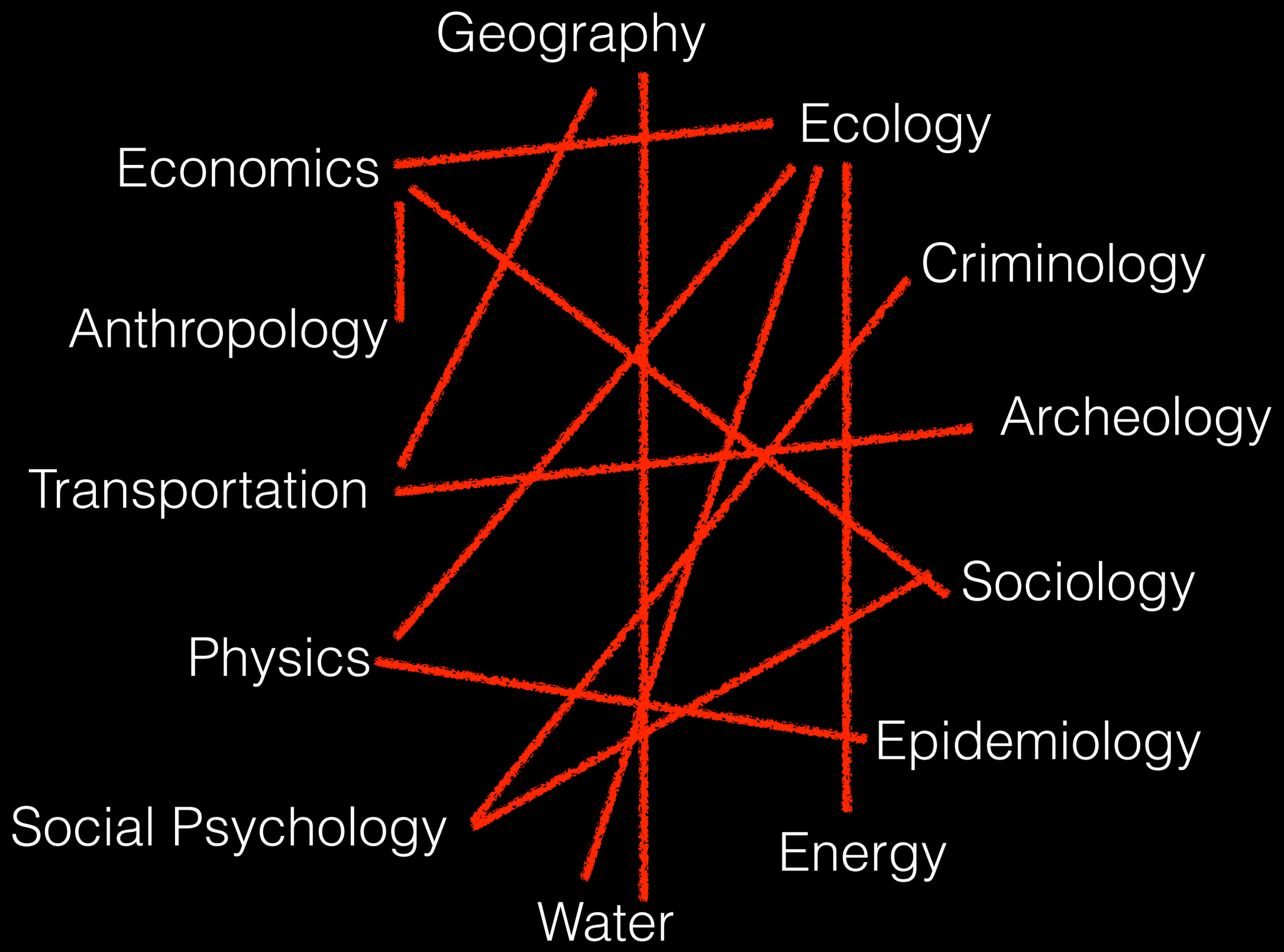

CROSS MANHATTAN ARTERIALS
AND RELATED
IMPROVEMENTS
LOWER MANHATTAN EXPRESSWAY
MID MANHATTAN EXPRESSWAY
DOWNTOWN REDEVELOPMENT
THE CENTRAL CITY
POST OFFICE
THE NEW EAST SIDE
Looking east along route of Lower Manhattan Expressway from Hudson River to East River
EAST RIVER
WILLIAMSBURG BRIDGE
MANHATTAN BRIDGE
HOLLAND TUNNEL
WEST SIDE ELEVATED HIGHWAY
HUDSON RIVER

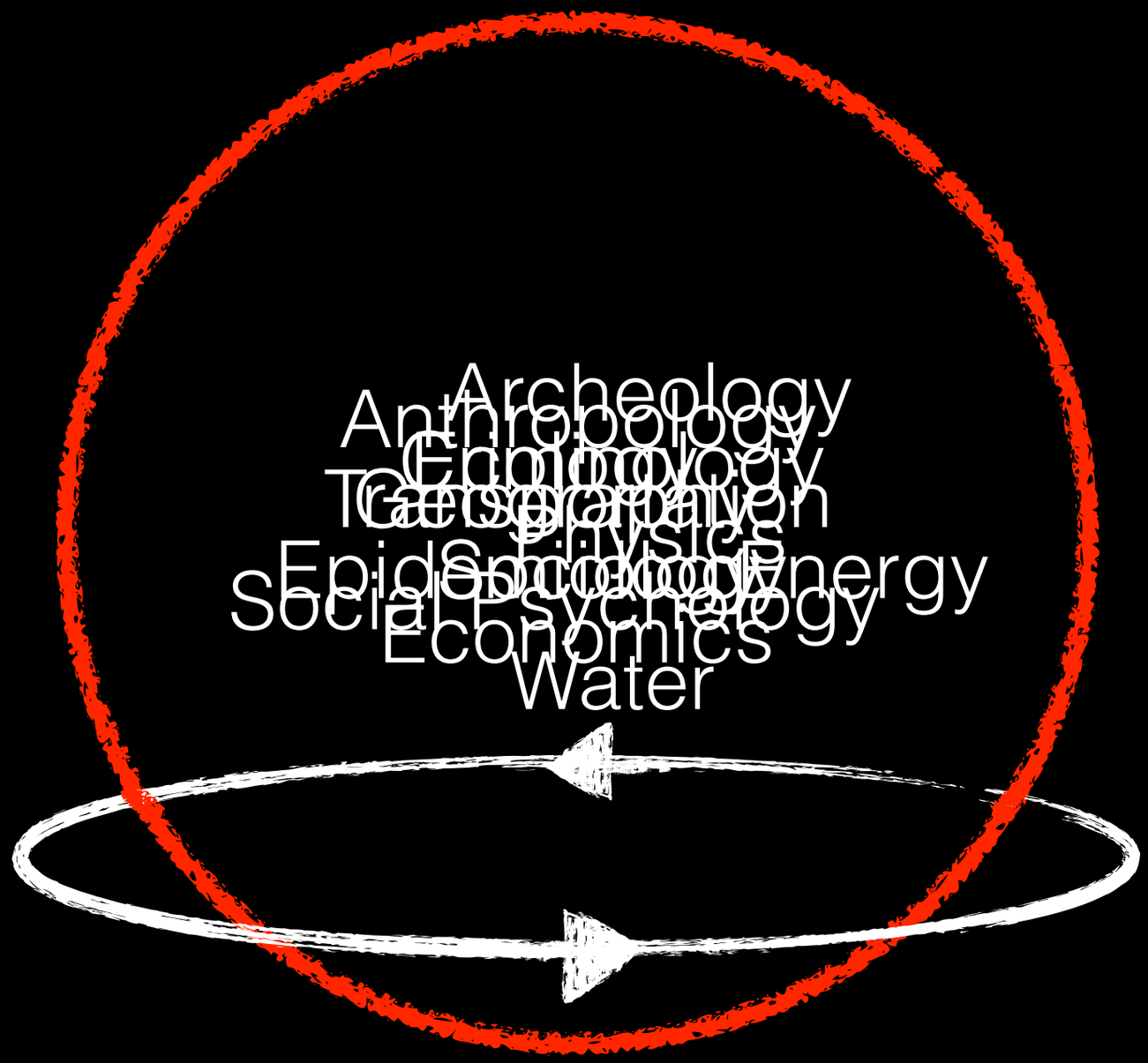
[Real cities, present] situations in which several dozen quantities are all varying simultaneously and in subtly connected ways

Jane Jacobs
The death and life of great American cities 1961

Cities are complex adaptive systems

- **Heterogeneity:** Diversity of people, organizations, land uses
- **Interconnectivity:** Everything is connected in networks
- **Scaling:** Cities of different sizes have different dis(advantages)
- **Circular-Causality:** Cause and effect are mixed
- **Development:** Cities change in open-ended ways


Geography
Ecology
Economics
Criminology
Anthropology
Archeology
Transportation
Sociology
Physics
Epidemiology
Social Psychology
Energy
Water



Data & Experiments

J. MANN/CORBIS

THE URBAN EQUATION

After spending tens of thousands of years living mostly in small settlements, humans have entered an urban stage of evolution. As of 2008, more than half the world's people live in cities, and the urban population is swelling by 1 million every week. By 2030, almost 6 in 10 people will live in metropolitan areas, which exert a powerful pull as economic and social magnets.

That concentration of people gives rise to some of the world's greatest problems, such as air and water pollution, poverty-stricken slums and epidemics of violence and illness. Yet throughout history, urbanites have produced soaring achievements, ranging from Notre Dame Cathedral to the mobile-phone networks that have revolutionized communication.

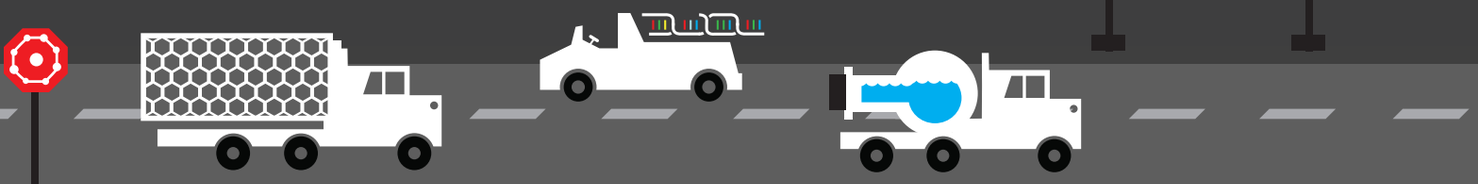
Cities are also home to considerable scientific capital; they hold most of the world's top universities and the vast majority of its researchers (see page 900). This week, *Nature* examines that special relationship between scientists and cities and how each can bring out the best in the other. The resources that cities offer can stimulate outstanding science for reasons that researchers are just starting to explore (see page 906). On the other side of the equation, scientists can assist cities in tackling their biggest problems. The Nobel laureate Mario Molina sets a good example, having redirected his research to improving the environment in Mexico City, one of the world's biggest megacities (see page 902).

Scientists are also helping cities to assume a lead position in combating global warming. With nations largely paralysed on this front, cities have emerged as a testing ground for cutting greenhouse-gas emissions and for adapting to the changes that warming will bring (see page 909). But these efforts are hampered by a disproportionate lack of data at the city level (see page 883). Cities must find a way to grow sustainably, which will require scientists across many disciplines to collaborate with leaders in other sectors of society to develop general rules for urban expansion (see page 912).

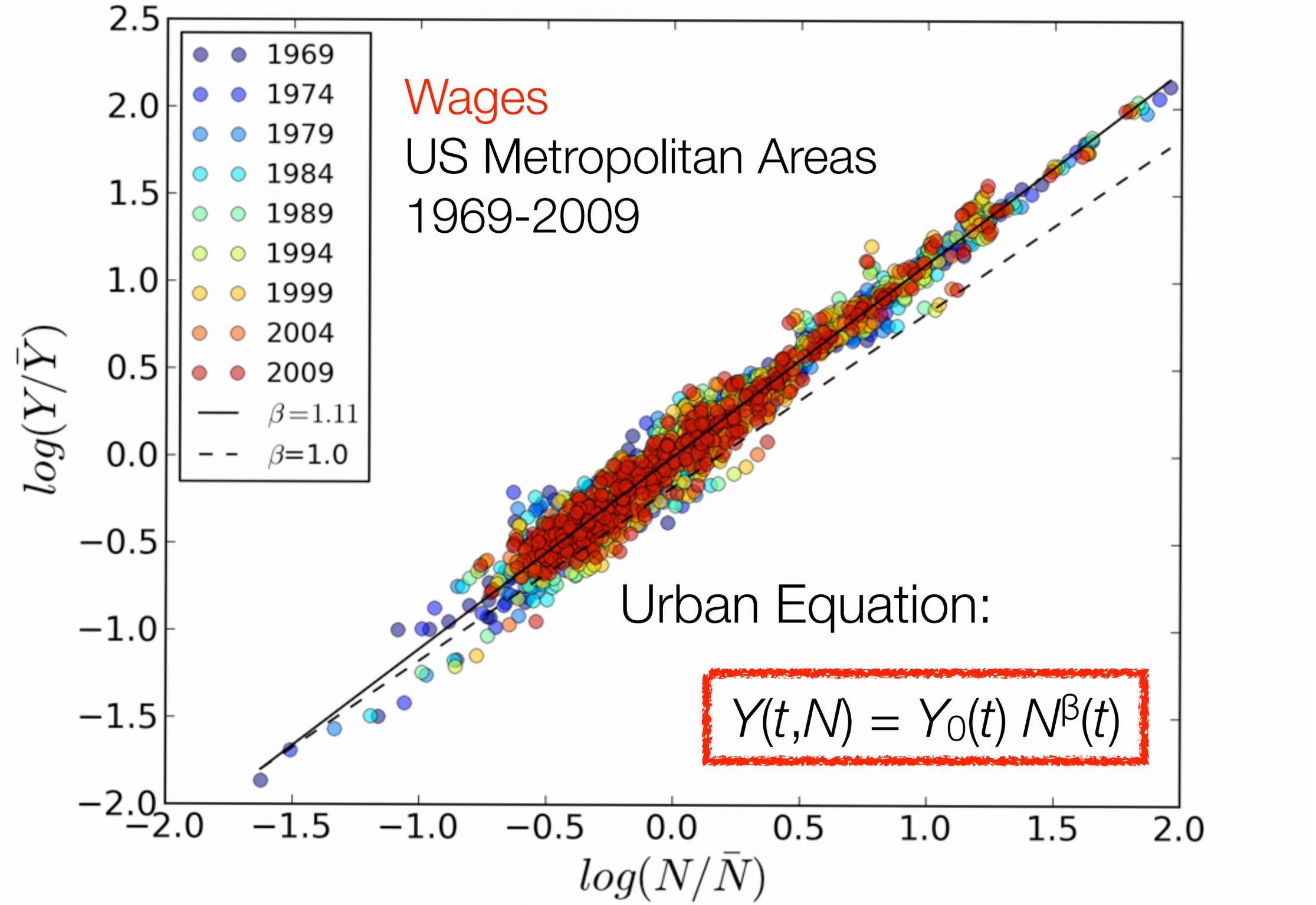
The threats to cities and the opportunities they present are attracting increasing attention from researchers in many areas. Synthetic biologists, for example, are exploring molecules that could clad skyscrapers and trap carbon dioxide (see page 916). Scientists have a responsibility to supply many more advances of that nature to ensure the viability of humans as an urban species. ■

ILLUSTRATIONS BY OLIVER MUNDAY

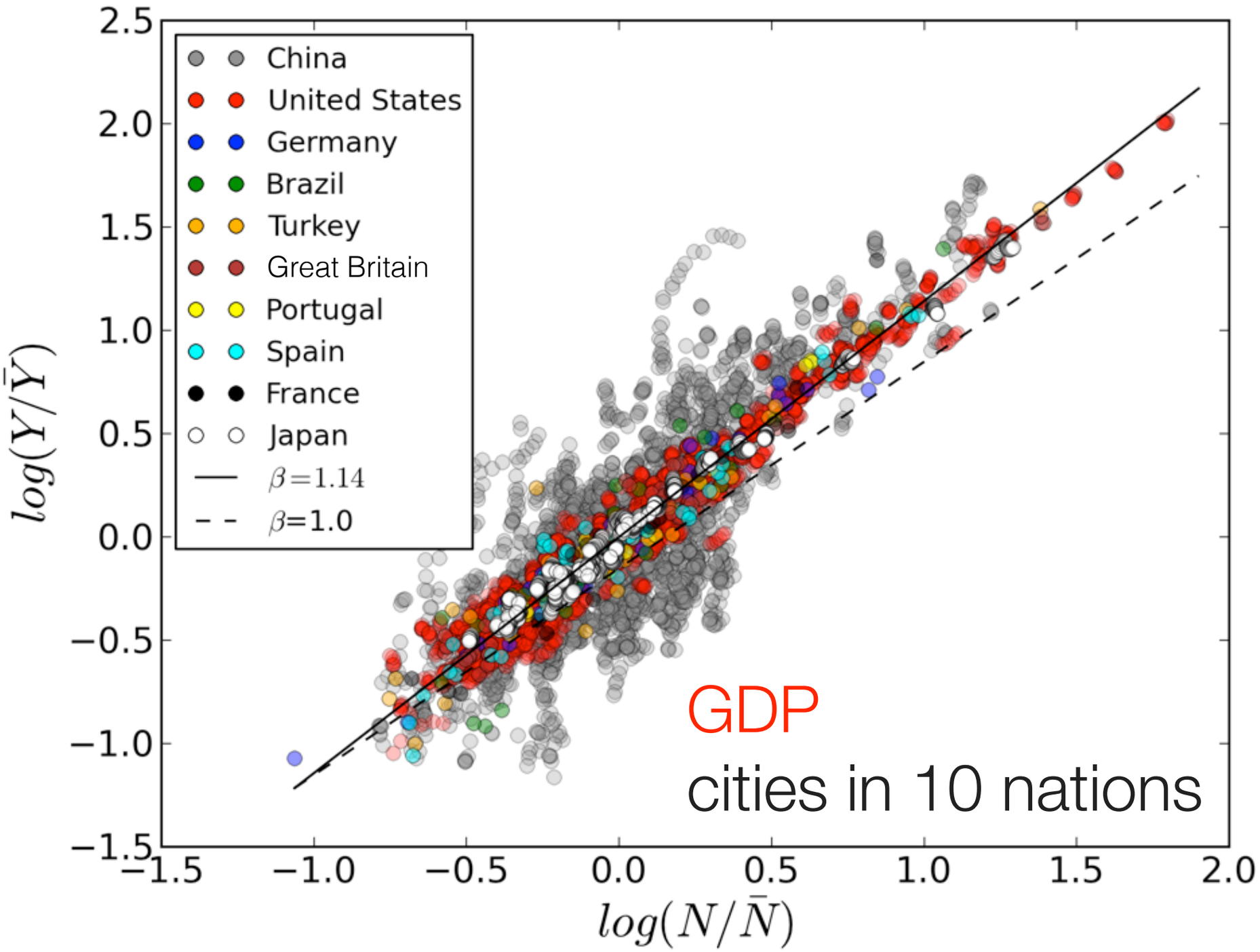
SCIENCE AND THE CITY
Full content and enhanced graphics at: **nature.com/cities**



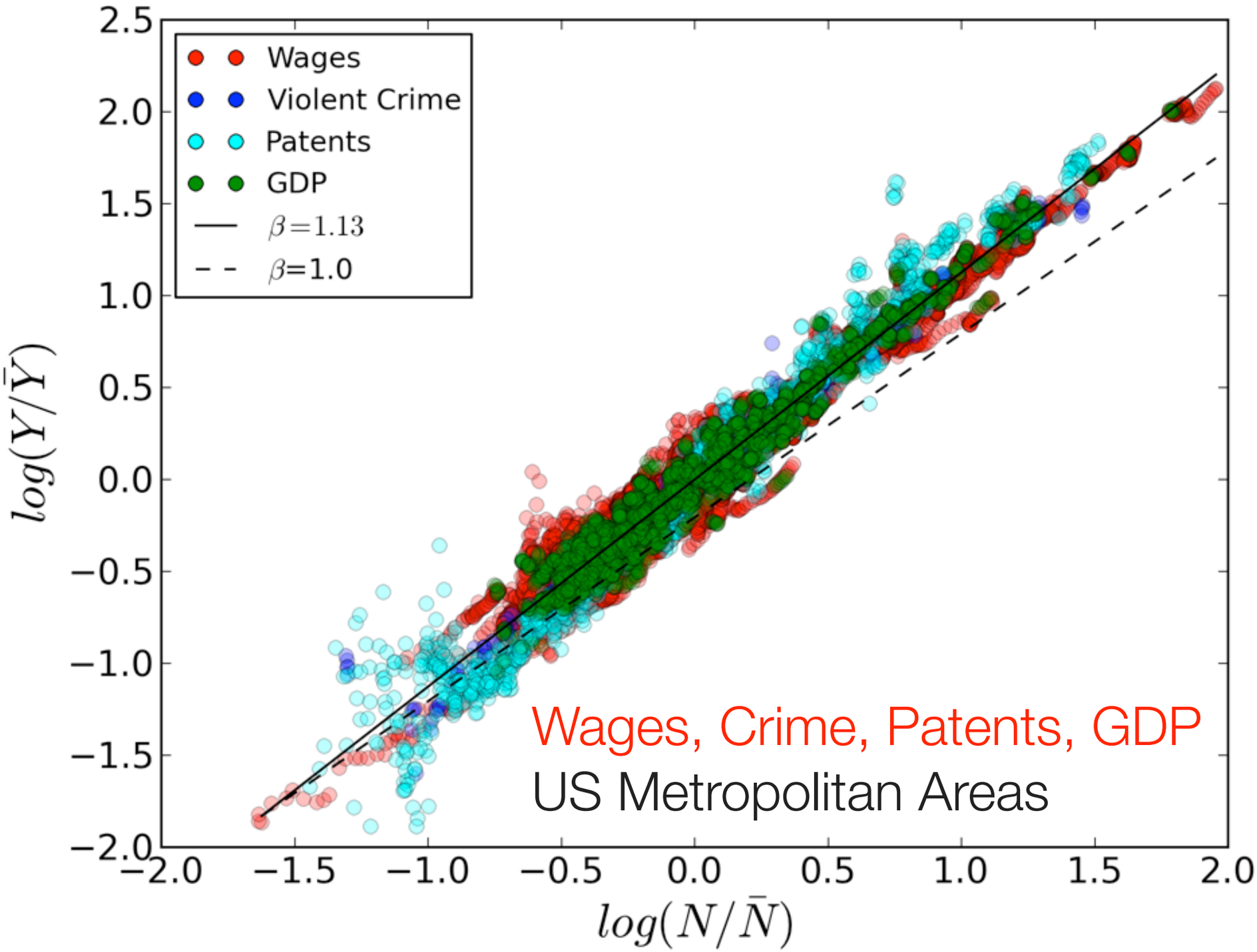
A law in **time** ...


1969
1974
1979
1984
1989
1994
1999
2004
2009
$\beta = 1.11$
$\beta = 1.0$
Wages
US Metropolitan Areas
1969-2009
Urban Equation:
$Y(t,N) = Y_0(t)\, N^{\beta}(t)$
$log(Y/\bar{Y})$
$log(N/\bar{N})$
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

A law in **across nations** ...



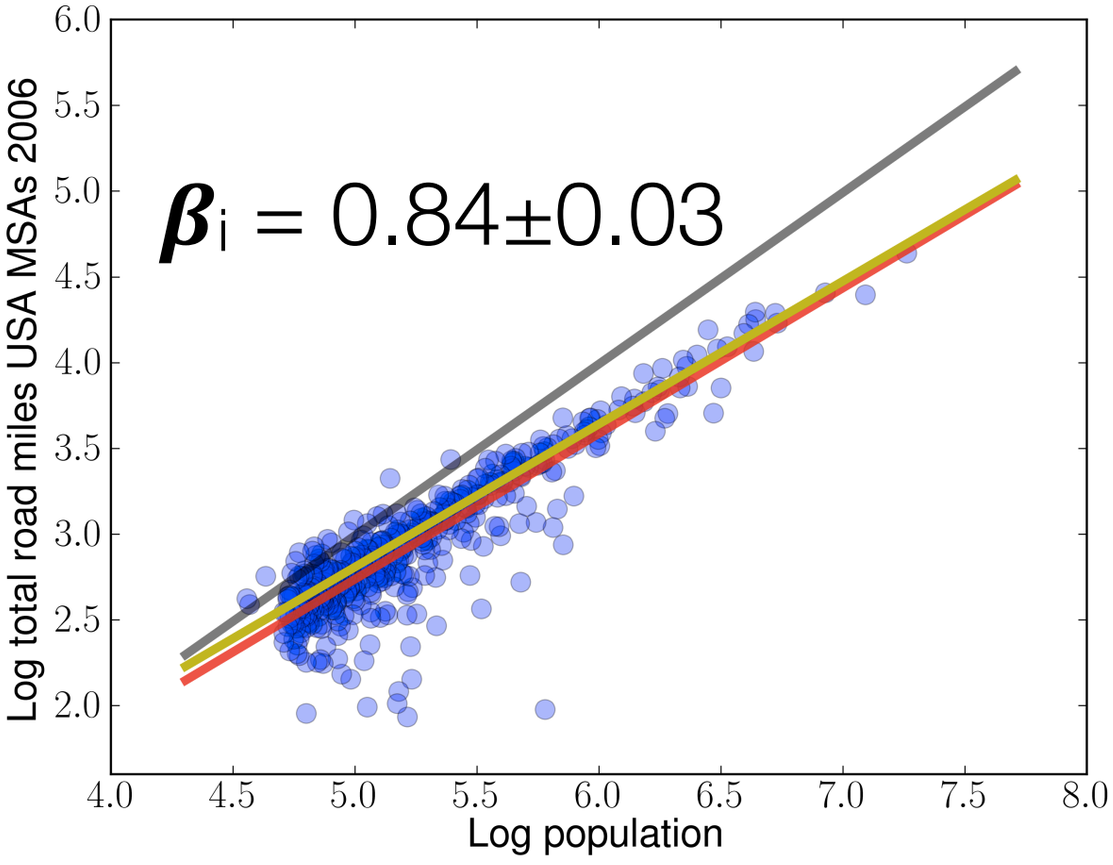
A law in **across quantities** ...

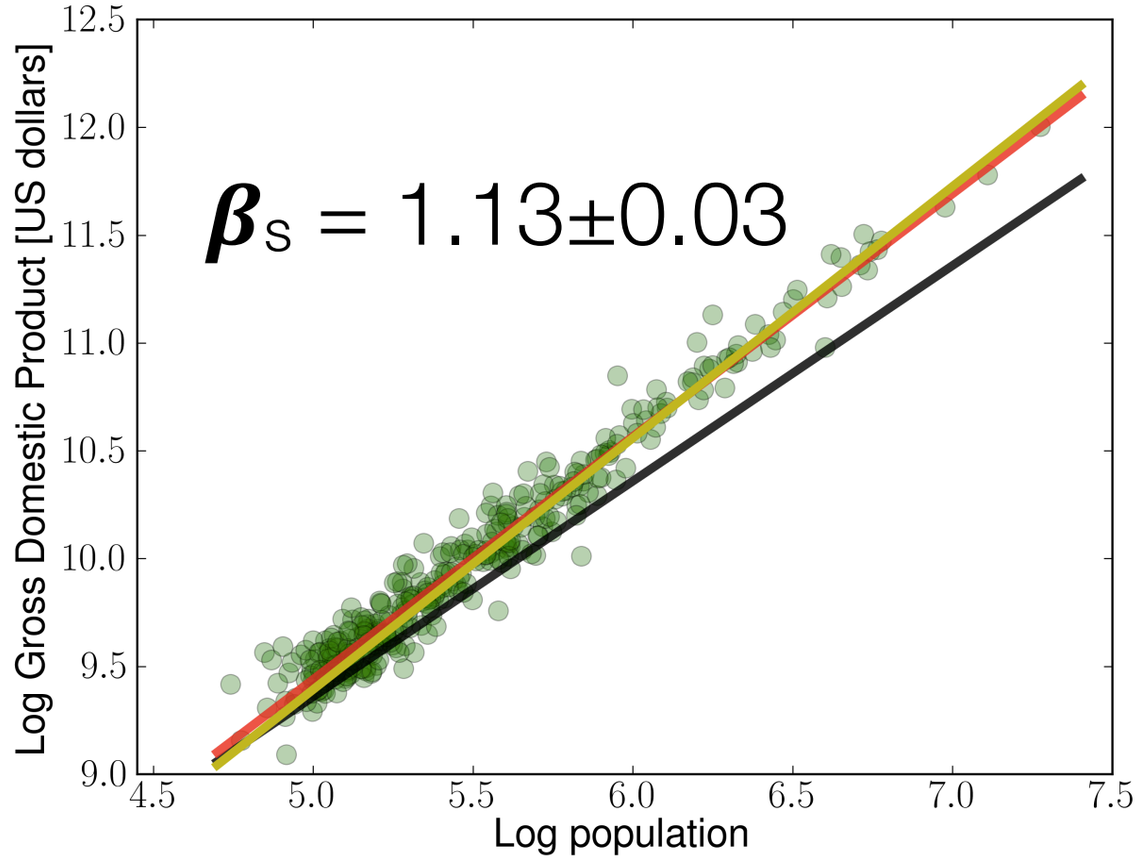


Wages, Crime, Patents, GDP

US Metropolitan Areas

Infrastructure & socioeconomic rates





Volume of Infrastructure

$$\sim N^{\beta_i}$$

$$\beta_i = 1 - \delta$$

$$\delta \simeq 0.15$$

Social Outputs

$$\sim N^{\beta_s}$$

$$\beta_s = 1 + \delta$$


15%
...need I say more?
SANTA FE INSTITUTE

The Origins of Scaling in Cities

Luís M. A. Bettencourt

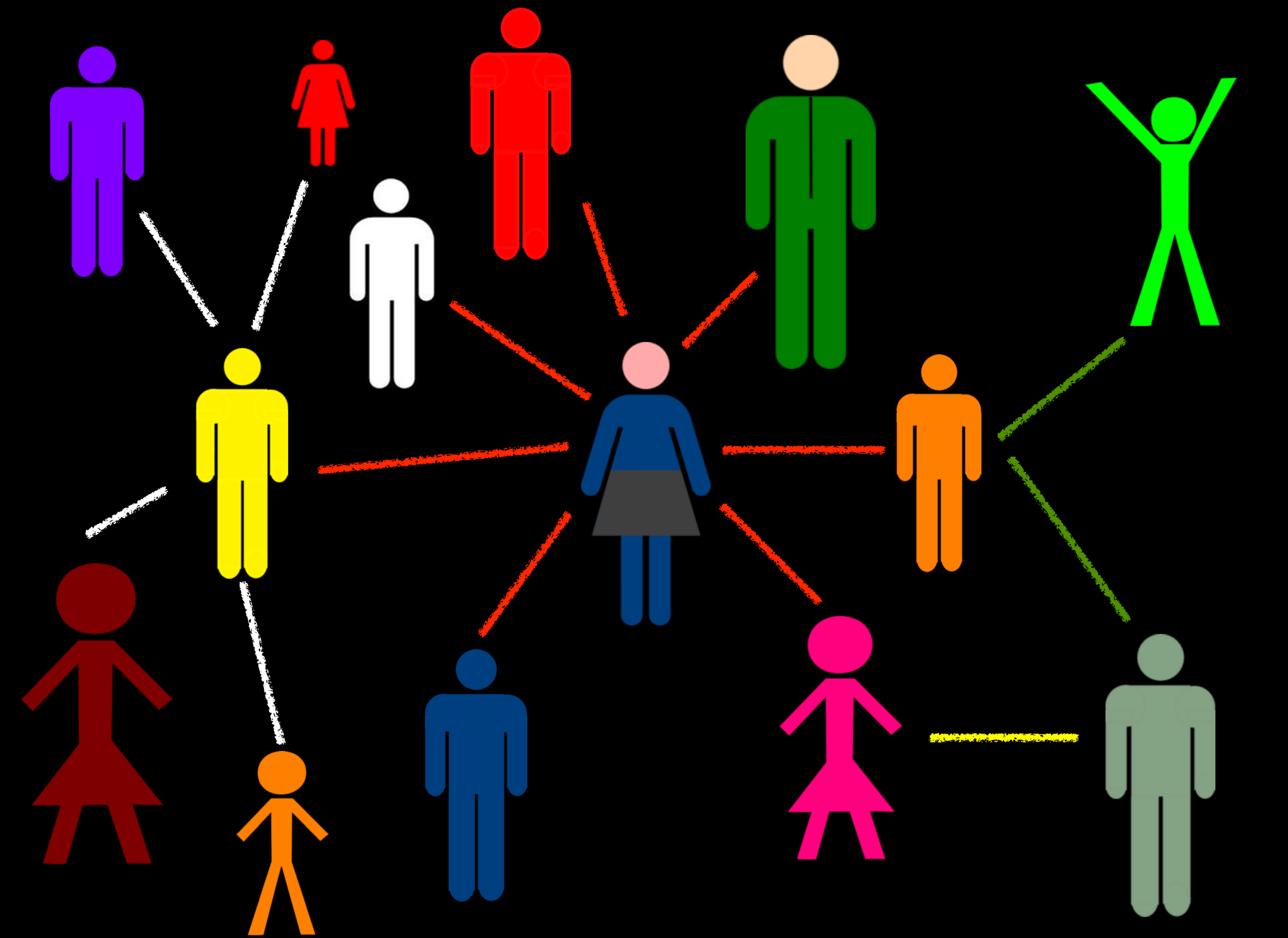
Despite the increasing importance of cities in human societies, our ability to understand them scientifically and manage them in practice has remained limited. The greatest difficulties to any scientific approach to cities have resulted from their many interdependent facets, as social, economic, infrastructural, and spatial complex systems that exist in similar but changing forms over a huge range of scales. Here, I show how all cities may evolve according to a small set of basic principles that operate locally. A theoretical framework was developed to predict the average social, spatial, and infrastructural properties of cities as a set of scaling relations that apply to all urban systems. Confirmation of these predictions was observed for thousands of cities worldwide, from many urban systems at different levels of development. Measures of urban efficiency, capturing the balance between socioeconomic outputs and infrastructural costs, were shown to be independent of city size and might be a useful means to evaluate urban planning strategies.

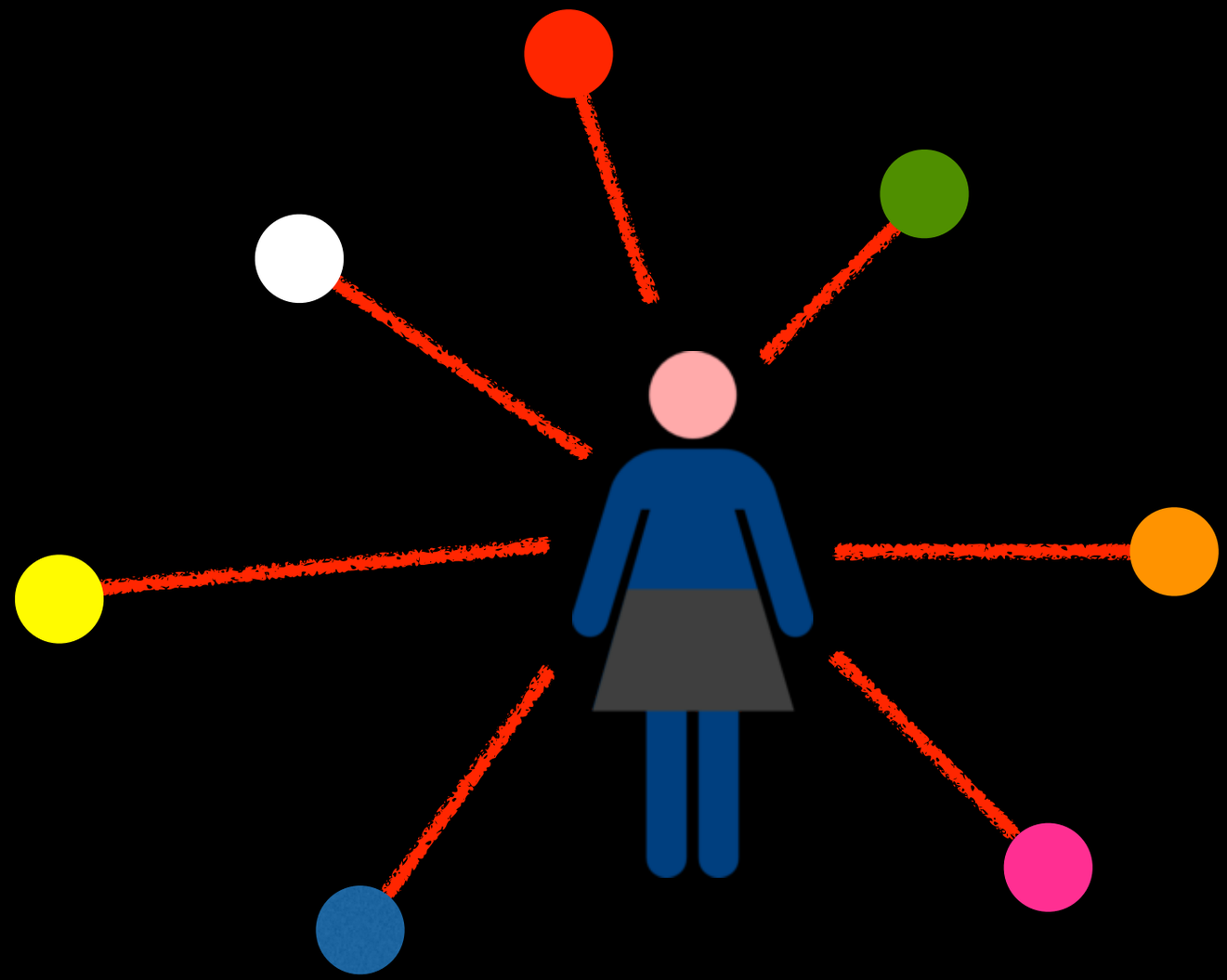
Cities exist, in recognizable but changing forms, over an enormous range of scales (*1*), from small towns with just a few

form rather than function, which limit their ability to help us understand and plan cities.

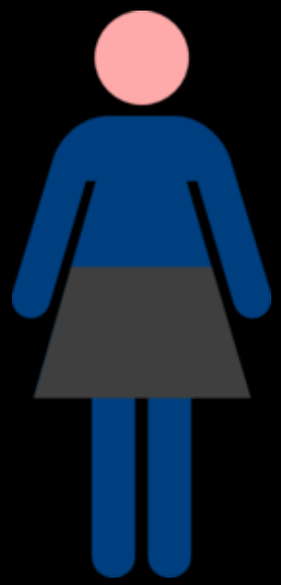
Recently, our increasing ability to collect and

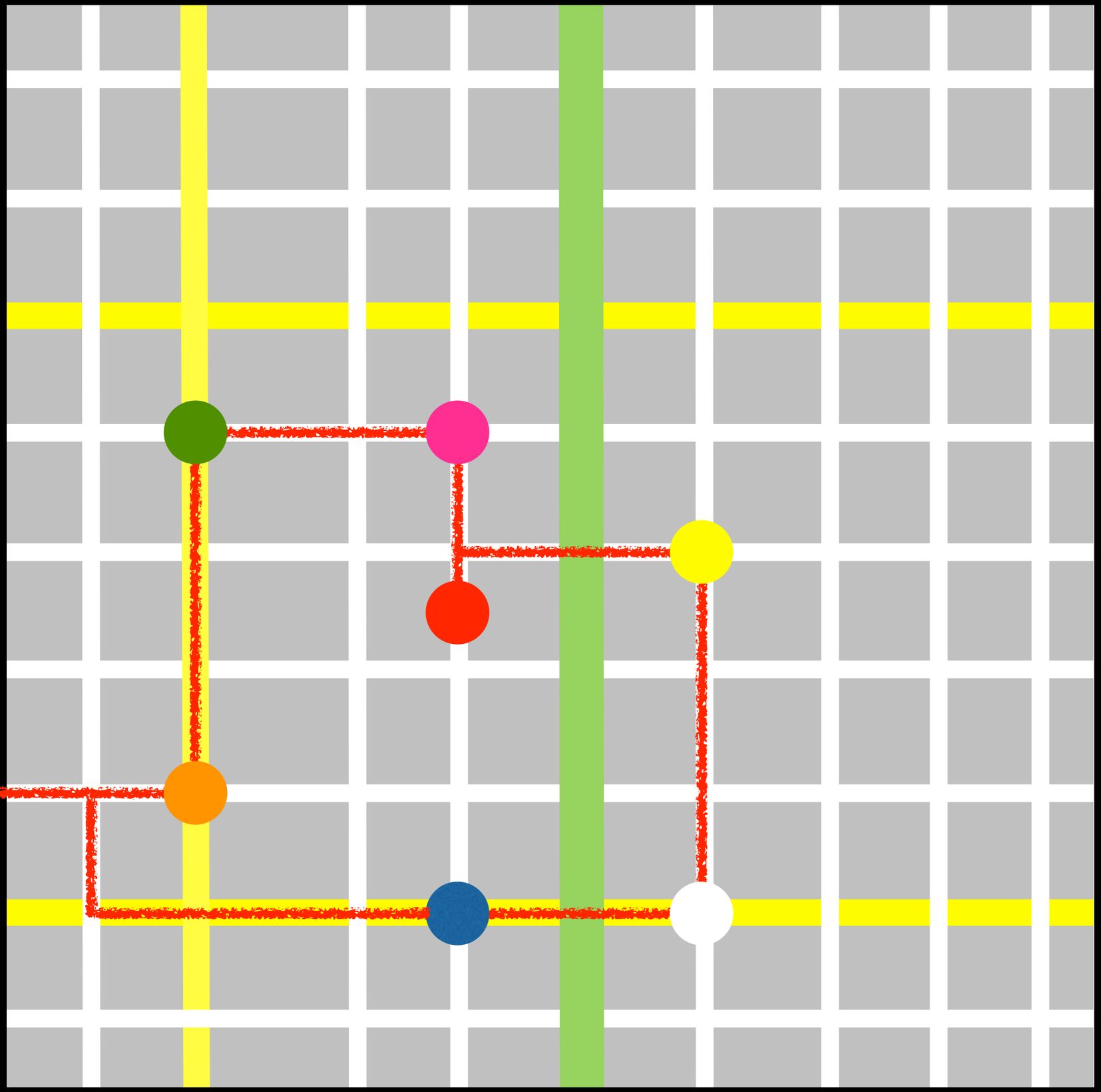
Mumbai Train Station
credit: Randy Olson





Health
Love
Money
Education
Fun
Food
Services
...





Health

Love

Money

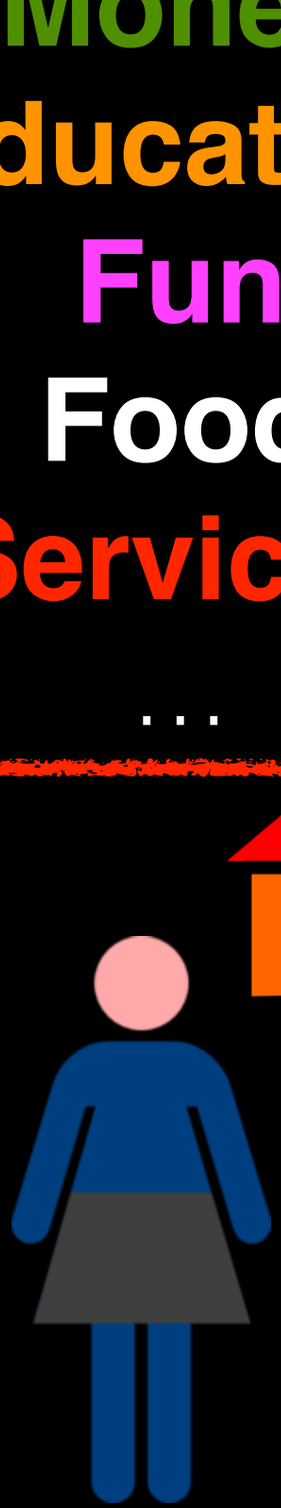
Education

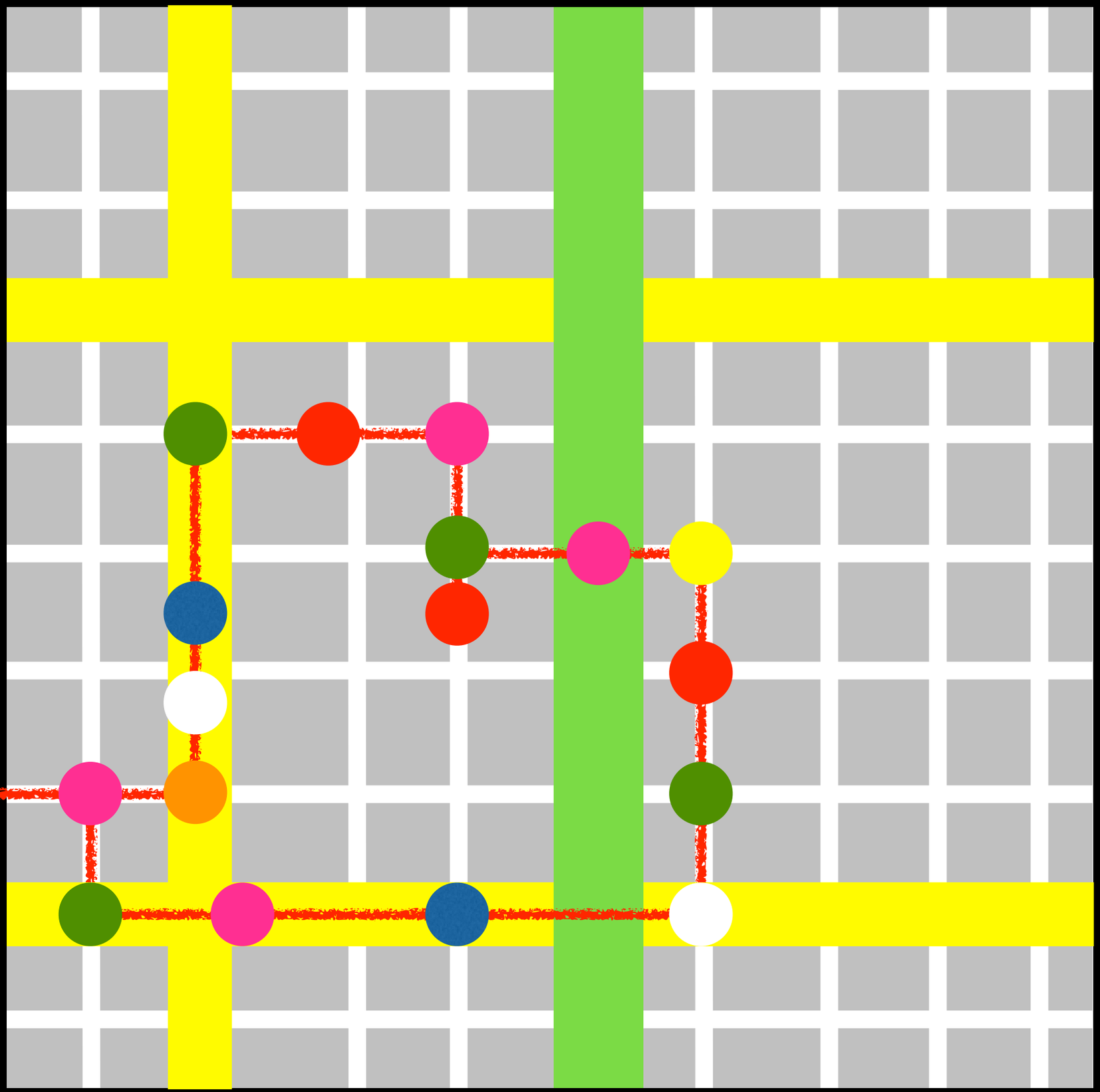
Fun

Food

Services

...





For Principles that Explain Urban Scaling

Cities are social networks

embedded in space-time through infrastructure

1. Mixing Populations

$$A(N) = aN^{\alpha}, \quad \alpha = \frac{D}{D+H} \sim \frac{2}{3}$$

2. Decentralized infrastructure networks

$$A_n(N) \sim \rho^{1/D} N \sim N^{5/6}$$

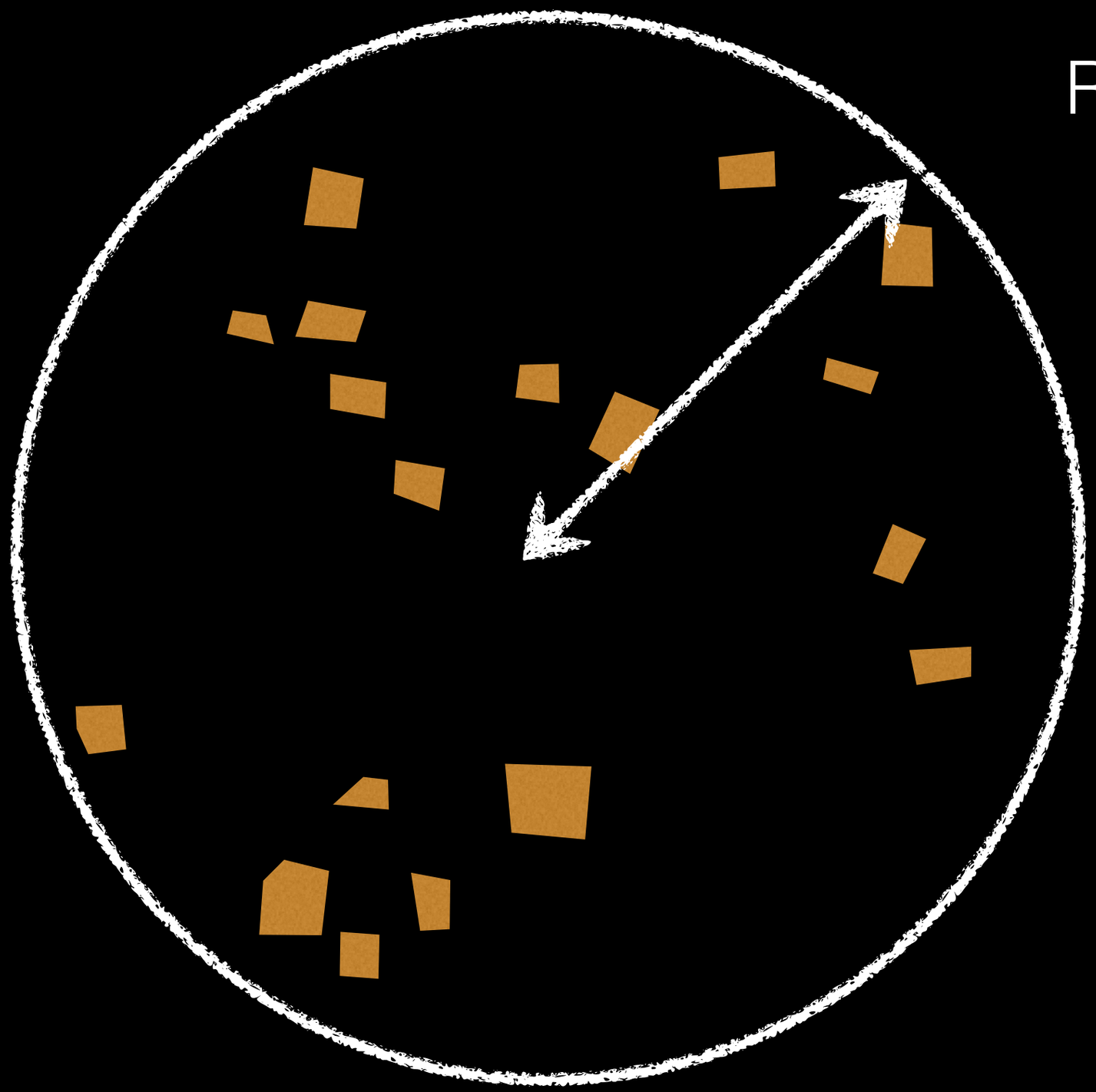
3. Bounded human effort

$$G = (ga_0 l) = \text{constant in } N$$

4. Social outputs reflect local human interactions

$$Y(N) \sim G \frac{N^2}{A_n(N)} \sim N^{7/6}$$

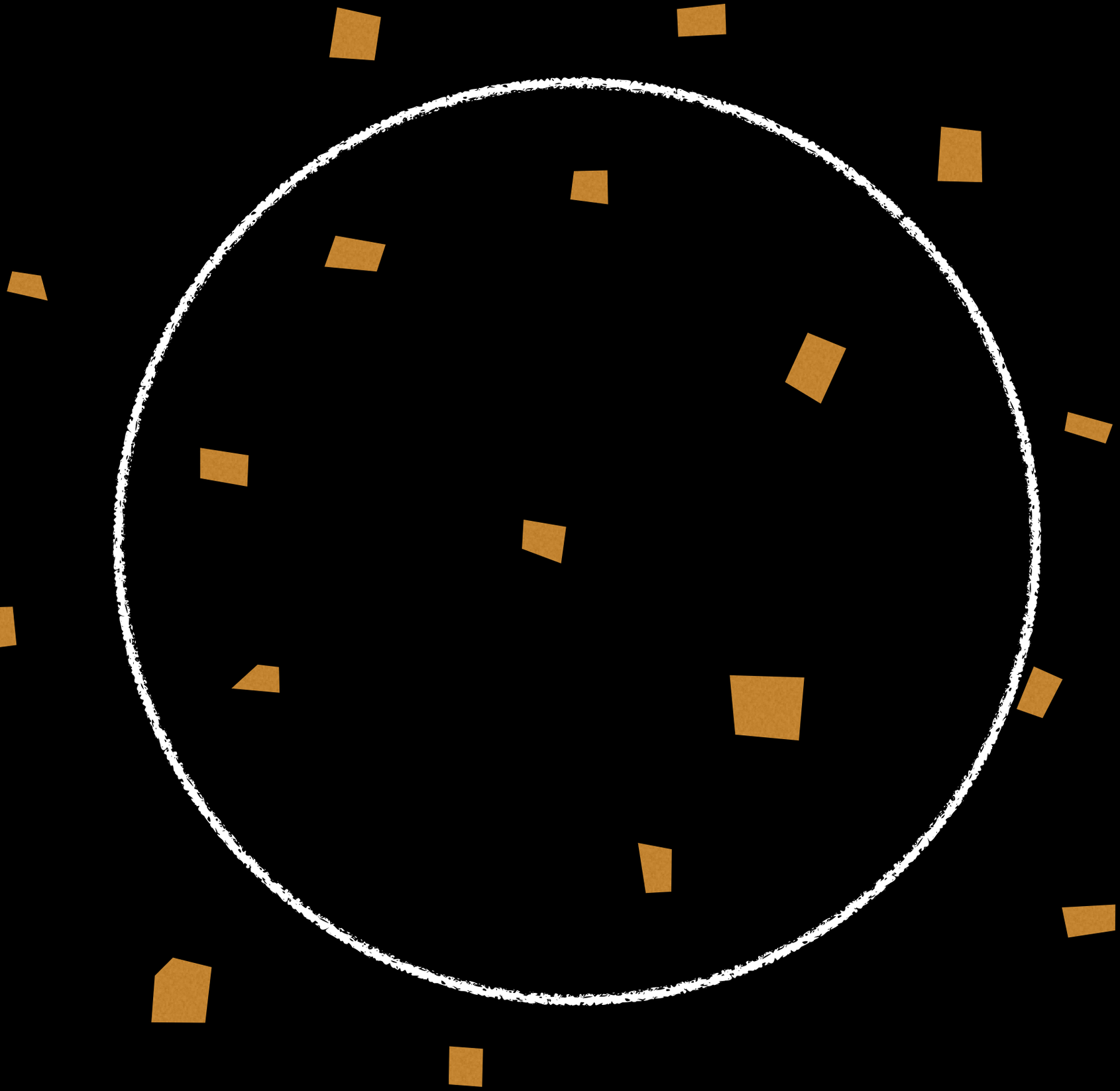
How Social Interactions determine Urban Space



R = Radius

Capilco
Aztec Rural Village

Ortman, Cabaniss, Sturm, Bettencourt, The Prehistory of Urban Scaling (2014)



$G\,N\,/\,A = \text{benefit} \sim \text{cost} = c\,R = c'\,A^{1/2}$

$$A = (\,G/c'\,N\,)^{2/3}$$

credit:Braun & Hogenberg / Ozgur Tufekci

1617

YORKE

Monke Bridge

Boudam

Gotly Gate

S. Maurs

S. Peters

The Lords place

Mary Gate

Laterop Poster

S. Iohns Grene

Ouse Au

Foſe Au.

Ouse Au.

A. Belfrey Church
B. S. Maryes Abbey
C. S. Martines
D. S. Helens
E. Trinite Church
F. S. Andrewes
G. S. Cuthberts
H. S. Sauiours
I. Chriſts Church
K. S. Sampſons
L. Croſe Church
M. Alhallowes
N. S. Michaels
O. S. Maryes
P. S. Denis
Q. S. Margarets
R. S. Georges
S. S. Laurence
T. Wow Gate
V. Trinity Abbey
X. S. Iohns
Y. Cliffords Towre
Z. The Caſtle
3. S. Loyes
4. S Martines
5. S. Mary BiſhopE
6. S. Mary BiſhopY
7. Boudam Baxe

8. Peter Gate
9. Collier Gate
10. Goodram Gate
11. Monke Gate
12. Aldwark
13. S Antonyes Hoſp
14. Connye ſtrete
15. Blake ſtrete
16. Stone Gate
17. Ouſe Bridge
18. Thurſdaies mark
19. Copper Gate
20. Fiſher gate
21. Foſe bridge
22. Iewe Bury

50 | 100 | 150 | 200
A Scale of Paſes

QUARTIER

DE LA SORBONNE

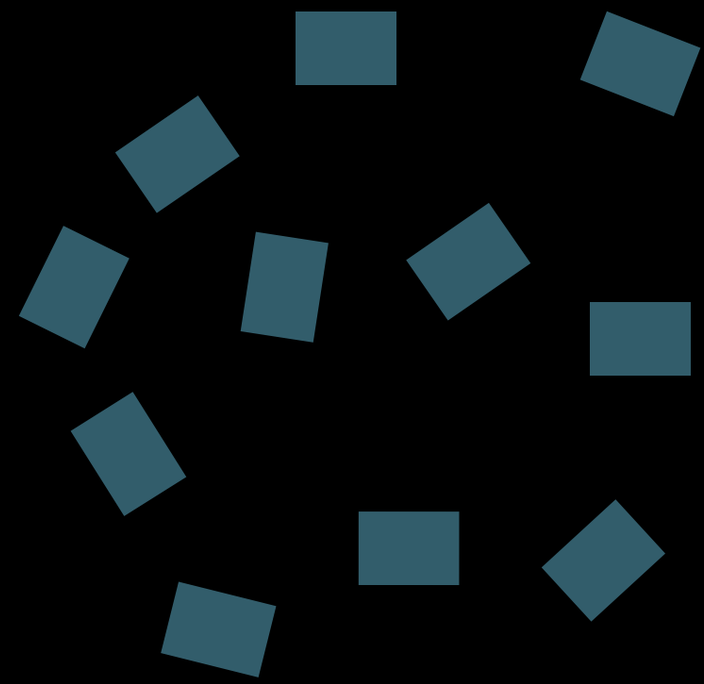
Ilots N.os 1, 2 et 3.

Napoleonic Cadaster of Paris 1810

The Topology of Cities (2015)

Shanghai credit:NASA

credit: bing



The volume of infrastructure grows faster than land

$A_n \sim d\ N \sim N^{5/6}$ $d = (A/N)^{1/2}$

$Y = G\ N\ N/A_n \sim G\ N^{7/6}$

Social interactions accelerate

How Urban Space determines Socioeconomic Outputs

Social Interactions in space and time

Social Interaction Types

$$Y = \sum_{i,j;k} g_k F^k_{ij},$$

Social Network

City “Production Function”

$$\bar{I}_{i,k} = p(k) \int d^D x \Gamma(x) \delta\left(x - x(t)\right) \simeq p(k) \Gamma_n \int dx^D \delta\left(x - x(t)\right)$$

$$= p(k) a_0 \ell \frac{N-1}{V_n} \simeq p(k) a_0 \ell \frac{N}{V_n},$$

Interactions per person increase with city size!

$$Y = G \frac{N^2}{A_n},$$

$$\bar{g} = \sum_k p(k) g_k \qquad G \equiv \bar{g} a_0 \ell.$$

The Scale-Independence of City Size

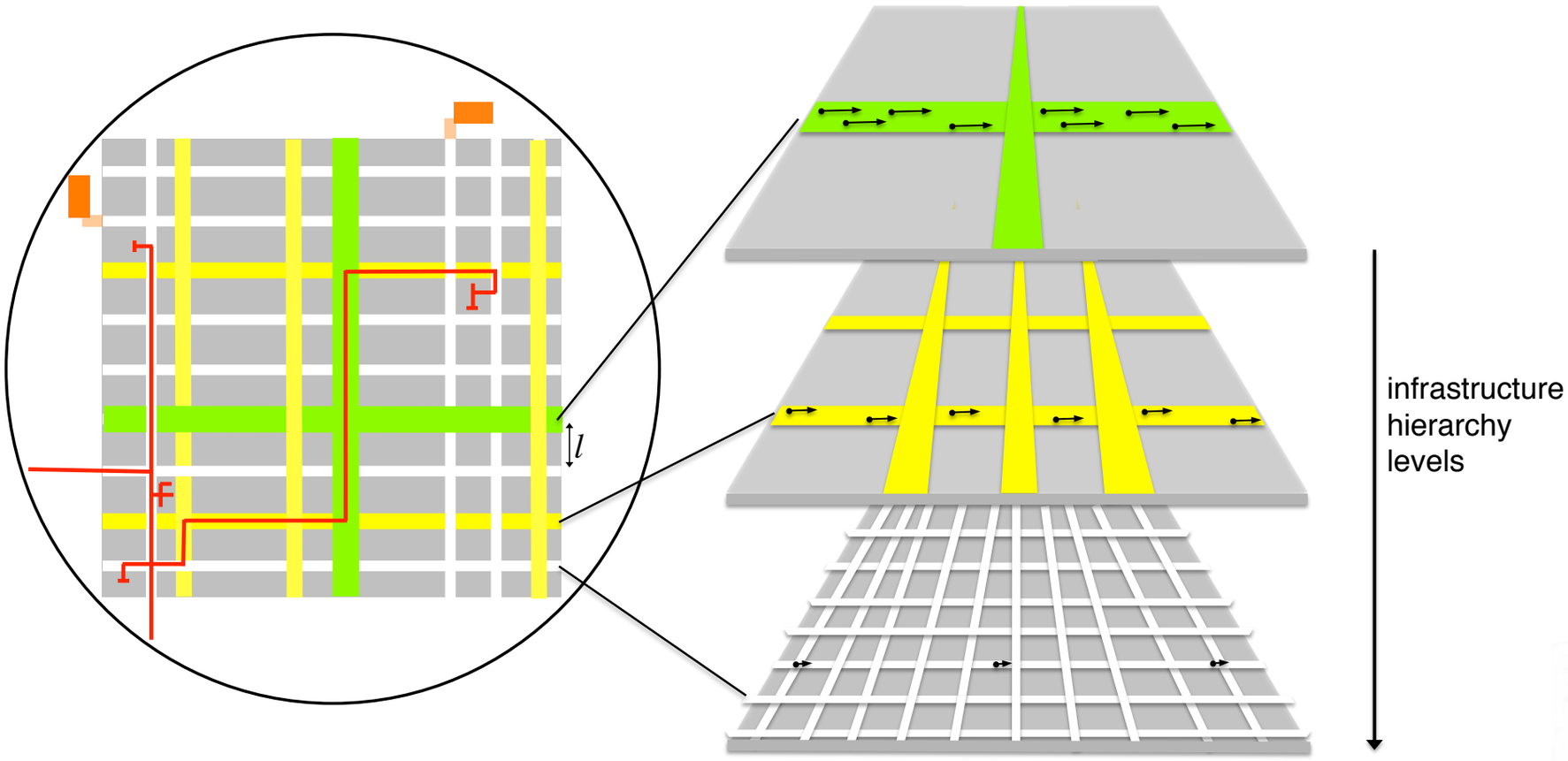
in the presence of increasing returns and transportation costs

1. Detailed Model of Urban Infrastructure

2. General Model of Cost of Transportation in Cities

3. The Properties of Scale-Independent Equilibrium

Infrastructure Networks in the City



$l_i = a_i / l$ length segments

$a_i = ab^{(\alpha-1)i}$ area segments

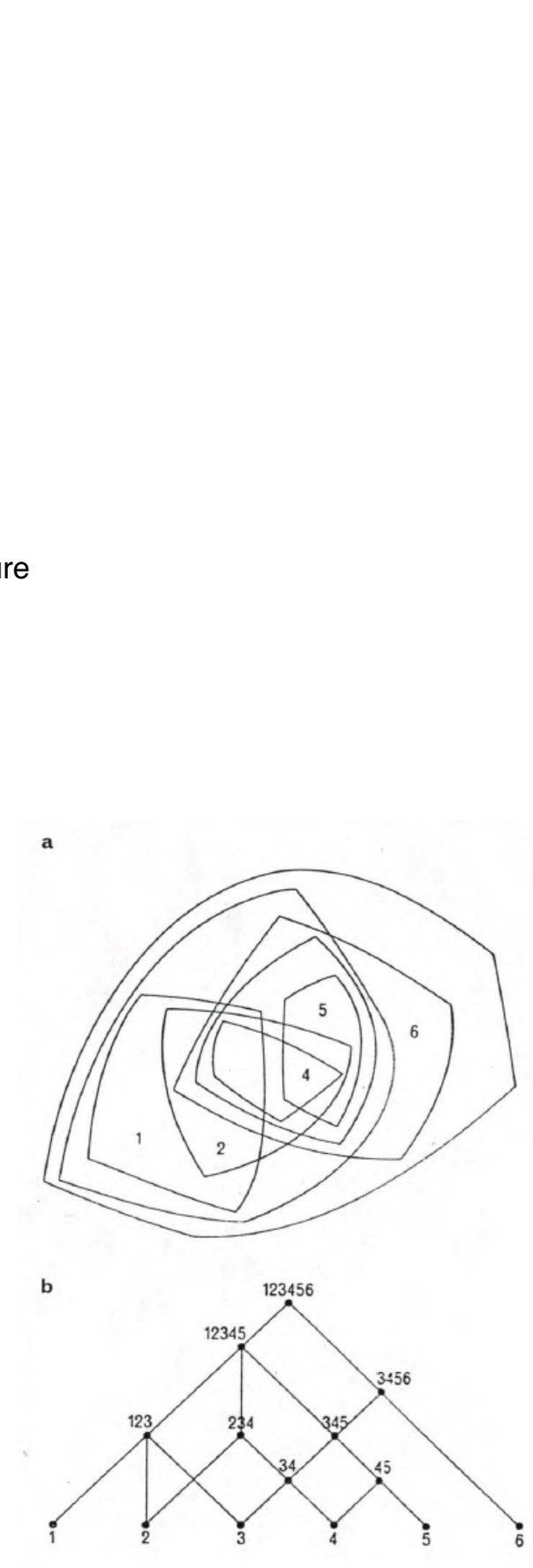
$s_i = s_* b^{(1-\delta)(h-i)}$ width segments

$s_0 >> s_h$

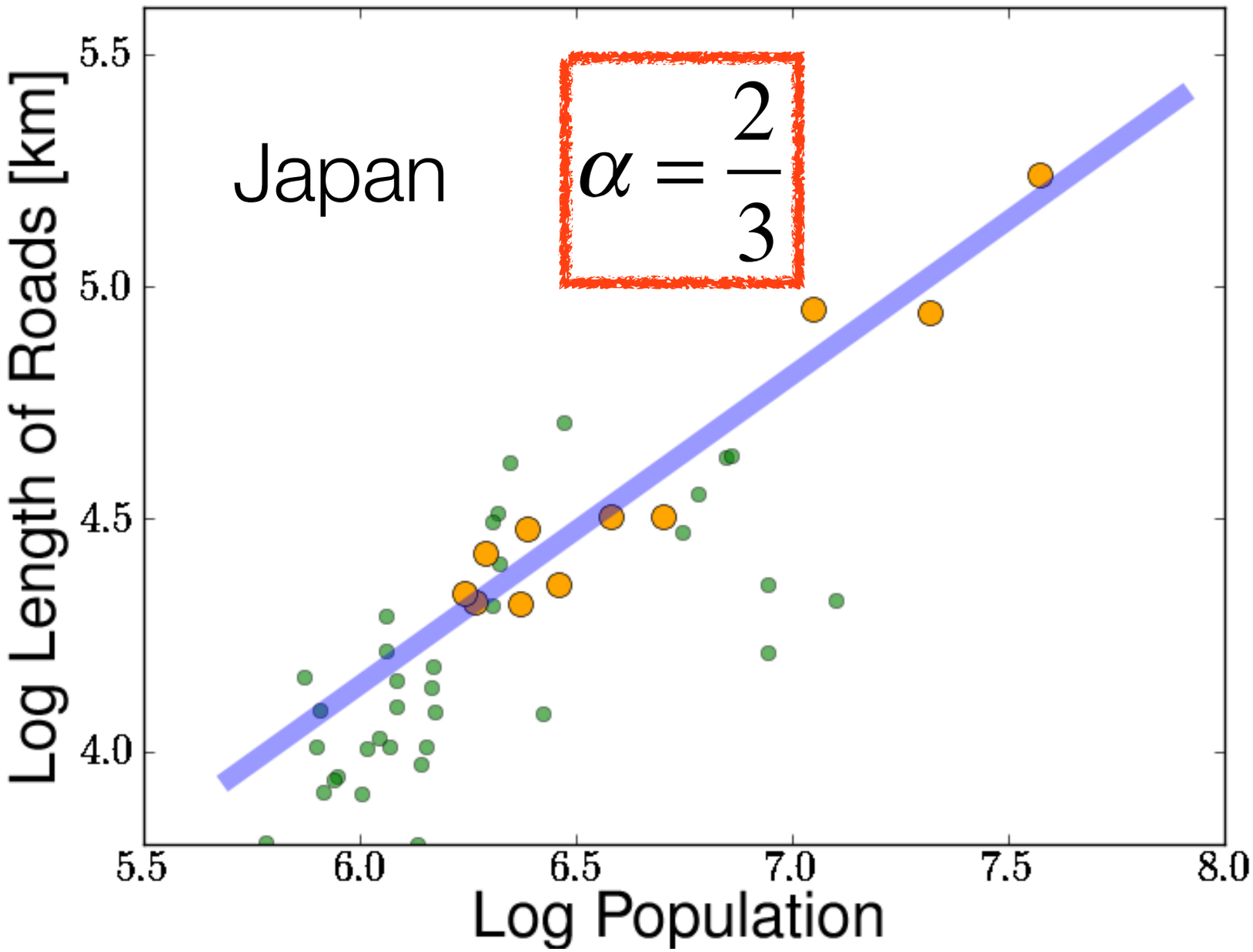
highways are much wider than paths, etc

$$L_n = \sum_{i=0}^{h} l_i N_i = \frac{a}{l} \sum_{i=0}^{h} b^{\alpha i} = \frac{a}{l} \frac{b^{\alpha(h+1)} - 1}{b^{\alpha} - 1} \simeq L_0 N^{\alpha}, \quad L_0 = a / l,$$

$$A_n = \sum_{i=0}^{h} s_i l_i N_i = s_* \frac{a}{l} b^{(1-\delta)h} \sum_{i=0}^{h} b^{(\alpha+\delta-1)i} \simeq A_0 N^{1-\delta}, \quad A_0 = \frac{s_* a}{l(1-b^{\alpha+\delta-1})},$$



C. Alexander, 1966


Japan
$\alpha = \frac{2}{3}$
Log Length of Roads [km]
Log Population
5.5
5.0
4.5
4.0
5.5
6.0
6.5
7.0
7.5
8.0

The Cost of Socializing in the City

Conservation of Current across infrastructural levels

$$J_i = s_i \rho_i v_i N_i = s_{i-1} \rho_{i-1} v_{i-1} N_{i-1} = J_{i-1}$$

$$\rho_i v_i = b^{\delta(h-i)} \rho_* v_*$$

$$J_i = J = J_0 N, \text{ with } J_0 = s_* \rho_* v_*$$

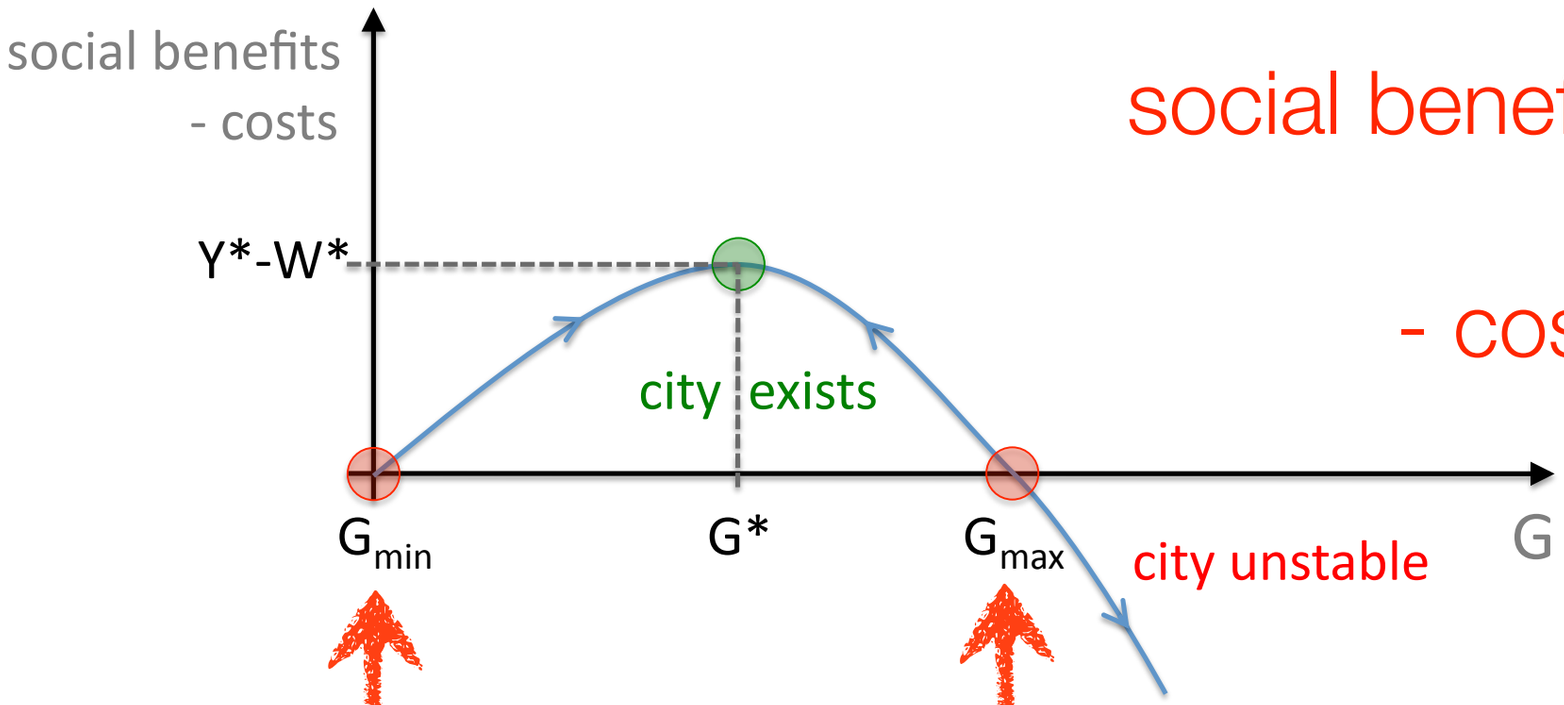
Resistance accounts for Cost of Movement:

$$r_i = r \frac{l_i}{s_i} \qquad R_i = \frac{r_i}{N_i} = \frac{ar}{ls_*} b^{-(1-\alpha+\delta)i-(1-\delta)h}$$

$$W = J^2 \sum_{i=1}^{h} R_i = J^2 \frac{ar}{ls_*} b^{-(1-\delta)h} \frac{1-b^{-(1-\alpha+\delta)(h+1)}}{1-b^{-1+\alpha-\delta}} \simeq W_0 N^{1+\delta}, \quad W_0 = \frac{arJ_0^2}{ls_*(1-b^{-1+\alpha-\delta})},$$

Cost of Transportation scales like Benefits !!

Equilibrium between social benefits and costs



social benefits: $Y(N) \sim G\frac{N^2}{A_n(N)} \sim N^{7/6}$

- costs: $W(N) \sim W_0\frac{I^2}{A_n(N)} \sim N^{7/6}$

$Y_0 \sim G^{1-\alpha}$ and $W_0 \sim G^{\alpha}$

Congestion/Danger

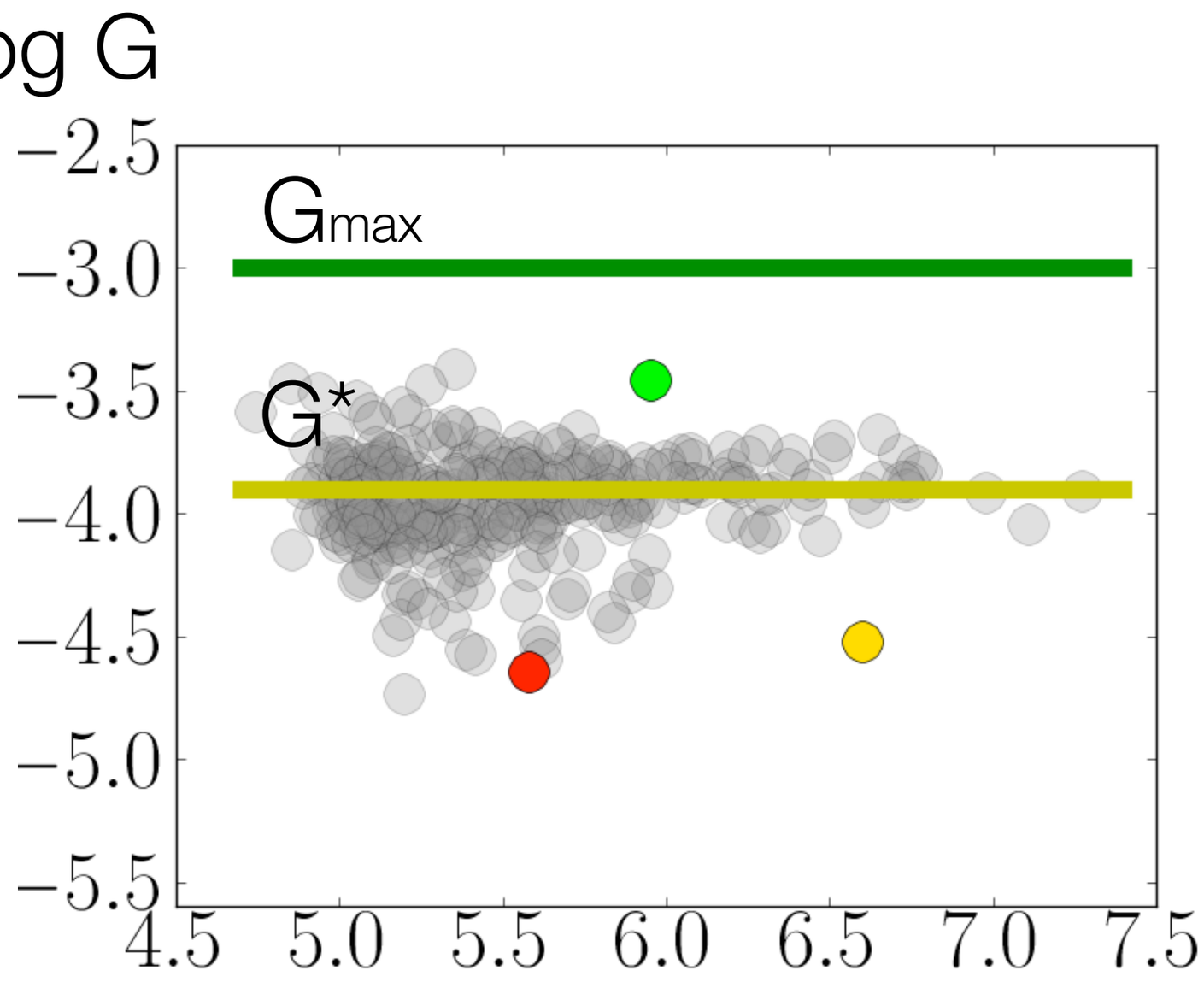
High Costs/Dispersion

Human Effort is conserved: Estimating G

The parameter G measures the integrated strength in interactions.

For a city it is:

$$G = \left(\frac{Y}{N}\right) \cdot \left(\frac{A_n}{N}\right)$$



Predictions and Consequences

Urban Scaling Relations	Model (D=2,H=1)	Model (D, H)	Effect
Land area $A = aN^{\alpha}$	$\alpha = \frac{2}{3}$	$\alpha = \frac{D}{D+H}$	spatial densification
Network volume $A_n = A_{n0}N^{\nu}$	$\nu = \frac{5}{6}$	$\nu = 1 - \delta = \frac{D^2+DH-H}{D(D+H)}$	growth of infrastructure
Network length $L = L_0N^{\lambda}$	$\lambda = \frac{2}{3}$	$\lambda = \alpha$	area filling networks
Average network width $\bar{S} = \bar{S}_0N^{\bar{\sigma}}$	$\bar{\sigma} = \frac{1}{6}$	$\bar{\sigma} = 1 - \delta$	widening of roads
Interactions per capita $y = Y_0N^{\delta}$	$\delta = \frac{1}{6}$	$\delta = \frac{H}{D(D+H)}$	increased interactions
Socioeconomic rates $Y = Y_0N^{\beta}$	$\beta = \frac{7}{6}$	$\beta = 1 + \delta = \frac{D^2+DH+H}{D(D+H)}$	acceleration of social rates
Power dissipation $W = W_0N^{\omega}$	$\omega = \frac{7}{6}$	$\omega = 1 + \delta$	increased congestion
Land Value $P_L = P_0N^{\delta_L}$	$\delta_L = \frac{1}{2}$	$\delta_L = \alpha - \delta$	increased land rents

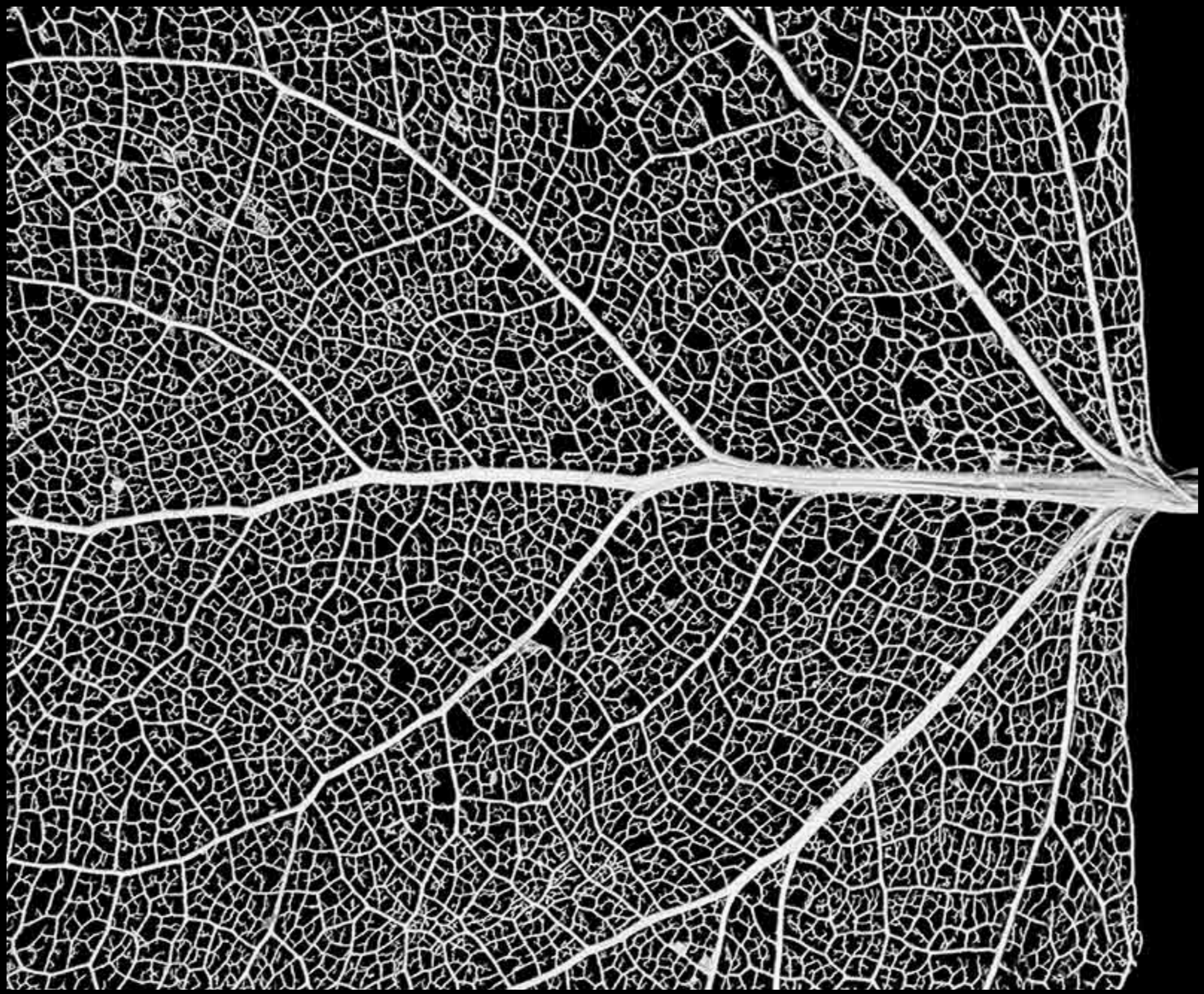
Scaling and Interdependence

of

Social, Economic, Infrastructural and Geographic factors

Bettencourt (2013) Science

Are cities like **other** complex systems?



credit: Royce Bair

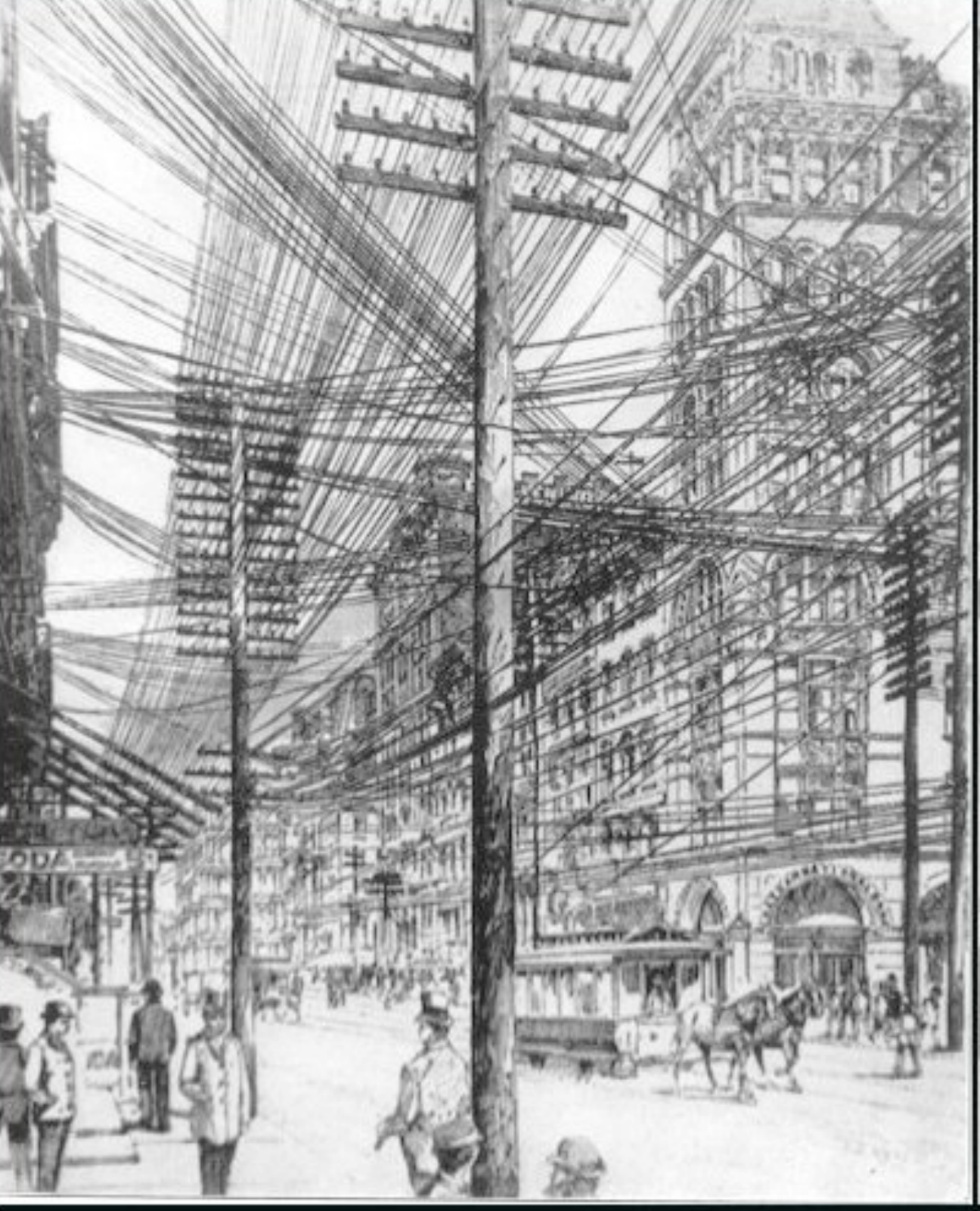

credit: NASA

credit: www.mostbeautifulthings.net


DEDMAXOPKA.LIVEJOURNAL.COM


credit: MacLean

credit: Robert Stone



1890s New York City
credit: www.loper-os.org

Hong Kong
credit:Guardian


1876
1932
1988
2013

New York, N = 19.6Mio.

Building heights [m]
200
100
50
20
10
5

N
0 5 10 km

Los Angeles, N = 12.8Mio.

N
0 5 10 km

Chicago, N = 9.5Mio.

N
0 5 10 km

Boston, N = 4.6Mio.

N
0 5 10 km

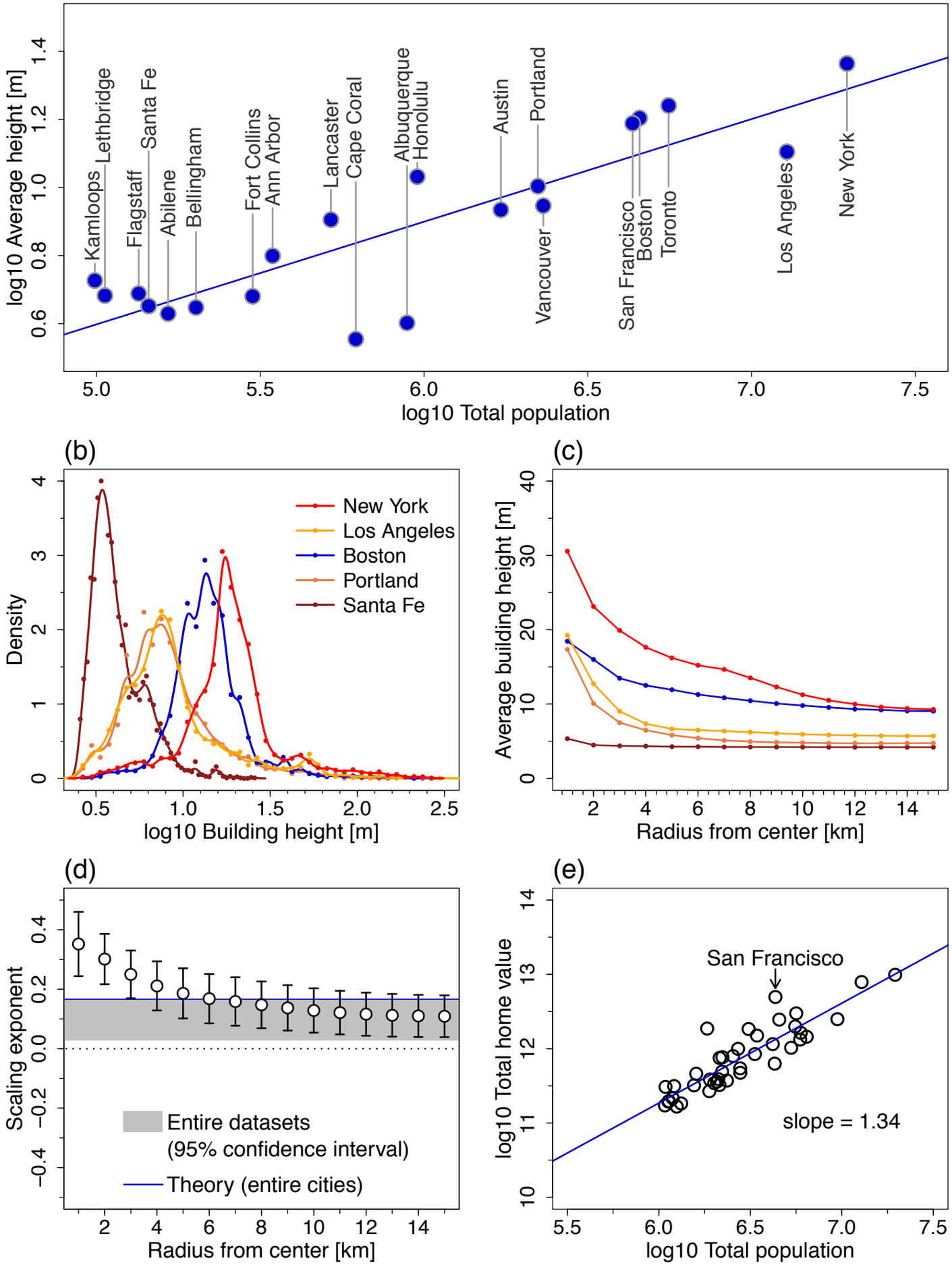
Portland, N = 2.2Mio.

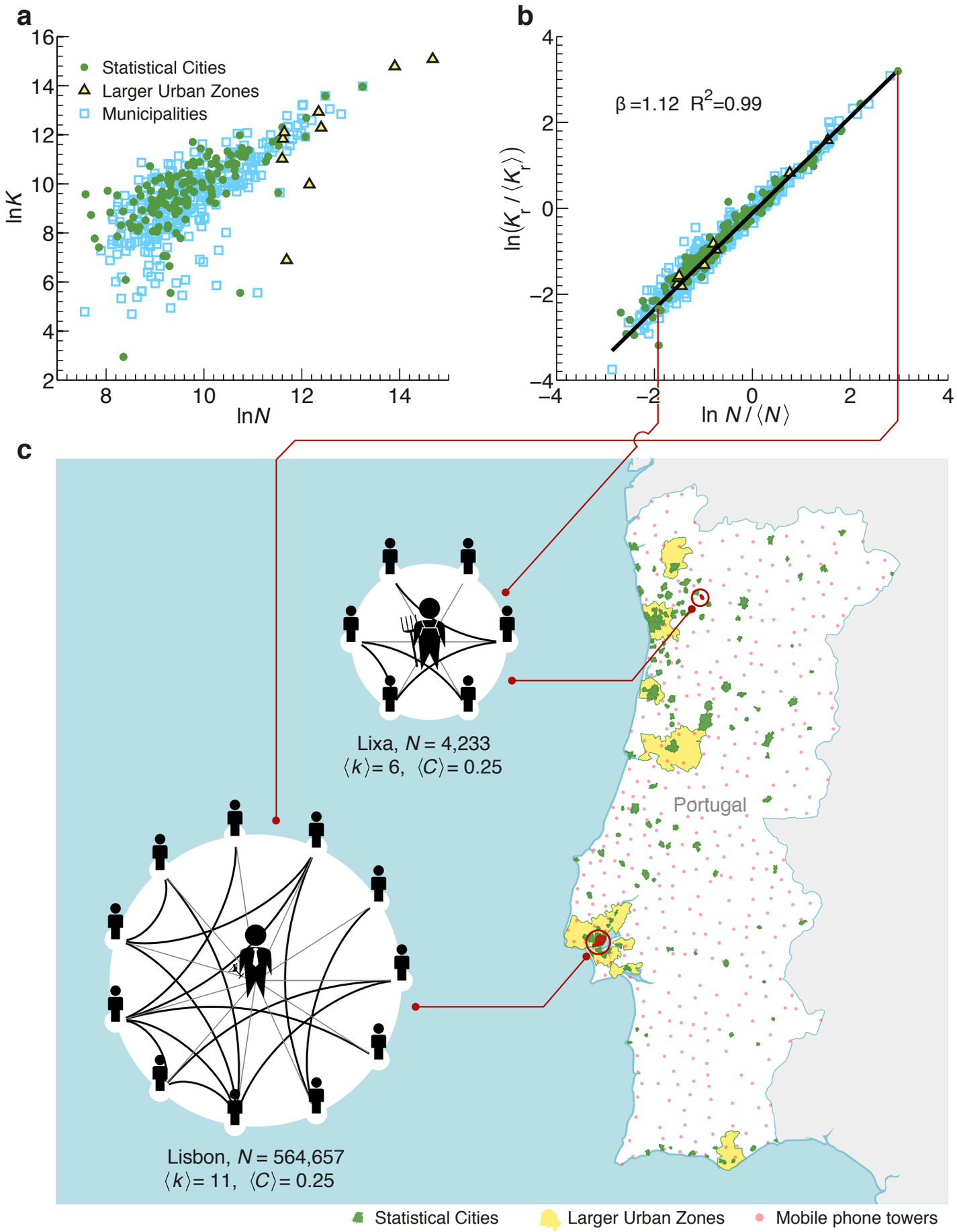
N
0 5 10 km

Santa Fe, N = 0.1Mio.

N
0 5 10 km

Schlaepfer, Lee, Bettencourt (2016)
Urban Skylines: building heights and shapes as measures of city size


log10 Average height [m]
log10 Total population
Kamloops
Lethbridge
Flagstaff
Santa Fe
Abilene
Bellingham
Fort Collins
Ann Arbor
Lancaster
Cape Coral
Albuquerque
Honolulu
Austin
Portland
Vancouver
San Francisco
Boston
Toronto
Los Angeles
New York
(b)
Density
log10 Building height [m]
New York
Los Angeles
Boston
Portland
Santa Fe
(c)
Average building height [m]
Radius from center [km]
(d)
Scaling exponent
Entire datasets
(95% confidence interval)
Theory (entire cities)
Radius from center [km]
(e)
log10 Total home value
San Francisco
slope = 1.34
log10 Total population



Urban cellphone networks

$\beta = 1.12\text{-}1.19$

in agreement with theory.

Network clustering is preserved:

same sense of community in town and country!

Schläpfer et al, 2014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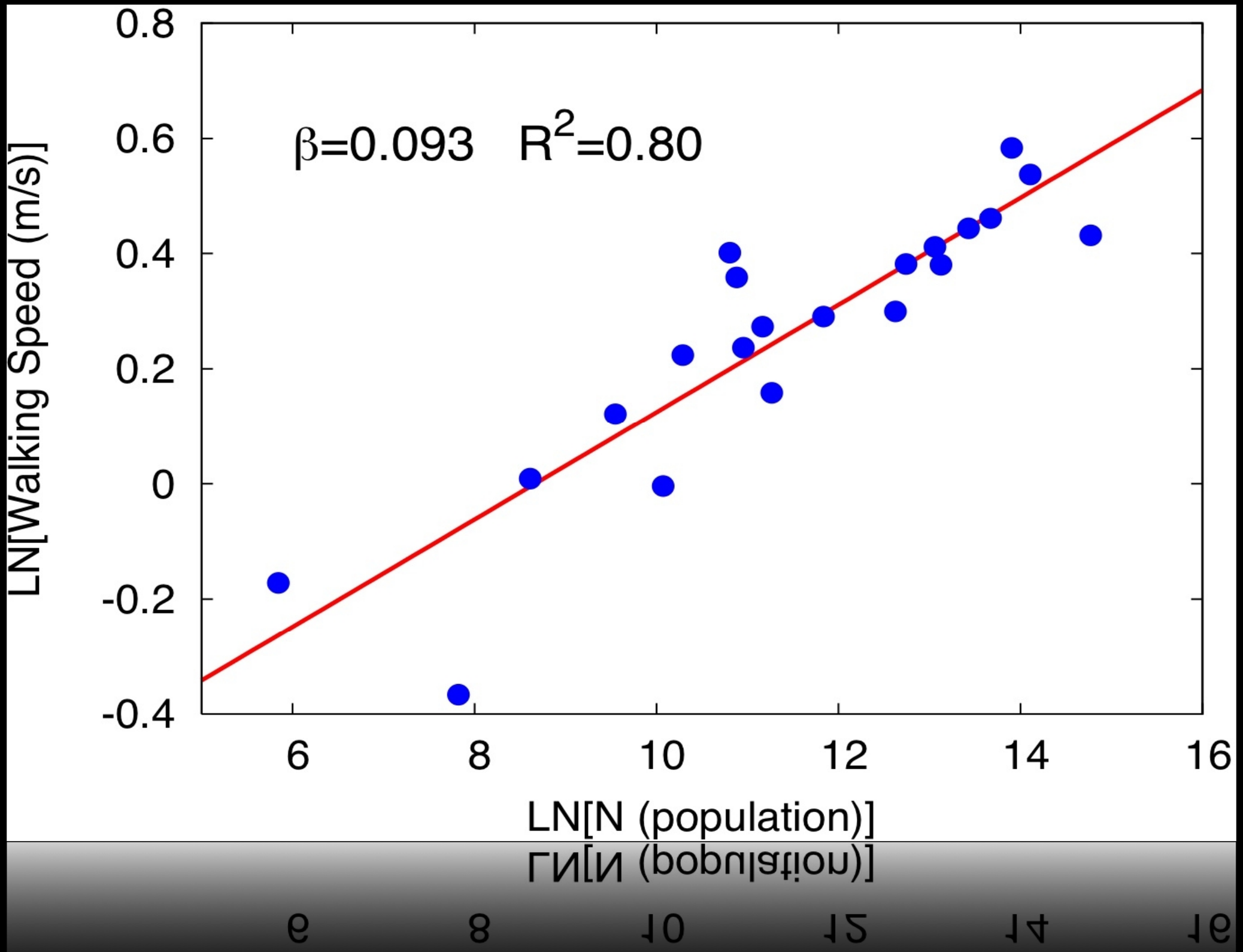
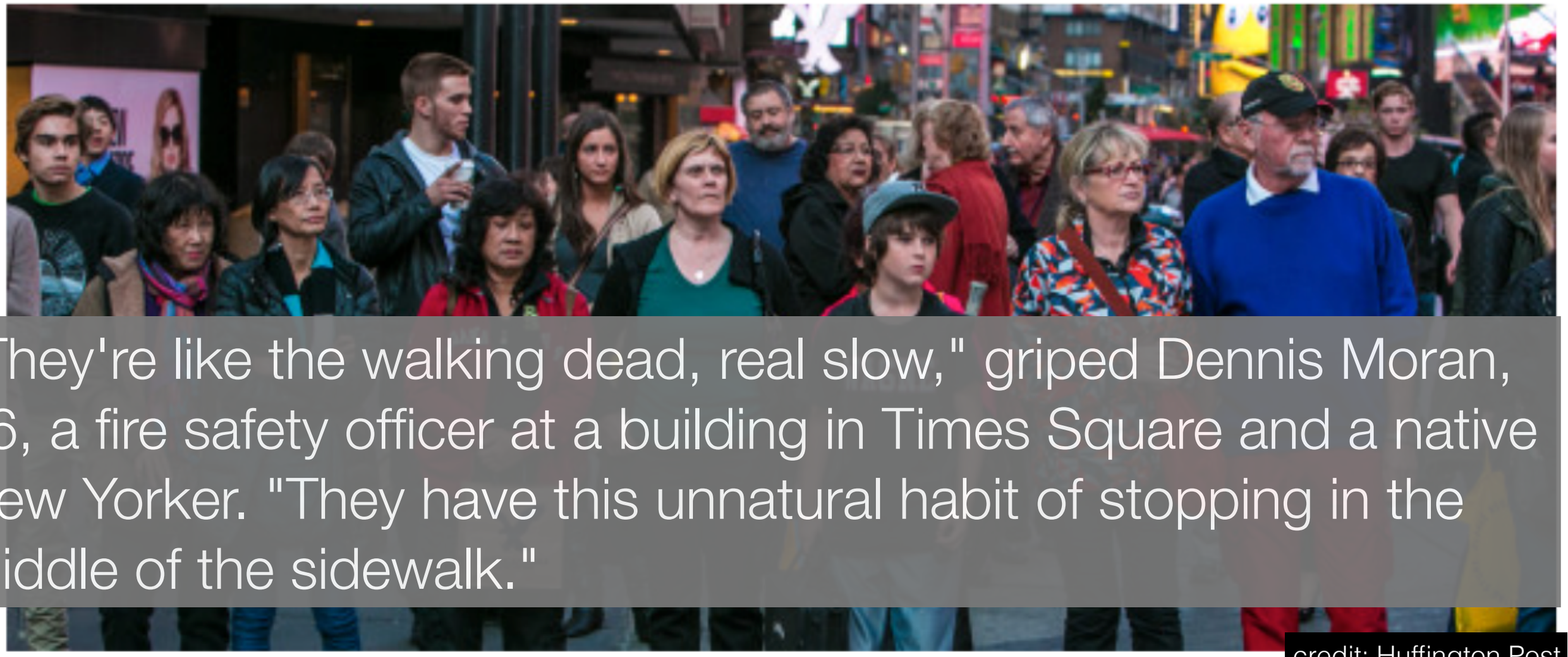
Walking Speed vs Population Size



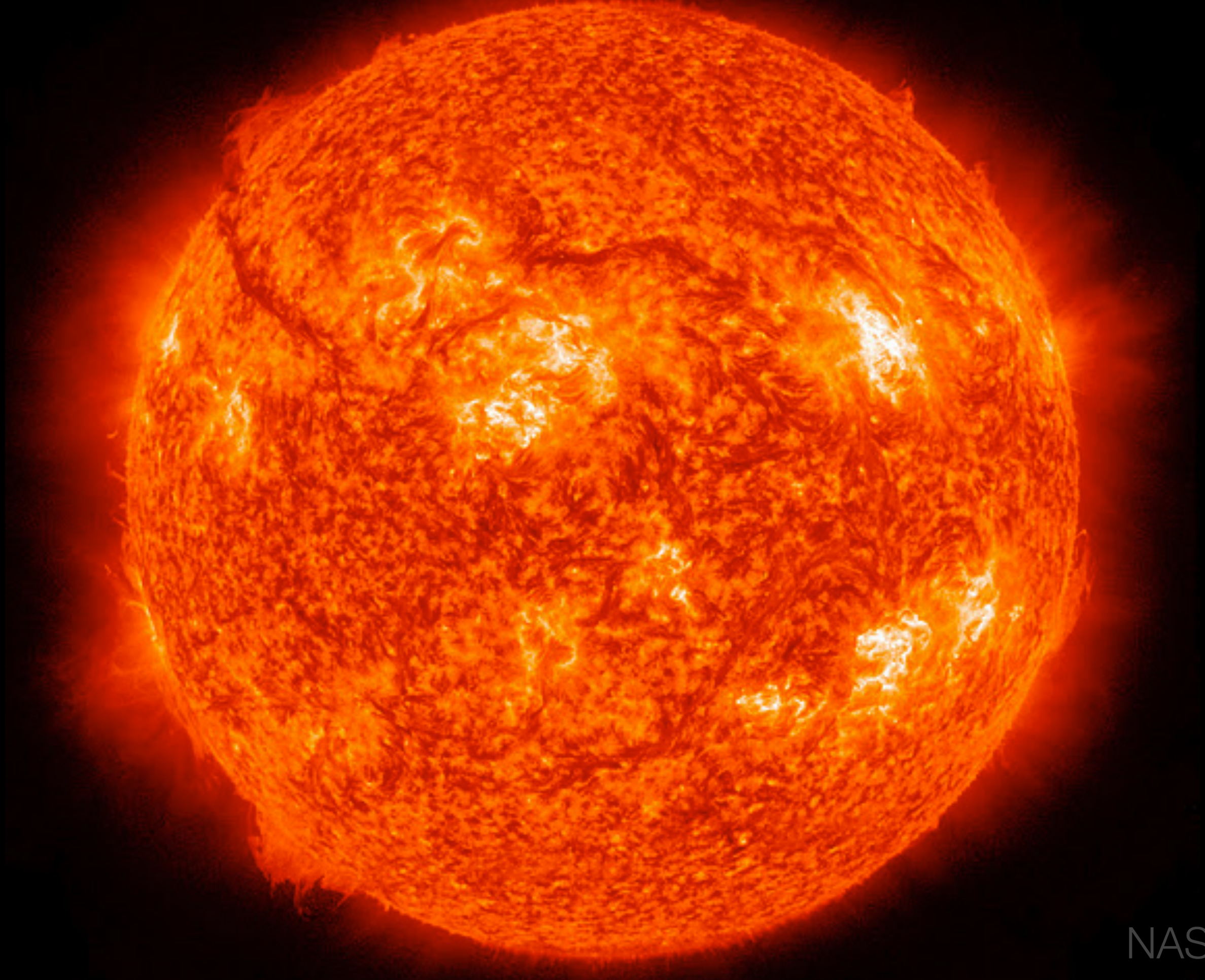
NYC Tourists Are 'Like Walking Dead,' Anger Fast-Paced New Yorkers During Holiday Season

By JAKE PEARSON 12/12/13 01:07 PM ET EST AP



"They're like the walking dead, real slow," griped Dennis Moran, 46, a fire safety officer at a building in Times Square and a native New Yorker. "They have this unnatural habit of stopping in the middle of the sidewalk."

credit: Huffington Post


NASA

Cities are

general-purpose

Social Reactors

“Super-Superlinear" Scaling of the Internet

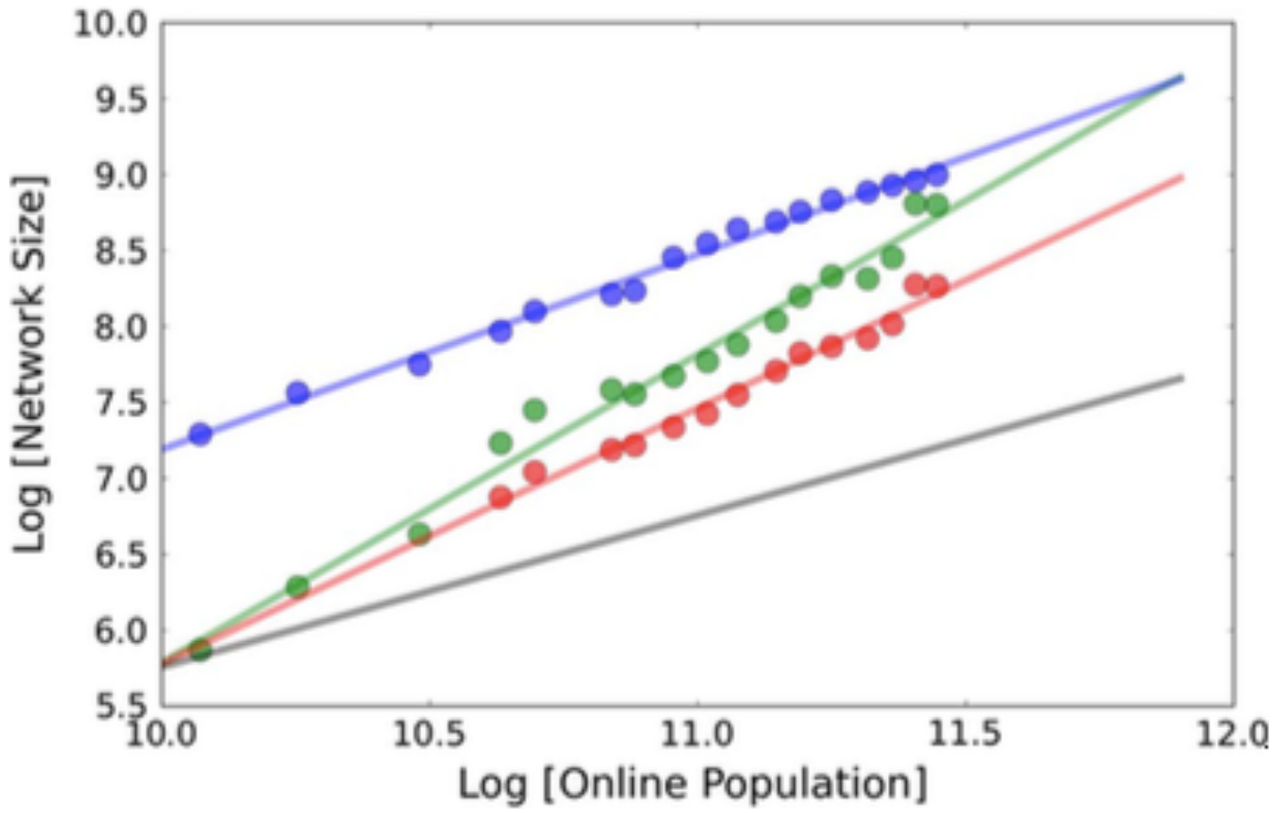


Fig. 3. ***Scaling of global computer networks with online population size. The size of the Internet, measured in terms of DNS hosts (blue) is characterized by an exponent 1.28 (95% CI*** $= [1.22, 1.34]$***, $R^2 = 0.99$), while the growth of the WWW, in terms of an estimate of total webpages (green), is characterized by an exponent 2.03 (95% CI*** $= [1.88, 2.17]$***, $R^2 = 0.98$) and of active pages (red) by an exponent 1.68 (95% CI*** $= [1.55, 1.82]$ ***$R^2 = 0.98$). In all cases, the size of online networks has been growing superlinearly with the number of Internet users, indicating that more pages and more computation is effectively used per capita as the network grows, much like in other open-ended social systems (e.g., cities). Exponents are manifestly different from those observed for cities.***

Bettencourt, IEEE, 2014

THE INTERNET DOESN'T MAT-
TER, YOU LIVE IN NEW YORK

86/100

credit: GODMODE