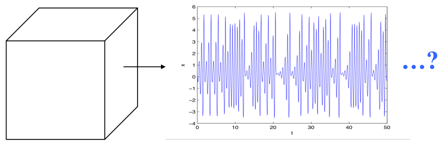
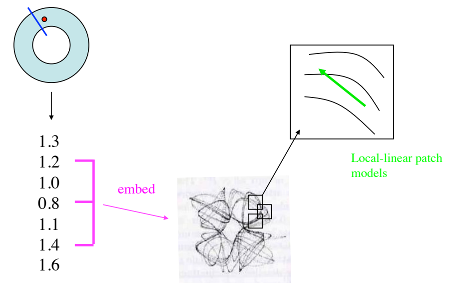
Prediction



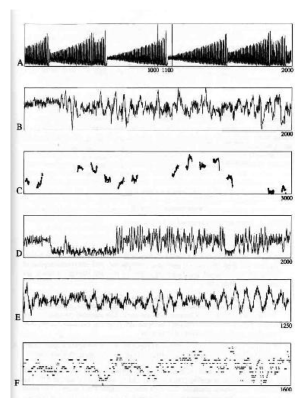
Predicting the path of a roulette ball…



The Eudaemonic Pie
(or The Newtonian Casino)

The Santa Fe competition

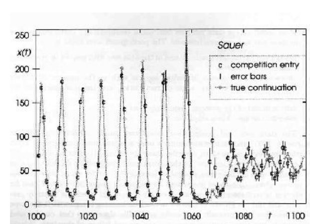
- Weigend & Gershenfeld, 1992
- put a bunch of data sets up on an ftp server
- and invited all comers to predict their future
- chronicled in *Time Series Prediction: Forecasting the Future and Understanding the Past*, Santa Fe Institute, 1993 (from which the images on the following half-dozen slides were reproduced)



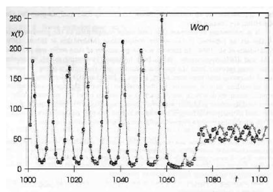
The Santa Fe competition: data

- Laboratory laser
- Medical data (sleep apnea)
- Currency rate exchange
- RK4 on some chaotic ODE
- Intensity of some star
- A Bach fugue

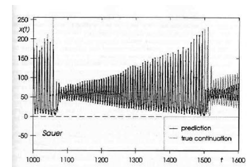
Embedding + patch models: (Sauer)



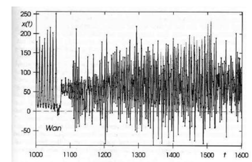
Neural net: (Wan)



Further out:

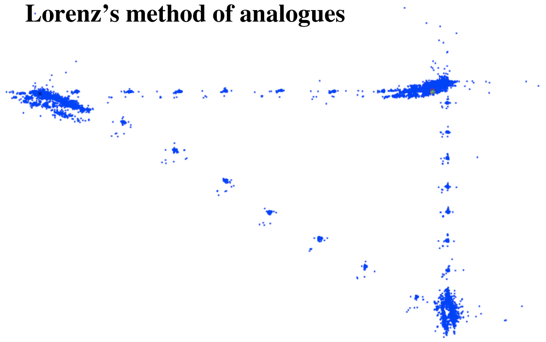


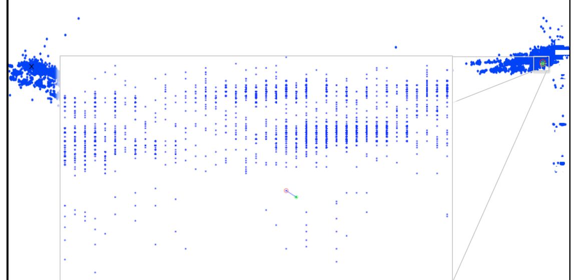
Sauer



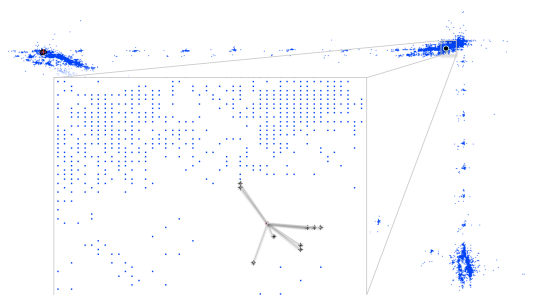
Wan

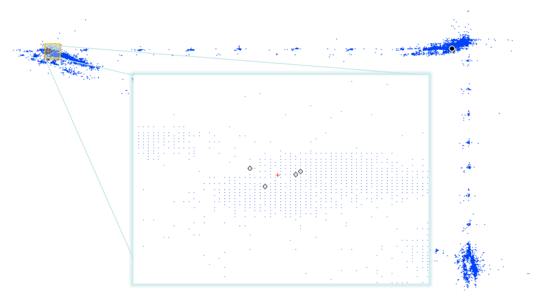
An even simpler prediction method:
Lorenz's method of analogues

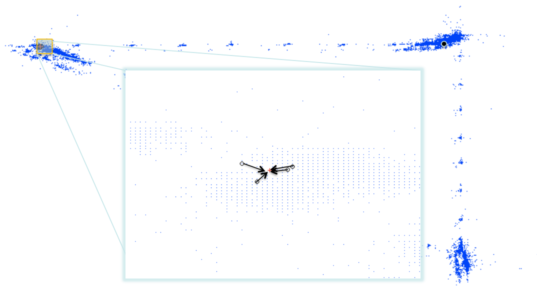




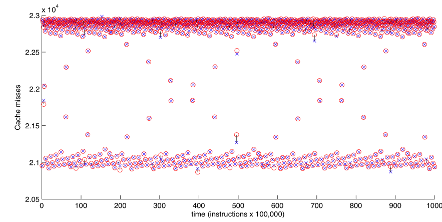
A *k*-nearest neighbor modification of LMA







Using *k*LMA to predict computer dynamics



RMSPE = 11.566
(.07% of the average)

`column_major`
cache misses

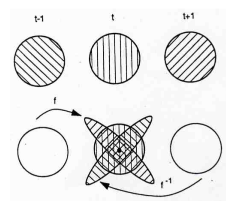
Garland/Bradley *Intelligent Data Analysis* 2011

Noise…

Linear filtering: a bad idea if the system is chaotic

Nonlinear alternatives:

- use the stable and unstable manifold structure on a chaotic attractor…

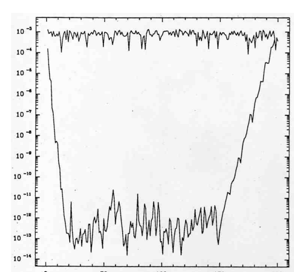


Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

Idea:

- If you have a model of the system, you can simulate what happens to each point in forward *and backward* time
- If your system has transverse stable and unstable manifolds, that does useful things to the noise balls
- Since all three versions of that data should be identical at the middle time, can average them
- ➡ noise reduction!
- Works best if manifolds are perpendicular, but requires only transversality

Results:



Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

Noise…

Linear filtering: a bad idea if the system is chaotic

Nonlinear alternatives:

- use the stable and unstable manifold geometry on a chaotic attractor
- what about using the *topology* of the attractor?

Computational Topology

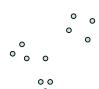
Why: this is the fundamental mathematics of shape. complements geometry.

What: compute topological properties *from finite data*

How:
• introduce resolution parameter
• count components and holes at different resolutions
• deduce topology from patterns therein

V. Robins Ph.D. thesis, UColorado, 1999

Connectedness: definitions

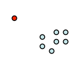


• how many "lumps" in a data set:

• ε-connectedness (after Cantor)

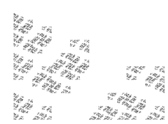
• ε-connected components

• ε-isolated points:

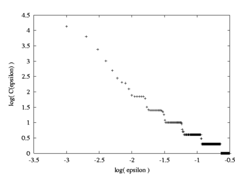


Connectedness: examples

If the data points are samples of a disconnected fractal like this:



The number of connected components looks like this:

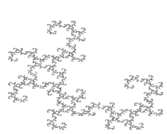


(note obvious tie-in to fractal dimension…)

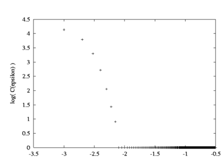
Robins *et al.*, *Physica D* **139**:276, *Nonlinearity* **11**:913

Connectedness: examples

If the data points are samples of a connected set like this:



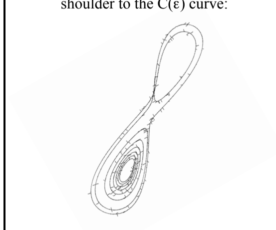
The number of connected components looks like this:

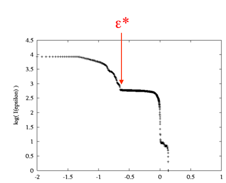


Robins *et al.*, *Physica D* **139**:276, *Nonlinearity* **11**:913

Connectedness and filtering

The effect of noise is to add isolated points to the set and a shoulder to the C(ε) curve:

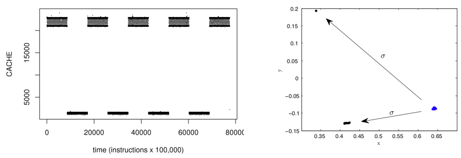




So if you know that the object is connected — like the attractor of a flow — you can reasonably assume that any isolated points are noisy, and remove them by pruning with ε = ε*

Robins *et al.*, *Intelligent Data Analysis* **8**:505, *Chaos* **14**:305

Continuity and filtering



Idea:
- deterministic, differentiable dynamics (maps & flows) are *continuous*

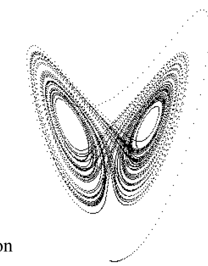
Conjecture:
- if the image of a connected set is not connected, more than one dynamics is at work

Approach:
- track connectedness over time

Applications:
- pulling apart interleaved dynamics, removing noise…

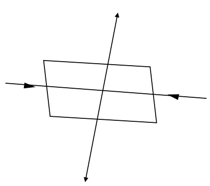
Alexander *et al., CHAOS*, 2012

Chaos and control…



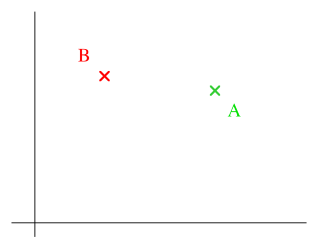
key concepts:
- dense attractor coverage
- exponential trajectory separation
- un/stable manifold structure
- local-linear control

Recall: local-linear control of a saddle point works in a region defined by the cross-sectional eigenstructure, together with the actuator capabilities



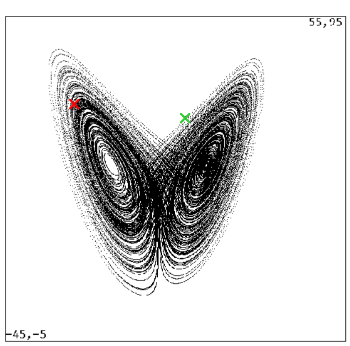
Control:

getting from A to B, minimizing some cost functional…



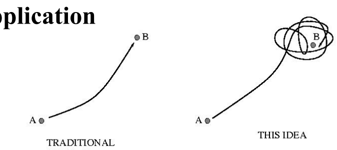
Lorenz System:

denseness, reachability, and control

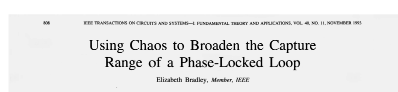


R = 50

Denseness & reachability in a real engineering application

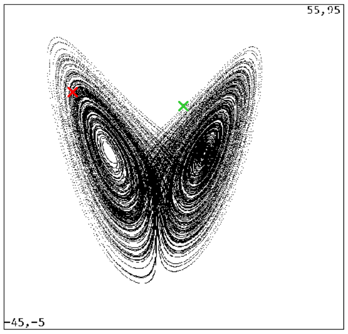


- **can control position/volume/density of attractor — *within limits***
- **possibly not reachable any other way**
- **not for time-critical applications (that "eventually")**


Using Chaos to Broaden the Capture Range of a Phase-Locked Loop

Elizabeth Bradley, *Member, IEEE*

Now what?



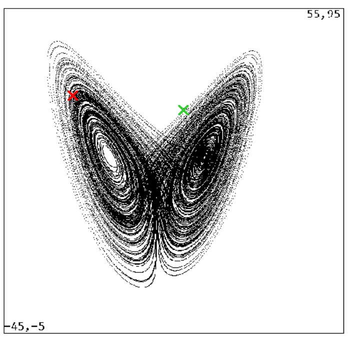
R = 50

OGY control

- dense attractor coverage → reachability
- un/stable manifold structure + UPO denseness + local-linear control → controllability

Ott *et al.*, *PRL* **64**:1196

Use local-linear control, designed using the eigen-values and eigen-vectors at that point × to balance a chaotic system on a UPO passing through that point.



But you're relying on denseness to get you into the controllable region, and that may take a while…

- dense attractor coverage → reachability
- un/stable manifold structure + UPO denseness + local-linear control → controllability
- exploit sensitive dependence, too???

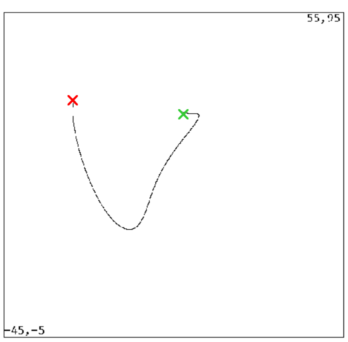
⟹ "targeting"

Lorenz System:

SDOIC-based targeting

OGY & co. have been used in *tons* of systems; see Shinbrot review paper.

Alfred Hubler has done a lot of cool stuff in this area as well.



Four R switches; 240X faster

Bradley, *Cybernetics & Systems* **26**:299

Program in Applied Mathematics

Erik Bollt

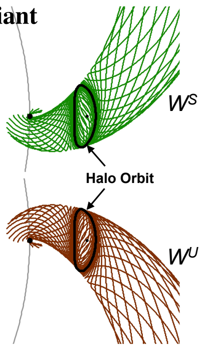
University of Colorado at Boulder
Boulder CO 80309-0526
(303) 492-4668

Other cool ways to use invariant manifolds

Want to get a spacecraft onto a "halo orbit," which is a UPO of the dynamics.

Unstable Periodic Orbits (UPOs) have invariant manifolds:

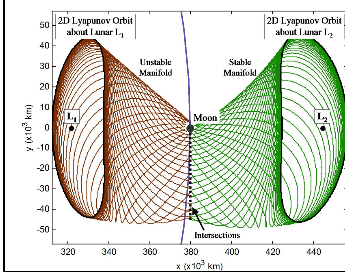
- Stable Invariant Manifold (W^S)
  - *The set of all trajectories a particle could use to arrive onto the UPO.*
- Unstable Invariant Manifold (W^U)
  - *The set of all trajectories a particle could take after a small perturbation from the UPO.*

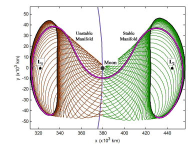


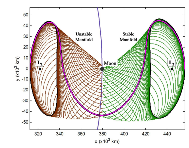
Jeff Parker, PhD thesis, UColorado 2008

Low-energy (cheap) orbit transfers

- Depart along W^U_{L1} & arrive on W^S_{L2}



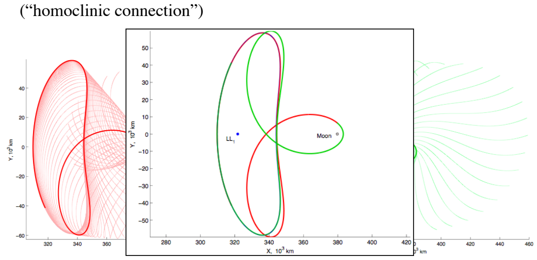




Jeff Parker, PhD thesis, UColorado 2008

Homoclinic orbits - The best case

- If a trajectory in Stable and Unstable intersect ("homoclinic connection")



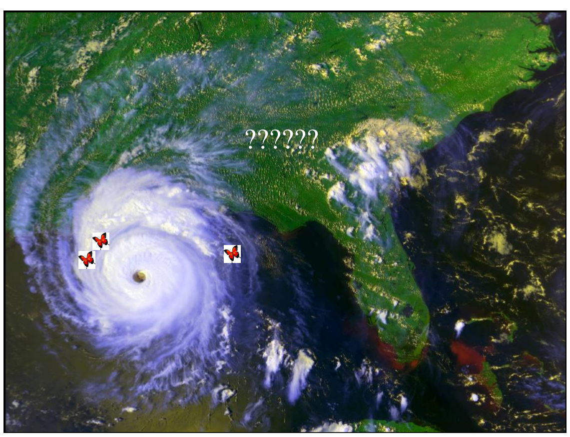
Unstable Manifold of an LL_1 Lyapunov Orbit

Stable Manifold of an LL_1 Lyapunov Orbit

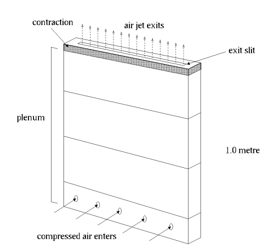
Jeff Parker, PhD thesis, UColorado 2008

Can we do any of that in spatially extended systems?

(i.e. harness the butterfly effect, exploit un/stable manifold geometry?)

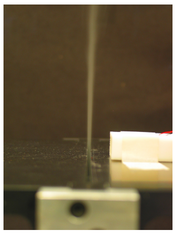


A 2D jet

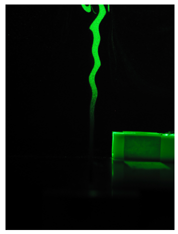


Peacock *et al.*, *Exp. Fluids* **37**:22

End view



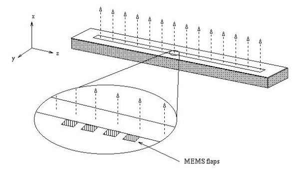
room lighting



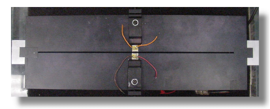
stop-action laser "slice"

aerosolized canola oil

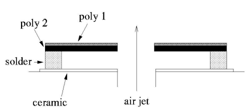
Forcing the jet flow

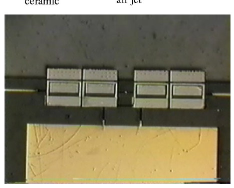


Slit: 2.5 X 400 mm



MEMS actuators





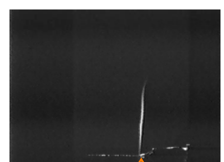
Video : overhead view at 2Hz, 10V

Area of individual flap is 1.0 x 0.25mm

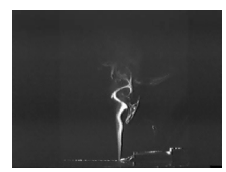
Ma *et al.*, *IEEE Trans. Adv. Packaging* **26**:268

The Butterfly effect in action…

no forcing



6Hz forcing

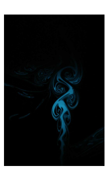


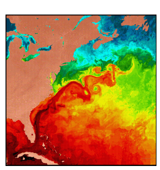
MEMS flap

Forcing generates coherent structures that enhance entrainment and mixing

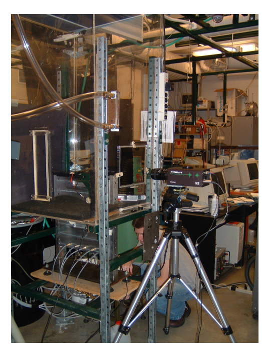
Peacock *et al.*, *Exp. Fluids* **37**:22

Does this have anything to do with reality?





Measurement & isolation:



Communication and chaos:

- Two coupled Lorenz systems will synchronize
- Robust to a small amount of noise
- Use this to transmit & receive information

$x' = a(y-x)$
$y' = rx - y - xz$
$z' = xy - bz$

→

$x' = a(y-x)$
$y' = r(x+\varepsilon x) - y - xz$
$z' = xy - bz$

- Chaotic carrier wave, so hard to intercept or jam

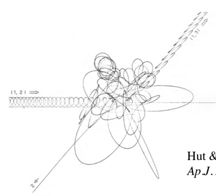
Pecora & Carroll *Phys. Rev. Lett* **64**:821

Another interesting application: chaos in the solar system

- orbits of Pluto, Mars
- Kirkwood gaps
- rotation of Hyperion & other satellites
- …

Solar system stability:

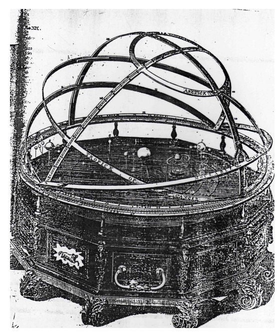
- recall: two-body problem not chaotic
- but three (or more) can be…



Hut & Bahcall
Ap.J. ***268**:319*

Exploring that issue, circa 1880:

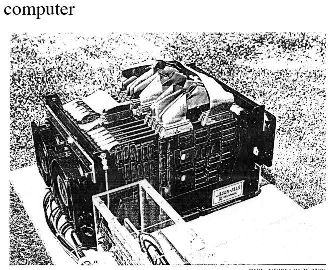
An *orrery*, which is a *mechanical computer* whose gear ratios and circular platters simulate the orbits of the planets



Exploring that issue, circa 1980:

- write the *n*-body equations for the solar system
- solve them using symplectic ODE solvers on a special-purpose computer

The *digital orrery*
(Wisdom & Sussman)



ONR N00014-86-K-0180

Numerical Evidence That the Motion of Pluto Is Chaotic

Gerald Jay Sussman and Jack Wisdom

The Digital Orrery has been used to perform an integration of the motion of the outer planets for 845 million years. This integration indicates that the long-term motion of the planet Pluto is chaotic. Nearby trajectories diverge exponentially with an *e*-folding time of only about 20 million years.

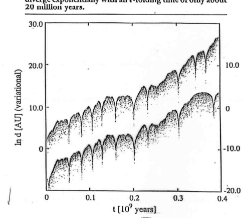


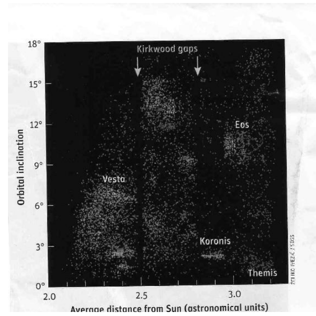
Figure 3: The exponential divergence of nearby trajectories is indicated by the average linear growth of the logarithms of the distance measures as a function of time. In the upper trace we see the growth of the variational distance around a reference trajectory. In the lower trace we see how two Plutos diverge with time. The distance saturates near 80AU when the Plutos are on opposite sides of the Sun. The variational method of studying neighboring trajectories does not have the problem of saturation. Note that the two methods are in excellent agreement until the two-trajectory method has nearly saturated.

Science **241**:433

Should we worry?

- No.

Kirkwood gaps:



From *Sky & Telescope*

Chaos and the Kirkwood gaps

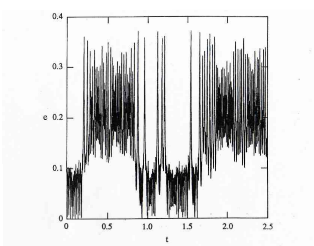


FIGURE 5. Eccentricity of a typical chaotic trajectory over a longer time interval. the time is now measured in millions of years. Bursts of high eccentricity behavior are interspersed with intervals of irregular low eccentricity behavior, broken by occasional spikes.

Wisdom, *Nuclear Phys. B* **2**:391

Evidence in favor of the conjecture:

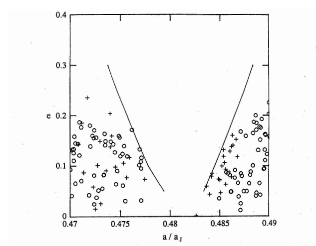
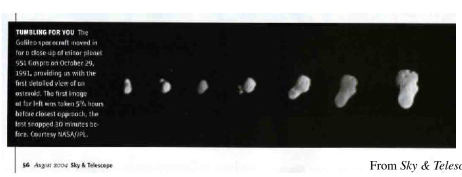


FIGURE 9. Comparison of the actual distribution of asteroids with the outer boundaries of the chaotic zone. There is both a chaotic region and quasiperiodic region in the gap, but trajectories of both types are planet crossing.

Wisdom, *Nuclear Phys. B* **2**:391

Chaotic tumbling of satellites:

Voyager and Galileo saw this…



From *Sky & Telescope*

Ap. J. **97**:570
Ap. J. **98**:1855

www.nasa.gov/mission_pages/cassini/multimedia/pia06243.html

Chaotic tumbling of satellites:

This happens for **all** satellites at some point in their history, unless they are perfectly spherical and in perfectly circular orbits (pf: KAM theorem; see Wisdom paper on syllabus)

Some of them are still tumbling chaotically because of their geometry, but most (like the earth and its moon) have settled down into tidal equilibria

More chaos in the solar system:

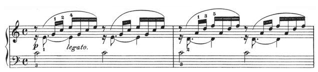
- obliquity of Mars (Touma & Wisdom, *Science* **259**:1294)

www.solarviews.com

- etc.

Musical Variations from a Chaotic Mapping

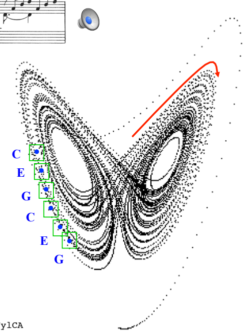
Dabby *Chaos* **6**:95



Pitch sequence:
C, E, G, C, E, G, C, E…

C **symbol dynamics**

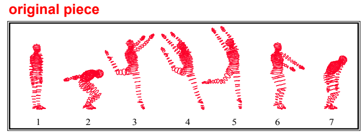
variation!



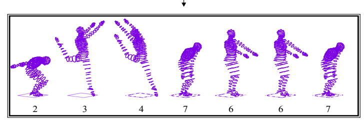
Also fun: http://www.youtube.com/watch?v=B2XtE9Yy1CA

Chaotic variations on movement sequences

original piece

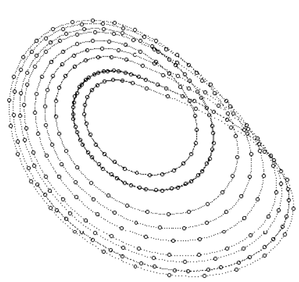


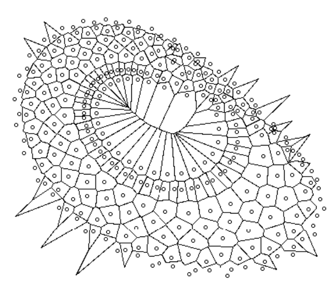
chaotic **mapping**

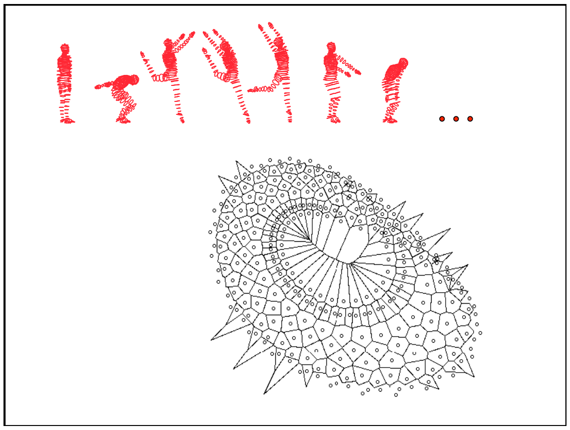


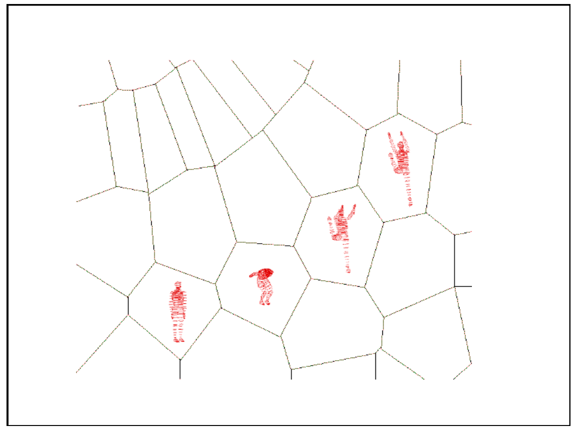
chaotic variation

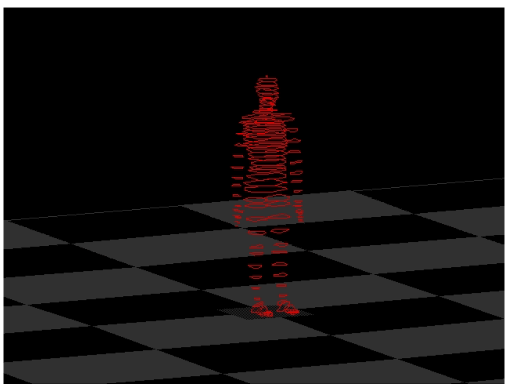
Bradley & Stuart, *Chaos* **8**:800

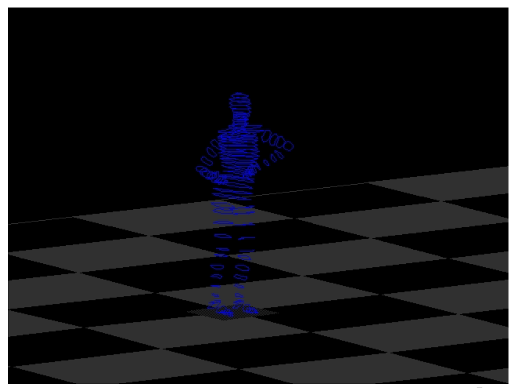




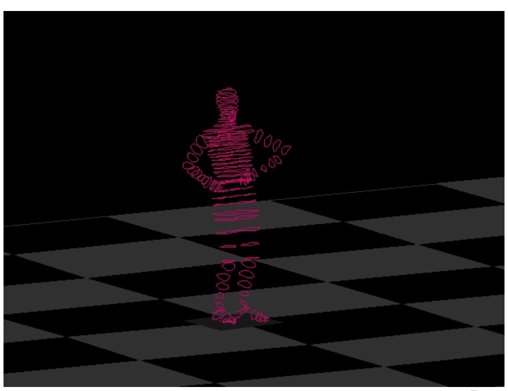




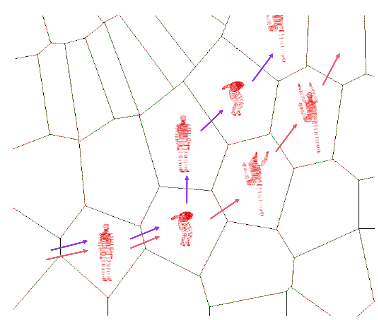


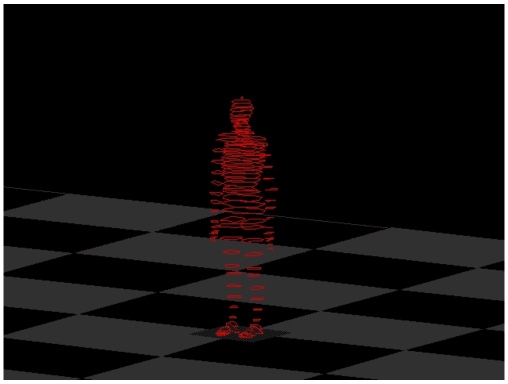


Lorenz

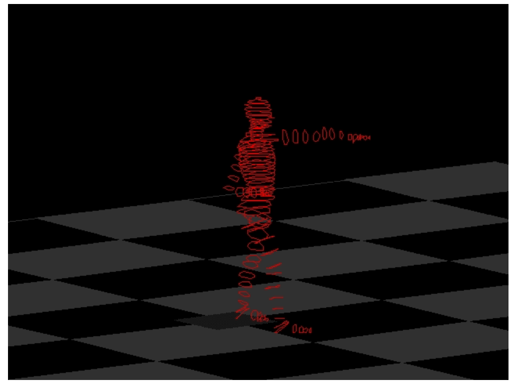


Rossler

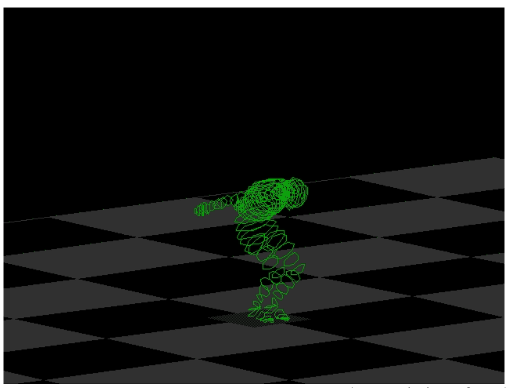




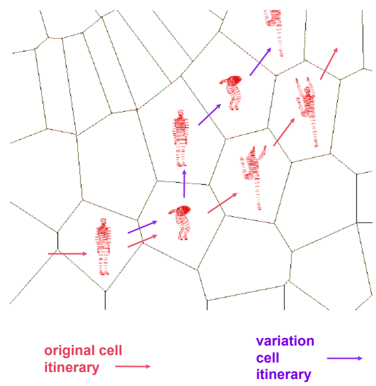
medley—original

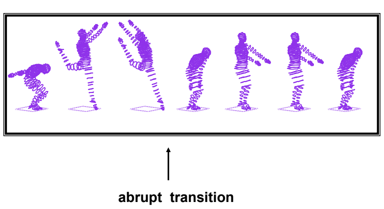


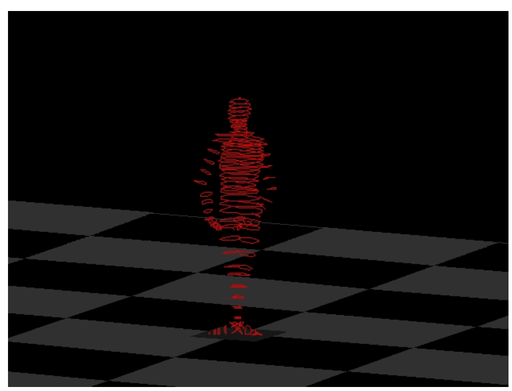
Rossler variation of medley

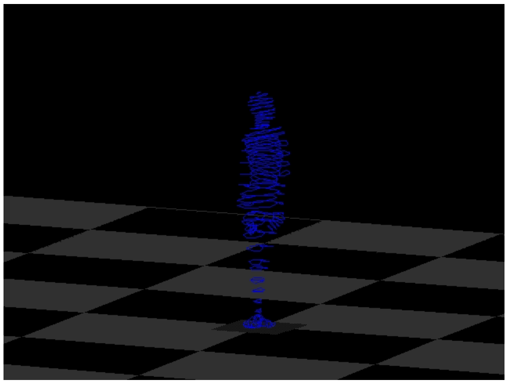


random variation of medley

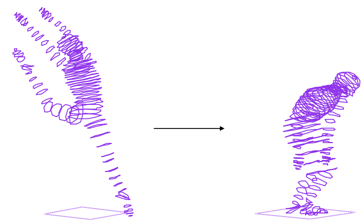






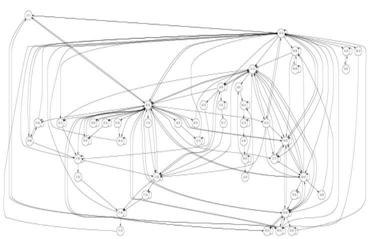


Interpolation



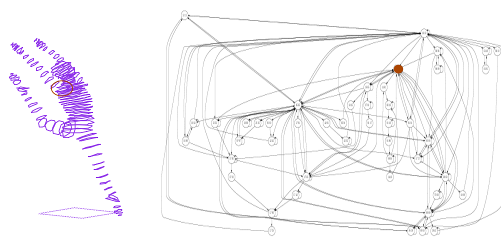
Stuart & Bradley, *ICML* 1998

Corpus-based approach

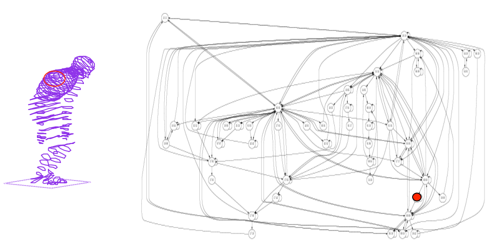


- **graph captures motions of one joint**
- **note: specific to the genre of the corpus!**

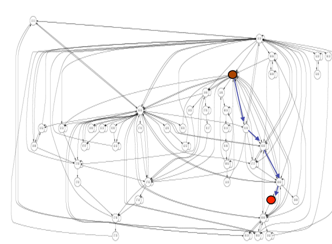
Initial state



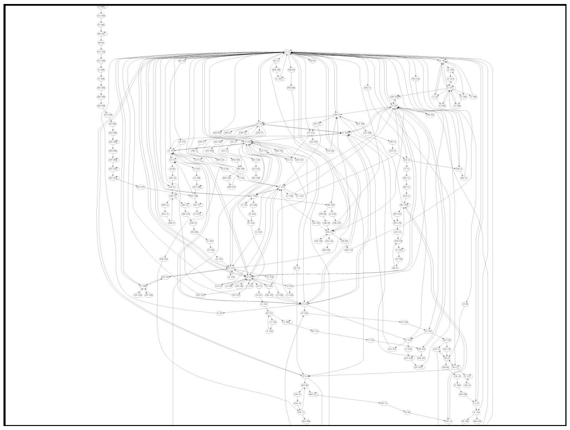
Target state

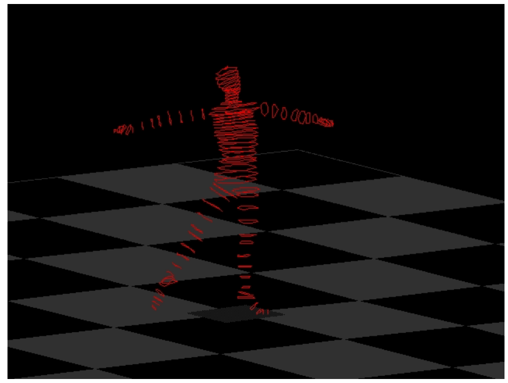


Graph search

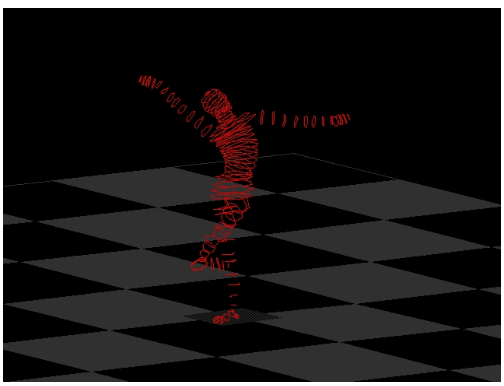


…for 44 joints in parallel!

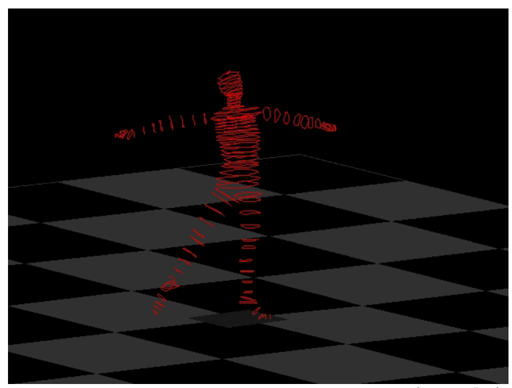




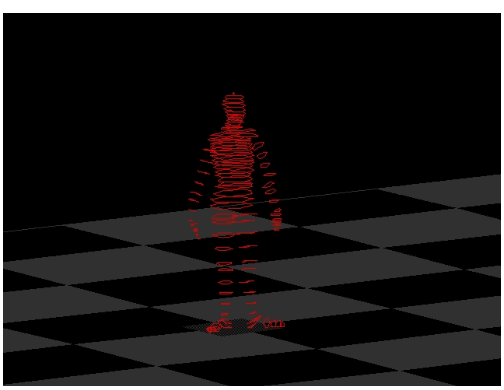
initial



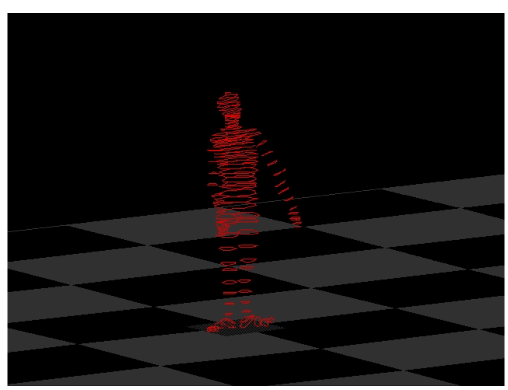
target



interpolation 1



initial



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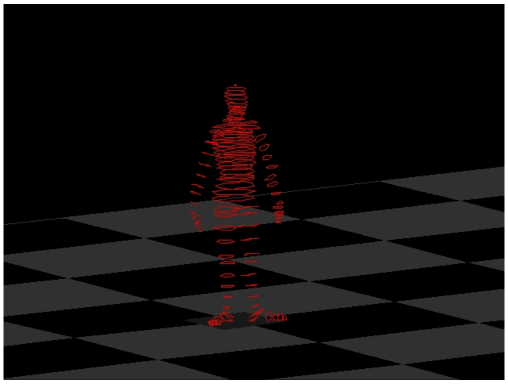
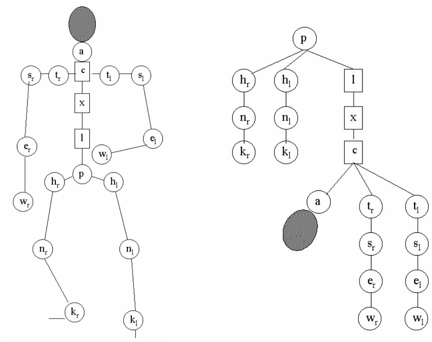
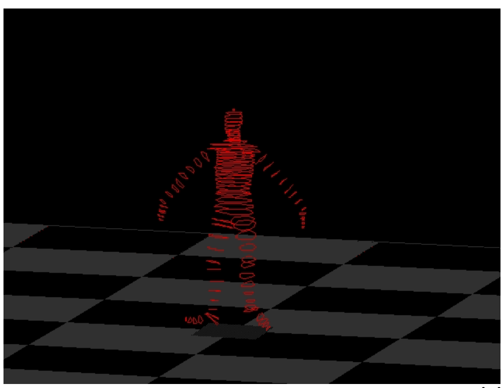
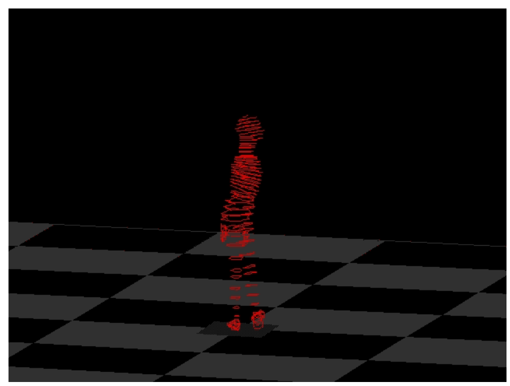


interpolation 2





initial



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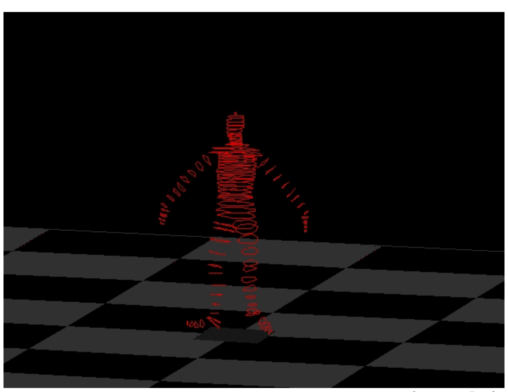
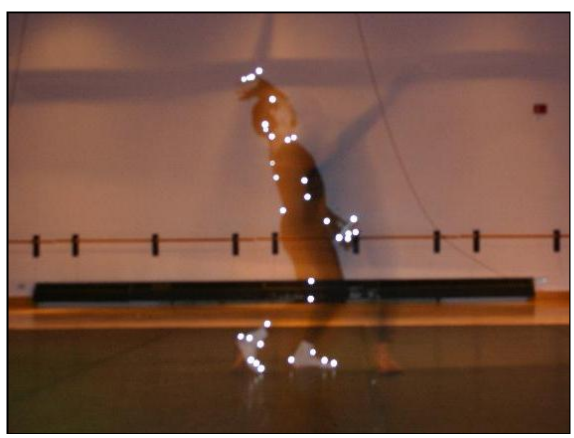
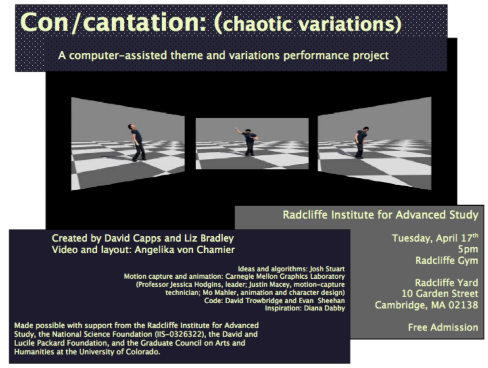
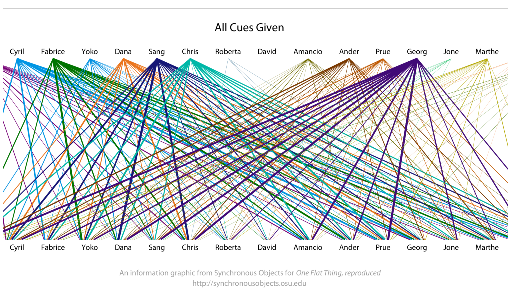


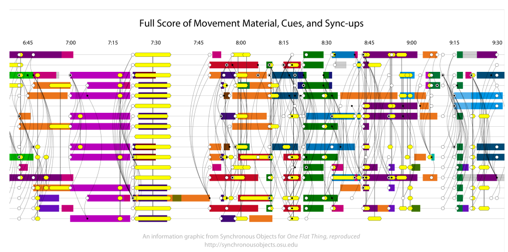
interpolation 3







synchronousobjects.osu.edu/content.html



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