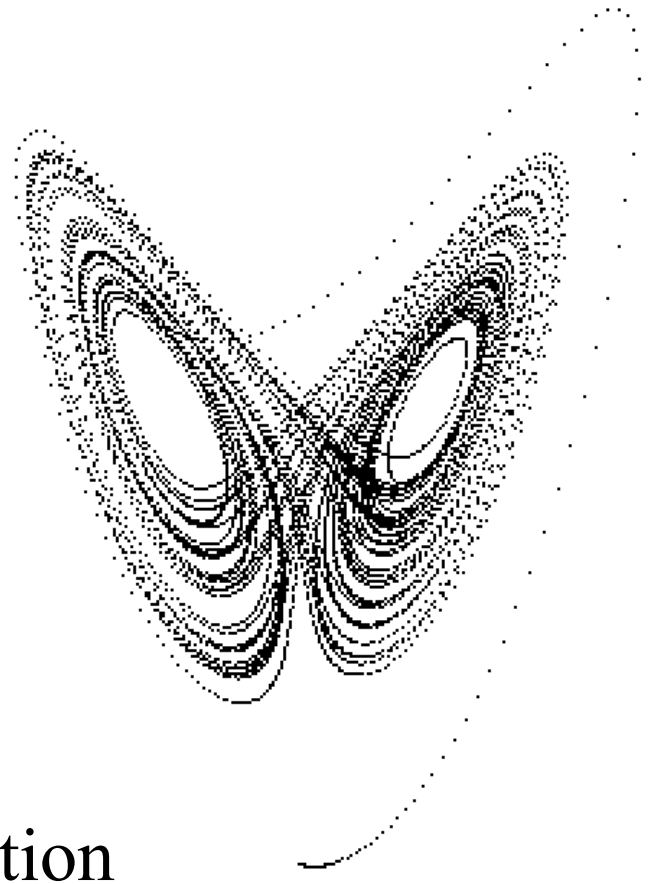
Chaos and control…

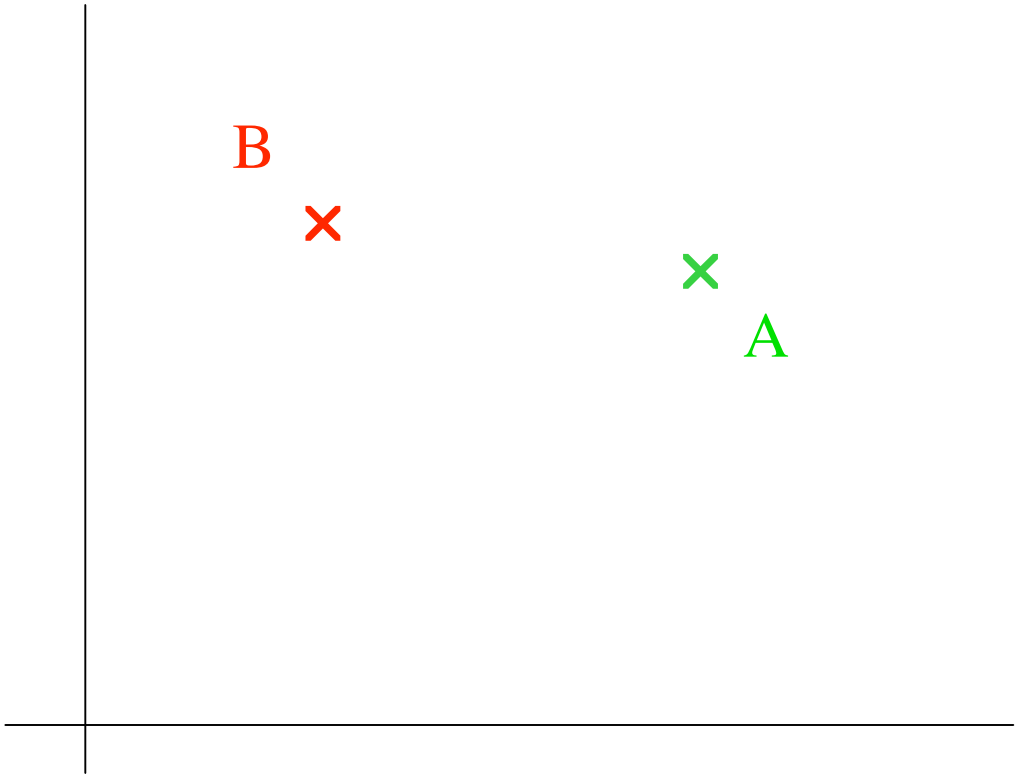


key concepts:

- dense attractor coverage
- exponential trajectory separation
- un/stable manifold structure
- local-linear control

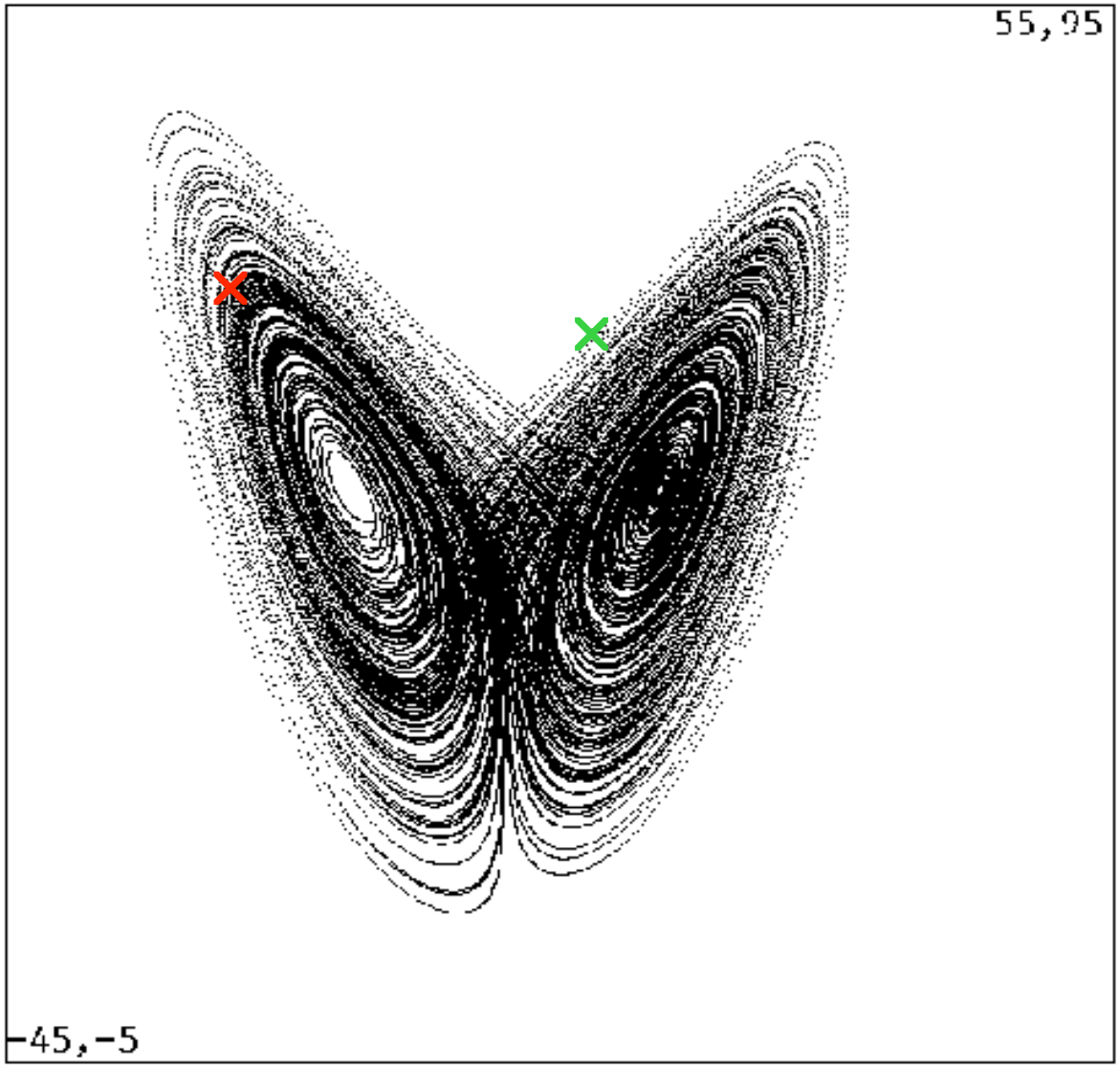
Control:

getting from A to B, minimizing some cost functional…



Lorenz System:

denseness, reachability, and control



R = 50

OYG control:

- dense attractor coverage → reachability
- un/stable manifold structure → controllability

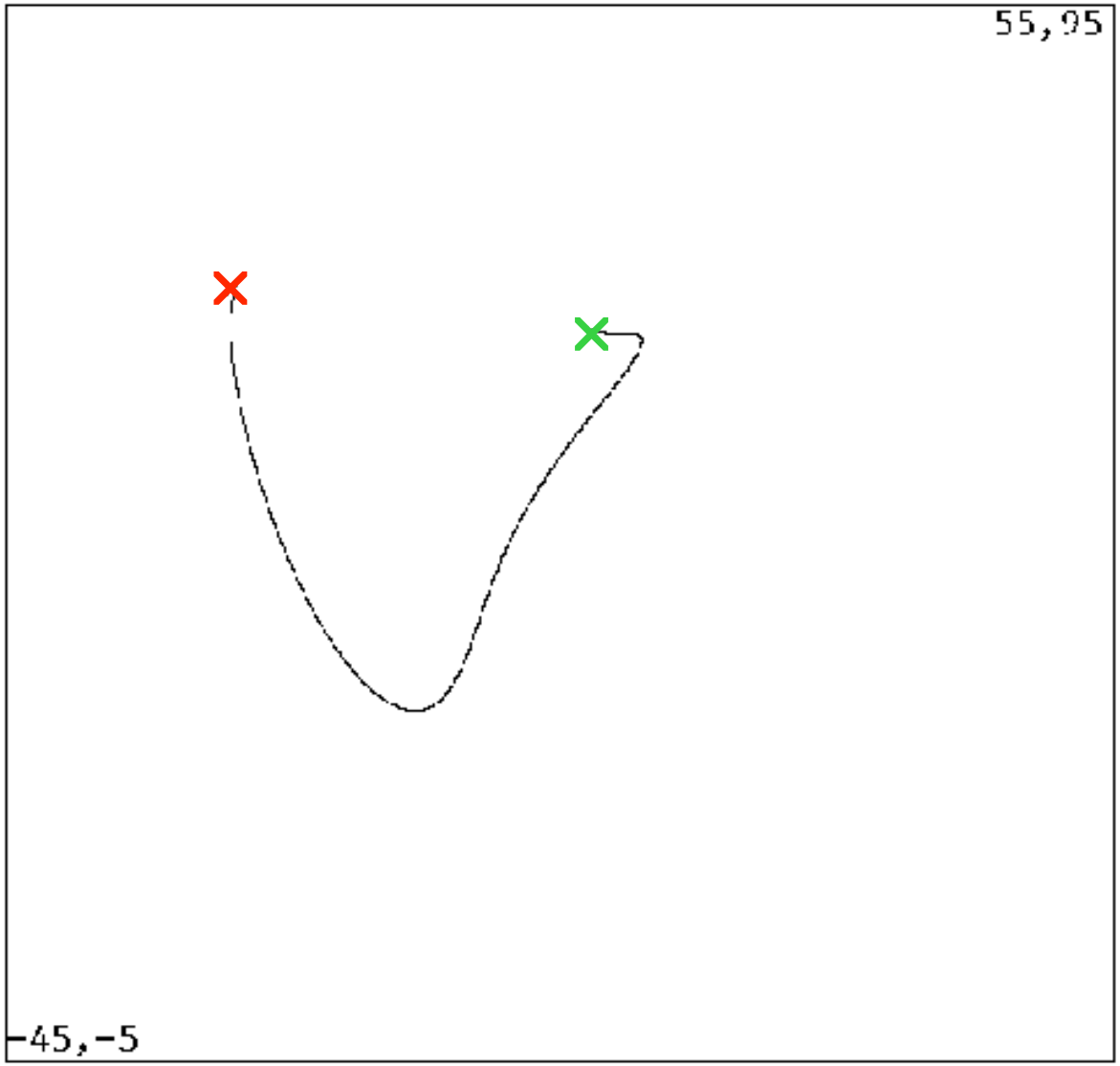
Ott *et al.*, *PRL* **64**:1196

- dense attractor coverage → reachability
- un/stable manifold structure → controllability
- exploit sensitive dependence, too???

  ⟹ "targeting"

Lorenz System:

SDOIC-based targeting



Four R switches; 240X faster

Shinbrot review paper: [47]

Program in Applied Mathematics

University of Colorado at Boulder
Boulder CO 80309-0526
(303) 492-4668

Can we do that in spatially extended systems, too?

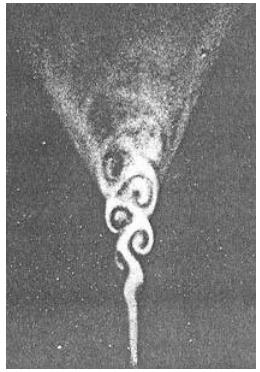
(i.e. harness the butterfly effect?)

- Sensitive flames (1856 – 1930's)

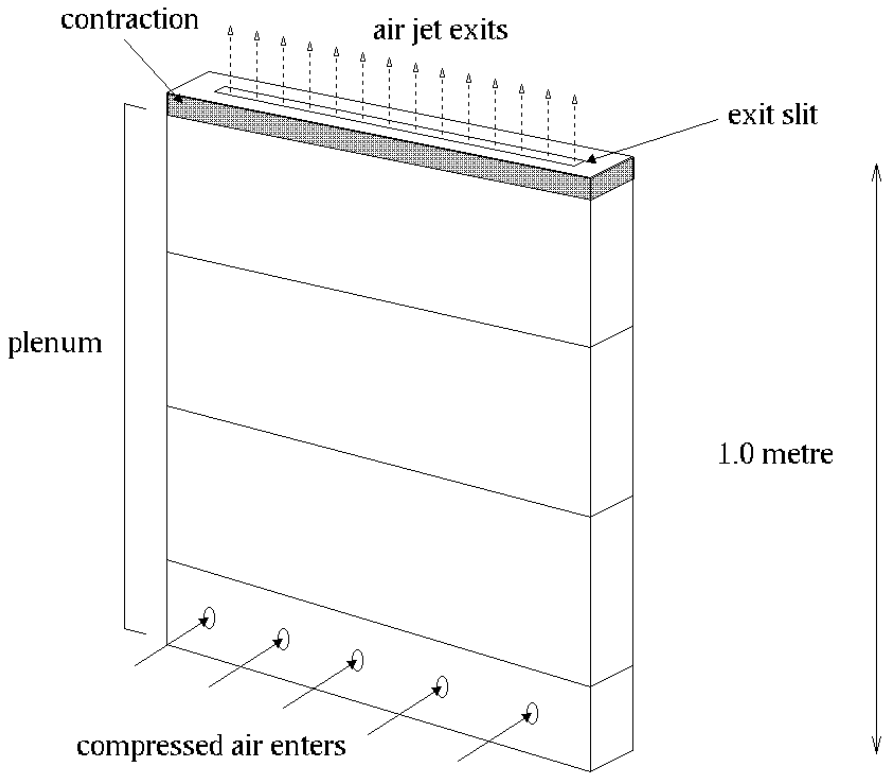
I repeat a passage from Spenser:

"Her ivory forehead full of bounty brave,
Like a broad table did itself dispread;
For love his lofty triumphs to engrave,
And write the battles of his great godhead.
All truth and goodness might therein be read,
For there their dwelling was, and when she spake,
Sweet words, like dropping honey she did shed;
And through the pearls and rubies softly brake
A silver sound, which heavenly music seemed to make."

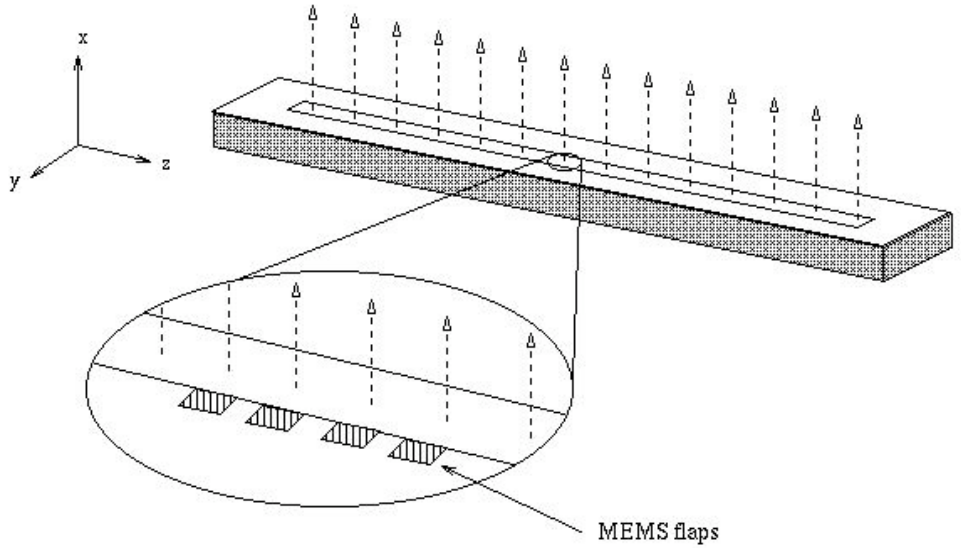
The flame selects from the sounds those to which it can respond. It notices some by the slightest nod, to others it bows more distinctly, to some its obeisance is very profound, while to many sounds it turns an entirely deaf ear.



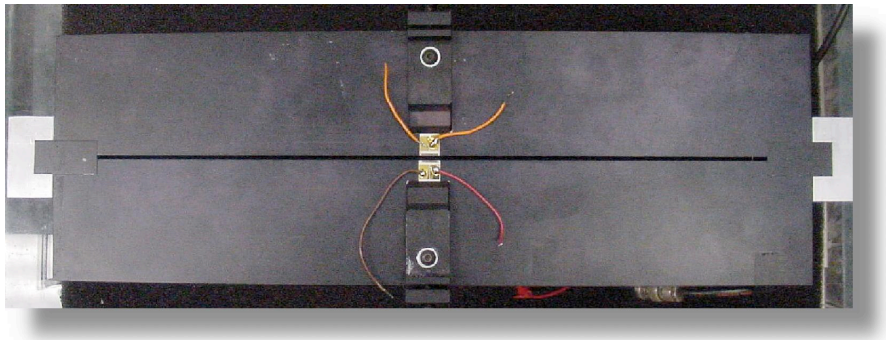
2D jet apparatus



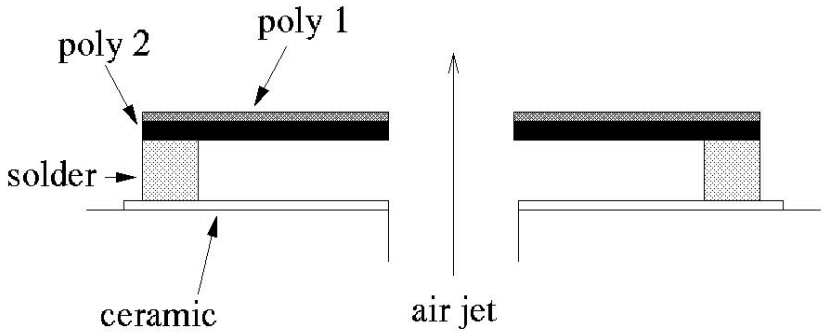
Forcing the jet flow

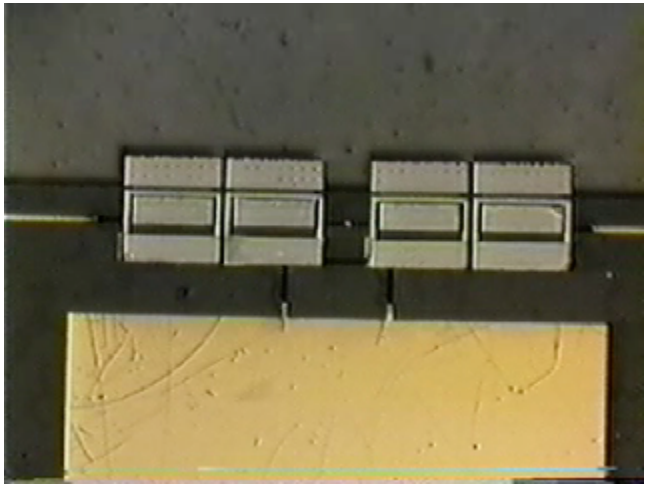


Slit: 2.5 X 400 mm



MEMS actuators

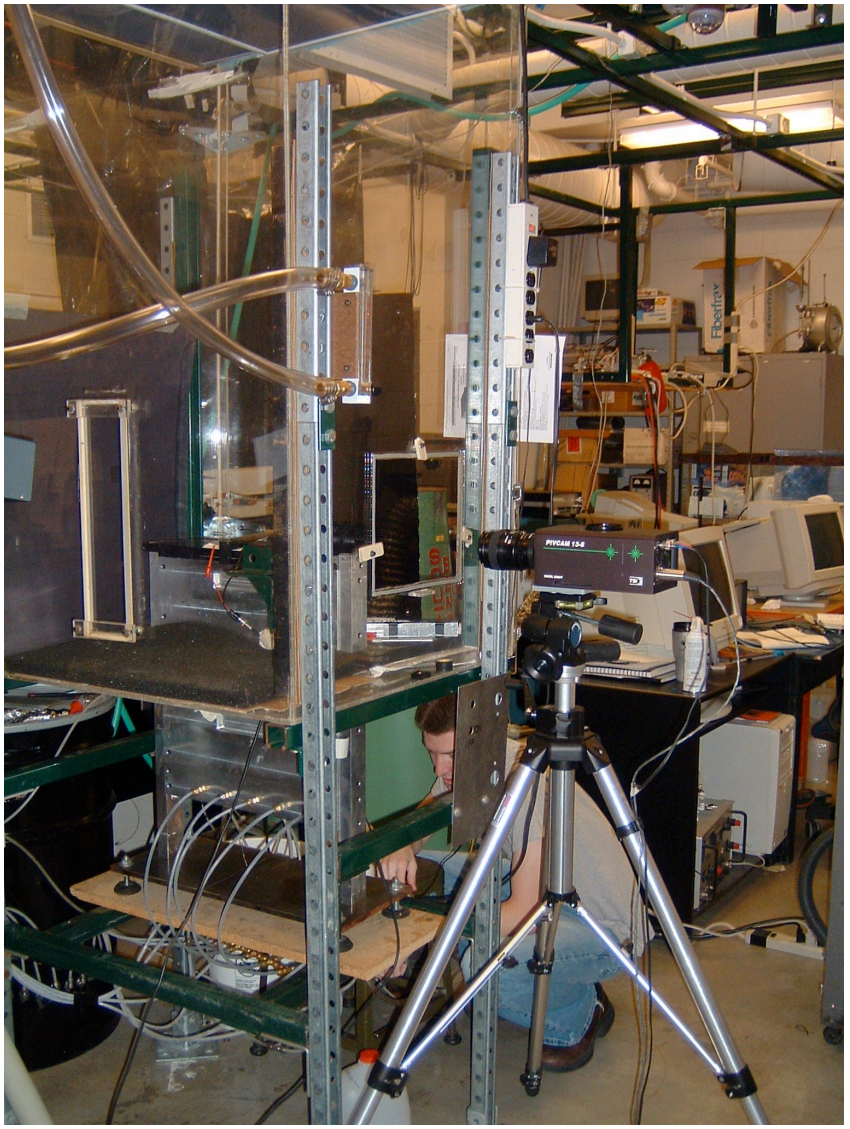




Video : overhead view at 2Hz, 10V

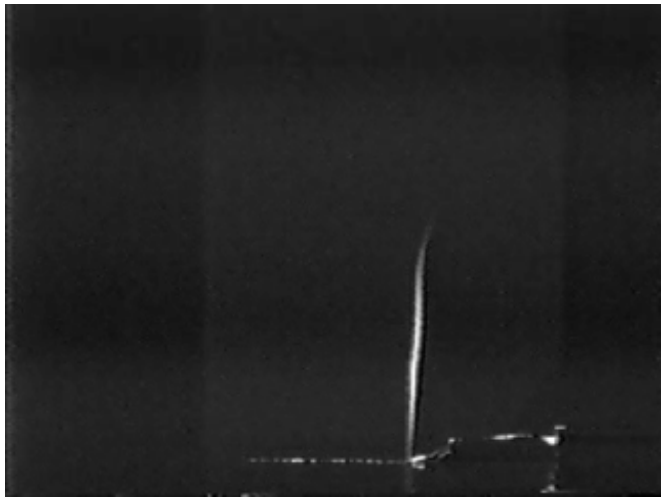
Area of individual flap is 1.0 x 0.25mm

Measurement & Isolation:



The Butterfly effect in action…

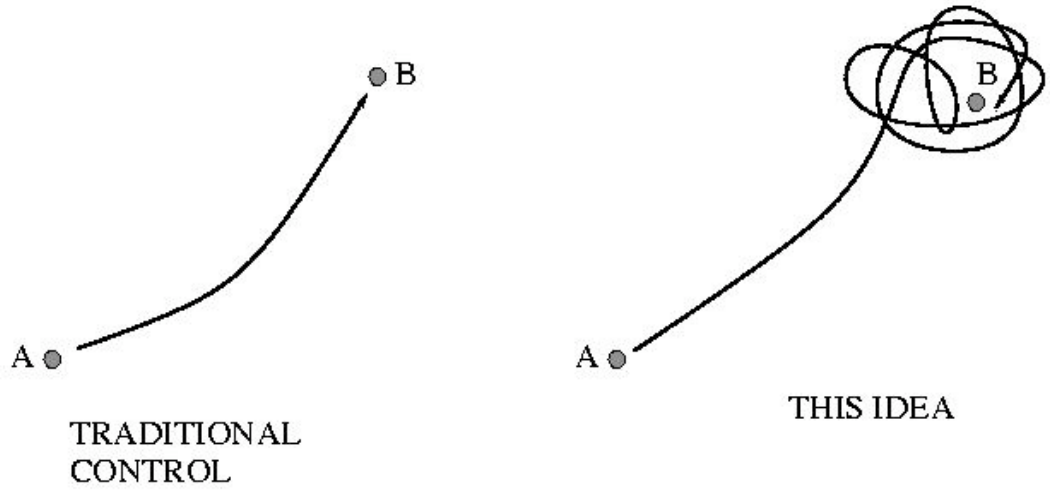
no forcing



6Hz forcing

Forcing generates coherent structures that enhance entrainment and mixing

"Chaos-enhanced reachability"



- **can control position/volume/density of attractor –*within limits***
- **possibly not reachable any other way**
- **nondeterministic – not for time-critical applications**

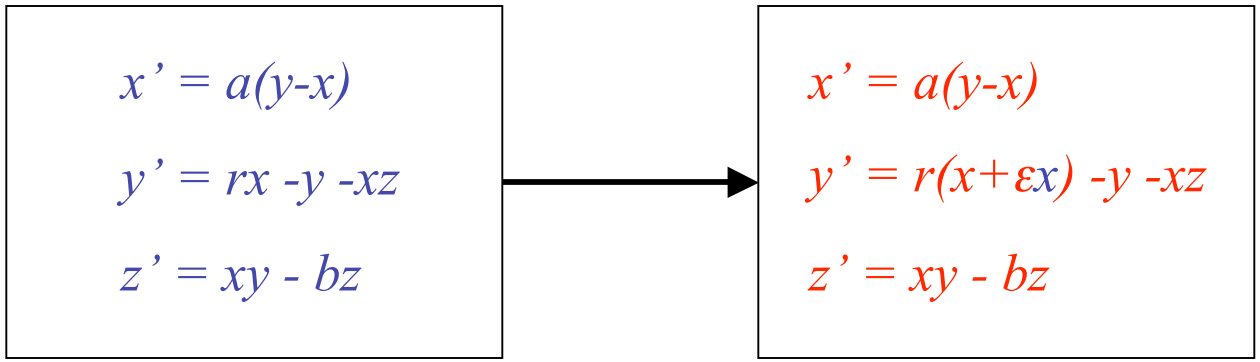
808 IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS, VOL. 40, NO. 11, NOVEMBER 1993

Using Chaos to Broaden the Capture Range of a Phase-Locked Loop

Elizabeth Bradley, *Member, IEEE*

Communication and chaos:

- Pecora & Carroll

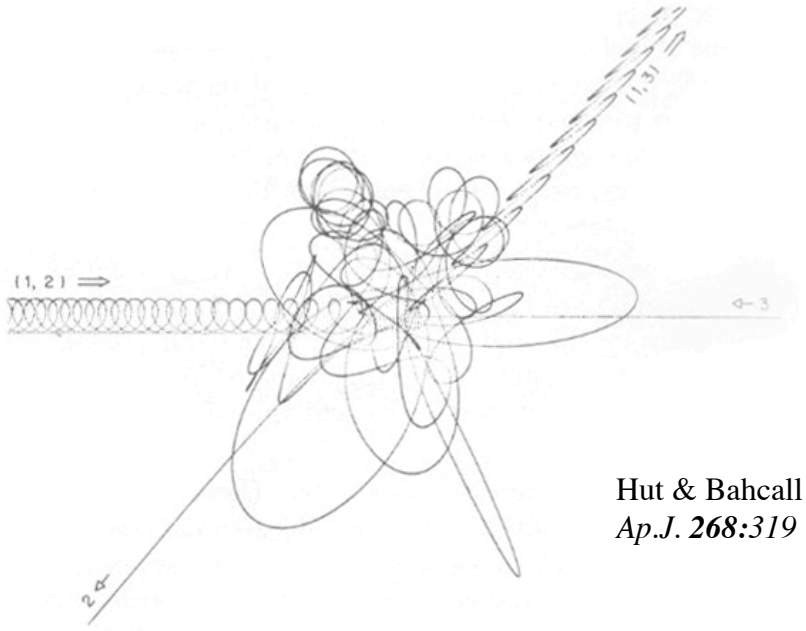


Another interesting application: chaos in the solar system

- orbits of Pluto, Mars
- Kirkwood gaps
- rotation of Hyperion & other satellites
- …

Solar system stability:

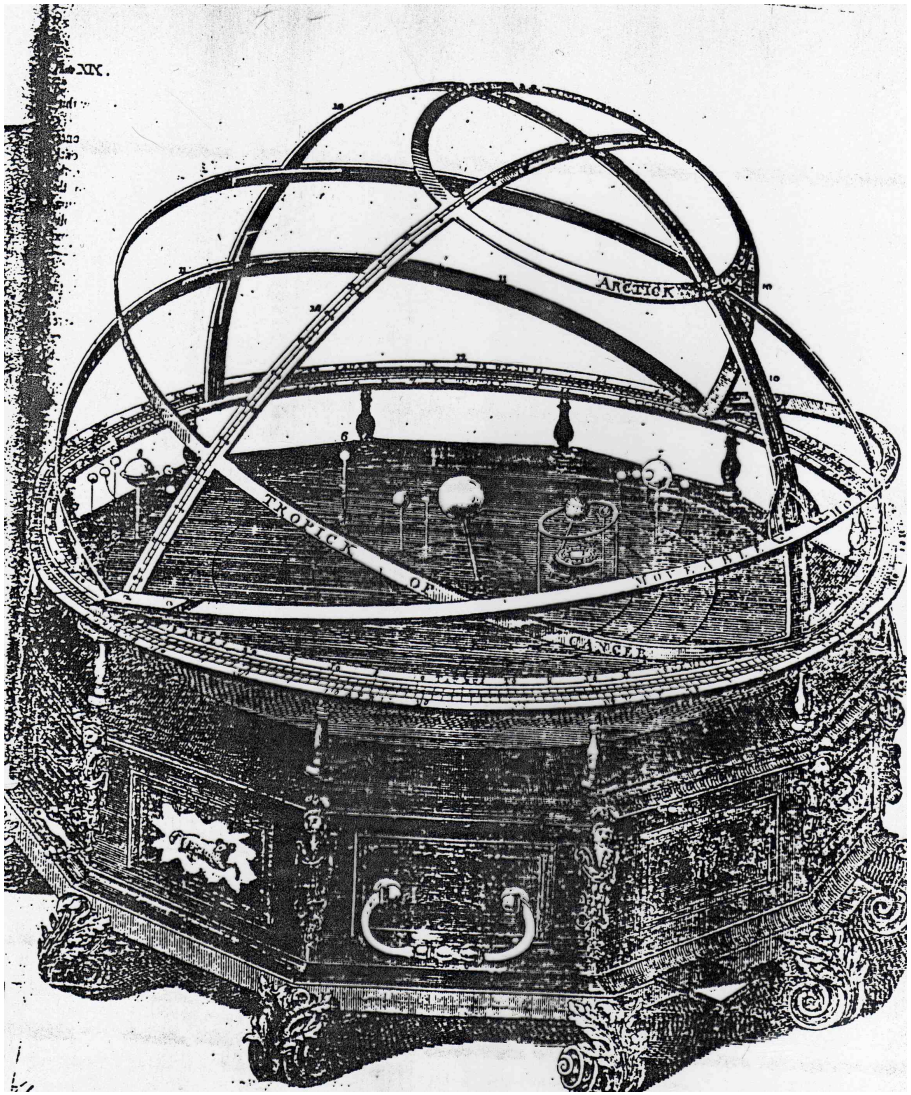
- recall: two-body problem not chaotic
- but three (or more) can be…



Hut & Bahcall
Ap.J. ***268:****319*

Exploring that issue, circa 1880:

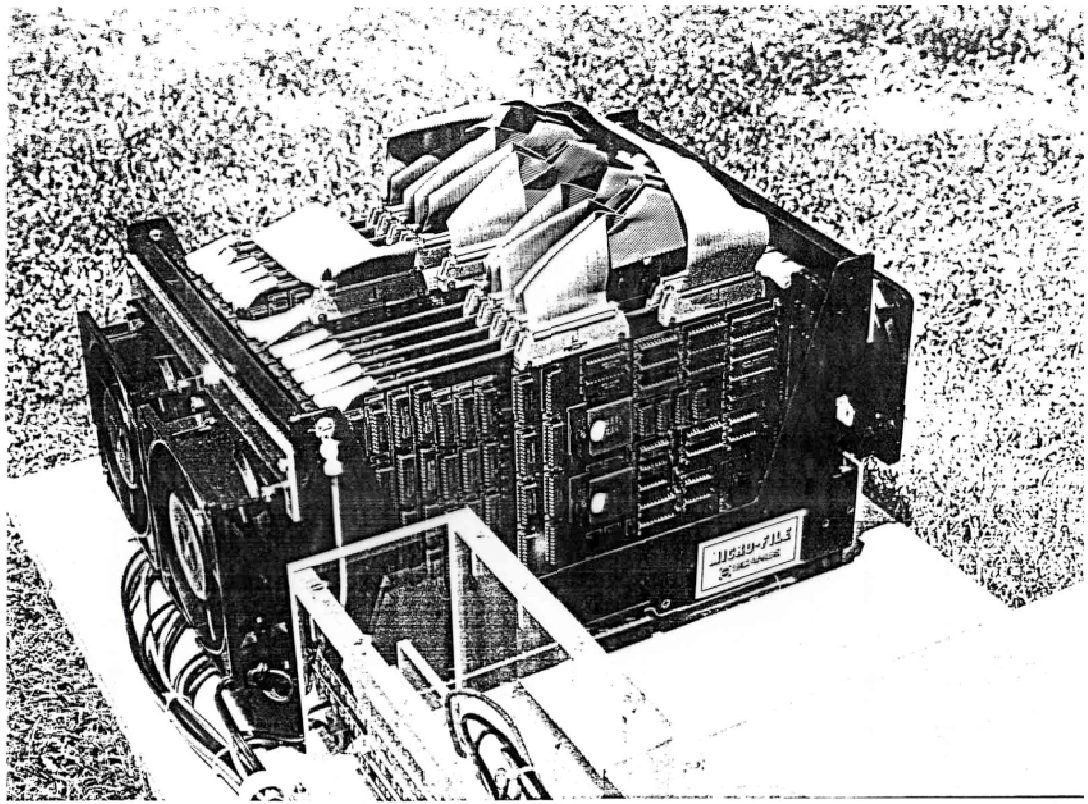
- an *orrery*



Exploring that issue, circa 1980:

- write the *n*-body equations for the solar system
- solve them on a special-purpose computer

The *digital orrery*
(Wisdom & Sussman)



ONR N00014-86-K-0180

Numerical Evidence That the Motion of Pluto Is Chaotic

GERALD JAY SUSSMAN AND JACK WISDOM

The Digital Orrery has been used to perform an integration of the motion of the outer planets for 845 million years. This integration indicates that the long-term motion of the planet Pluto is chaotic. Nearby trajectories diverge exponentially with an *e*-folding time of only about 20 million years.

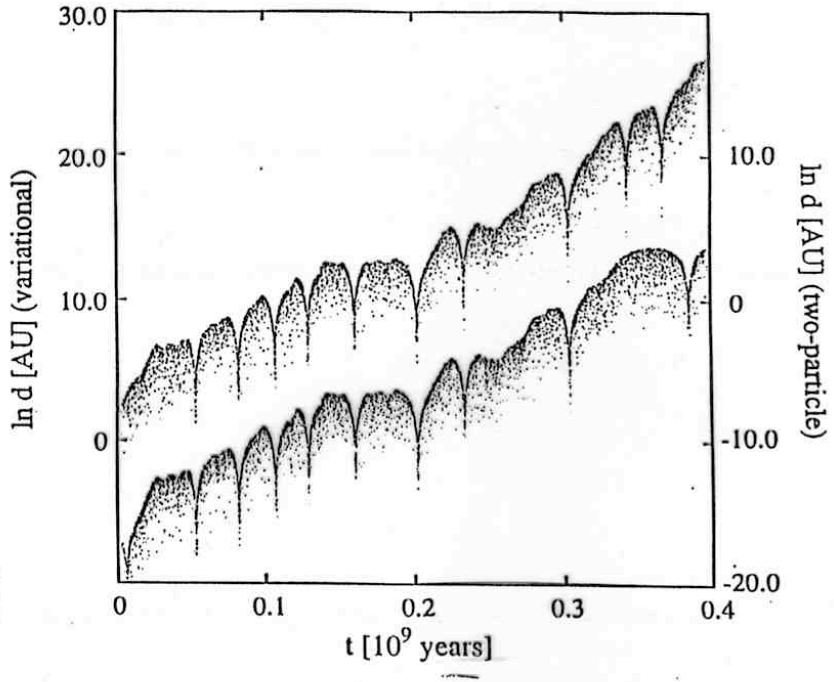
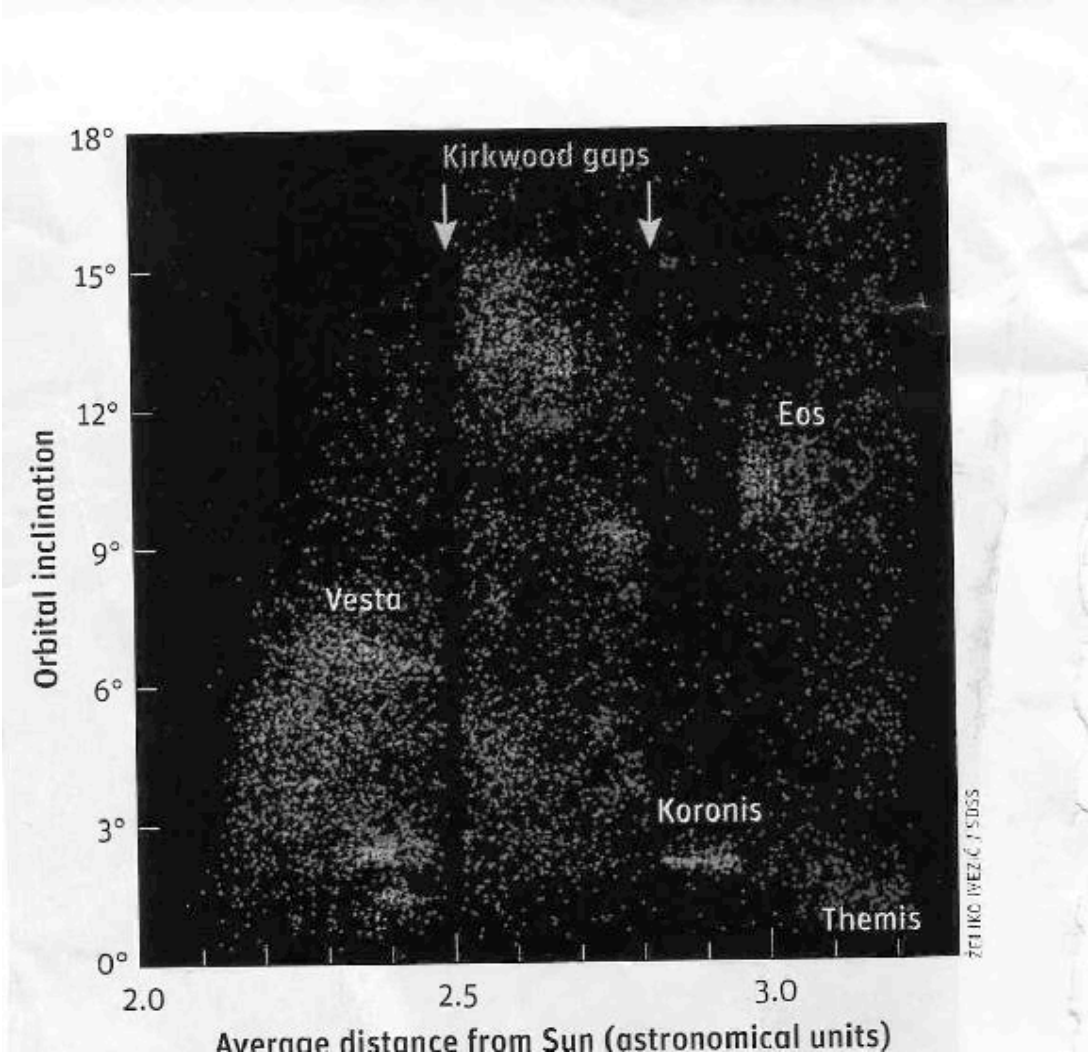


Figure 3: The exponential divergence of nearby trajectories is indicated by the average linear growth of the logarithms of the distance measures as a function of time. In the upper trace we see the growth of the variational distance around a reference trajectory. In the lower trace we see how two Plutos diverge with time. The distance saturates near 80AU when the Plutos are on opposite sides of the Sun. The variational method of studying neighboring trajectories does not have the problem of saturation. Note that the two methods are in excellent agreement until the two-trajectory method has nearly saturated.

Should we worry?

- No.

Kirkwood gaps:



Sky & Telescope

Chaos and the Kirkwood gaps

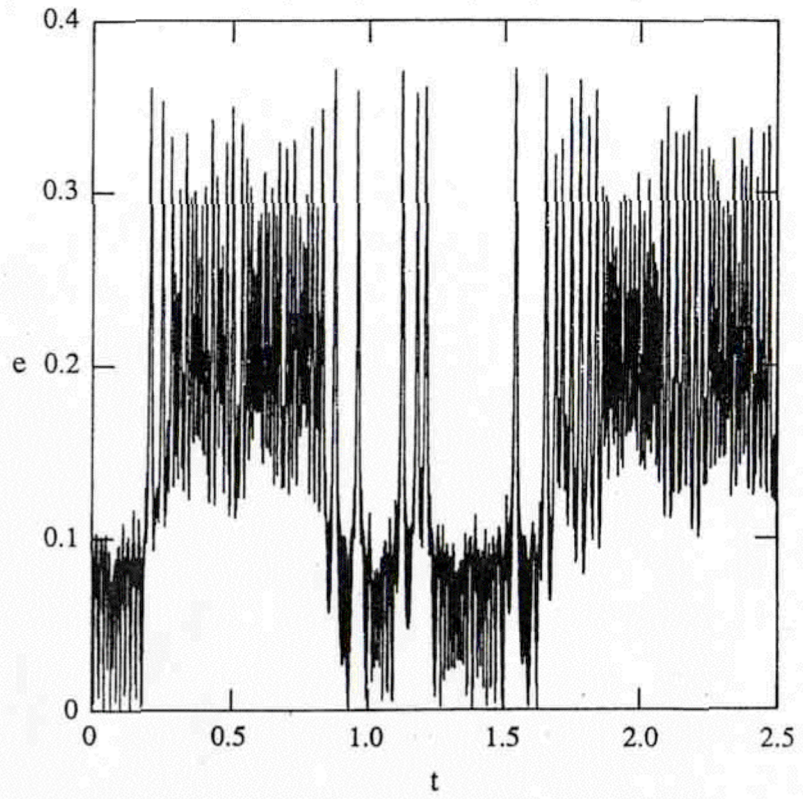
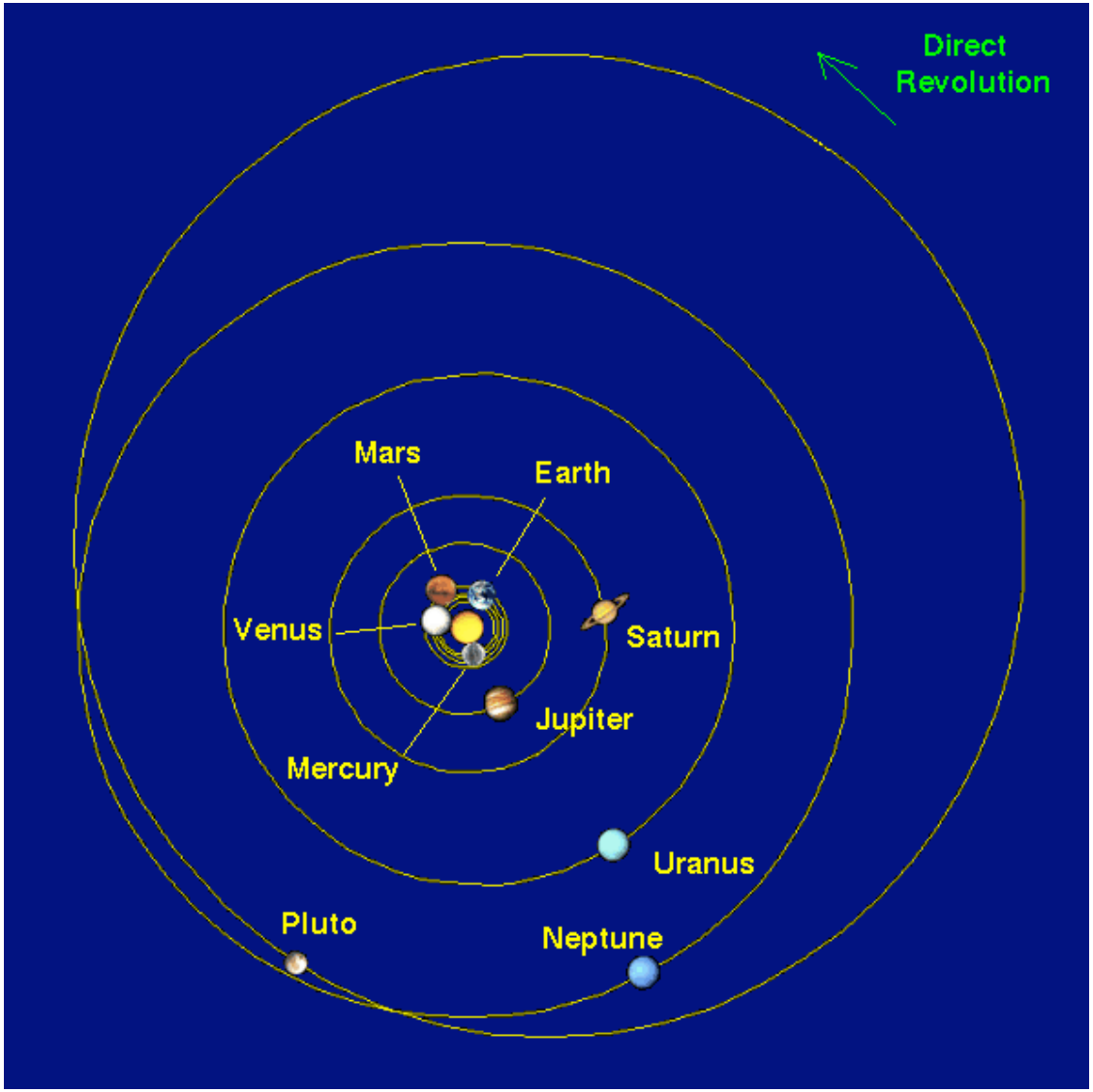


FIGURE 5. Eccentricity of a typical chaotic trajectory over a longer time interval. the time is now measured in millions of years. Bursts of high eccentricity behavior are interspersed with intervals of irregular low eccentricity behavior, broken by occasional spikes.

Wisdom, *Nuclear Phys. B* **2:***391*



http://csep10.phys.utk.edu/astr161/lect/solarsys/revolution.html

Evidence in favor of the conjecture:

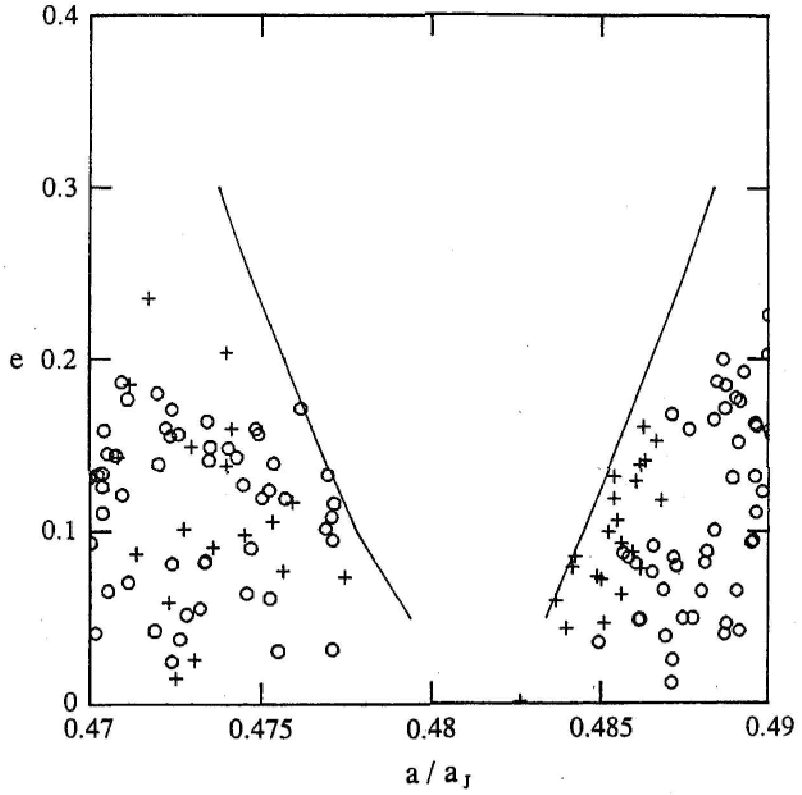


FIGURE 9. Comparison of the actual distribution of asteroids with the outer boundaries of the chaotic zone. There is both a chaotic region and quasiperiodic region in the gap, but trajectories of both types are planet crossing.

Wisdom, *Nuclear Phys. B* **2:***391*

Chaotic tumbling of satellites:

Voyager and Galileo saw this…

Ap. J. **97**:570
Ap. J. **98**:1855

TUMBLING FOR YOU The Galileo spacecraft moved in for a close-up of minor planet 951 Gaspra on October 29, 1991, providing us with the first detailed view of an asteroid. The first image at far left was taken 5¼ hours before closest approach, the last snapped 30 minutes before. Courtesy NASA/JPL.

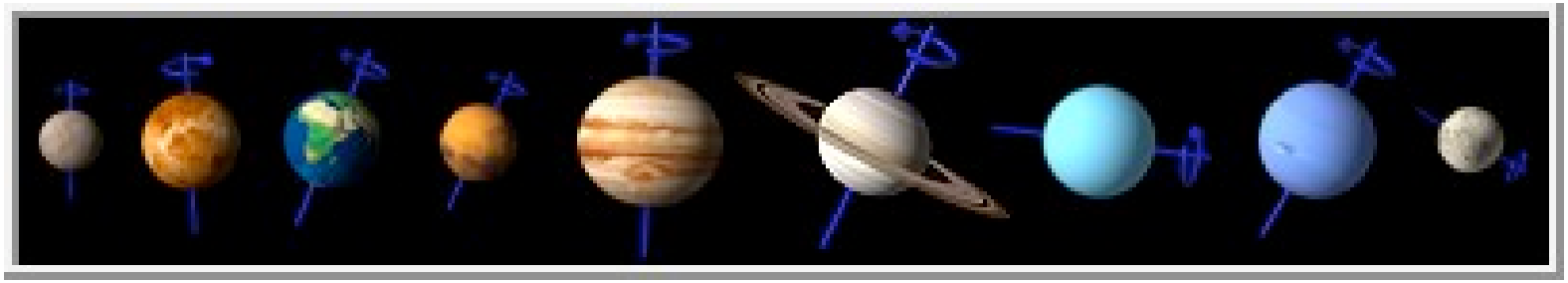
…and it happens for all satellites at some point in their history, unless they are perfectly spherical and in perfectly circular orbits (pf: KAM theorem; see [53] on syllabus.)

Show NASA movie of Hyperion tumbling

http://www.nasa.gov/mission_pages/cassini/multimedia/pia06243.html

More chaos in the solar system:

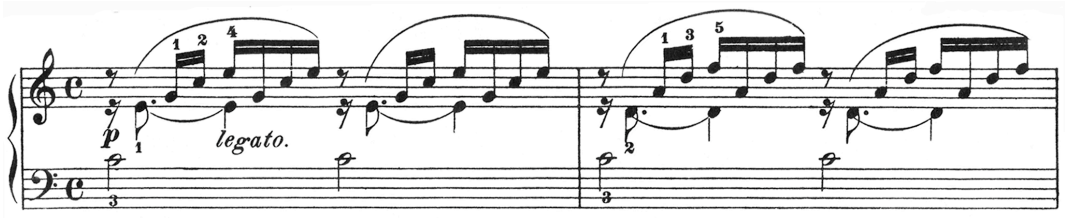
- obliquity of Mars (Touma & Wisdom, *Science* **259***:1294)*



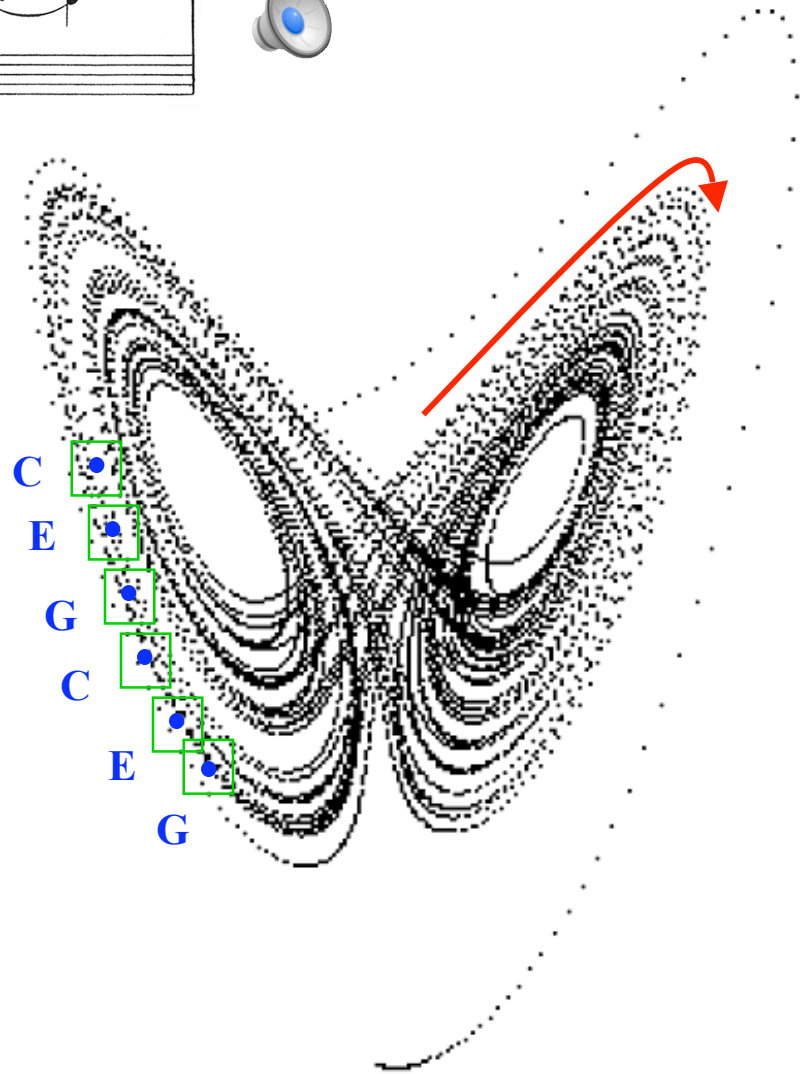
www.solarviews.com

- etc.

Musical Variations from a Chaotic Mapping



Pitch sequence:
C, E, G, C, E, G, C, E…



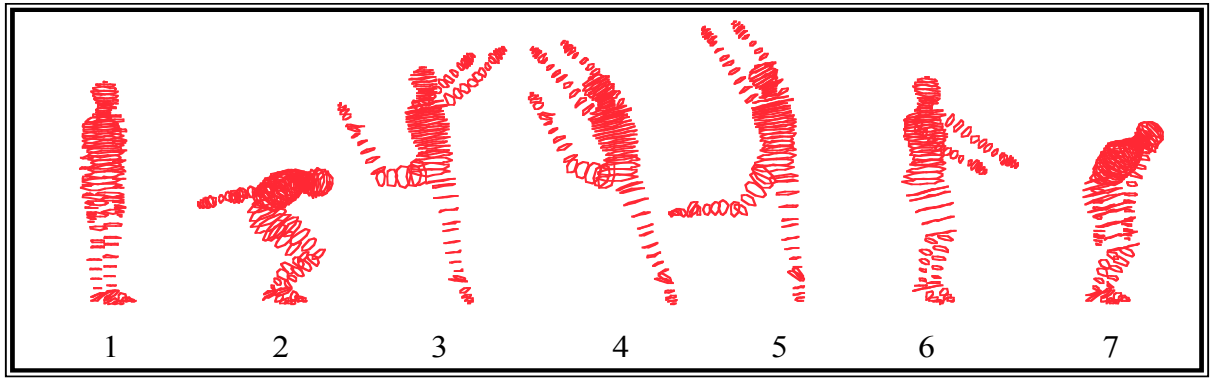
C **symbol dynamics**

variation!

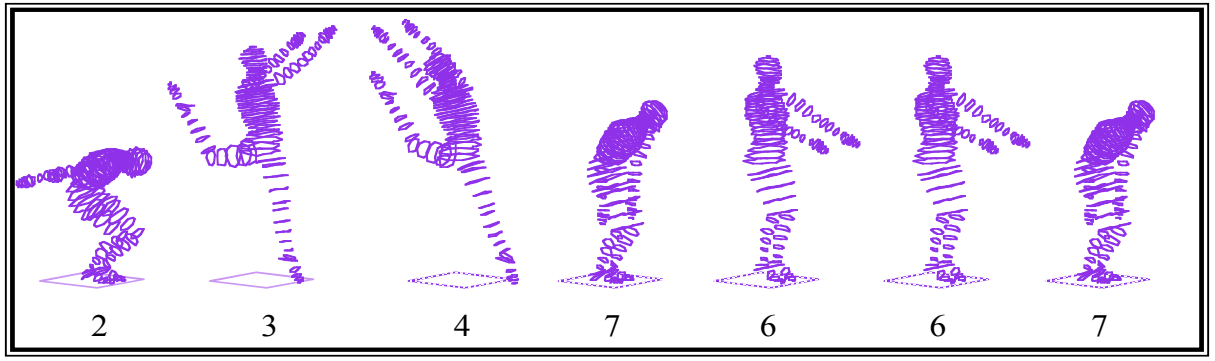
Dabby *Chaos* **6**:95

Chaotic variations on movement sequences

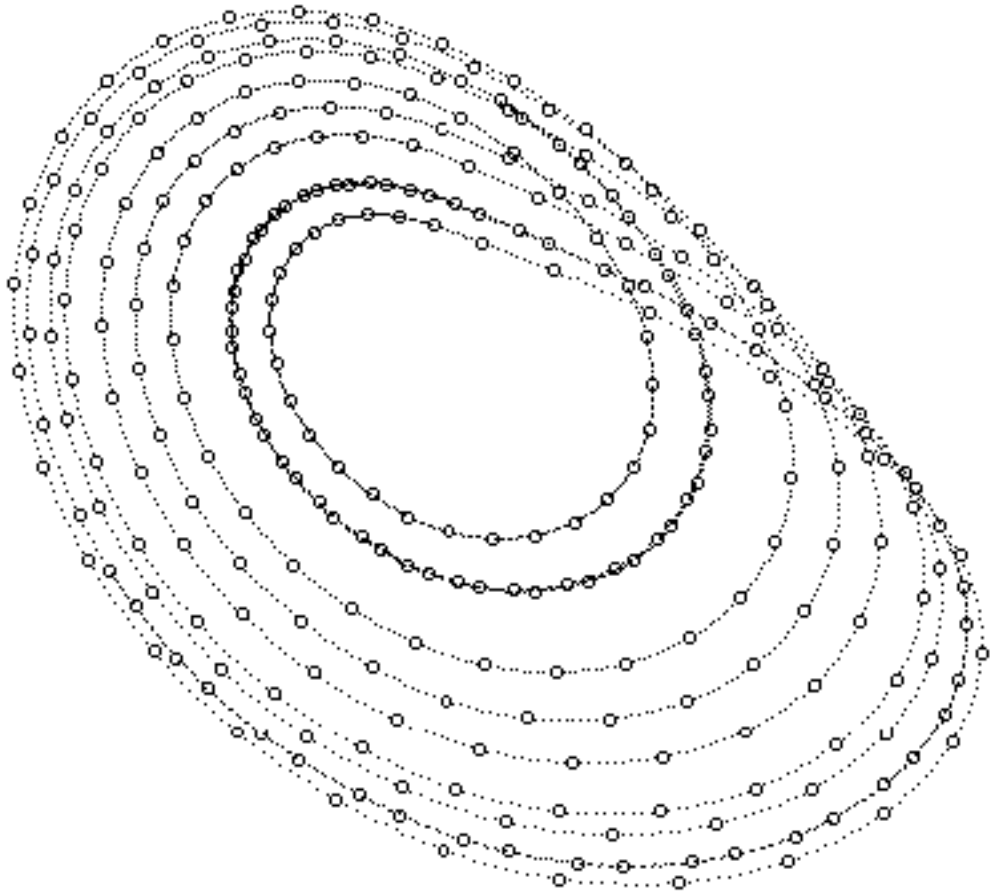
original piece

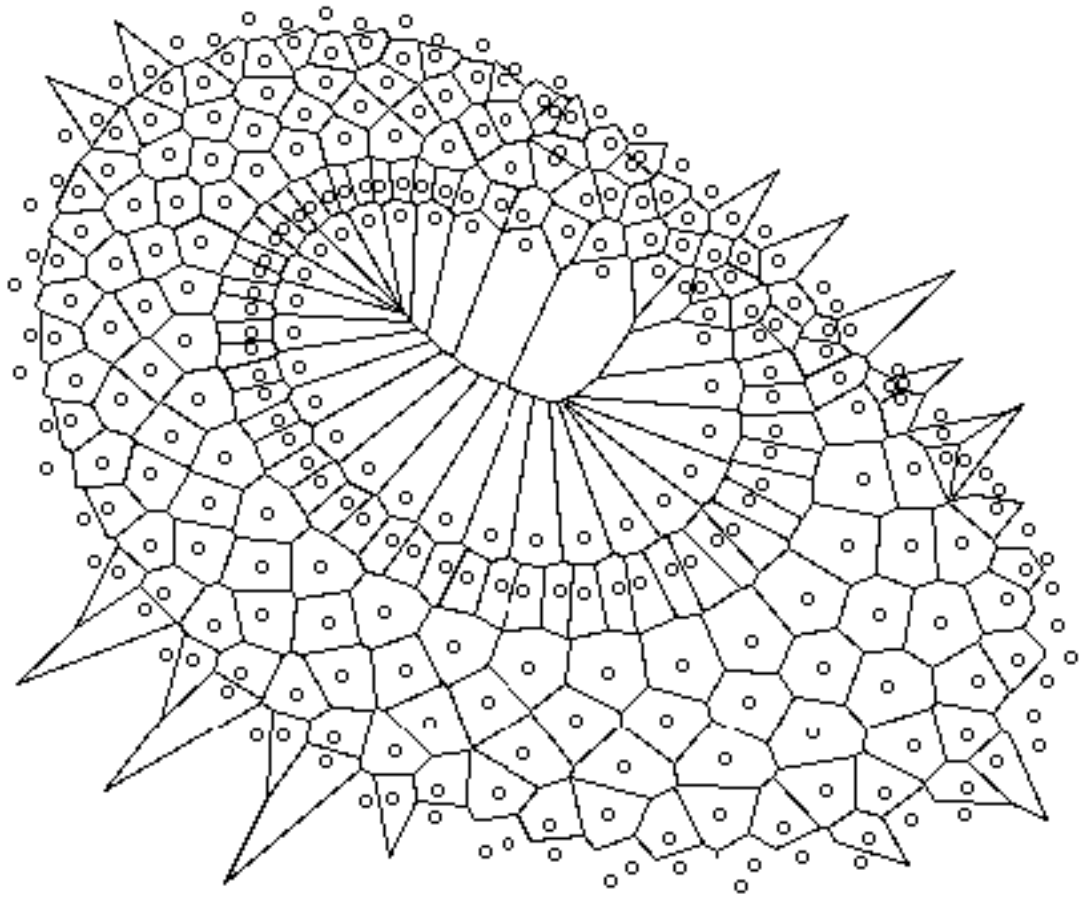


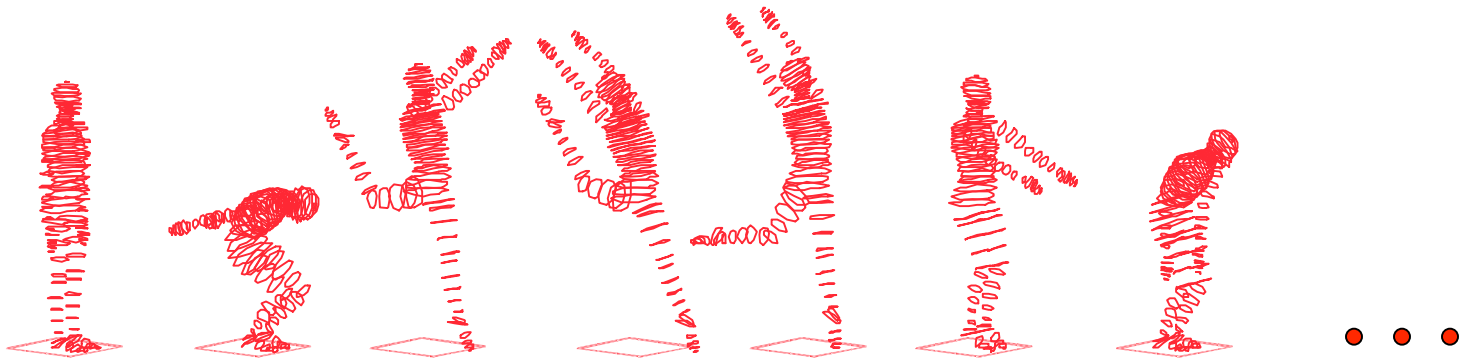
chaotic mapping

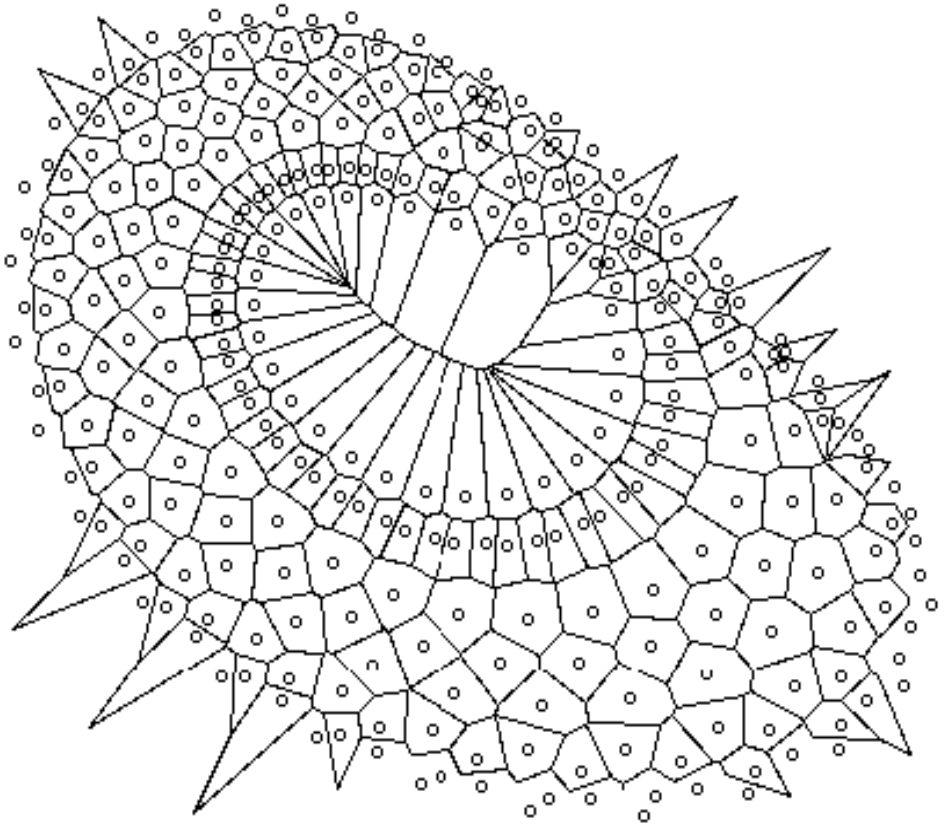


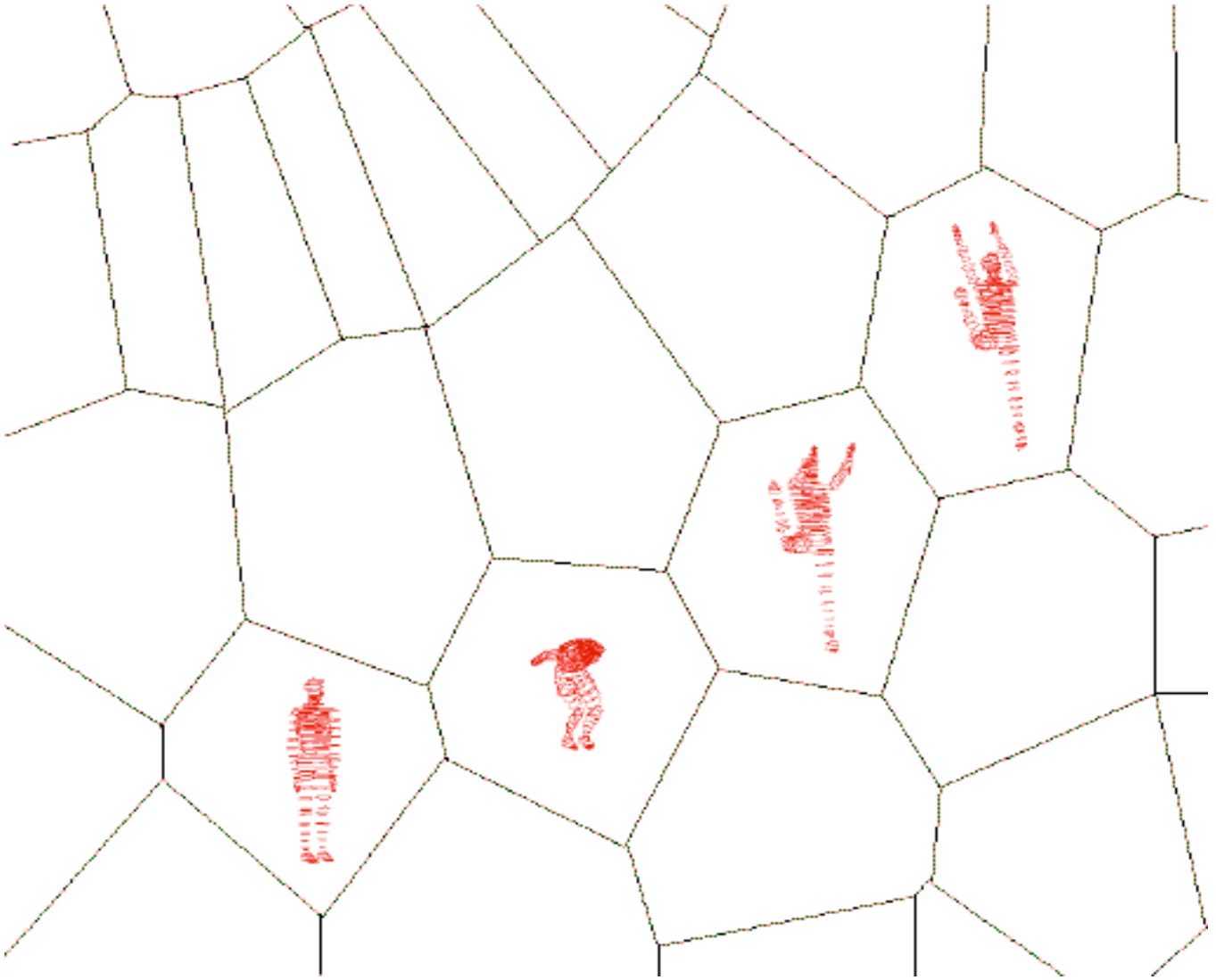
chaotic variation

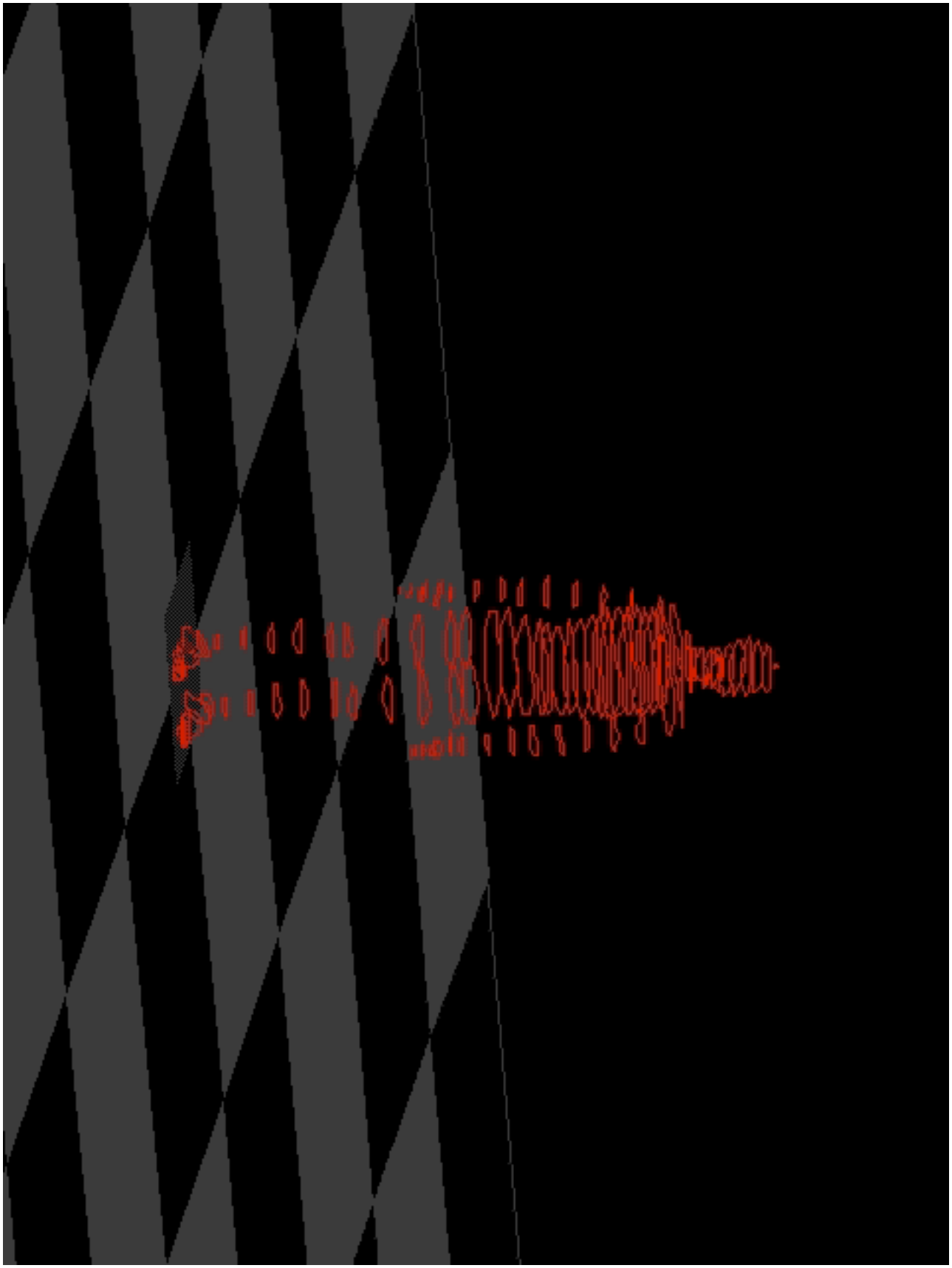


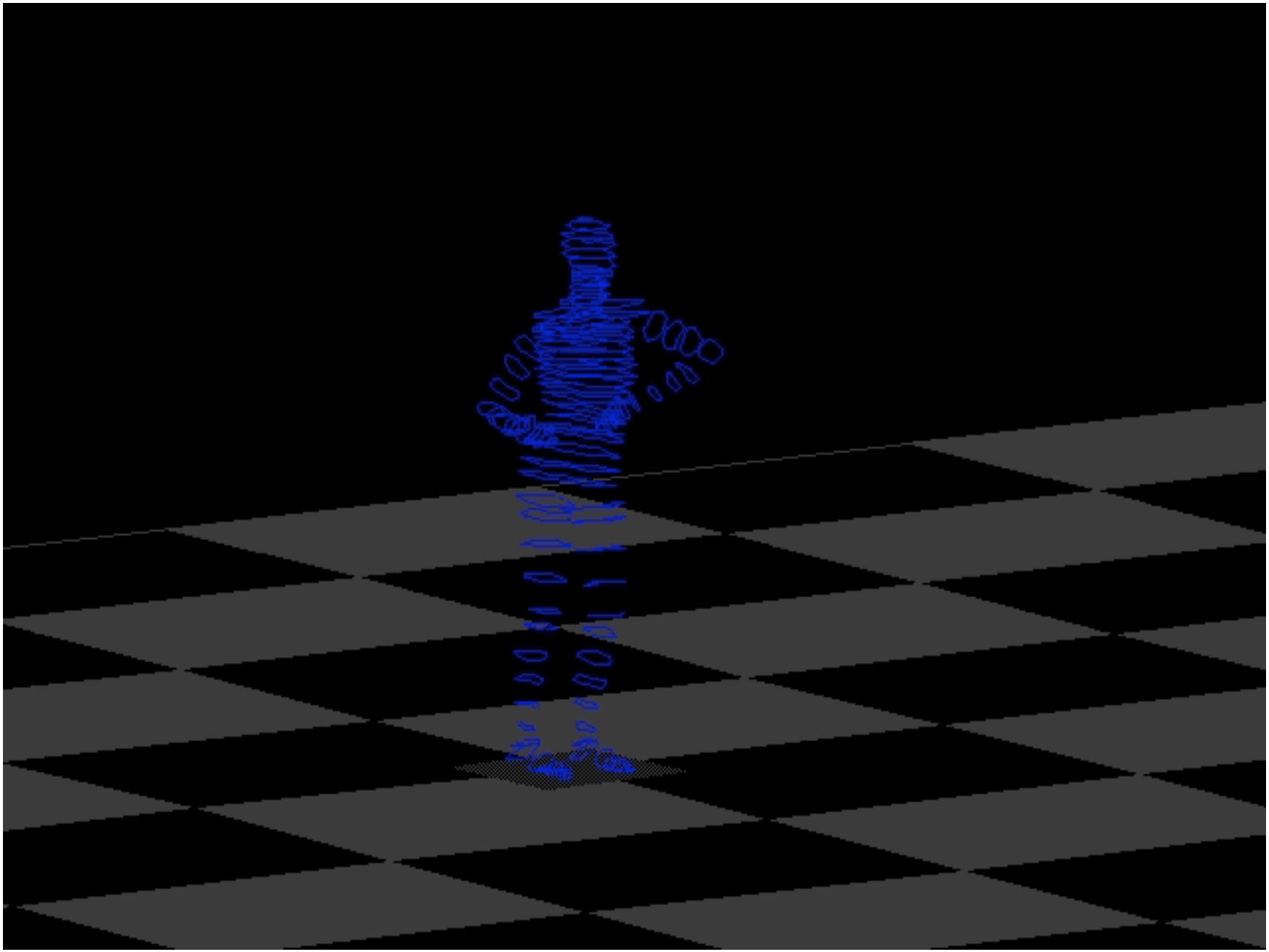




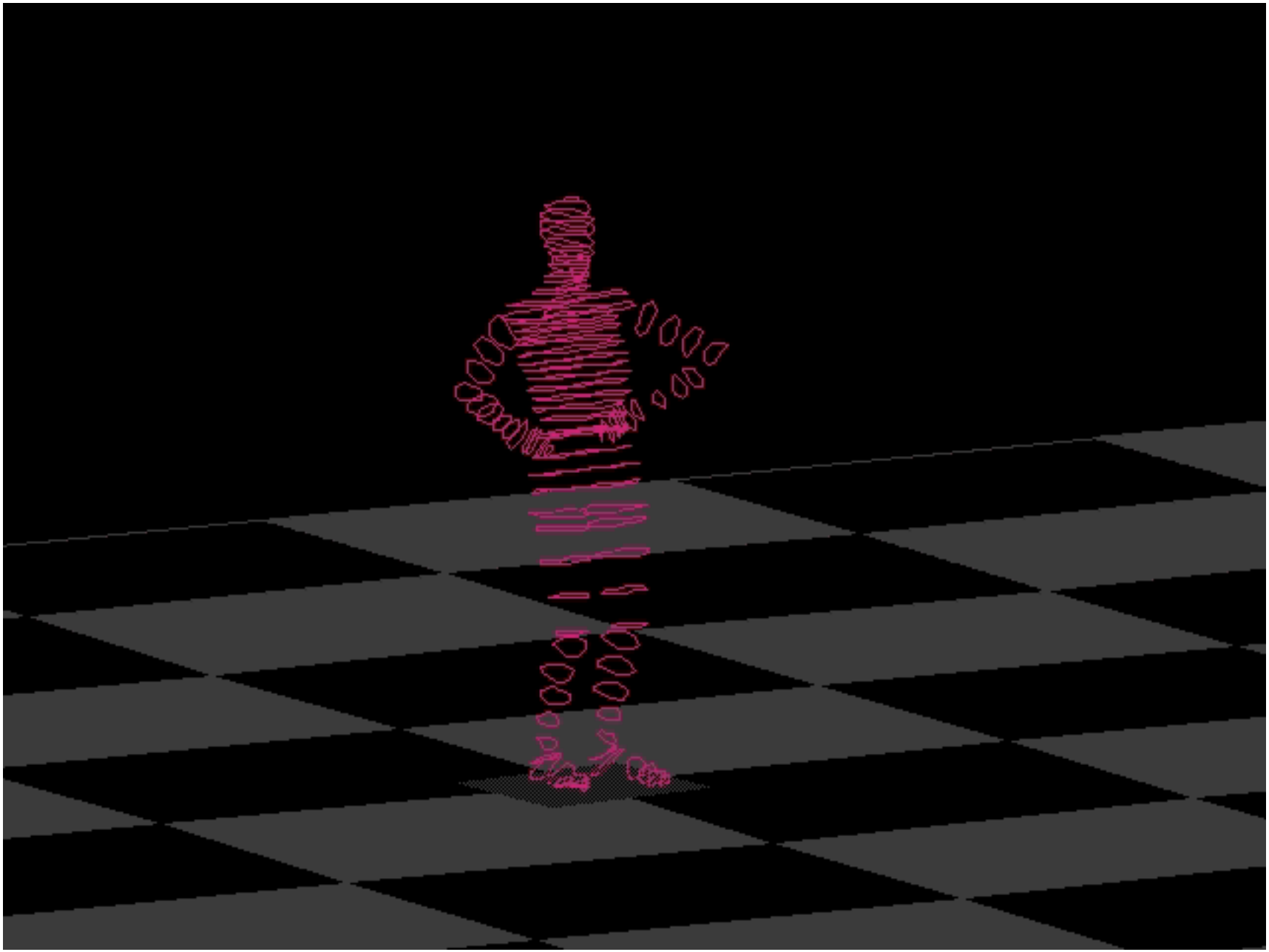




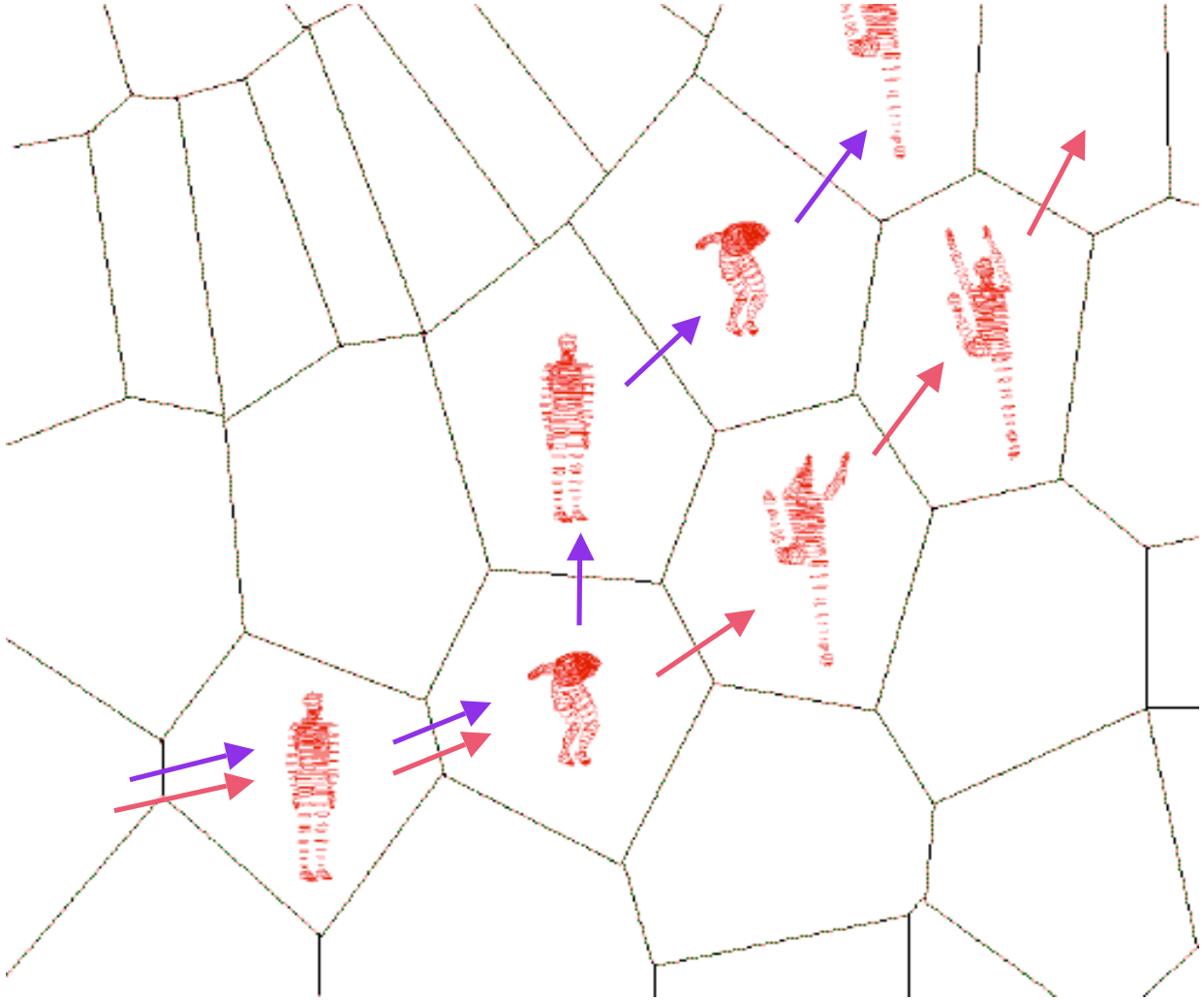




Lorenz

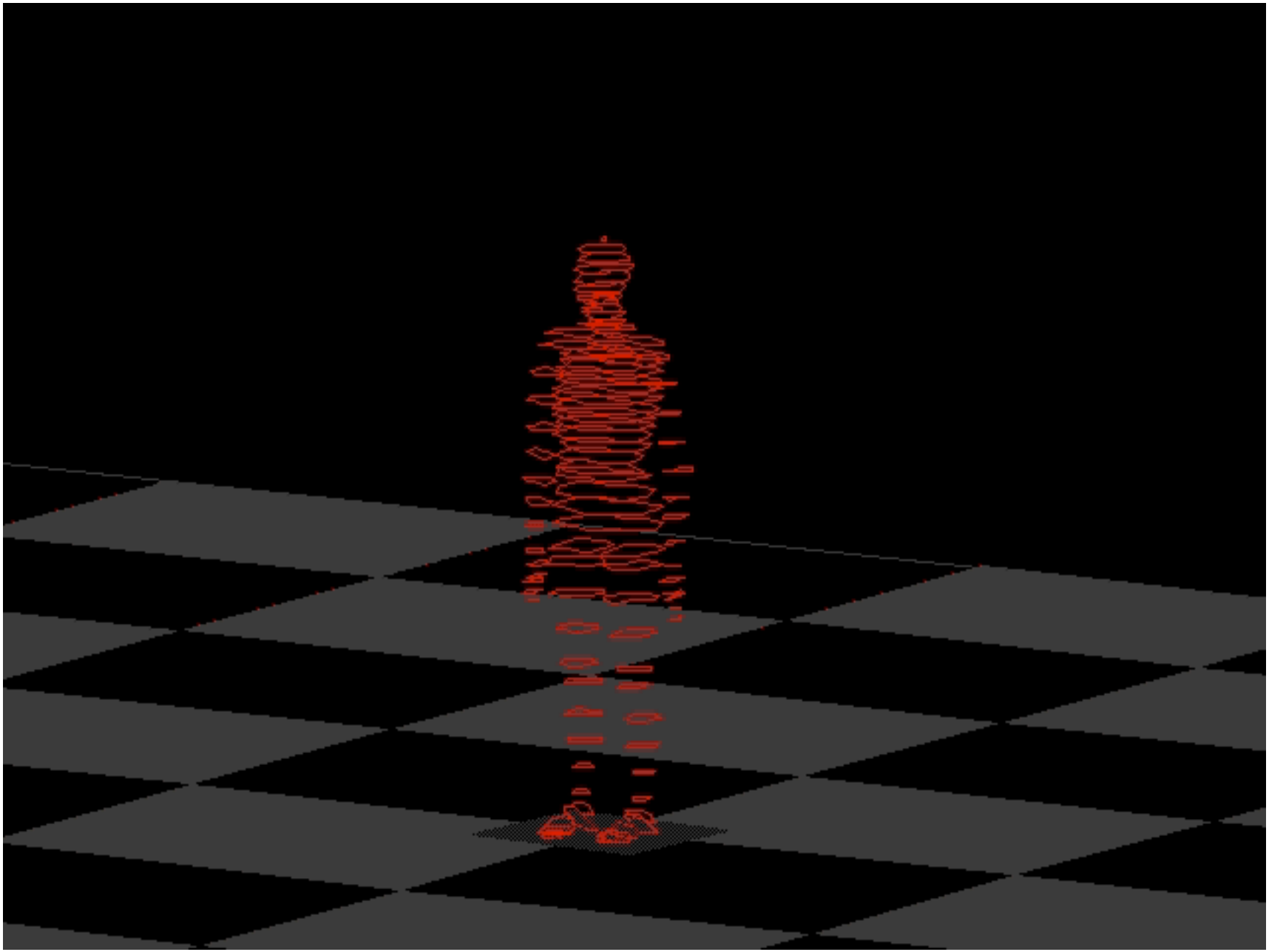


Rossler

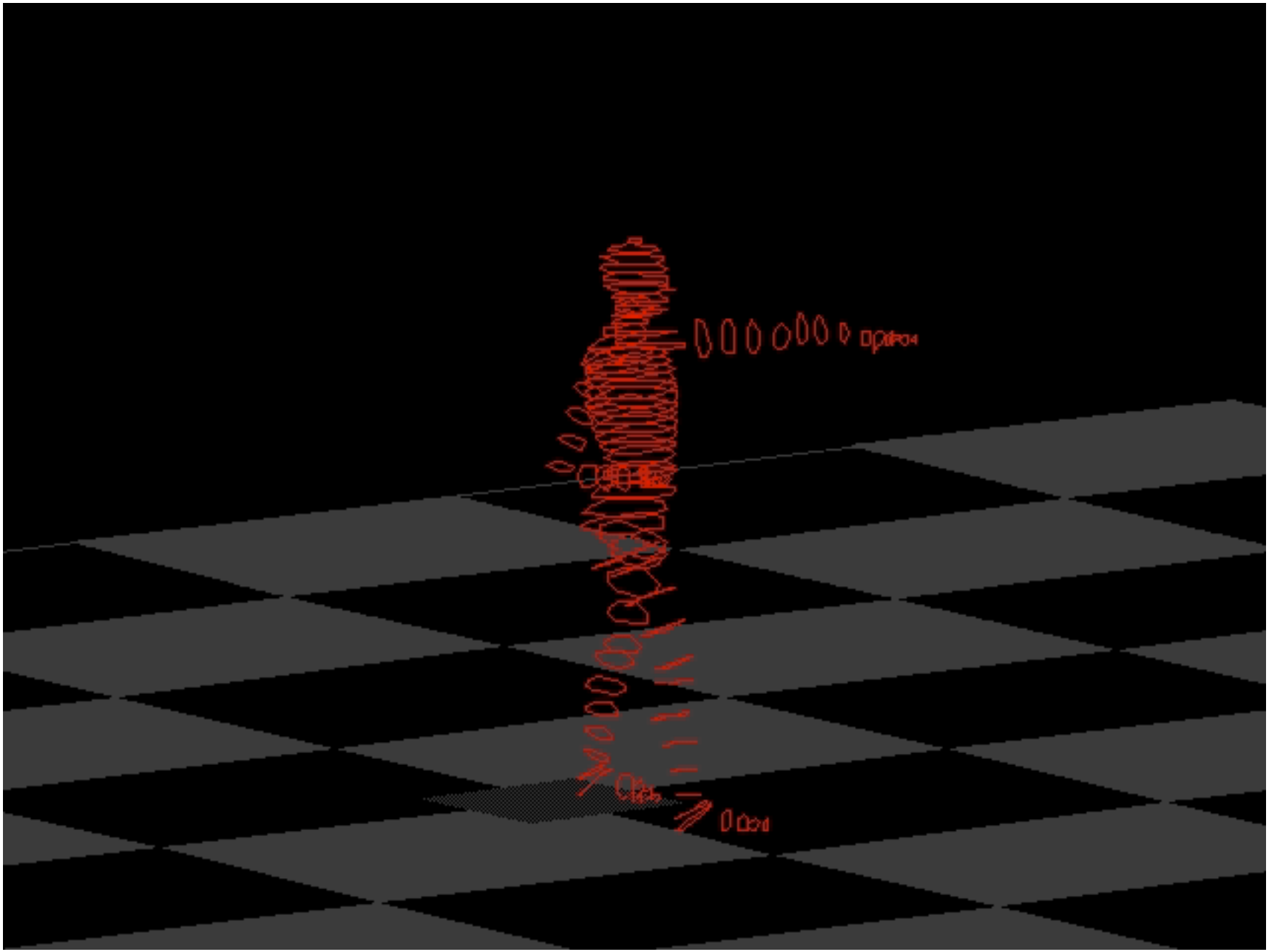


original cell itinerary

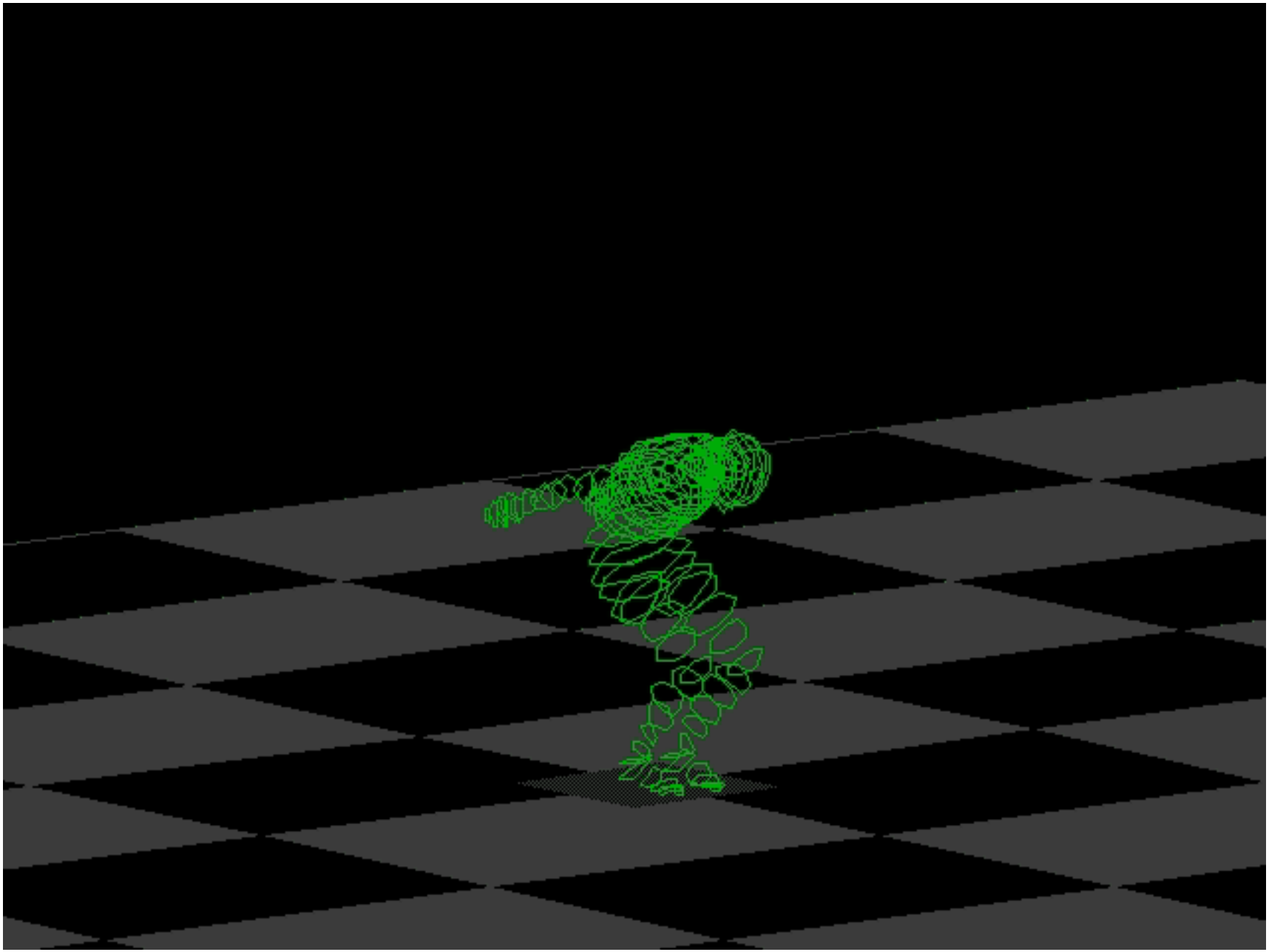
variation cell itinerary



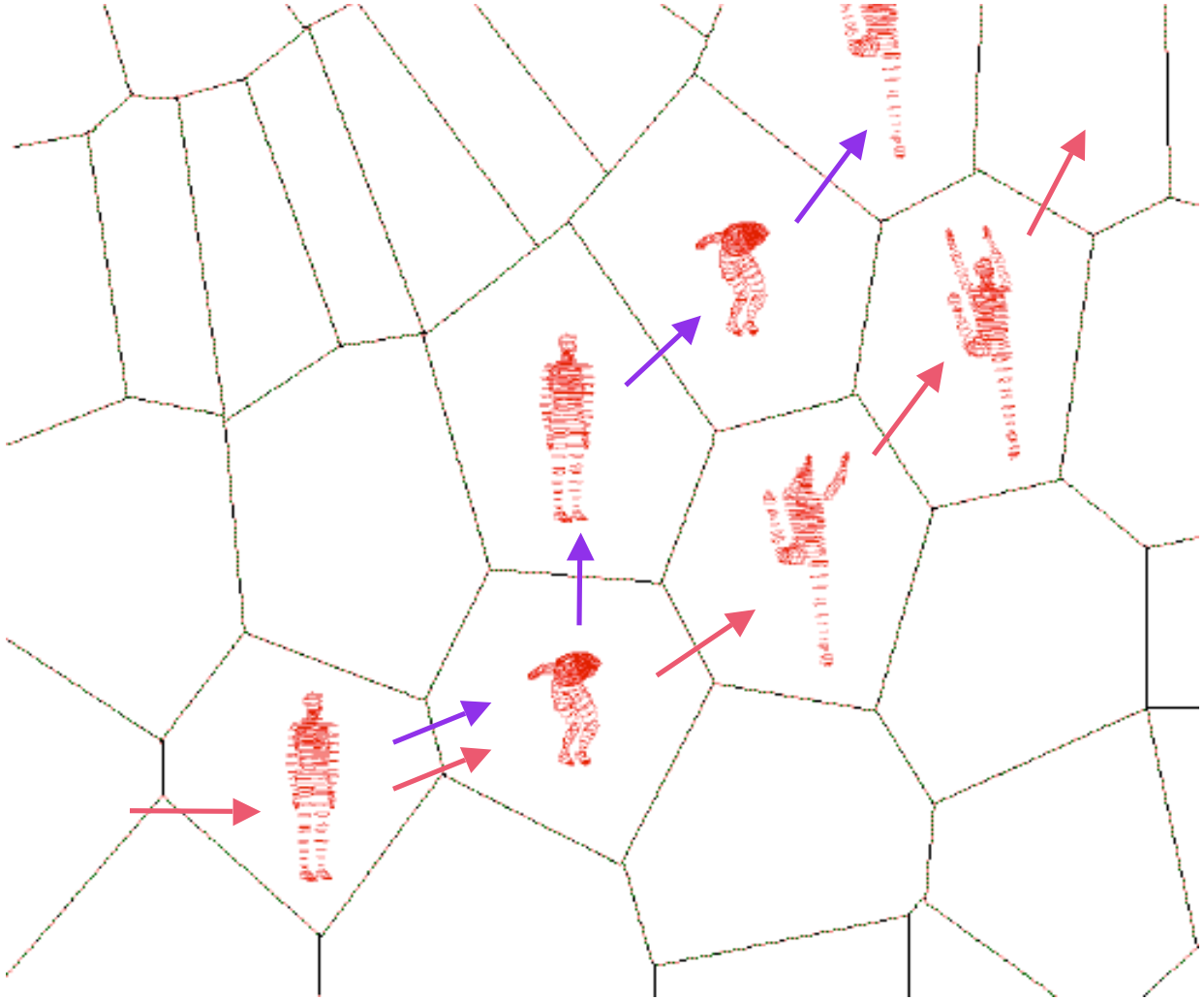
medley



Rossler medley

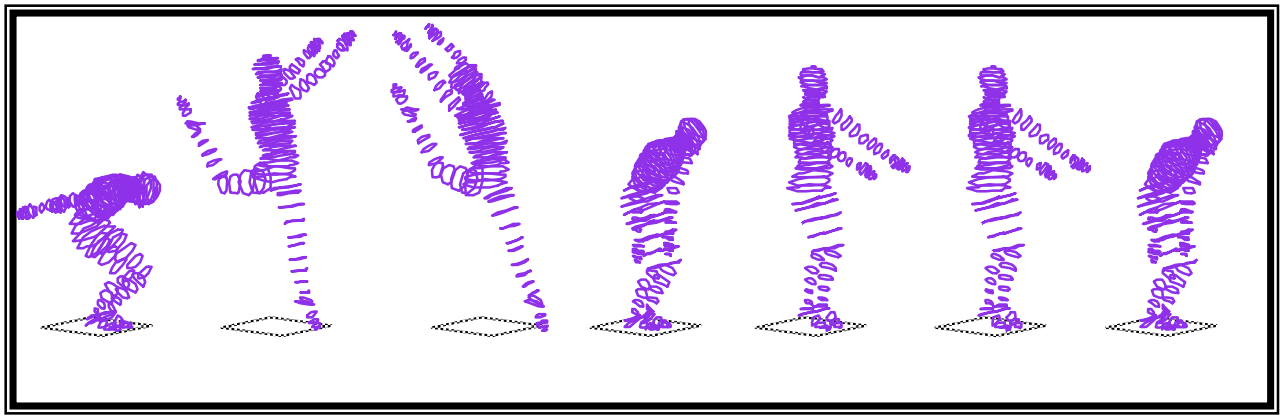


random medley

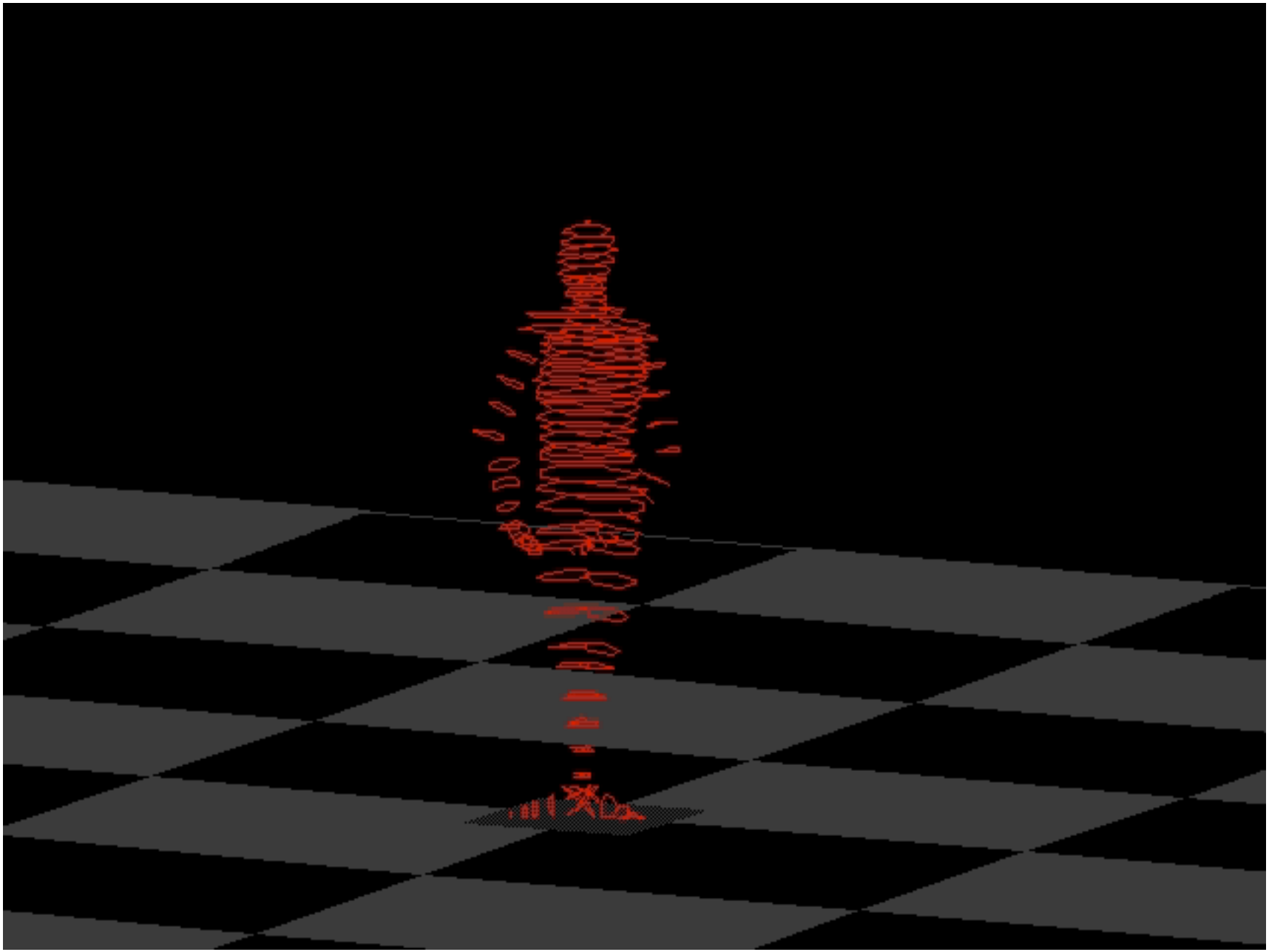


original cell itinerary →

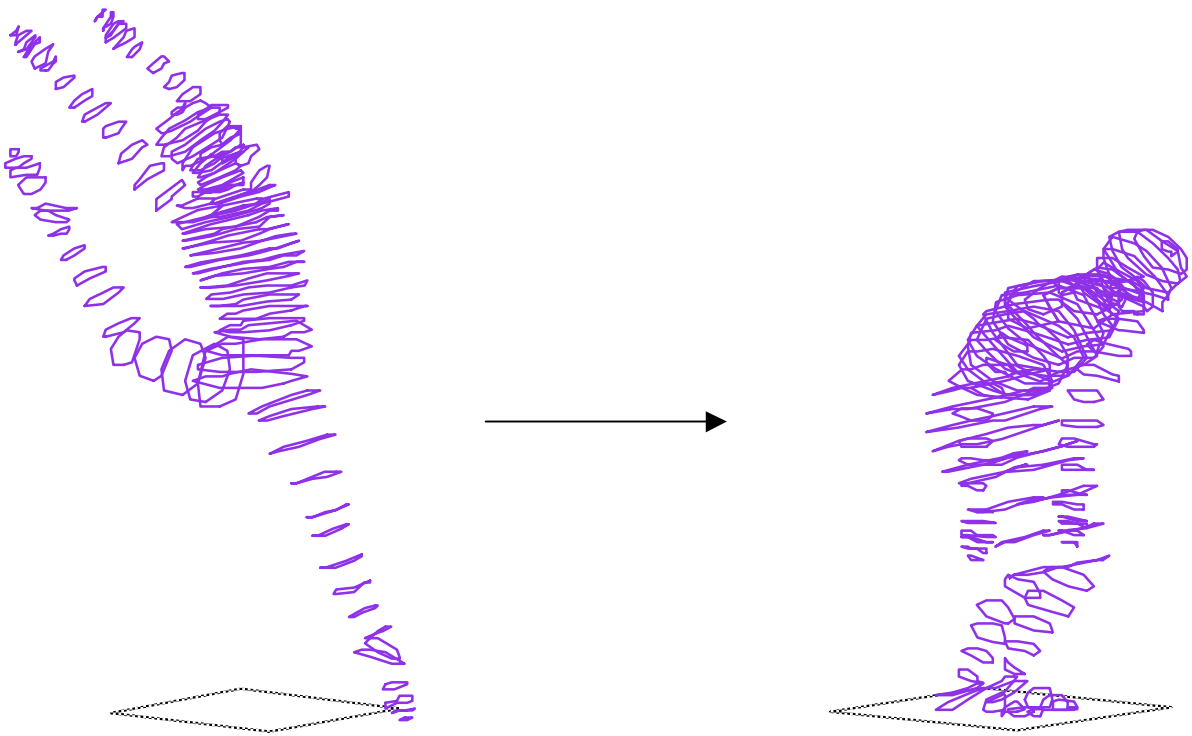
variation cell itinerary →



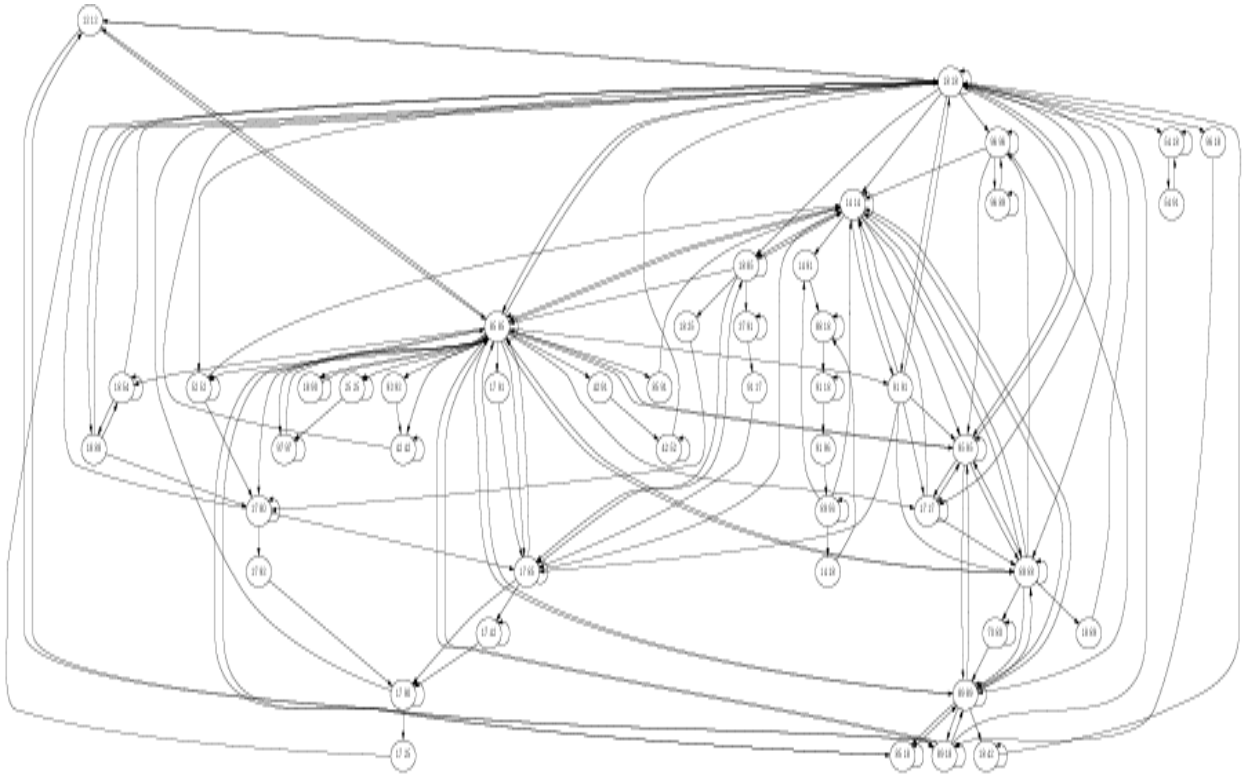
abrupt transition



Interpolation

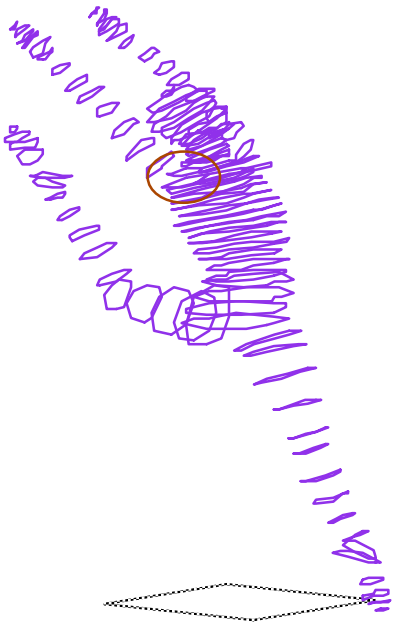


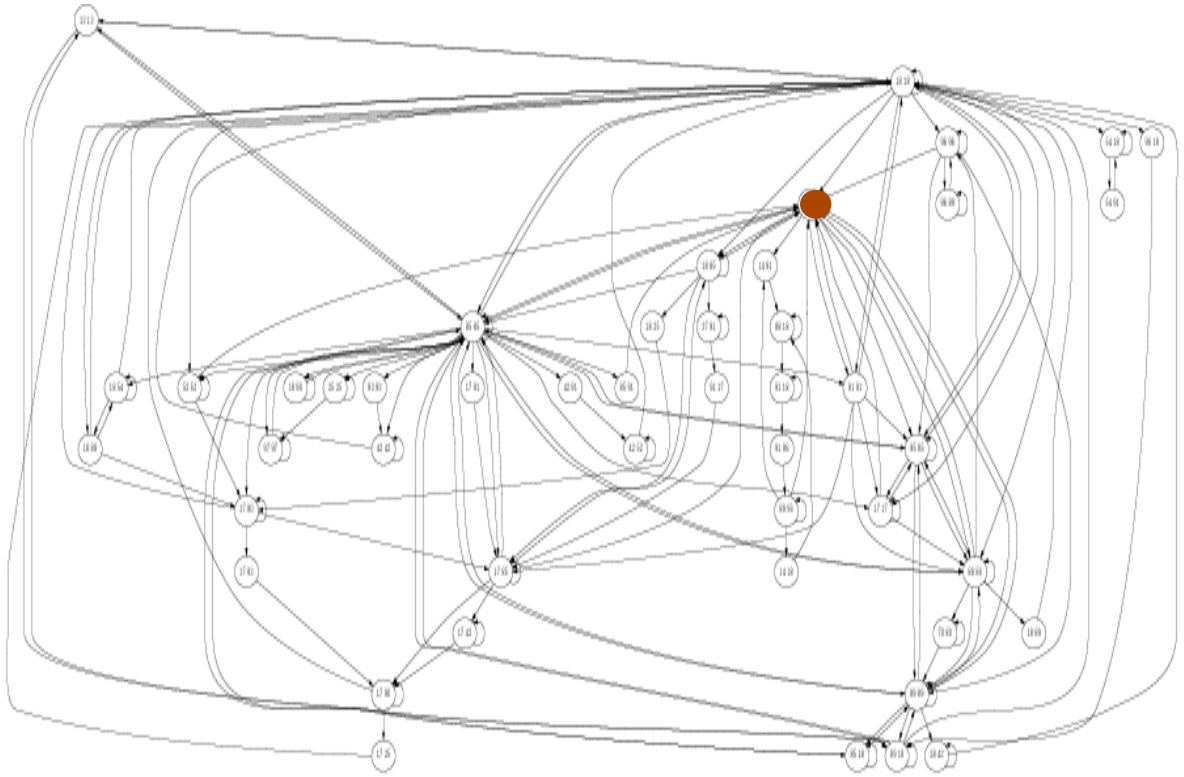
Corpus-based approach



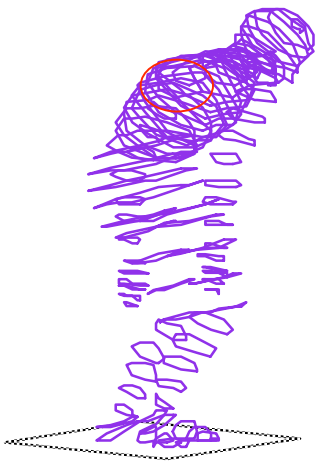
- graph captures motions of one joint
- note: specific to the genre of the corpus!

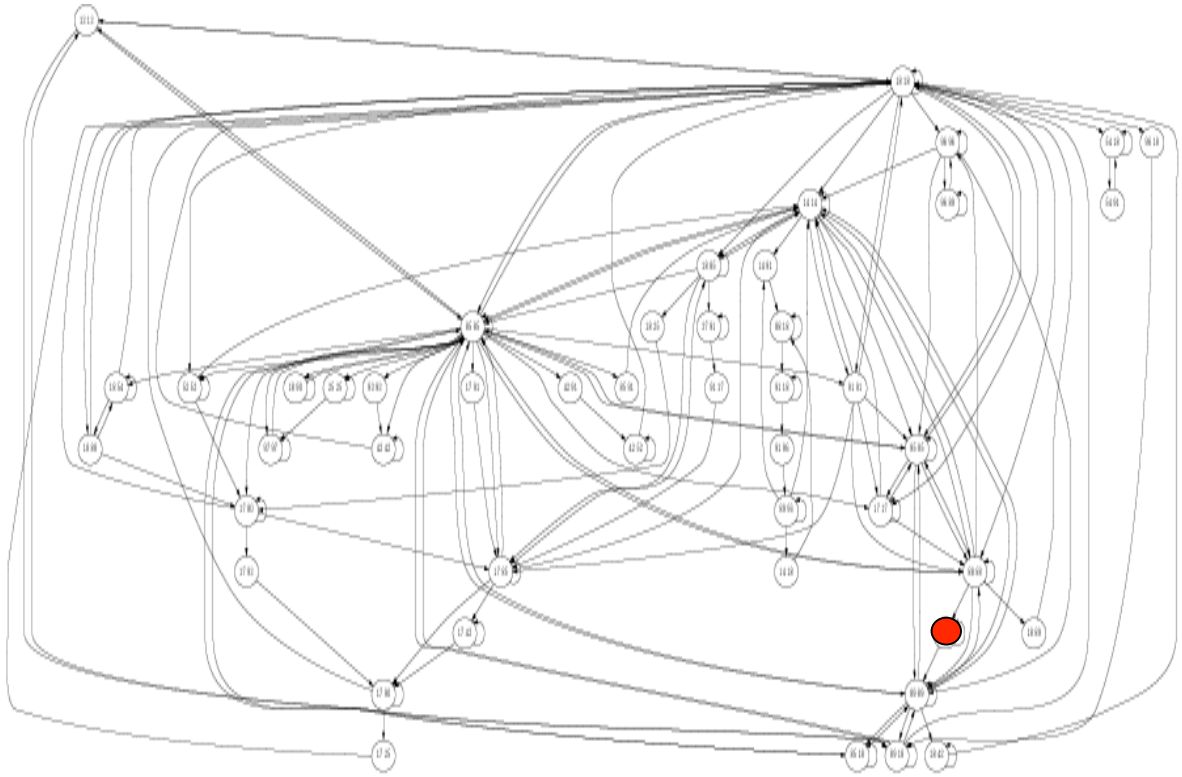
Initial state



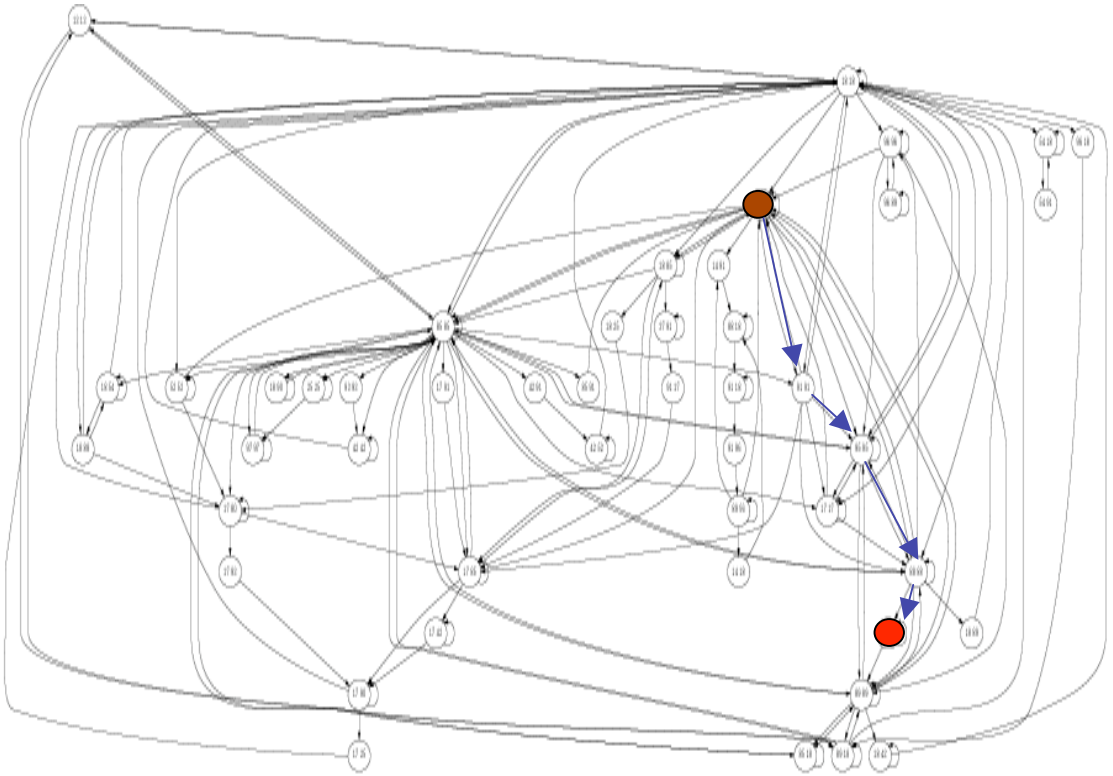


Target state

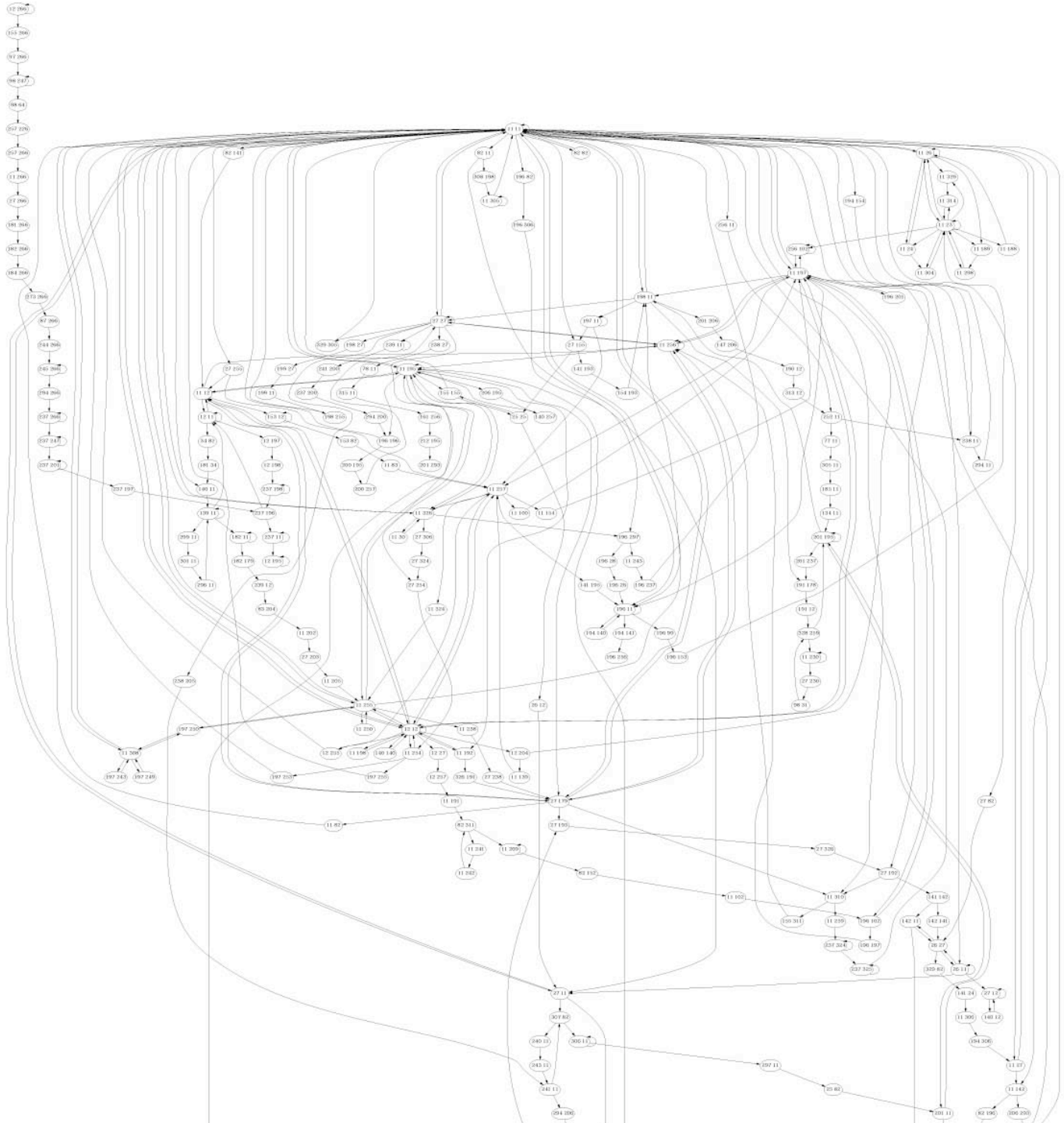




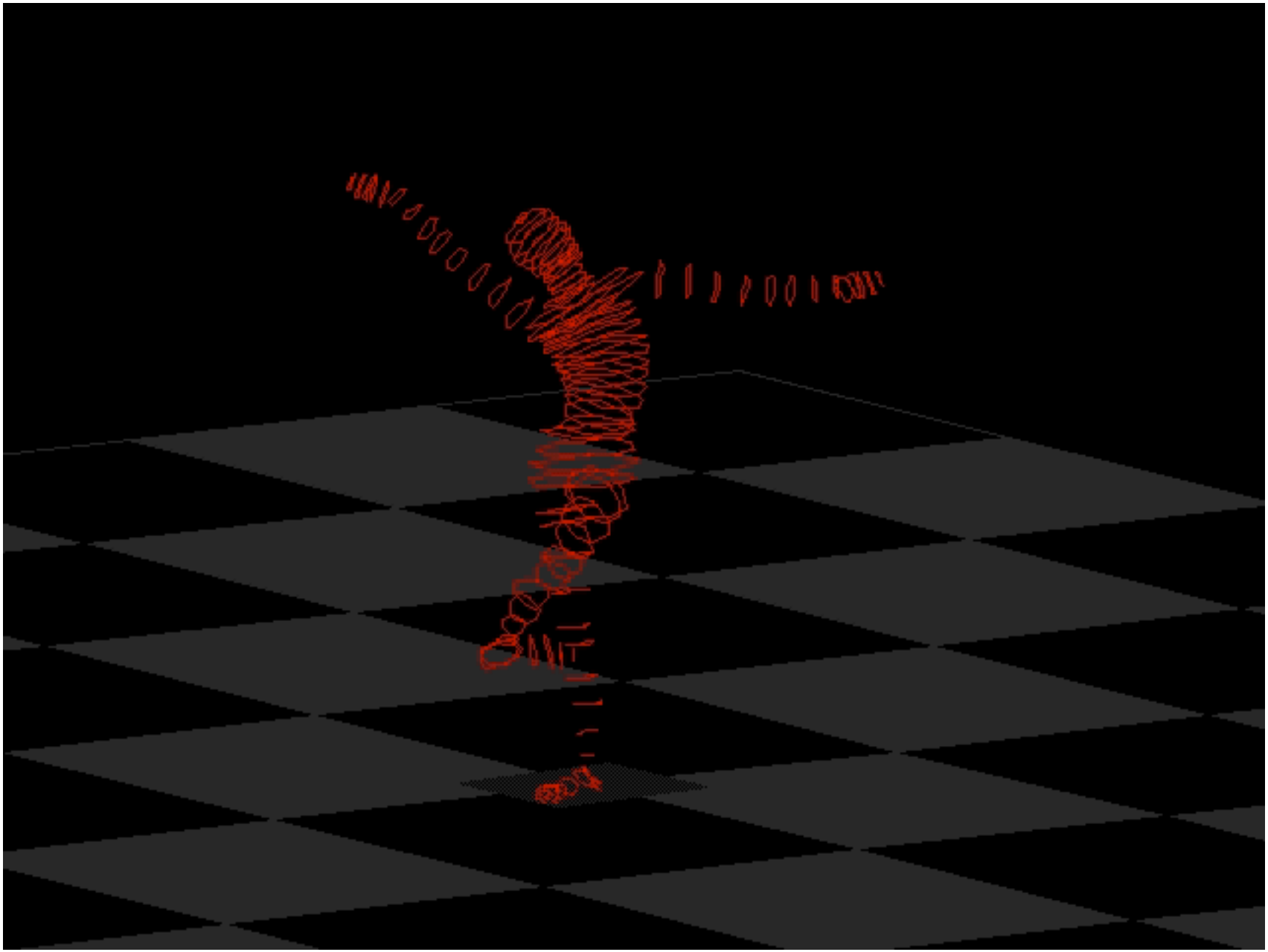
Graph search



…for 44 joints in parallel!



initial



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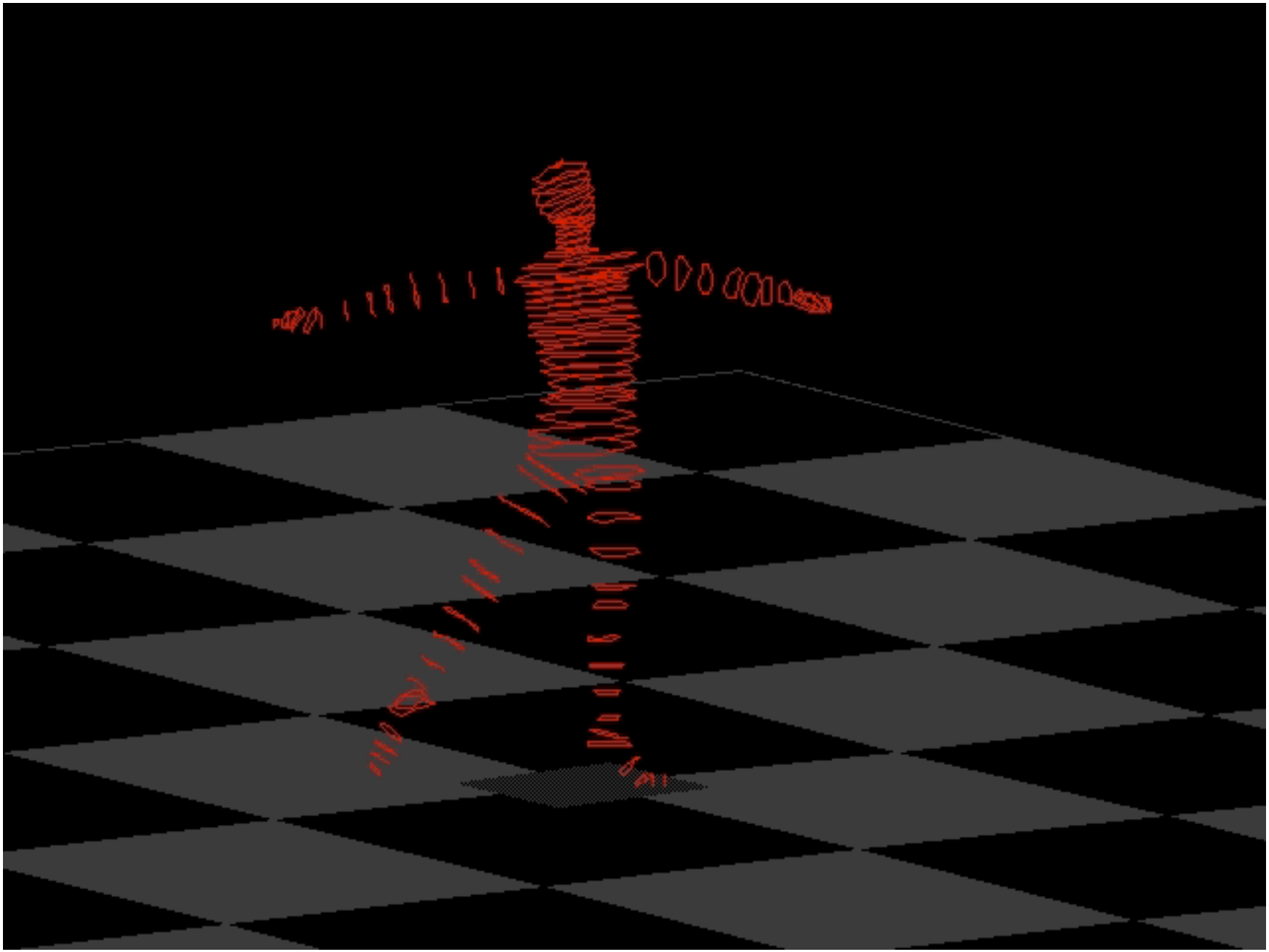
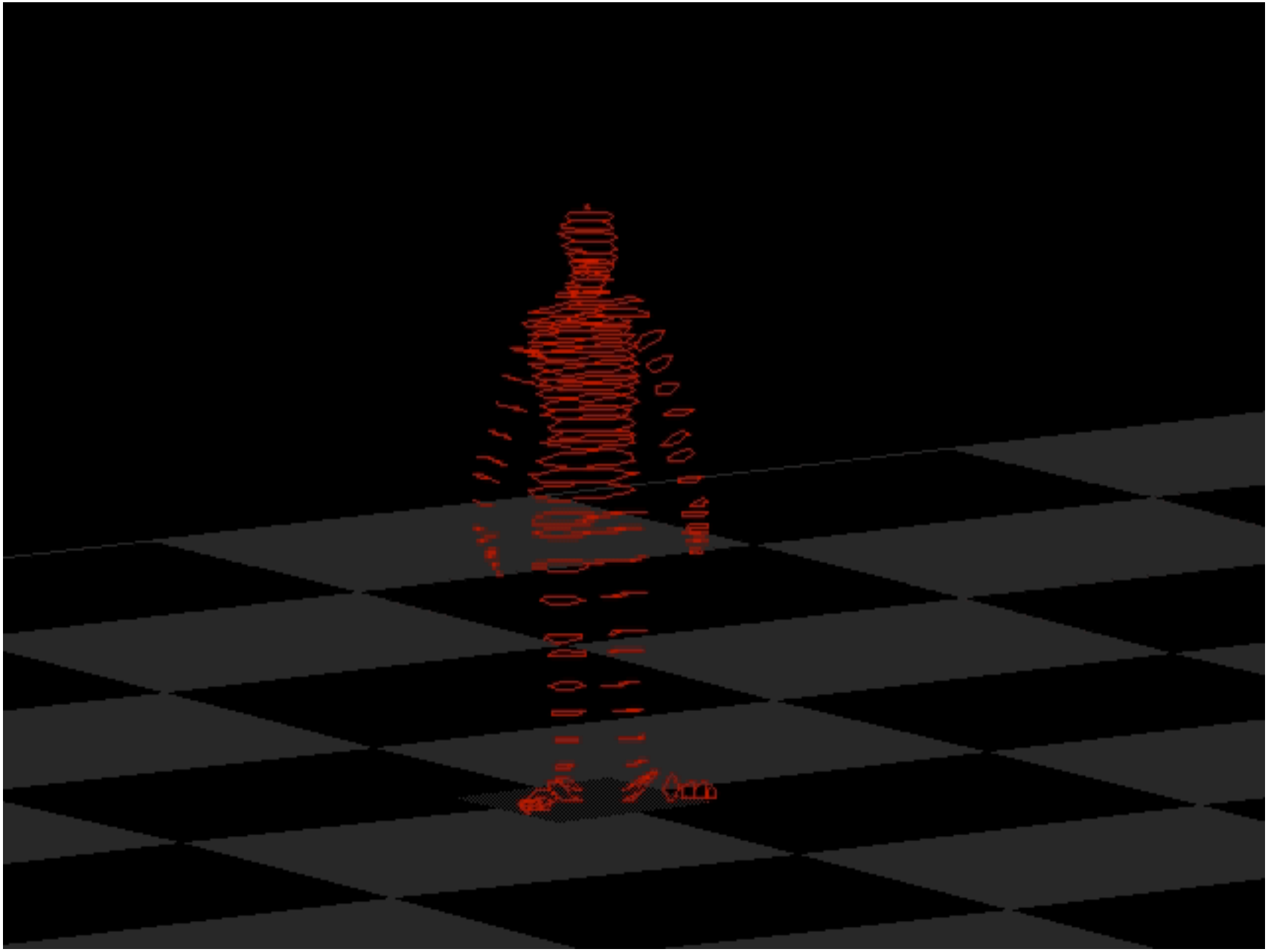


interpolation 1



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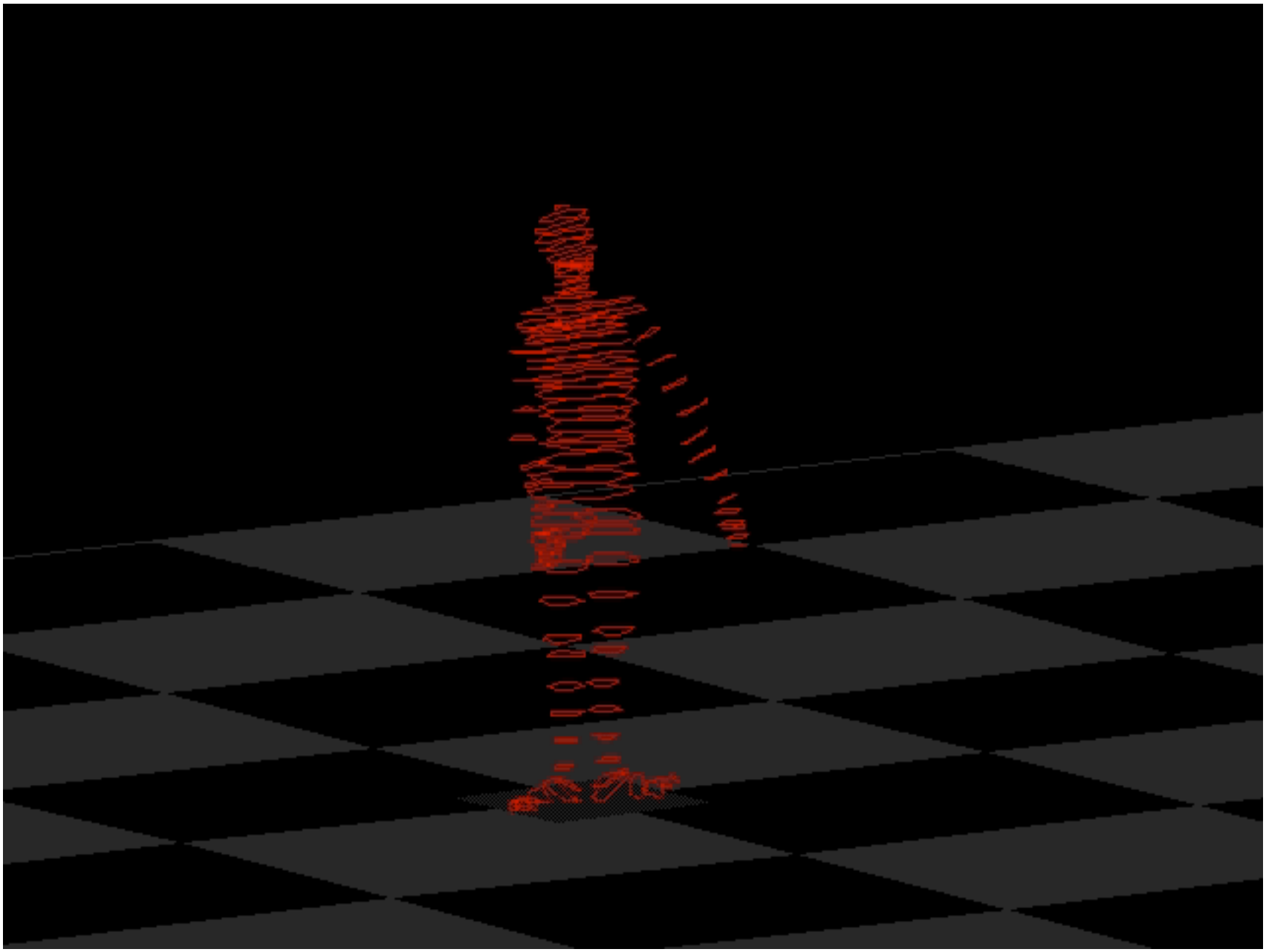

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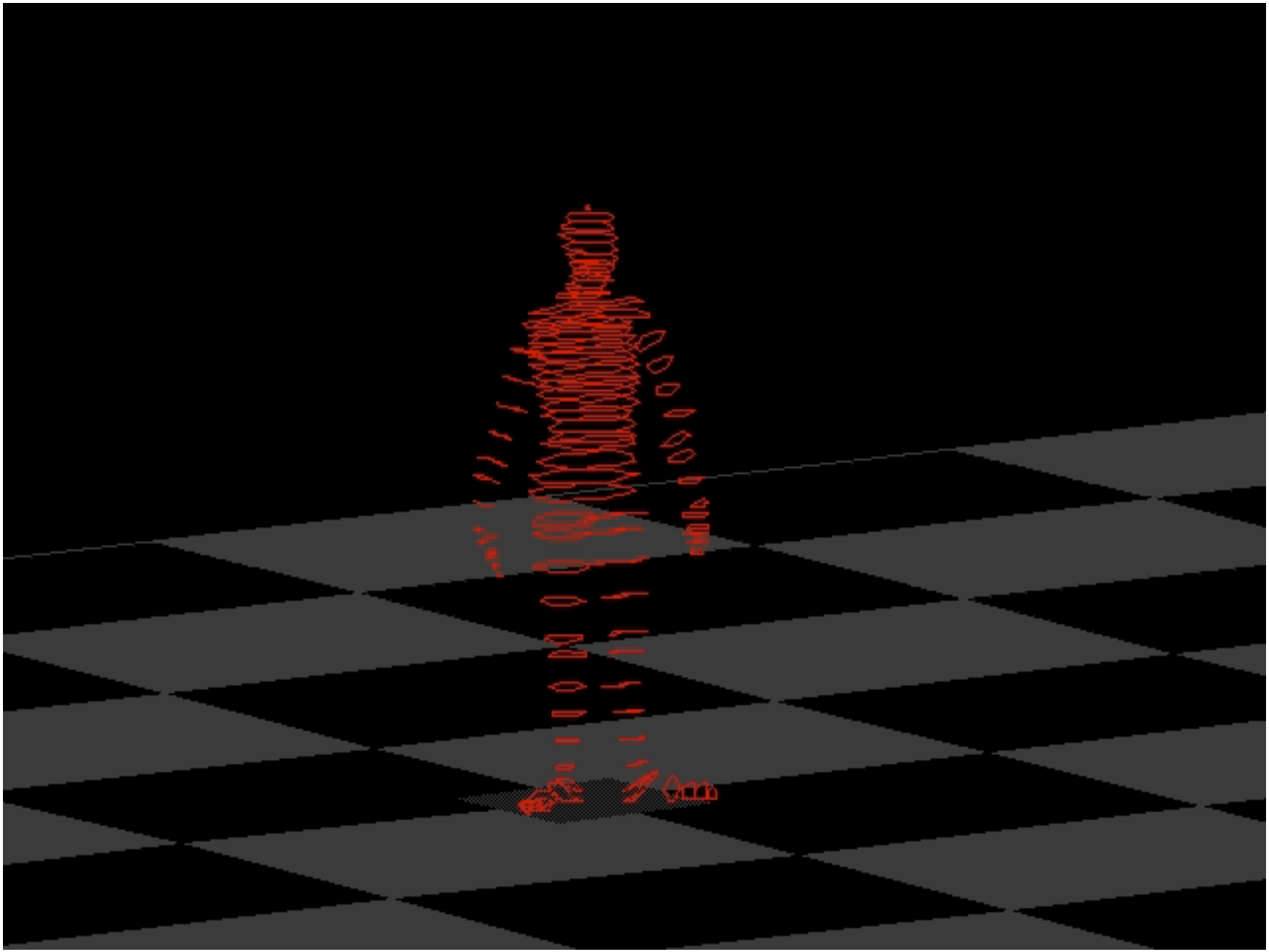
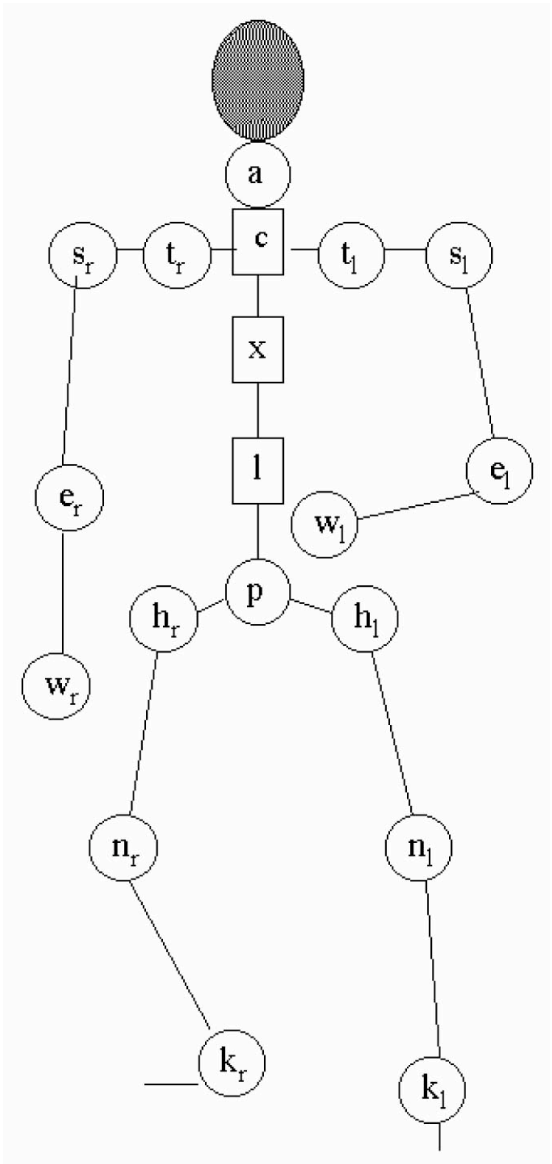


interpolation 2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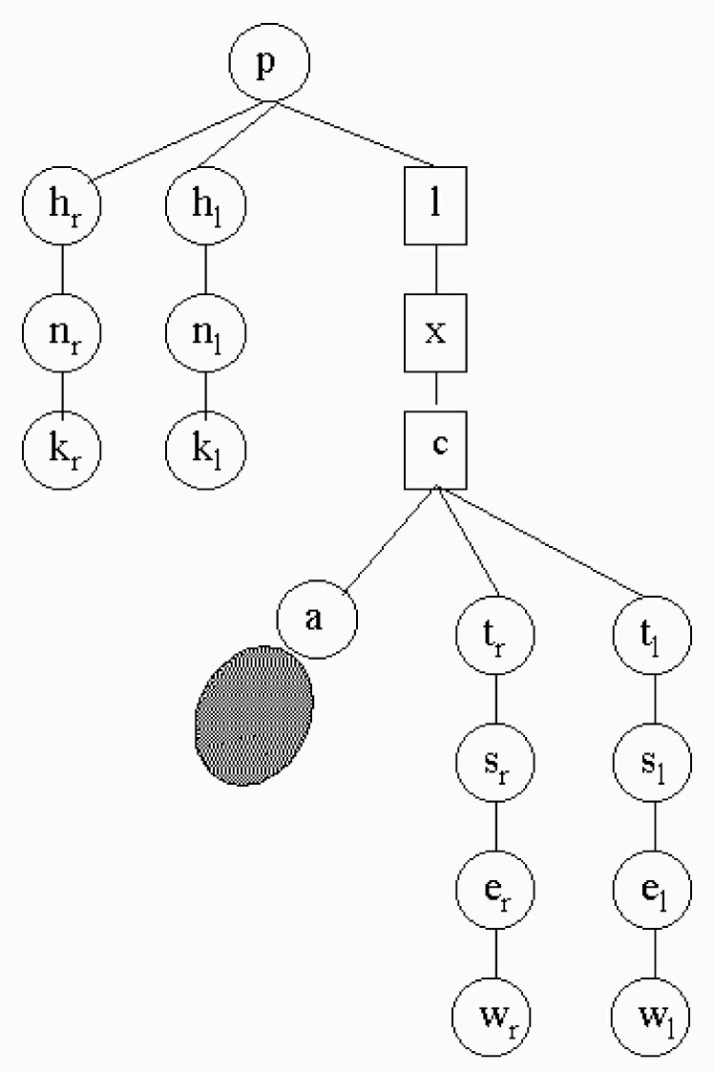
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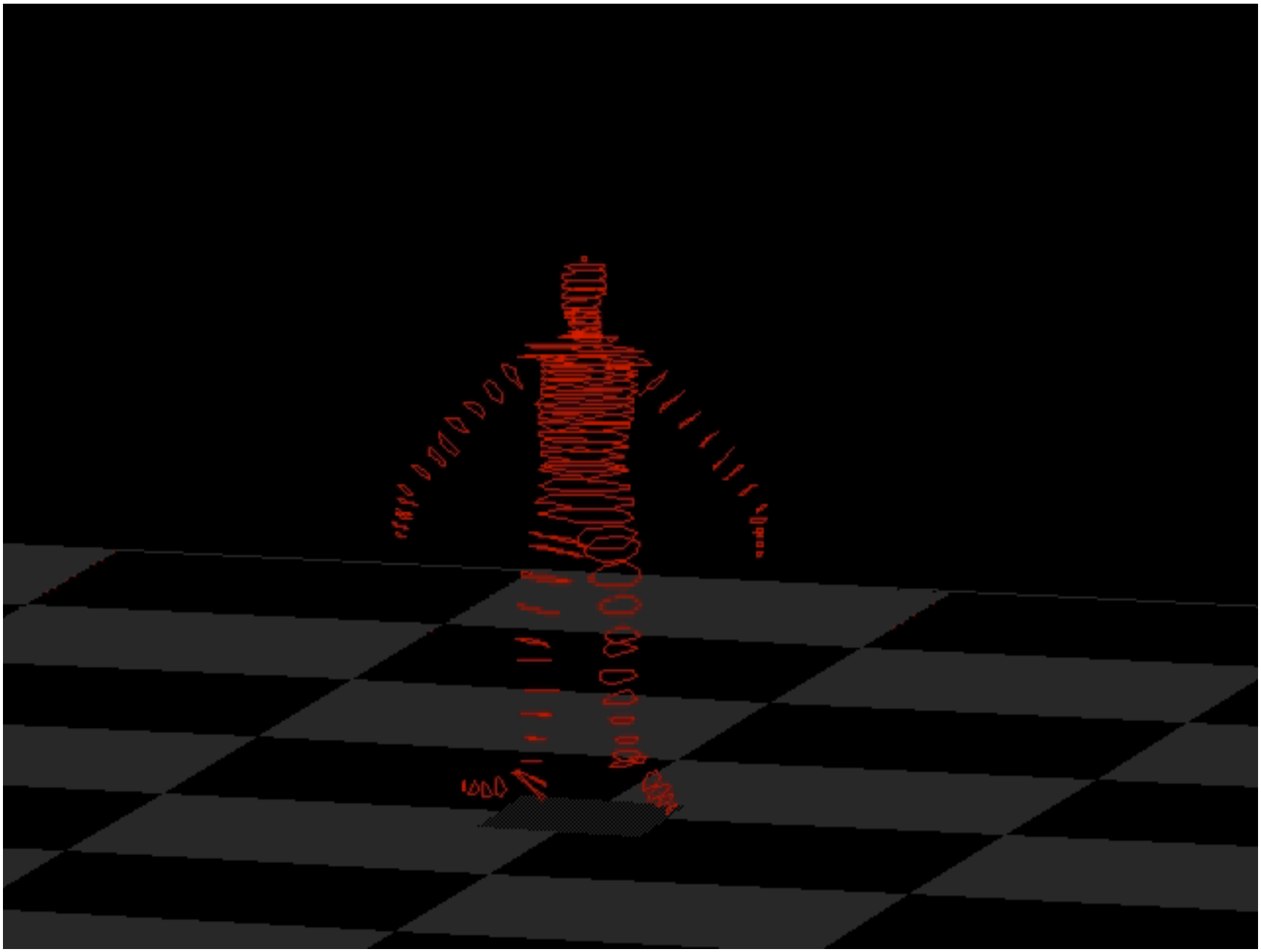

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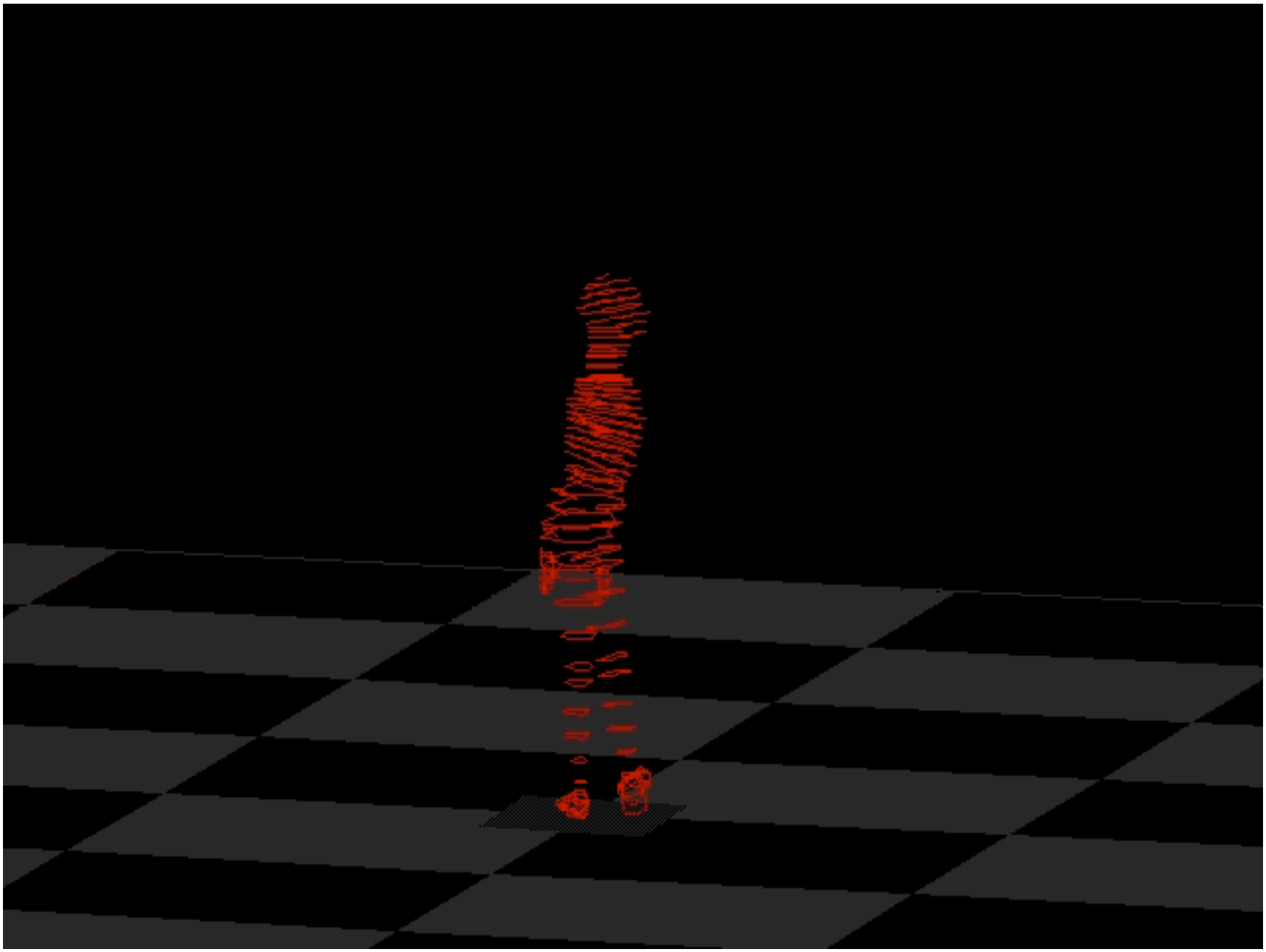
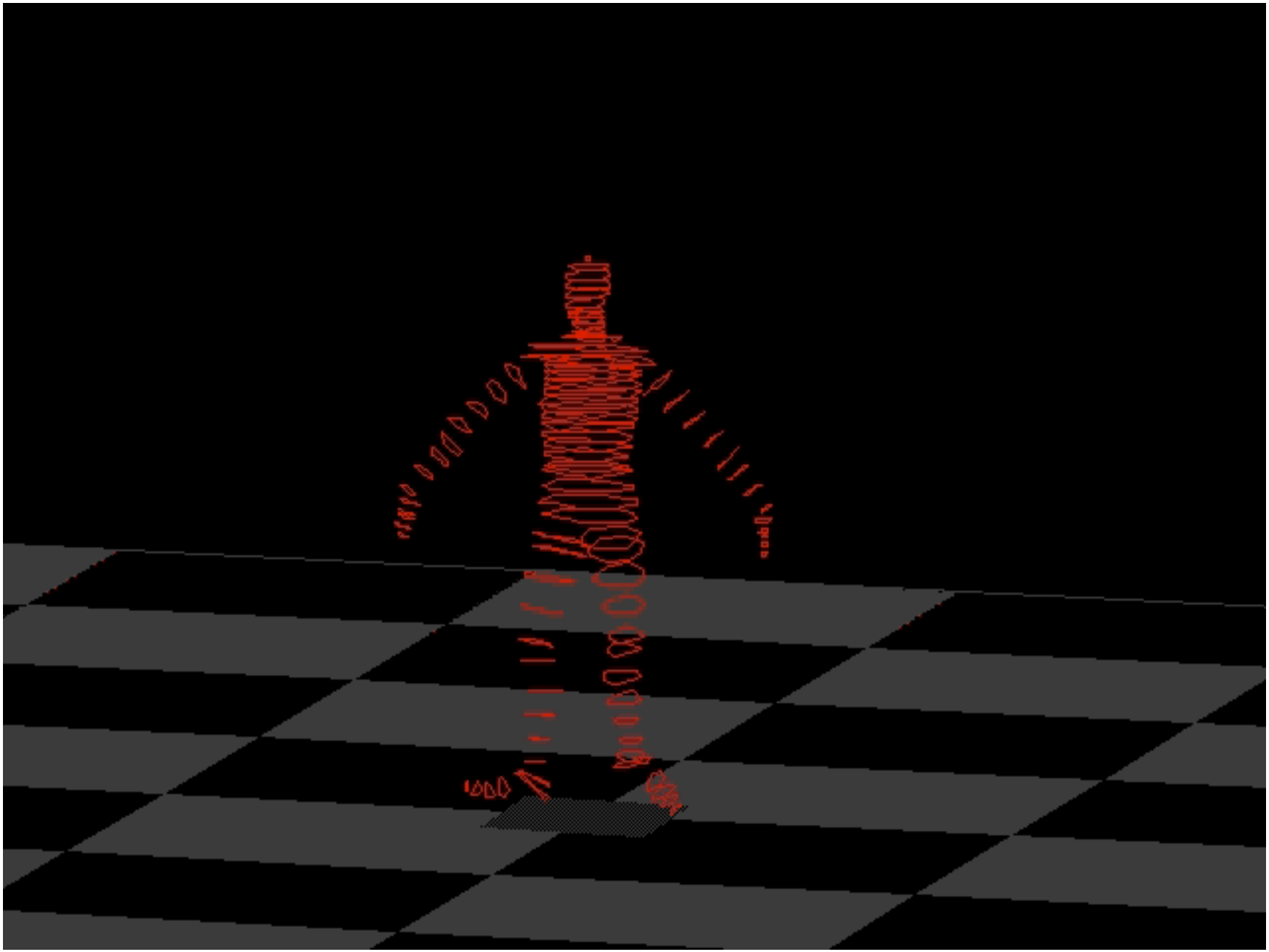
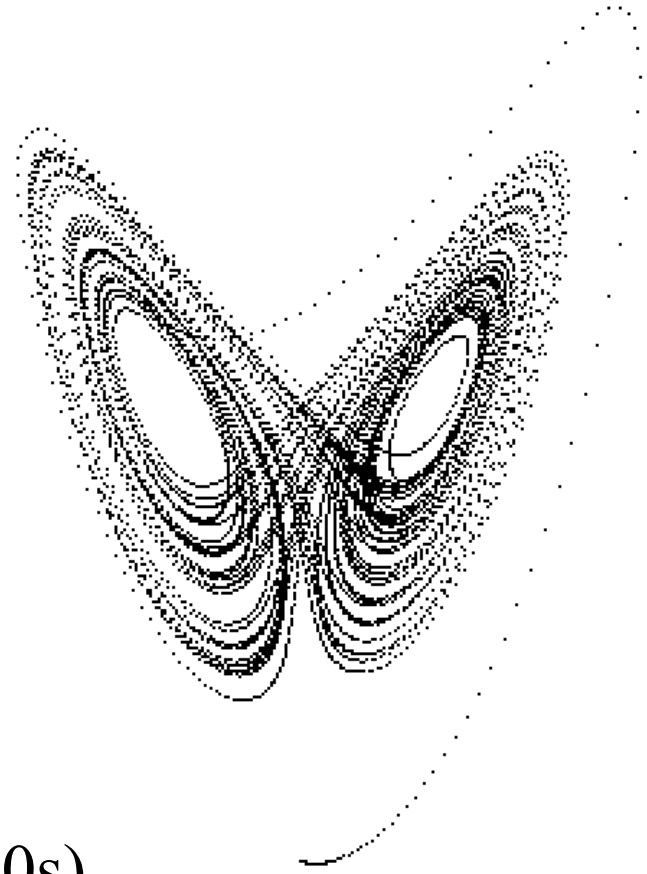


target



interpolation 3

Some history:



- Huygens (late 1600s)
- Poincare (late 1800s)
- Lorenz (early 1960s)
- Yorke (early 1970s)
- Chaos Cabal at UCSC (mid 1970s)
- Wisdom (early 1980s)
- Strogatz (late 1980s)