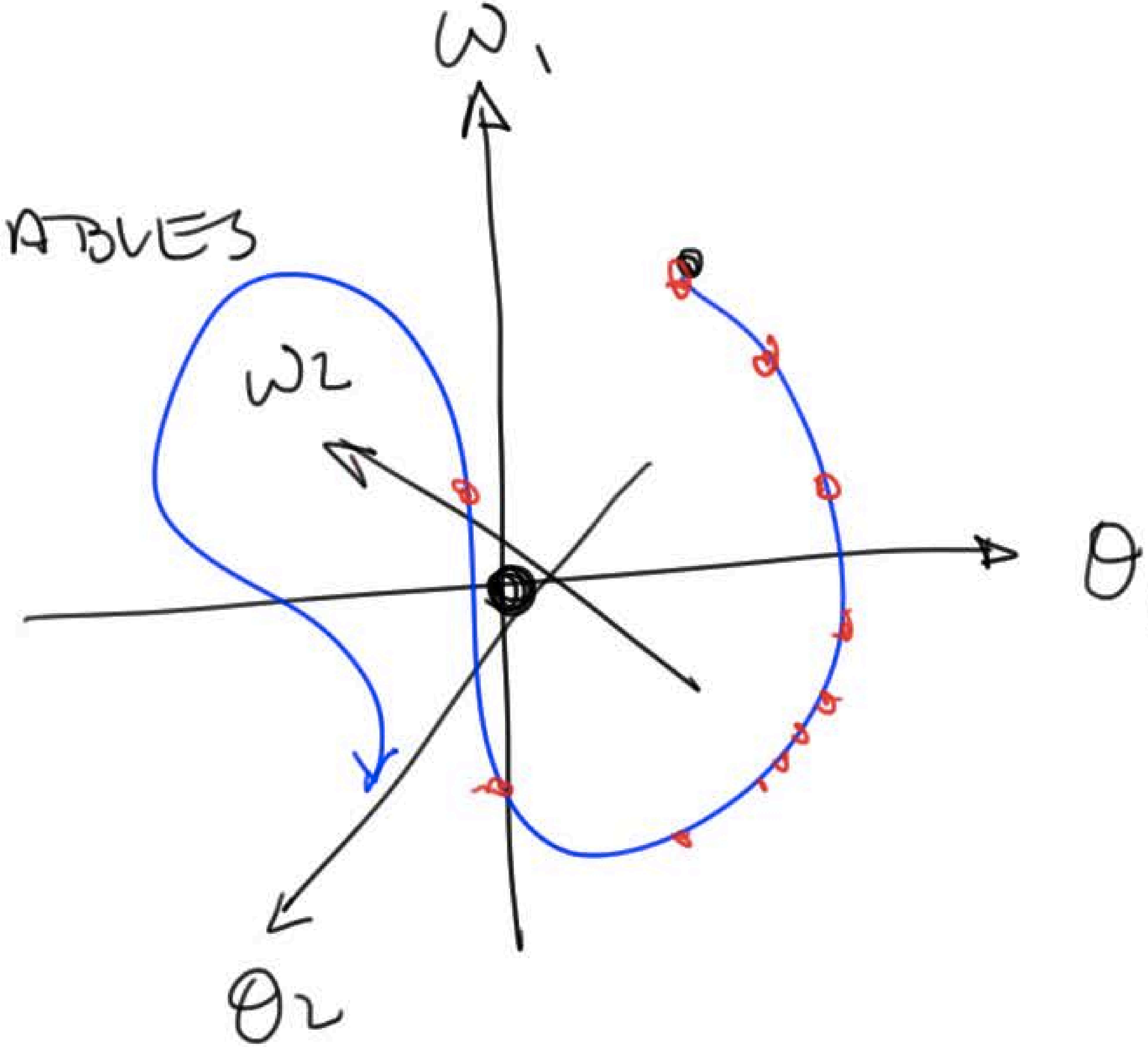
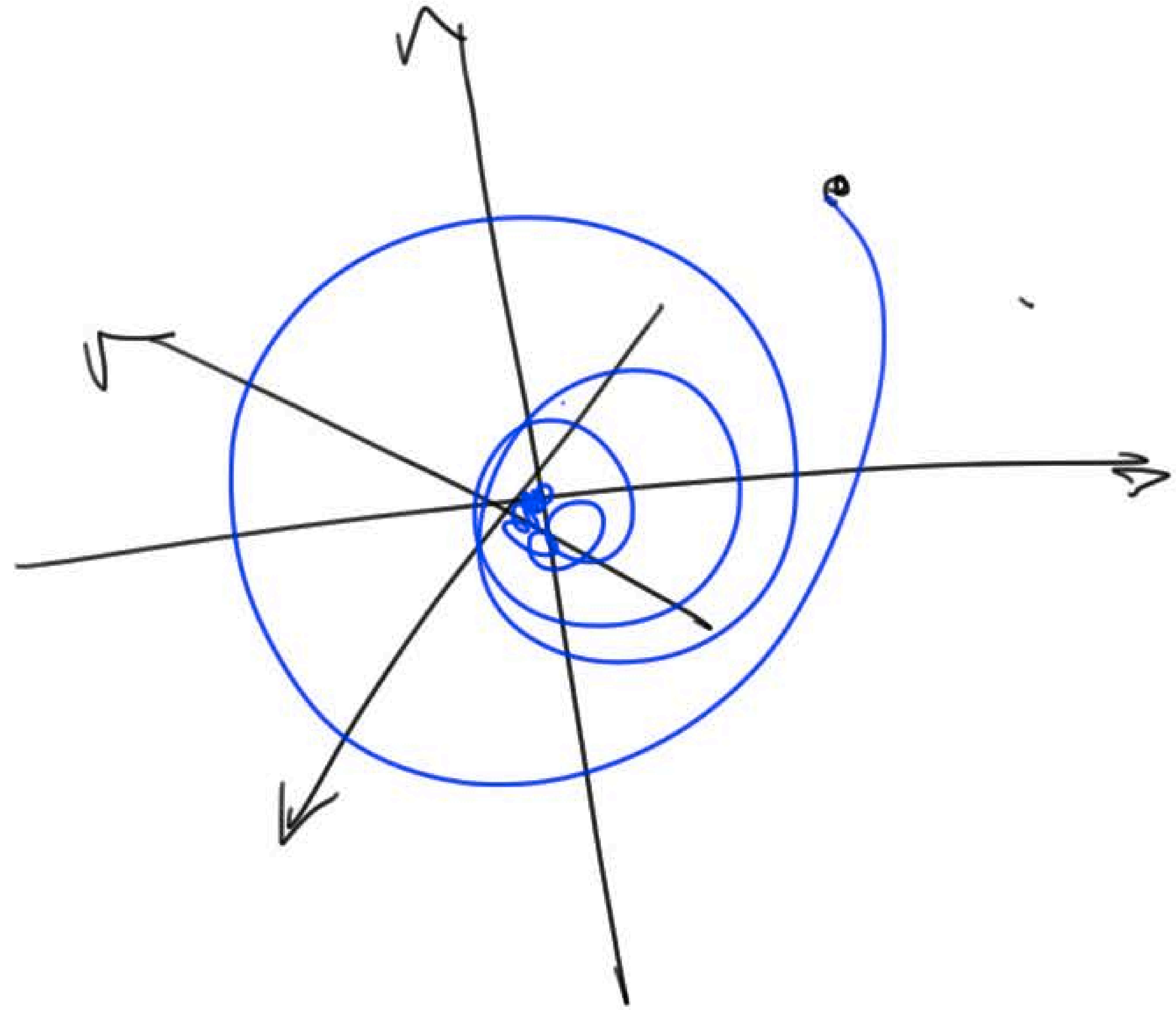
5

STATE SPACE

AXES = STATE VARIABLES





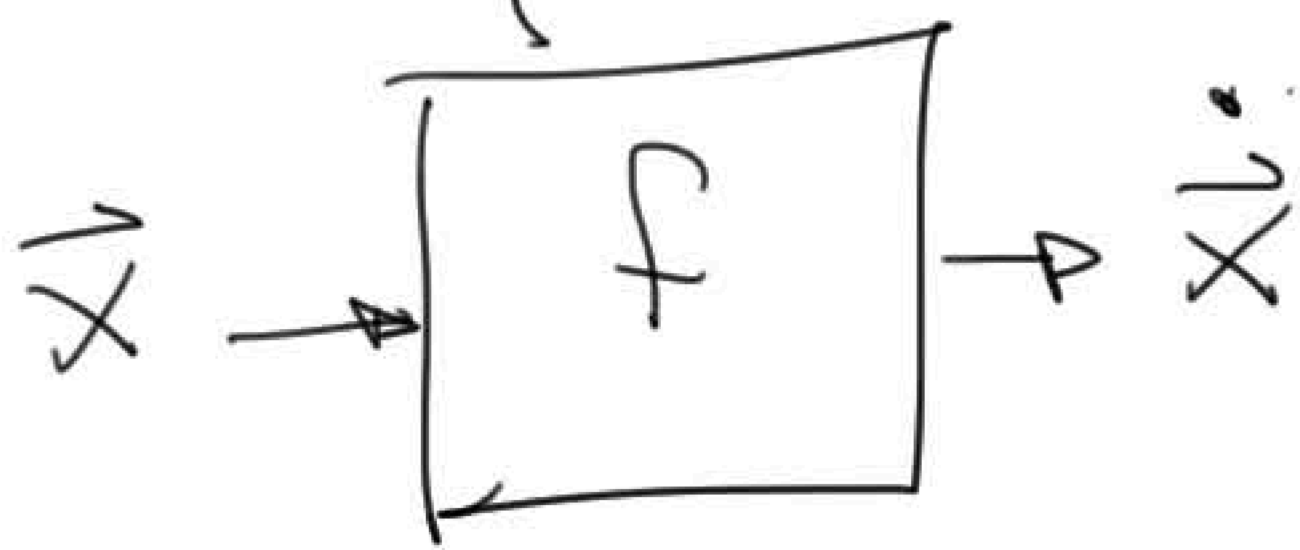
ORDINARY DIFFERENTIAL EQUATIONS   ODEs

MAPS $x_{n+1} = f(x_n)$

$$x_{n+1} = f(x_n, y_n)$$
$$y_{n+1} = g(x_n, y_n)$$

FLOW $\dot{\vec{x}} = \vec{f}(\vec{x})$ $\vec{x}(t)$ $\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \end{bmatrix}$

$$x' = \dot{x} = \frac{d}{dt}x(t)$$



$$\begin{bmatrix} \theta_1(t) \\ \omega_1(t) \\ \theta_2(t) \\ \omega_2(t) \end{bmatrix}$$

STATE VECTOR

$$\frac{d}{dt} x(t) = 1$$ ODE

$$x(t=0) = 1$$ IC

$$x(t) \ \forall \ t > 0$$ SOLN

$$x(t) = t + c$$

GENERAL SOLN

IC $\Rightarrow$ $c = 1$

$$x(t) = t + 1.$$

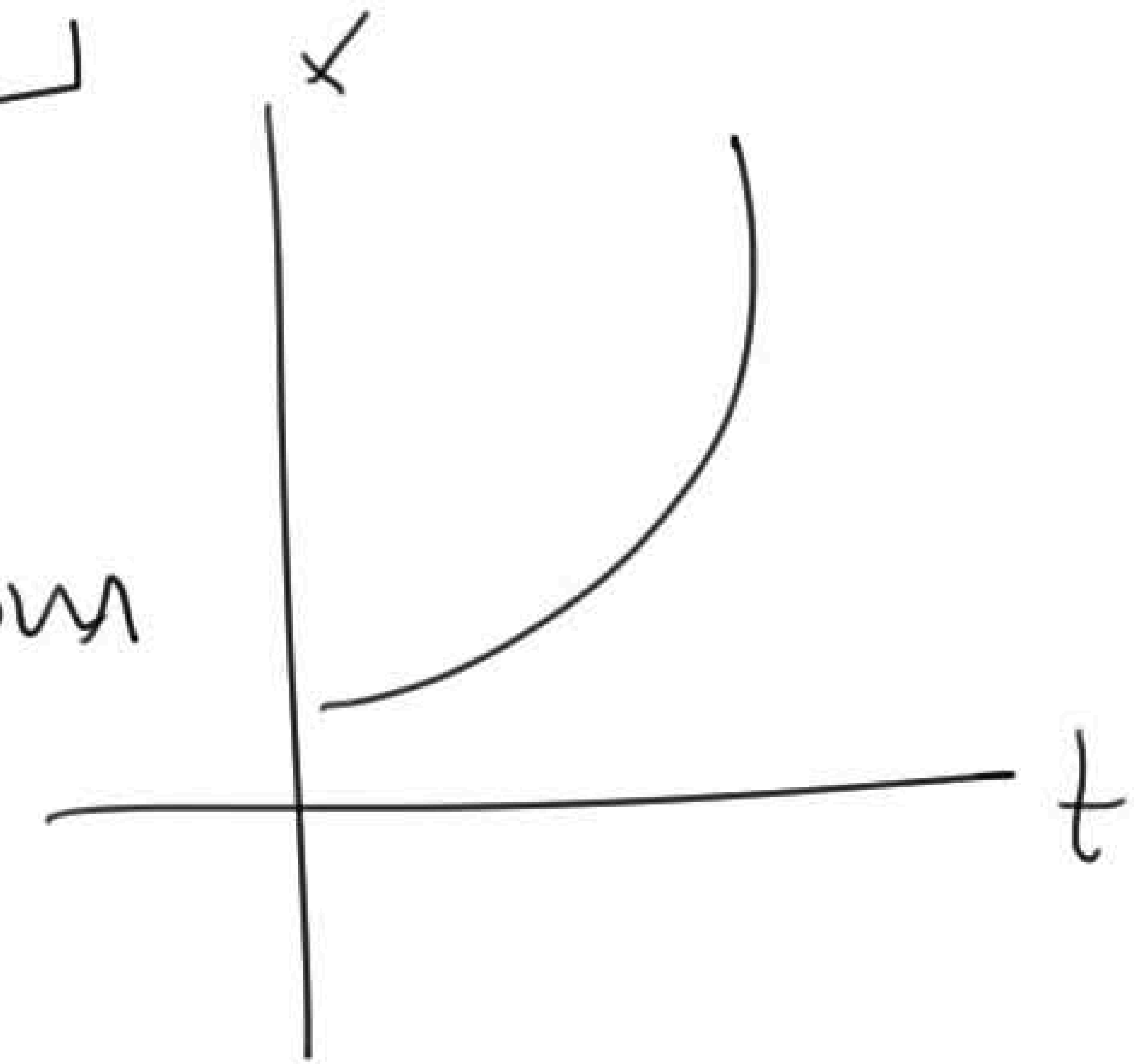
$$x' = x$$

$$x(t=0) = 1$$

$$x = e^t + c$$

$$x = e^t$$

ANALYTIC
CLOSED FORM } SOLN



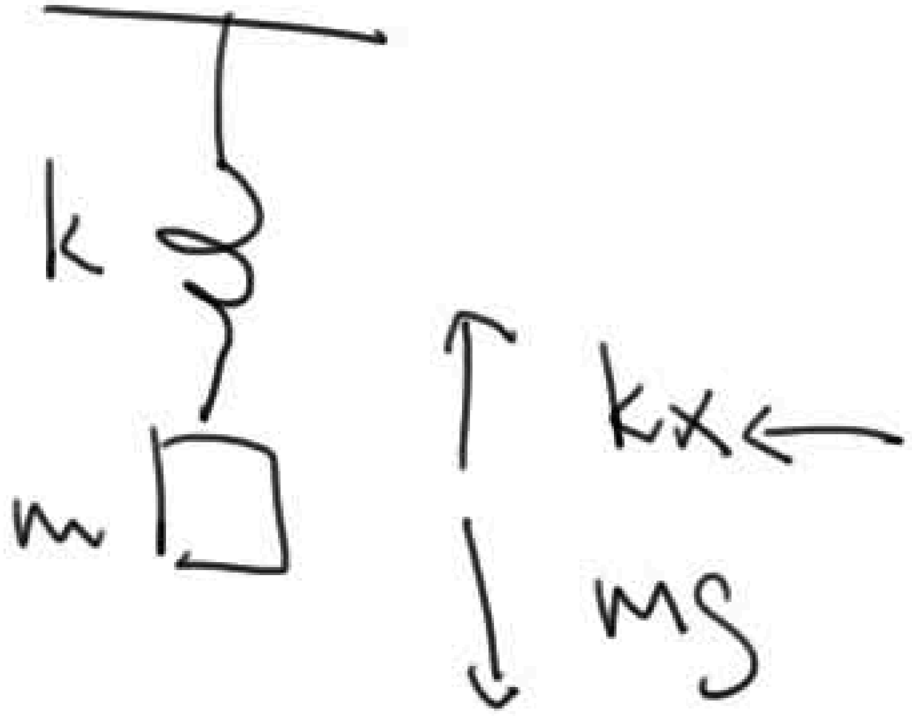
$\Updownarrow$

$\overline{\text{CHAOS}}$

WHY ODES?

- $\frac{d}{dt}$

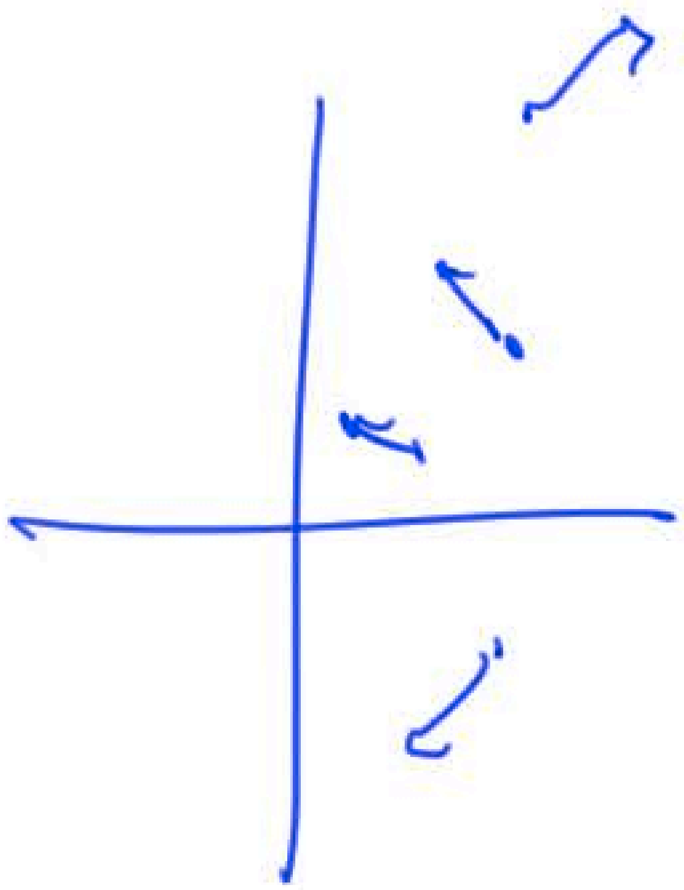
- GOOD MODELS!



$$ma = mg - kx$$

$$m\ddot{x} = mg - kx$$

$$\ddot{x} = g - \frac{k}{m}x$$

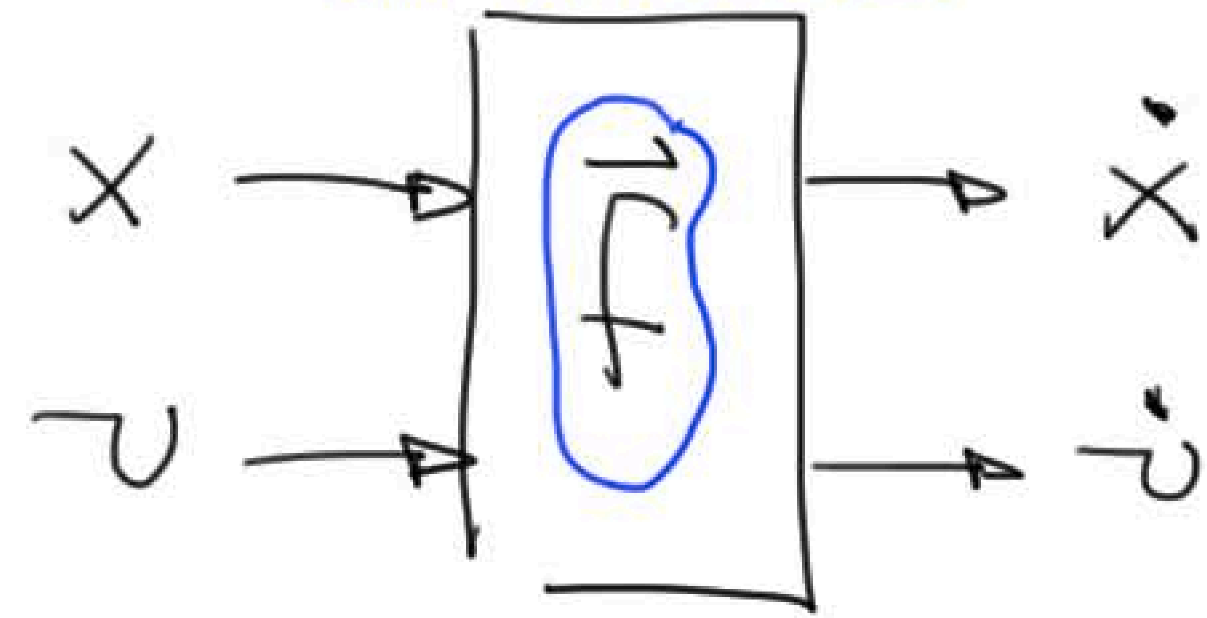


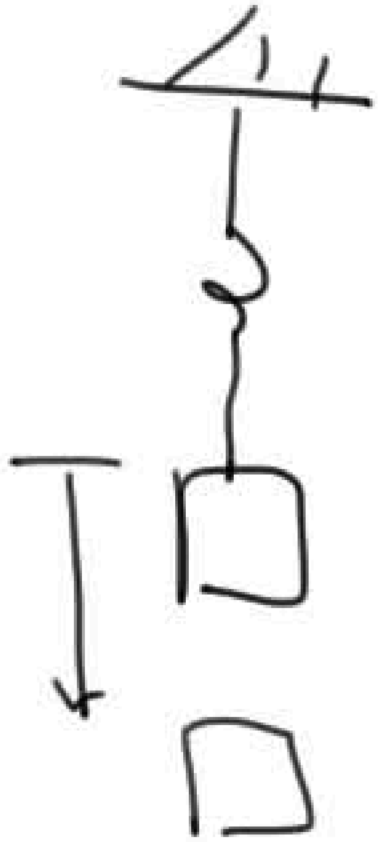
2nd ORDER ODE → set of 1st order ODES

$$\dot{x} = v$$
$$\dot{v} = g - \frac{k}{m}x$$

no dots

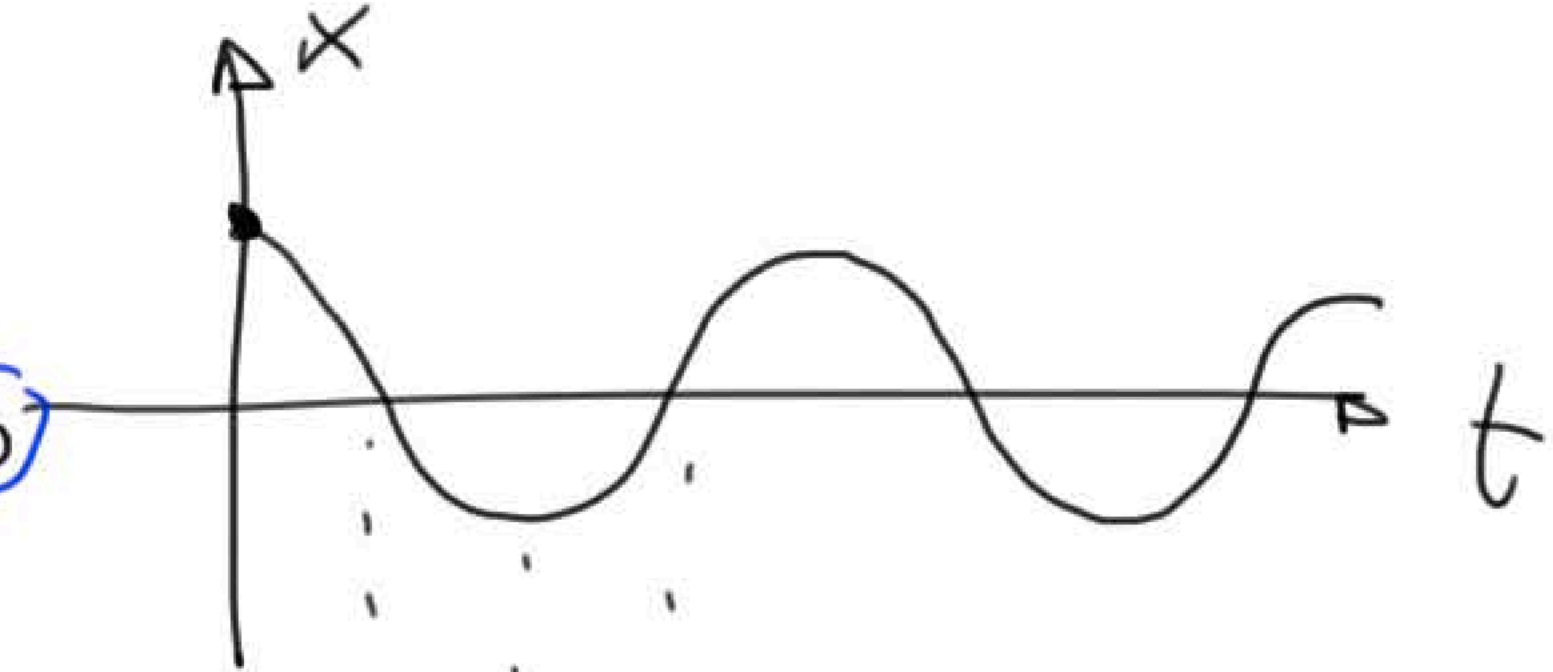
"THE DYNAMICS"

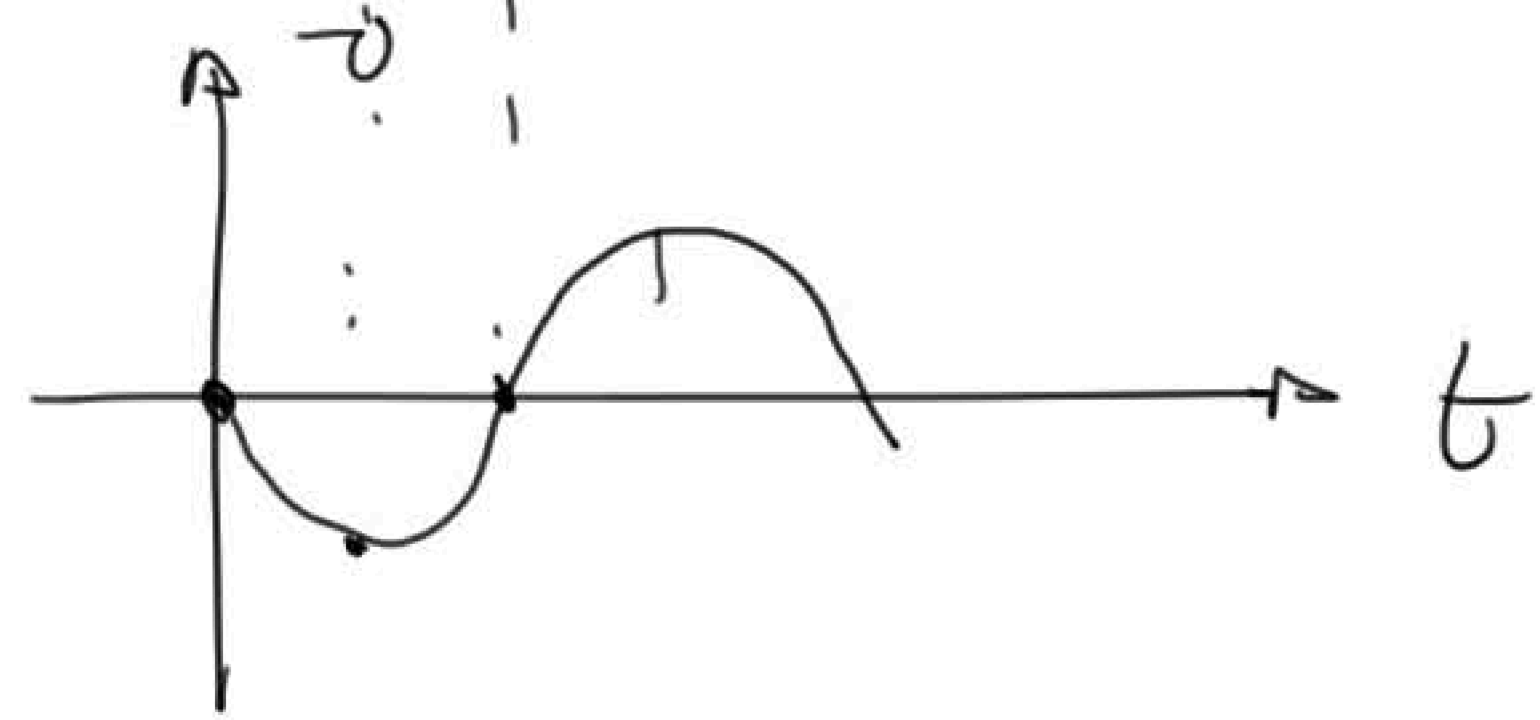




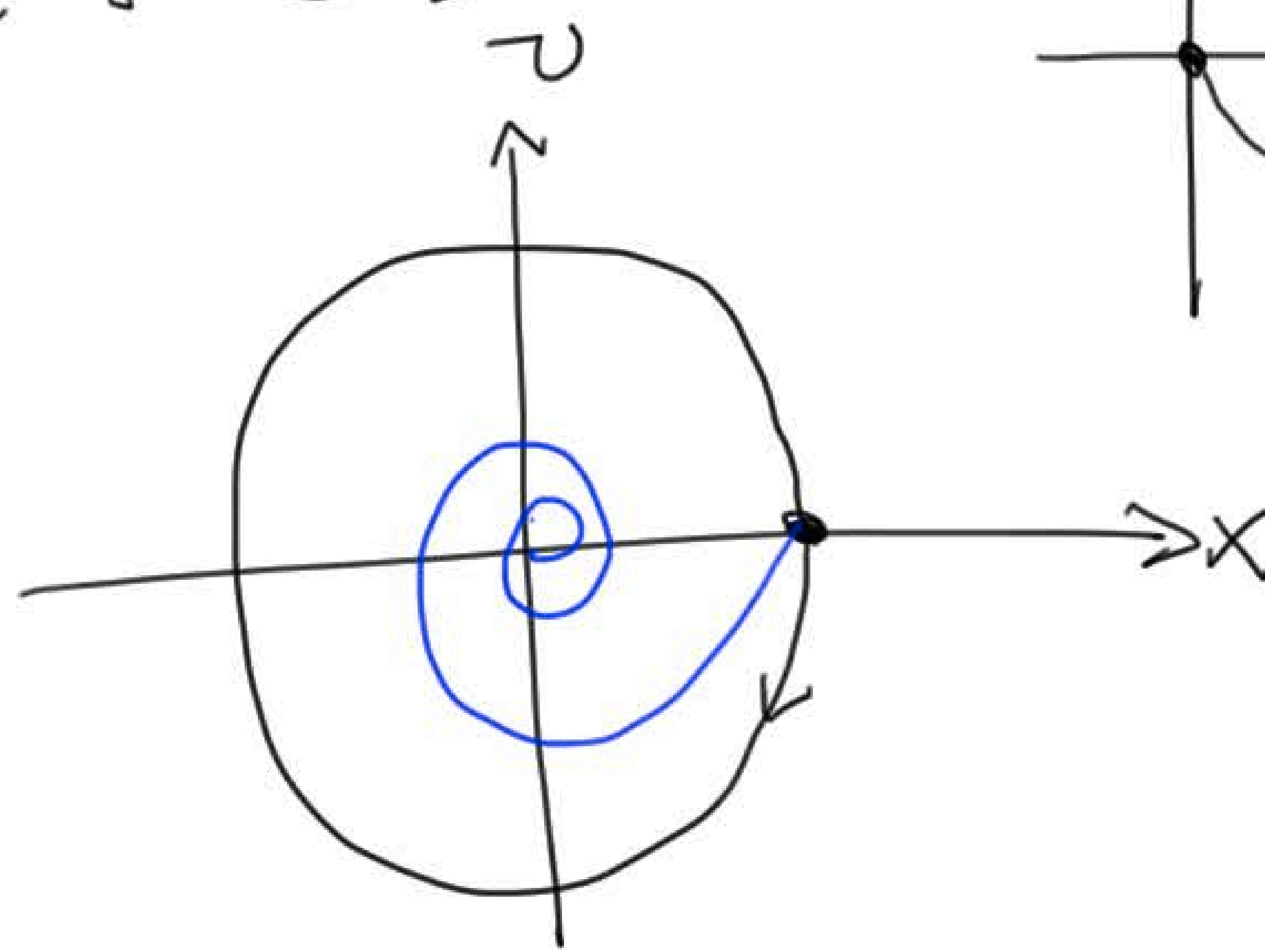
$$x'' = -x + \beta v$$

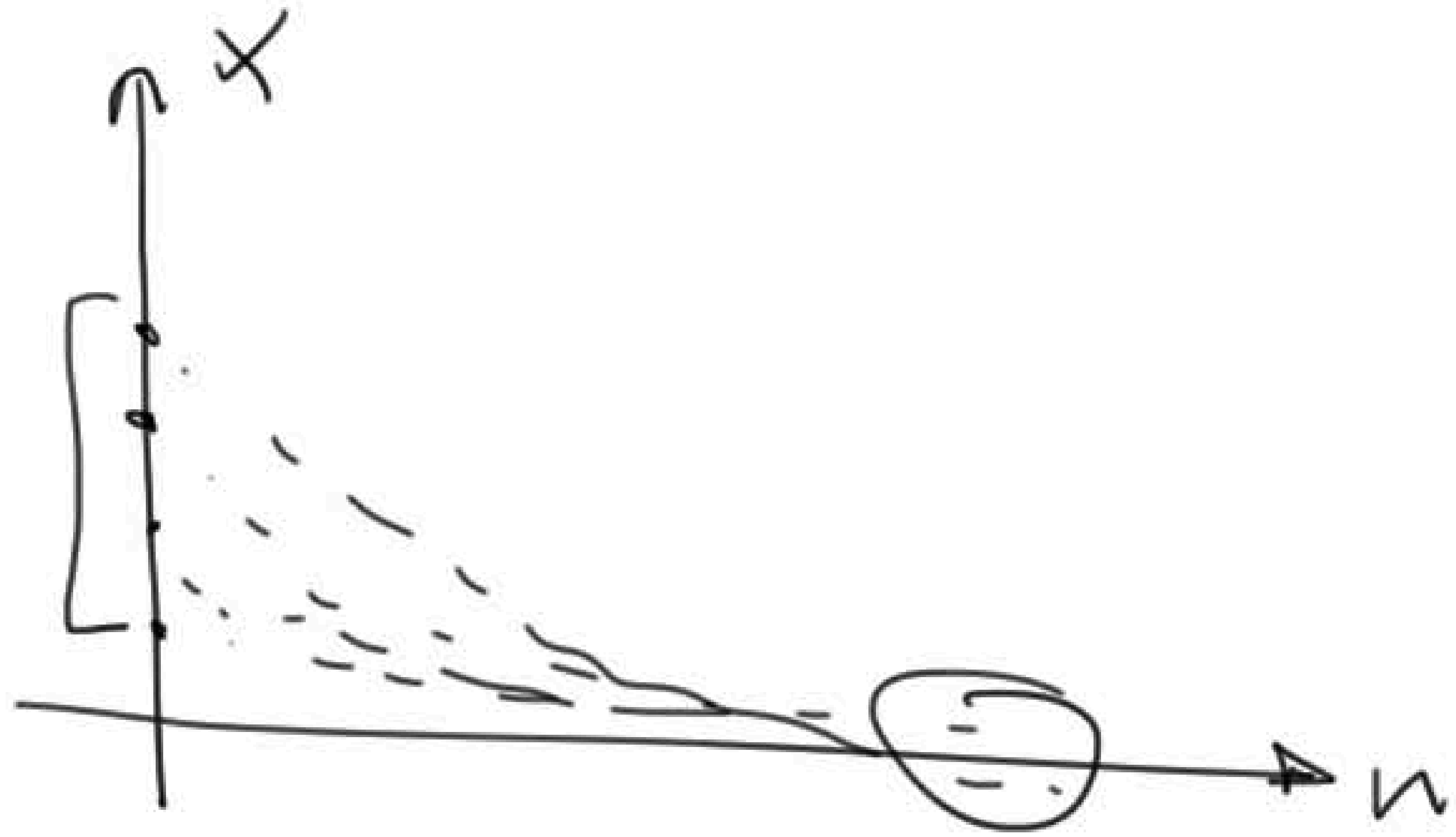
FRICTION

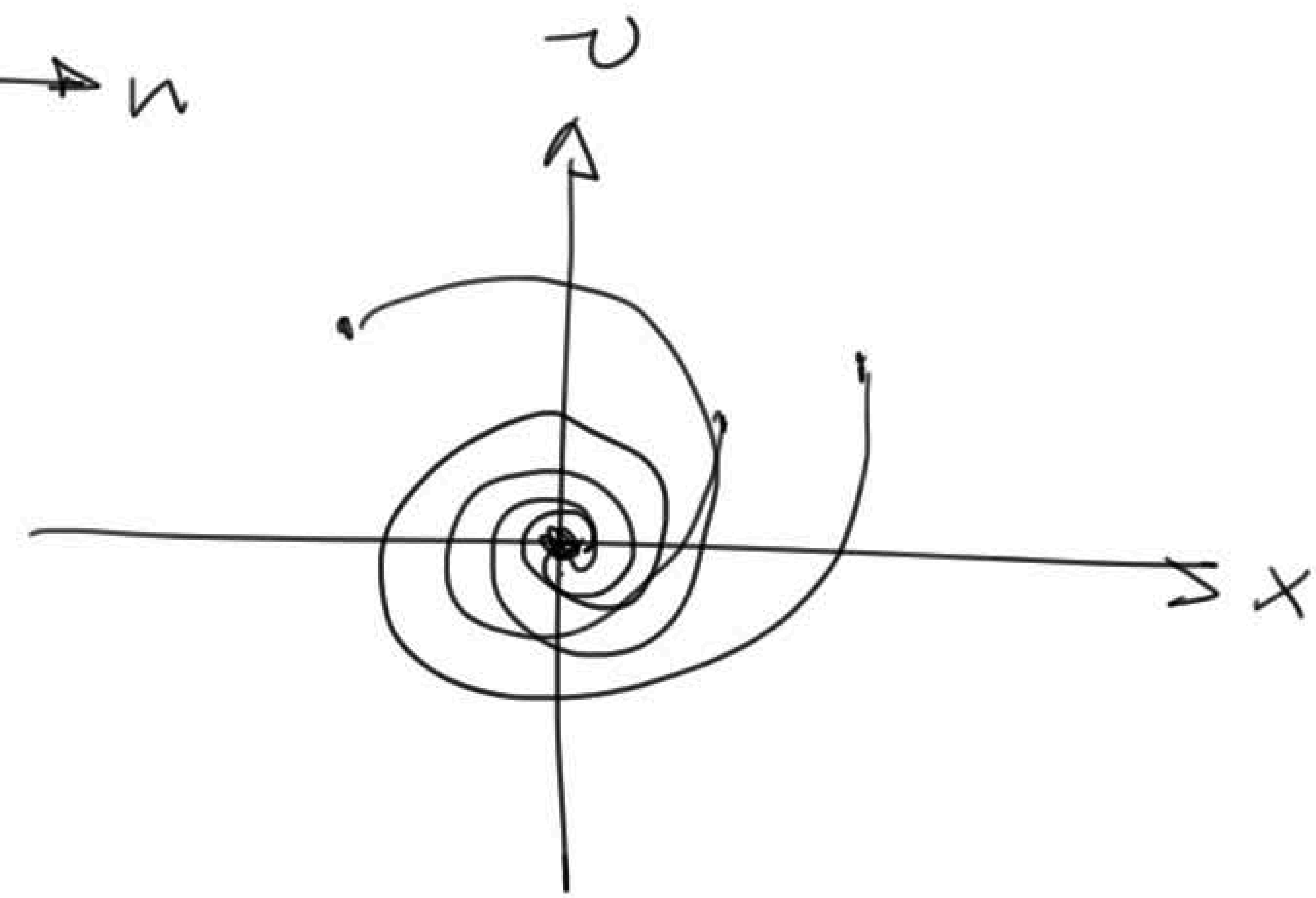




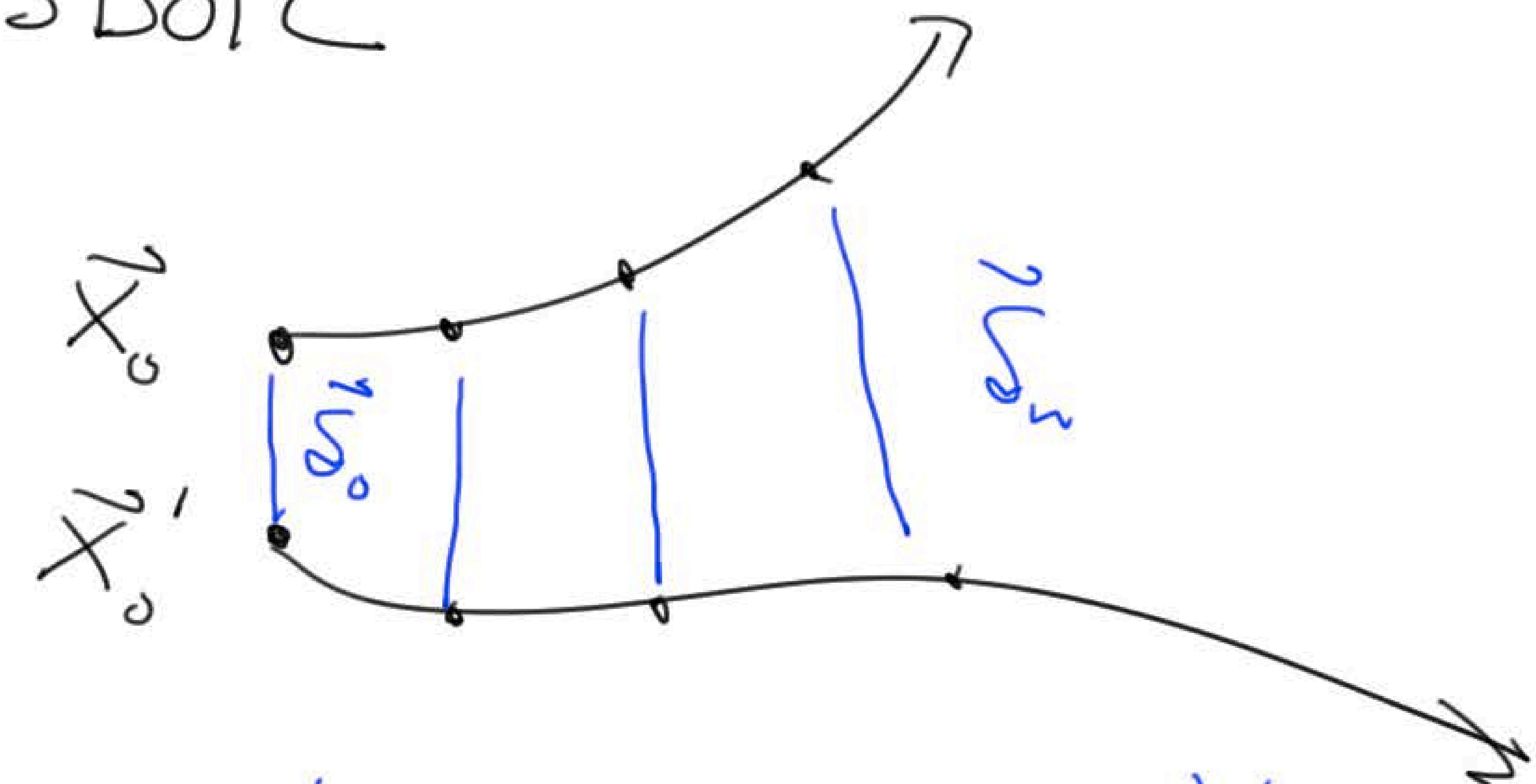
$$\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$




x
n


v
x

SDOIC



$$|\delta(t)| = |\delta_0|\, e^{\lambda t}$$