The Market as a Learning Organism

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The Market is Smart

Ultimately, the economy always follows the stock market so, somehow, the stock market knows.

– Ted Andros,
Wall Street Plus

Amazingly, the stock market knows better than the analysts do.

– Harry Herrmann,
CIO, Waddell & Reed

The Market is Mad

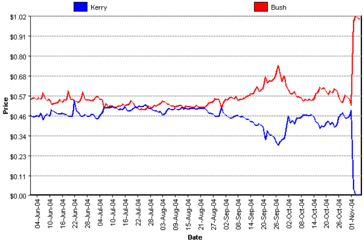
The professors overlooked the fact that people, traders included, are not always reasonable. This is the true lesson of Long-Term's demise. No matter what the models say, traders are not machines guided by silicon chips; they are impressionable and imitative; they run in flocks and retreat in hordes.

– Roger Lowenstein
When Genius Failed: The Rise and Fall of LTCM

The stock market has predicted nine out of the last five recessions.

– Paul Samuelson
Newsweek, 19 Sept. 1966

How Markets Learn



Iowa Presidential Futures Market

Vista Delivery

Three mechanisms for market learning:

- Markets exchange information among traders, and traders come to learn what others know from market prices.
- Markets select traders with the most accurate beliefs.
- Markets balance the beliefs of differentially informed traders.

What We Do

Our analysis builds on Blume and Easley (2006), Sandroni (2000) and Blume and Easley (1992).

- Infinite horizon exchange economy
- Complete markets, Arrow securities
- No learning

We characterize Pareto optimal consumption paths and their supporting prices.

The Environment:

- S States, $\{1, \ldots, \mathrm{s}\}$.
- Σ Paths, $\sigma = (\sigma_0, \ldots)$. $\sigma^t = (\sigma_0, \ldots, \sigma_t)$.
- n_t^s State counts, $n_t^s = \#\{\tau \leq t : \sigma_\tau = s\}$.
- ρ True probability distribution on S; State process is iid.

Traders:

- c A *consumption plan* $\{c_t\}_{t=1}^{\infty}$, $c_t : \sigma^t \mapsto \mathbf{R}_{++}$.
- e^i Trader i's *endowment*. ...
- ρ^i Trader i's iid beliefs on S.
- β_i Trader i's discount factor.
- $u^i : \mathbf{R}_+ \to \mathbf{R}$ Trader i's payoff to consumption at any partial history.

$$U_i(c) = E_{p^i}\left\{\sum_{t=0}^{\infty} \beta_i^t u_i\big(c_t(\sigma^t)\big)\right\}$$

Assumptions

A.1. The payoff functions u_i are C^1, strictly concave, strictly monotonic, and satisfy an Inada condition at 0.

A.2. Each trader has a strictly positive at every partial history, and the aggregate endowment is uniformly bounded, below away from zero and from above.

A.3. At every partial history, each trader i believes all truly possible states to be possible.

Pareto Optimality

If c^* is a Pareto optimal allocation of resources, then there is a strictly positive vector of welfare weights $(\lambda^1, \ldots, \lambda^I)$ such that c^* solves

$$\begin{aligned} \max_{(c^1,\ldots,c^I)} \quad & \sum_i \lambda^i U_i(c^i) \\ \text{such that} \quad & \sum_i c^i - e \leq \mathbf{0} \\ & \forall t, \sigma \; c_t^i(\sigma^t) \geq 0 \end{aligned}$$

where $e_t = \sum_i e_t^i$.

For all t there is a random variable $\eta_t : \sigma^t \mapsto \mathbf{R}_{++}$ such that

$$\lambda^i \beta_i^t u_i'(c_t^i(\sigma^t)) \prod_s \rho_i(s)^{n_t^s(\sigma)} - \eta_t(\sigma^t) = 0$$

almost surely, and

$$\sum_i c_t^i(\sigma^t) = e_t(\sigma^t)$$

Competitive Equilibrium

Def. A present value price system is $p = \{p_t\}_{t=0}^{\infty}$, $p_t : \sigma^t \mapsto \mathbf{R}_{++}$ such that, for each trader i, $p \cdot e^i < \infty$.

Def. A competitive equilibrium is a present value price system p^* and a consumption plan c^{i*} for each trader such that ...

Existence is due to Peleg and Yaari (1970), and the first welfare theorem is elementary.

Def. $q_t^s(\sigma^t)$ is the price of the Arrow security that pays off in partial history (σ^t, s) in terms of consumption at partial history σ^t. That is, q is the current value price system.

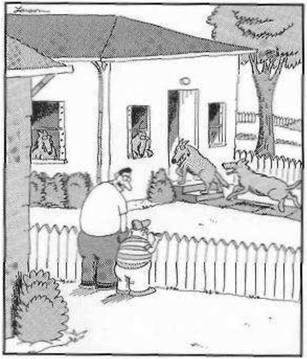
Correct Asset Pricing?

A.4. The aggregate endowment is history-independent. $e_t(\sigma) \equiv e \gg 0$. For interpretative purposes only.

Proposition: Assume A.1–3. Then n each path σ at each date t and for all $\epsilon > 0$ there is a $\delta > 0$ such that if $|c_t^i(\sigma^t) - e| < \delta$, then $||q_t(\sigma^t) - \rho^i|| < \epsilon$. If, in addition, A.4 holds, then if all traders have identical beliefs ρ', for all dates t and paths σ, $q_t^s(\sigma^t) = \rho'$.

If all traders have correct beliefs, then the price at σ^t of the asset that pays off in state s at date $t+1$ is ρ_s, the probability state s occurs.

Selection



"I know you miss the Wainrights, Bobby, but they were weak and stupid people—and that's why we have wolves and other large predators."

Given the uncertainty of the real world, the many actual and virtual traders will have many, perhaps equally many, forecasts. . . If any group of traders was consistently better than average in forecasting stock prices, they would accumulate wealth and give their forecasts greater and greater weight. In this process, they would bring the present price closer to the true value.

– R. Cootner, 1967

Selecting Bettors

$$\log \frac{u_i'\left(c_t^i(\sigma^t)\right)}{u_j'\left(c_t^j(\sigma^t)\right)} = t \log \frac{\beta_j}{\beta_i} + \sum_s n_t^s(\sigma) \log \frac{\rho_j(s)}{\rho^i(s)}$$

This is something to which the law of large numbers can be applied.

Bettor Asymptotics

How far are *i*s beliefs from the truth?

Relative Entropy $$I_i = \sum_s \rho^s \log \frac{\rho^s}{\rho_i^s}$$

Limit MU ratios:

$$\frac{1}{t} \log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} = \log \frac{\beta_j}{\beta_i} - \sum_s \rho_s \left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j} \right) -$$
$$\frac{1}{t} \sum_s \big(n_t^s(\sigma) - t\rho_s\big) \left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j} \right)$$
$$\to (\log \beta_j - I_j) - (\log \beta_i - I_i)$$

Define the Fitness Index $f_i = \log \beta_i - I_i$.

Assume A.1–3. If $\mathbf{f_i < \max_j f_j}$, then trader i vanishes.

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If there is a unique trader i with maximal survival index f_i among the trader population, then market prices converge to ρ^i almost surely.

What is the logical basis for interpreting the price of an all-or-nothing futures contract as a market probability that the event will occur?

– Manski (2004)

Balance

Suppose several traders have maximal fitness index.

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} = -\sum_s \big(n_t^s(\sigma) - t\rho_s\big)\Big(\log \rho_s^j - \log \rho_s^i\Big)$$

The RHS is a random walk. $\limsup \text{RHS} = +\infty$, so

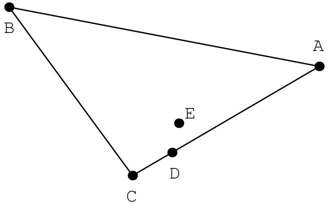
$$\mathbf{\liminf c_t^i = 0}.$$

Are traders with maximal fitness disappearing, or is the market volatile?

FOC

For distribution θ on S,

$$\mathrm{lo}(\theta) = \big(\log \theta(s)/\theta(1)\big)_{s=2}^{\mathbf{s}}$$



Trader i is interior if $\mathrm{lo}(\rho^i)$ is in the relative interior of $\mathrm{Conv}\{\mathrm{lo}(\rho^j)\}_{j=1}^{I}$. She is extremal if $\mathrm{lo}(\rho^i)$ is an extreme point, that is, not a non-negative linear combination of the other $\mathrm{lo}(\rho^j)$, and boundary otherwise.

If $s \leq 3$,

- All maximally fit traders survive.
- For extremal traders, $\limsup_t c_t^i/e = 1$.

If $s > 3$,

- Interior traders vanish.
- For extremal traders, $\limsup_t c_t^i/e = 1$.

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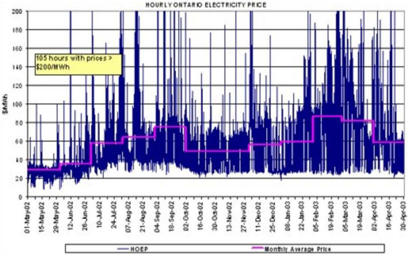
Survival requires enormous patience and extreme beliefs.



If multiple traders are maximally fit, then for all extremal traders i and all $\epsilon > 0$, $|q_t - \rho^i| < \epsilon$ infinitely often. If $s > 3$ it is possible that for $\epsilon > 0$ sufficiently small, the event $|q_t - \rho| < \epsilon$ is transient, even if some survivor has rational expectations.

PICS

Even with corners, $f_i < f_j$ implies $c_t^i \to 0$ a.s.

CARA utility: $u_i(c) = -\frac{1}{\gamma_i} \exp -c/\gamma_i$, $I_i = I_j$.

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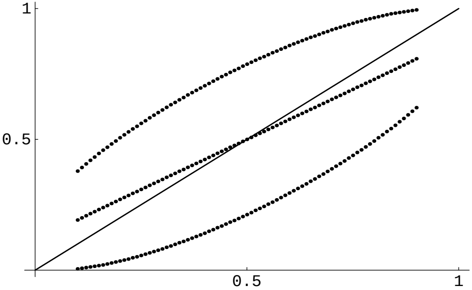
CARA utility: $u_i(c) = -\frac{1}{\gamma_i} \exp -c/\gamma_i$, $I_i = I_j$.

$$\frac{c_j}{\gamma_j} - \frac{c_i}{\gamma_i} = -\sum_s \big(n_t^s(\sigma) - t\rho_s\big)\left(\log \rho_s^j - \log \rho_s^i\right)$$



True probability versus beliefs such that $I_i = 0.2$.

Long run average price is correct only when $\rho = 1/2$.

Concluding Comments

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  - Evstigneev, Hens and Schenk-Hoppé, "Globally Evolutionarily Stable Portfolio Rules", 2005 Working Paper.
  - Kogan, Ross, Wang and Westerfield, "The Price Impact and Survival of Irrational Traders", *Journal of Finance 61:1 (Feb. 2006).*
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- Other Pricing Implications
  - Malamud and Trubowitz, "The Long Horizon Behavior of Asset Prices in Heterogeneous Economies", 2006 Working Paper.
- Other Selection Results
  - Condie, "Living with Ambiguity: Prices and Survival when Investors have Heterogeneous Preferences", 2006 Working Paper.

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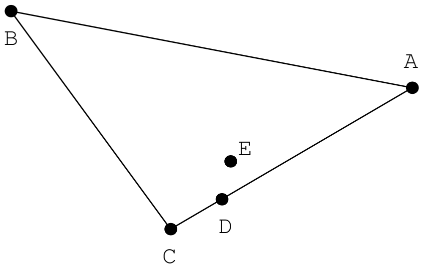
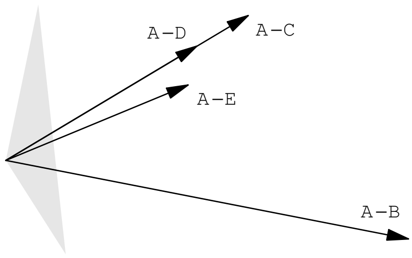
Random Walks





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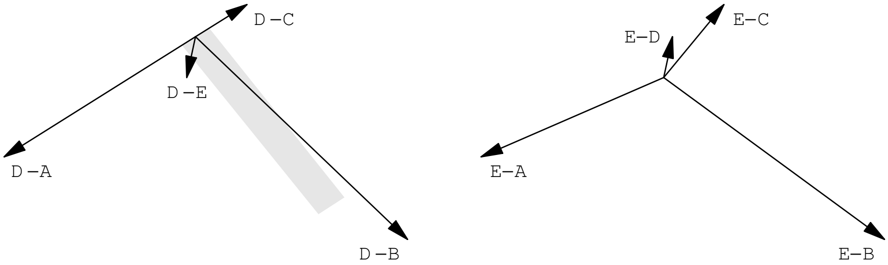
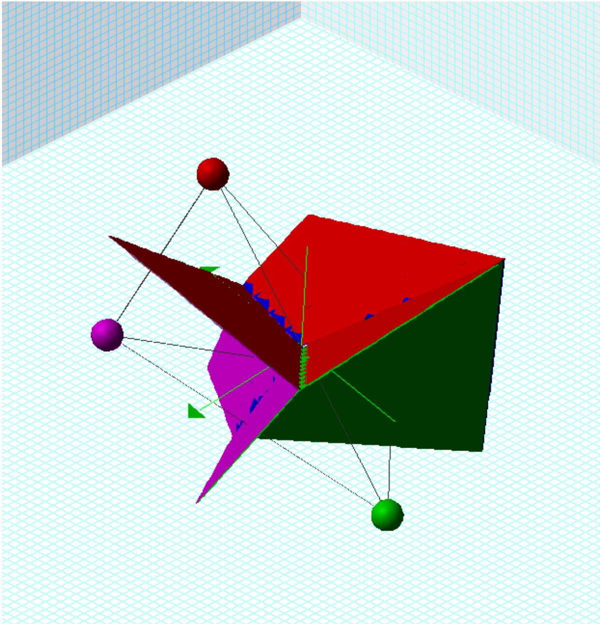


More Random Walks



The Bayesian Market

$u^i(c) = \log c$, identical discount factors.

$$\log \frac{c_t^i(\sigma^t)}{c_t^j(\sigma^t)} = \log \frac{\lambda_i}{\lambda_j} + \sum_s n_t^s(\sigma^t) \log \frac{\rho_i(s)}{\rho_j(s)}$$

Consumption shares evolve as do Bayesian posteriors.

BACK

Multiple Survivors

Limit log MU ratios:

$$\log \frac{u_i'\big(c_t^i(\sigma^t)\big)}{u_j'\big(c_t^j(\sigma^t)\big)} = \log \frac{\beta_j}{\beta_i} - \sum_s \rho_s \left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j} \right) - \sum_s \big(n_t^s(\sigma) - t\rho_s \big) \left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j} \right)$$

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Multiple Survivors

Limit log MU ratios:

$$\log \frac{u_i'\left(c_t^i(\sigma^t)\right)}{u_j'\left(c_t^j(\sigma^t)\right)} = \qquad 0$$

$$- \sum_s \left(n_t^s(\sigma) - t\rho_s\right)\left(\log \frac{\rho_s}{\rho_s^i} - \log \frac{\rho_s}{\rho_s^j}\right)$$

BACK