Metabolic Scaling in Plant Growth

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Abstract

In this work we use metabolic scaling theory (MST) principles to develop a model for tree growth. Our model is constructed using a fractal generating technique called L-Systems. From this approach we show that a difference in the scaling of the number of leaves in the plant and its total biomass leads to an energetic constraint that ultimately bounds its growth.

Introduction

"many of the most fundamental biological processes manifest an extraordinary simplicity when viewed as a function of size, regardless of the class or taxonomic group being considered…"

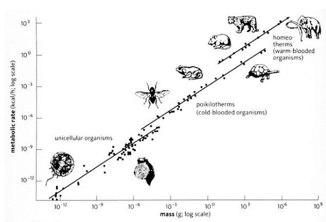
West and Brown (2005)

Scaling with size typically follows a simple power-law behavior of the form:

$$Y = Y0 * M^b$$

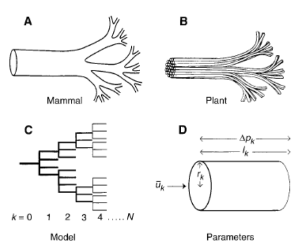
Y: observable biological quantity
Y0: normalization constant
M: mass of the organism
b: exponencial constant (in biological systems...typically multiples of 1/4)

Metabolic Scaling:



"…metabolism and the consequent distribution of energy and resources play a central, universal role in constraining the structure and organization of all life at all scales, and that the principles governing this are manifested in the pervasive quarter-power scaling laws."

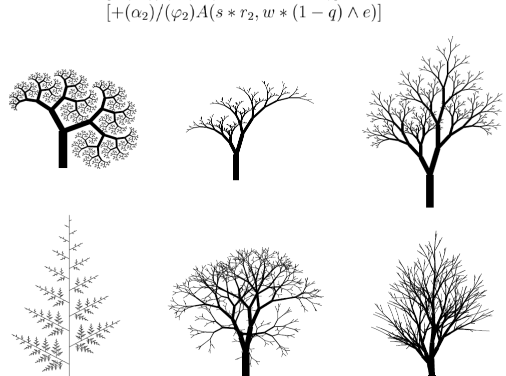
West and Brown (2005)



L-Systems

Rule-based systems used to generate fractals.

$$\omega: \quad A(100, w_0)$$
$$p_1: \quad A(s,w) \quad : s >= min \;\rightarrow\; !(w)F(s)$$
$$[+(\alpha_1)/(\varphi_1)A(s*r_1, w*q \wedge e)]$$
$$[+(\alpha_2)/(\varphi_2)A(s*r_2, w*(1-q) \wedge e)]$$



The model

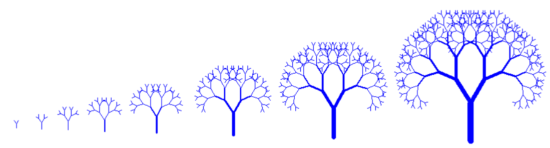
Structure:

- Area preserving branching
- Terminal units are size invariant

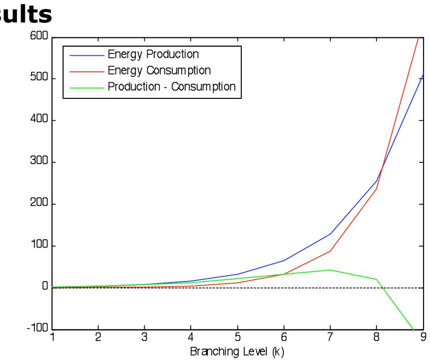
Conservation of Energy:

Production = Metabolism + Growth

- Production ∝ Number_of_leaves
- Metabolism ∝ Tree_volume
- Growth rule: branch if Growth > 0



Results

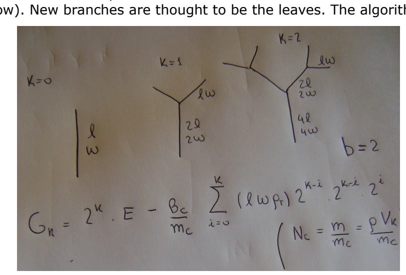


Analysis of the Model

We assume that the total energy income of the tree comes from the leaves absorption of light. Furthermore, we assume that the total energy income is proportional to the number of leaves. The total outcome of energy is assumed to be proportional to the number of cells, the later being proportional to the volume of the tree.
At each step of the algorithm we compute the following quantity.

G = N_leaves . E – N_cells . B_c

If G>0 a branching occurs. When a branching occurs, the length and width of the older branches doubles its values, while the new branches are born with unitary length and width (see fig below). New branches are thought to be the leaves. The algorithm runs until G<0.



In general this can be written as (for b = 2, 3, 4 . .)

$$0 < b^k . a - \frac{b^{2k+1}}{b-1}\left(1 - \frac{1}{b^{k+1}}\right)$$

Where b is the number of leaves that born from a branch and a is a parameter. Since the left term grows faster than the right term, the growth is bounded for all b.

References

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