

Santa Fe Institute 2011 Complex Systems Summer School

Introduction to Nonlinear Dynamics

Instructor:

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Syllabus:

1. Introduction; Dynamics of Maps chs 1 & 10 of [52]
 - a brief tour of nonlinear dynamics [34] (in [18])
 - an extended example: the logistic map
 - how to plot its behavior
 - initial conditions, transients, and fixed points
 - bifurcations and attractors
 - chaos: sensitive dependence on initial conditions, λ , and all that
 - pitchforks, Feigenbaum, and universality [23] (in [18])
 - the connection between chaos and fractals [24], ch 11 of [52]
 - period-3, chaos, and the u-sequence [33, 36] (latter is in [18])
 - *maybe*: unstable periodic orbits [3, 26, 51]
2. Dynamics of Flows [52], sections 2.0-2.3, 2.8, 5, and 6 (except 6.6 and 6.8)
 - maps vs. flows
 - time: discrete vs. continuous
 - axes: state/phase space [10]
 - an example: the simple harmonic oscillator
 - some math & physics review [9]
 - portraying & visualizing the dynamics [10]
 - trajectories, attractors, basins, and boundaries [10]
 - dissipation and attractors [44]
 - bifurcations

- how sensitive dependence and the Lyapunov exponent manifest in flows
 - anatomy of a prototypical chaotic attractor: [24]
 - stretching/folding and the un/stable manifolds
 - fractal structure and the fractal dimension ch 11 of [52]
 - unstable periodic orbits [3, 26, 51]
 - shadowing
 - *maybe*: symbol dynamics [27] (*in [14]*); [29]
 - *Lab*: the logistic map and the driven pendulum
3. Tools [2, 10, 39, 42]
- ODE solvers and their dynamics [9, 35, 37, 46]
 - *maybe*: PDE solvers [9, 46]
 - Poincaré sections [28]
 - stability, eigenstuff, un/stable manifolds and a bit of control theory
 - embedology [30, 31, 32, 41, 48, 49, 47, 54] (*[41] is in [39] and [47] is in [55];*)
 - *Lab*: nonlinear time series analysis with the TISEAN package [1, 32].
4. Applications [14, 39, 40]
- prediction [4, 5, 6, 15, 16, 55]
 - filtering [21, 22, 25]
 - control [8, 7, 12, 38, 50] (*[38] is in [39]*)
 - communication [17, 43]
 - classical mechanics [11, 45, 53, 56, 57]
 - music, dance, and image [13, 19, 20]

References

- [1] www.mpipks-dresden.mpg.de/~tisean/Tisean_3.0.1/index.html.
- [2] H. Abarbanel. *Analysis of Observed Chaotic Data*. Springer, 1995.
- [3] D. Auerbach, P. Cvitanovic, J.-P. Eckmann, G. Gunaratne, and I. Procaccia. Exploring chaotic motion through periodic orbits. *Physical Review Letters*, 58:2387–2389, 1987.
- [4] T. Bass. *The Eudaemonic Pie*. Penguin, New York, 1992.
- [5] T. Bass. *The Predictors*. Owl Books, 2000.

- [6] D. Berreby. Chaos hits Wall Street. *Discover*, 14:76–84, March 1993.
- [7] E. Bollt and J. Meiss. Targeting chaotic orbits to the moon. *Physics Letters A*, 204:373–378, 1995.
- [8] E. Bradley. Using chaos to broaden the capture range of a phase-locked loop. *IEEE Transactions on Circuits and Systems*, 40:808–818, 1993.
- [9] E. Bradley. Numerical solution of differential equations. Research Report on Curricula and Teaching CT003-98, University of Colorado, Department of Computer Science, 1998. www.cs.colorado.edu/~lizb/na/ode-notes.pdf.
- [10] E. Bradley. Analysis of time series. In *Intelligent Data Analysis: An Introduction*, pages 199–226. Springer-Verlag, 2nd edition, 2000. M. Berthold and D. Hand, eds.
- [11] E. Bradley. Classical mechanics. Research Report on Curricula and Teaching CT007-00, University of Colorado, Department of Computer Science, 2000. www.cs.colorado.edu/~lizb/chaos/classmech.pdf.
- [12] E. Bradley and D. Straub. Using chaos to broaden the capture range of a phase-locked loop: Experimental verification. *IEEE Transactions on Circuits and Systems*, 43:914–822, 1996.
- [13] E. Bradley and J. Stuart. Using chaos to generate variations on movement sequences. *Chaos*, 8:800–807, 1998.
- [14] D. Campbell, R. Ecke, and J. Hyman. *Nonlinear Science: The Next Decade*. M.I.T. Press, 1992.
- [15] M. Casdagli. Nonlinear prediction of chaotic time series. *Physica D*, 35:335–356, 1989.
- [16] M. Casdagli and S. Eubank, editors. *Nonlinear Modeling and Forecasting*. Addison Wesley, 1992.
- [17] K. M. Cuomo and A. V. Oppenheim. Circuit implementation of synchronized chaos with applications to communications. *Physical Review Letters*, 71:65–68, 1993.
- [18] P. Cvitanovic. Introduction. In *Universality in Chaos*. Adam Hilger, Bristol U.K., 1984.
- [19] D. Dabby. Musical variations from a chaotic mapping. *Chaos*, 6:95–107, 1996.
- [20] D. Dabby. A chaotic mapping for musical and image variation. In *Proceedings of the Fourth Experimental Chaos Conference*, 1997.
- [21] J. Farmer and J. Sidorowich. Predicting chaotic time series. *Physical Review Letters*, 59:845–848, 1987.
- [22] J. Farmer and J. Sidorowich. Exploiting chaos to predict the future and reduce noise. In *Evolution, Learning and Cognition*. World Scientific, 1988.
- [23] M. J. Feigenbaum. Universal behavior in nonlinear systems. *Los Alamos Science*, 1:4–27, 1980.

- [24] C. Grebogi, E. Ott, and J. A. Yorke. Chaos, strange attractors and fractal basin boundaries in nonlinear dynamics. *Science*, 238:632–638, 1987.
- [25] J. Guckenheimer. Noise in chaotic systems. *Nature*, 298:358–361, 1982.
- [26] G. Gunaratne, P. Linsay, and M. Vinson. Chaos beyond onset: A comparison of theory and experiment. *Physical Review Letters*, 63:1, 1989.
- [27] B.-L. Hao. Symbolic dynamics and characterization of complexity. *Physica D*, 51:161–176, 1991.
- [28] M. Hénon. On the numerical computation of Poincaré maps. *Physica D*, 5:412–414, 1982.
- [29] C. Hsu. *Cell-to-Cell Mapping*. Springer-Verlag, New York, 1987.
- [30] J. Huke. Embedding nonlinear dynamical systems: A guide to Takens theorem. Technical Report MIMS EPrint: 2006.26, University of Manchester, 2006. eprints.ma.man.ac.uk/175/.
- [31] J. Iwanski and E. Bradley. Recurrence plots of experimental data: To embed or not to embed? *Chaos*, 8:861–871, 1998.
- [32] H. Kantz and T. Schreiber. *Nonlinear Time Series Analysis*. Cambridge University Press, Cambridge, 1997.
- [33] T.-Y. Li and J. A. Yorke. Period three implies chaos. *American Mathematical Monthly*, 82:985–992, 1975.
- [34] E. N. Lorenz. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20:130–141, 1963.
- [35] E. N. Lorenz. Computational chaos – A prelude to computational instability. *Physica D*, 35:229–317, 1989.
- [36] N. Metropolis, M. Stein, and P. Stein. On finite limit sets for transformations of the unit interval. *J. Combinatorial Theory*, 15:25–44, 1973.
- [37] R. Miller. A horror story about integration methods. *J. Computational Physics*, 93:469–476, 1991.
- [38] E. Ott, C. Grebogi, and J. A. Yorke. Controlling chaos. *Physical Review Letters*, 64:1196–1199, 1990.
- [39] E. Ott, T. Sauer, and J. Yorke. *Coping with chaos*. Wiley, 1994.
- [40] J. M. Ottino. *The Kinematics of Mixing: Stretching, Chaos, and Transport*. Cambridge, Cambridge U.K., 1992.
- [41] N. Packard, J. Crutchfield, J. Farmer, and R. Shaw. Geometry from a time series. *Physical Review Letters*, 45:712, 1980.
- [42] T. S. Parker and L. O. Chua. *Practical Numerical Algorithms for Chaotic Systems*. Springer-Verlag, New York, 1989.

- [43] L. M. Pecora and T. L. Carroll. Synchronization in chaotic systems. *Physical Review Letters*, 64:821–824, 1990.
- [44] I. Percival. Chaos in Hamiltonian systems. *Proceedings of the Royal Society, London*, 413:131–144, 1987.
- [45] I. Peterson. *Newton’s Clock: Chaos in the Solar System*. W. H. Freeman, New York, 1993.
- [46] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. *Numerical Recipes: The Art of Scientific Computing*. Cambridge University Press, Cambridge U.K., 1988.
- [47] T. Sauer. Time-series prediction by using delay-coordinate embedding. In *Time Series Prediction: Forecasting the Future and Understanding the Past*. Santa Fe Institute Studies in the Sciences of Complexity, Santa Fe, NM, 1993.
- [48] T. Sauer. Interspike interval embedding of chaotic signals. *Chaos*, 5:127, 1995.
- [49] T. Sauer, J. Yorke, and M. Casdagli. Embedology. *Journal of Statistical Physics*, 65:579–616, 1991.
- [50] T. Shinbrot. Progress in the control of chaos. *Advances in Physics*, 44:73–111, 1995.
- [51] P. So, E. Ott, S. Schiff, D. Kaplan, T. Sauer, and C. Grebogi. Detecting unstable periodic orbits in chaotic experimental data. *Physical Review Letters*, 76:4705–4708, 1996.
- [52] S. Strogatz. *Nonlinear Dynamics and Chaos*. Addison-Wesley, Reading, MA, 1994.
- [53] G. J. Sussman and J. Wisdom. Chaotic evolution of the solar system. *Science*, 257:56–62, 1992.
- [54] F. Takens. Detecting strange attractors in fluid turbulence. In D. Rand and L.-S. Young, editors, *Dynamical Systems and Turbulence*, pages 366–381. Springer, Berlin, 1981.
- [55] A. S. Weigend and N. S. Gershenfeld, editors. *Time Series Prediction: Forecasting the Future and Understanding the Past*. Santa Fe Institute Studies in the Sciences of Complexity, Santa Fe, NM, 1993.
- [56] J. Wisdom. Chaotic behavior in the solar system. *Nuclear Physics B*, 2:391–414, 1987.
- [57] J. Wisdom. Is the solar system stable? and Can we use chaos to make measurements? In D. K. Campbell, editor, *Chaos/XAOC : Soviet-American perspectives on nonlinear science*. American Institute of Physics, New York, 1990.

References [2, 4, 5, 14, 16, 18, 29, 39, 52, 55] are in the CSSS library.

More Resources:

www.cs.colorado.edu/~lizb

amath.colorado.edu/faculty/jdm/faq.html

www.mpipks-dresden.mpg.de/~tisean