An Ecological Framework for Last-Tree Experiments

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1 Population dynamics

An experiment will be in discrete time. Thus we limit attention to models that represent standard formulations for modeling population dynamics in discrete time. In all cases, the population size of the resource species in $t + 1$ is proportional to the population size in t. This is a central idea in population biology. Individuals reproduce. Consequently models of population dynamics typically assume that in some interval of time the number of new individuals recruited into the population is proportional to the number of reproductive individuals present at the beginning of the time interval. This is one potential problem with the standard approach in CPR experiments. If we wish to think of the common-pool resource as some kind of biological resource, there's no biological justification for providing a fixed, per-period endowment. If we want to focus on other theoretical problems, then the fixed endowment might be a useful simplification, but it tells us little about decisions involving the dynamics of a resource species across multiple periods.

Here are three models of population dynamics for a single species with no harvesting. CME's impression is that altogether they represent three very different but important postulates about the nature of population growth. All have the generic form $N_{t+1} = f(N_t)N_t$.

- **Exponential:** In this case, $f(N_t) = \lambda = \exp\{r\}$, and so $N_{t+1} = \exp\{r\}N_t$. This is your basic model of exponential growth. We can all be happy it never works this way, or the world would have been swamped by bacteria long before we appeared on the scene. But, exponential growth is a good approximation to population dynamics when the population is at low densities. This can be seen by looking at the logistic model below an imagining that $N_t \ll K$. The problem is, if the population is growing exponentially, this state of affairs won't last long.

- **Logistic:** $N_{t+1} = \exp\{r(1 - N_t/K)\}N_t$. This is your basic model that assumes the environment, for some unspecified reason, has a finite capacity to sustain the species. In continuous time, the state variable always converges smoothly to K, the carrying capacity. In discrete time, this only happens when r is small enough. As r increases, the discrete-time logistic shown here shows cyclical behavior with periods of increasing length and then eventually chaos. This if the famous logistic bifurcation graph we've all seen about a million times.

- **Allee:** $N_{t+1} = \exp\{r((N_t - a)/K)(1 - N_t/K)\}N_t$. This model is similar to the logistic with one crucial difference. If the population drops below some threshold, $a < K$, the population starts to decline. The biological reasoning here usually takes some form like the following: at densities low enough, individuals cannot find mates, etc., and thus the already small population gets even smaller.

2   Harvesting and the optimization problem

A standard assumption when adding predation in a discrete-time setting is to assume that the predation rate (e.g. harvesting) is Poisson distributed as a function of a quantity we will call effort, e_t. This means the resources harvested in t are $h(N_t, e_t) =$

$(1 - \exp\{-qe_t\})f(N_t)N_t$, where q is a parameter with units $[\text{effort}]^{-1}$. Note that in this case we're assuming that between t and $t+1$, the resource species first reproduces and is then harvested. Reversing the order would mean that $f(\cdot)$ is instead a function of the post-harvest population level. The proportion of the resource population not harvested is $\exp\{-qe_t\}$, and the population dynamics are $N_{t+1} = \exp\{-qe_t\}f(N_t)N_t$. If we assume price taking at price p and a cost function, $c(e_t)$, the harvester's theoretical optimization problem is the following,

$$\max_{\{e_t\}_t} \sum_t \delta^t \left\{ ph(N_t, e_t) - c(e_t) \right\}$$

$$\text{subject to}$$

$$N_0 \geq 0$$

$$N_{t+1} = \exp\{-qe_t\}f(N_t)N_t$$

$$\forall t, e_t \geq 0.$$

At some point we might want to solve this for kicks.