

Statistical tests for contagion in observational social network studies

Greg Ver Steeg and Aram Galstyan

Networks Workshop, SFI 2013



July 25, 2007

Study Says Obesity Can Be Contagious

By [GINA KOLATA](#)

[Obesity](#) can spread from person to person, much like a virus, researchers are reporting today. When a person gains weight, close friends tend to gain weight, too.

[Subscribe](#)[News & Features](#)[Topics](#)[Blogs](#)[Videos & Podcasts](#)[Education](#)

Mind & Brain :: 60-Second Science :: December 5, 2008 :: 9 Comments :: Email :: Print



Happiness Is Contagious

A happy person within a social circle quickly influences those around him or her to be happy, extending to three degrees of separation. Adam Hinterthuer reports

Like happiness, loneliness is contagious

Updated 12/1/2009 5:14 AM | [Comment](#) | [Recommend](#)

[E-mail](#) | [Print](#) |

By Sharon Jayson, USA TODAY

Loneliness, like happiness, can be contagious, says research out today that shows how feeling lonely can make others lonely, too.

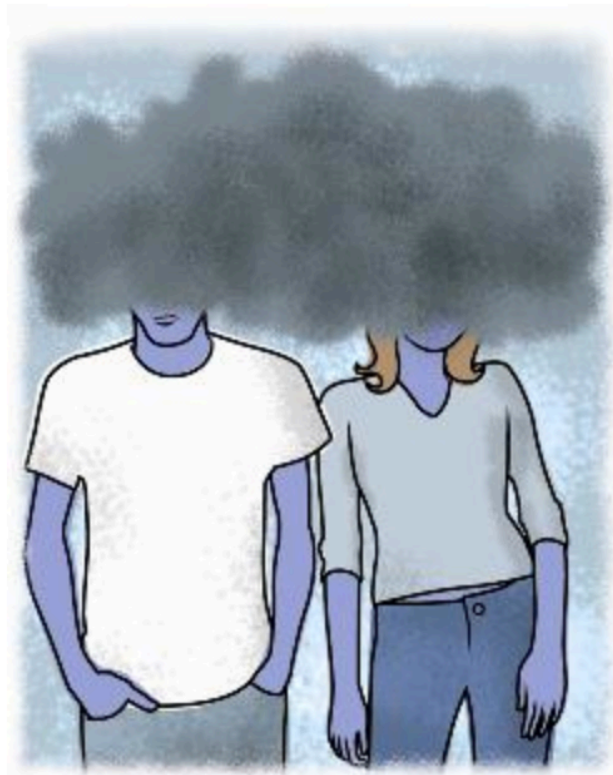
The study by John Cacioppo, a psychologist at the [University of Chicago](#), builds on recent research showing that happiness is contagious and spreads through social networks.

LONELINESS: [It's increasing and can harm your health](#)

HAPPINESS: [Staying positive in negative situations](#)

CO-HOUSING: [Seniors can be independent but not alone](#)

Cacioppo worked with the two researchers who did the earlier happiness research: Nicholas Christakis, a professor of medical sociology at Harvard Medical School, and James Fowler, associate professor of political science at the [University of California-San Diego](#).



[Enlarge](#)

Illustration by Alejandro Gonzalez,
USA TODAY

Share

[Add to Mixx](#)

[Facebook](#)

[Twitter](#)

[More](#)

Subscribe

[myYahoo](#)

[iGoogle](#)

[More](#)

The Telegraph

HOME NEWS WORLD SPORT FINANCE COMMENT BLOGS CULTURE TRAVEL **LIFE** FASHION
Women Motoring Health Property Gardening Food History Relationships Expat
Politics Business **Family** Sex Life Wonder Women Columnists


HOME » WOMEN » **MOTHER TONGUE**

Divorce 'is contagious'

Divorce can be contagious within groups of friends, according to a new study.

By Richard Savill

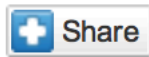
2:42PM BST 04 Jul 2010

 Print this article

Los Angeles Times

GPA can be contagious among high school students, study finds

February 13, 2013 | By Karen Kaplan



Researchers have some new advice for high school students who want to improve their grades: Become friends with academically oriented classmates.

It may sound obvious, but researchers went to considerable effort to prove it.

They surveyed all members of the junior class at Maine-Endwell High School in Endwell, N.Y., and asked students to rate each of their classmates as either a “best friend,” a “friend,” an “acquaintance” or someone they didn’t know.



High school students' grades were strongly influenced by the grades...
(Don Bartletti / Los Angeles...)

HOME / SCIENCE : THE STATE OF THE UNIVERSE.

Disconnected?

We've heard that obesity and divorce can be passed from one person to another. Critics now wonder how the "social contagion" studies ever passed peer review.

By Dave Johns

Why the interest in this case?

- Marketing Facebook applications?
A heuristic for contagion is fine.



- Making decisions about public health?
Let's double-check that.



Study Says Obesity Can Be Contagious

By GINA KOLATA
Published: July 25, 2007

Obesity can spread from person to person, much like a virus, researchers are reporting today. When a person gains weight, close friends tend to gain weight, too.



Catching Obesity From Friends May Not Be So Easy

By GINA KOLATA
Published: August 8, 2011

Why are friends' behaviors correlated?

We mimic our friends:
Contagion/influence

We befriend people who are already like us:
Selection/Homophily

Difficulty: HIDDEN variables
(aka “unobserved covariates”, “latent attributes”)

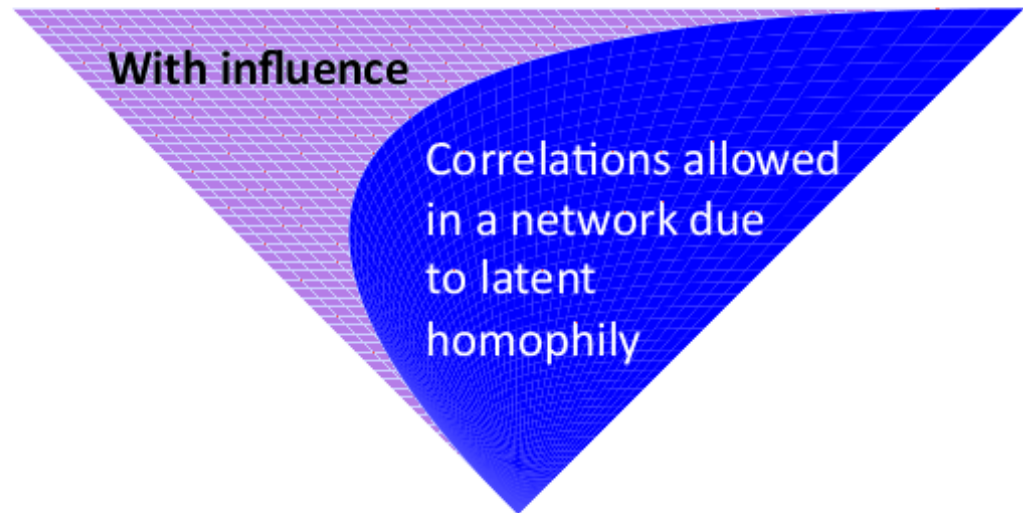
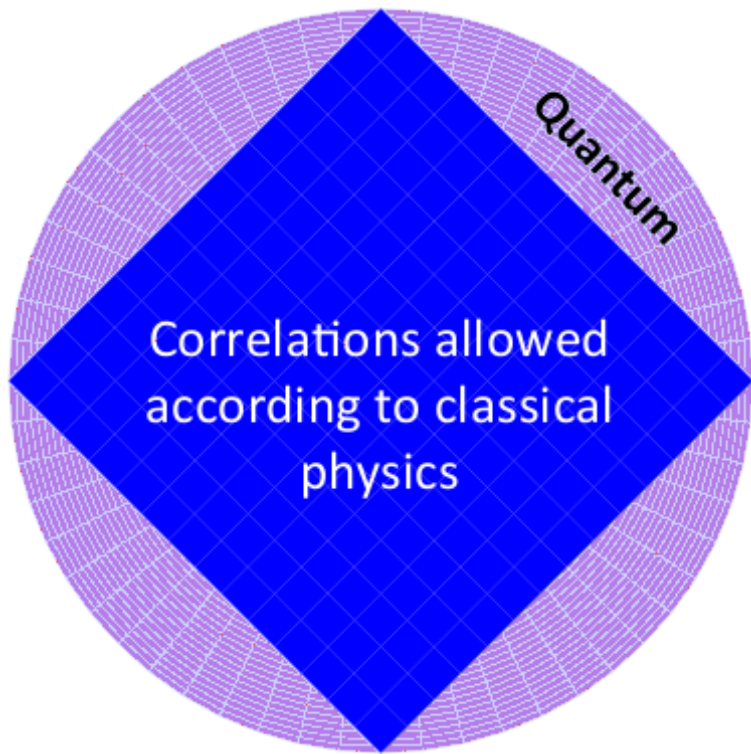
© DESPAIR.COM

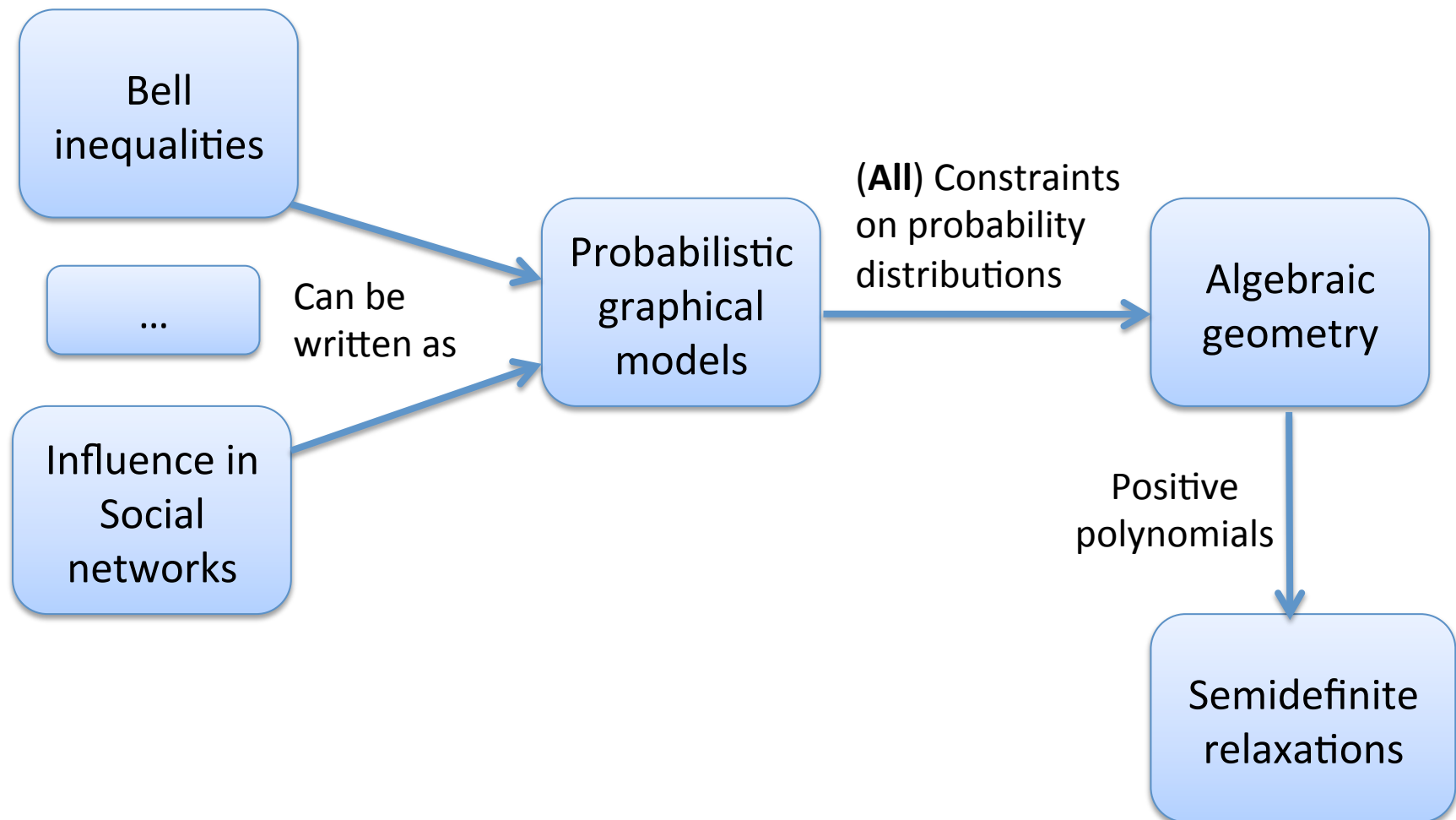


INDIVIDUALITY

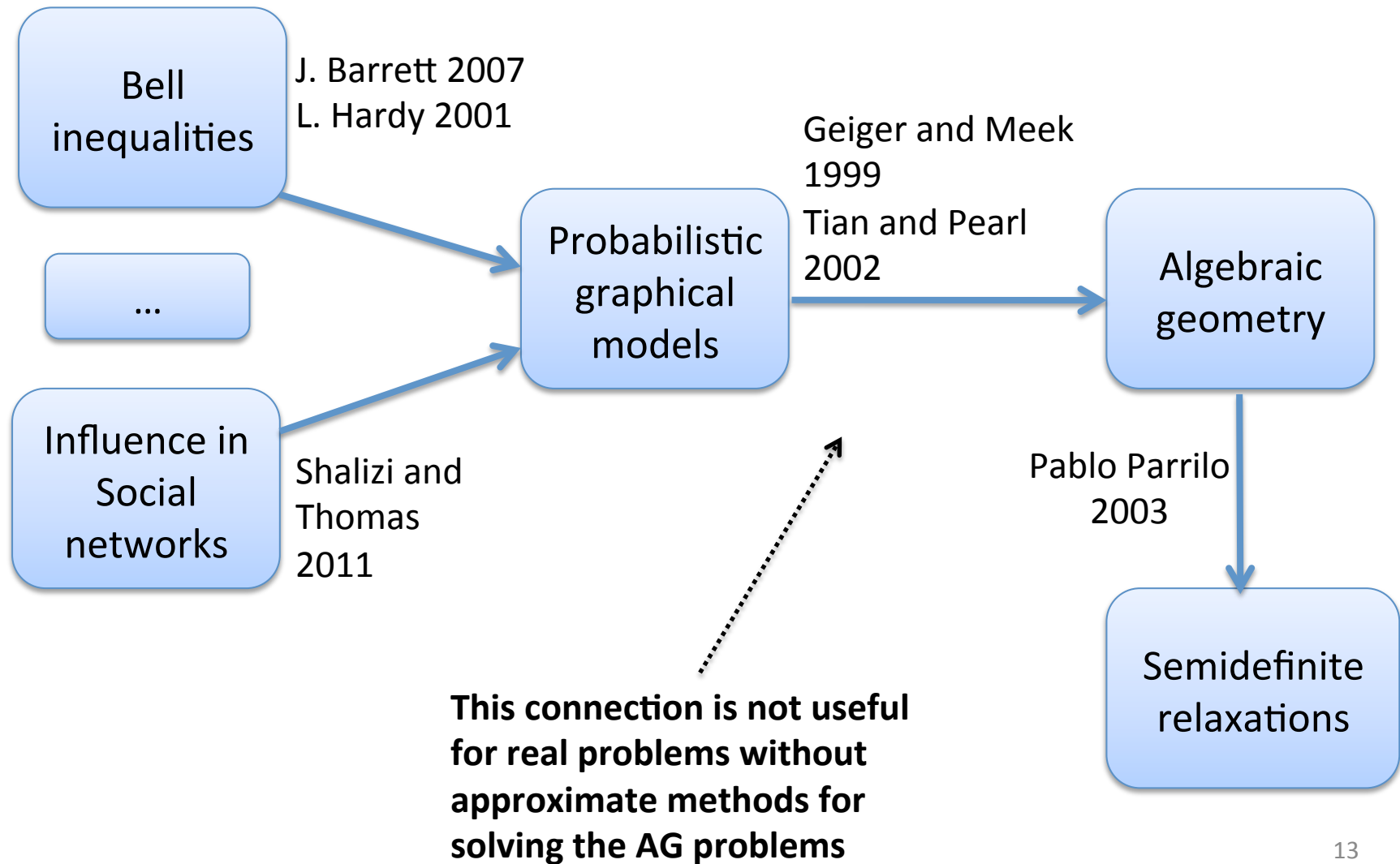
ALWAYS REMEMBER THAT YOU ARE UNIQUE. JUST LIKE EVERYBODY ELSE.

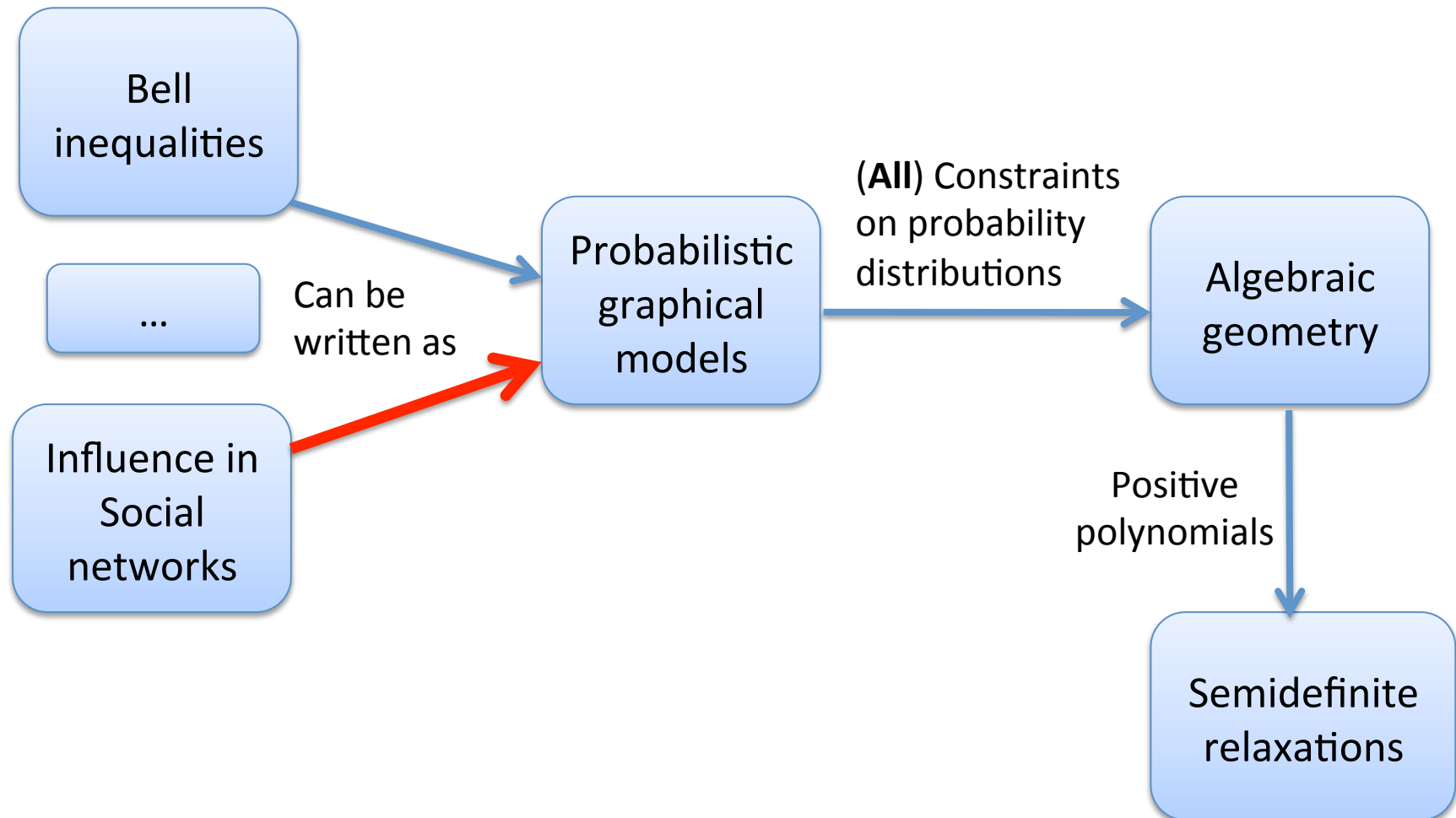
Ruling out hidden variables



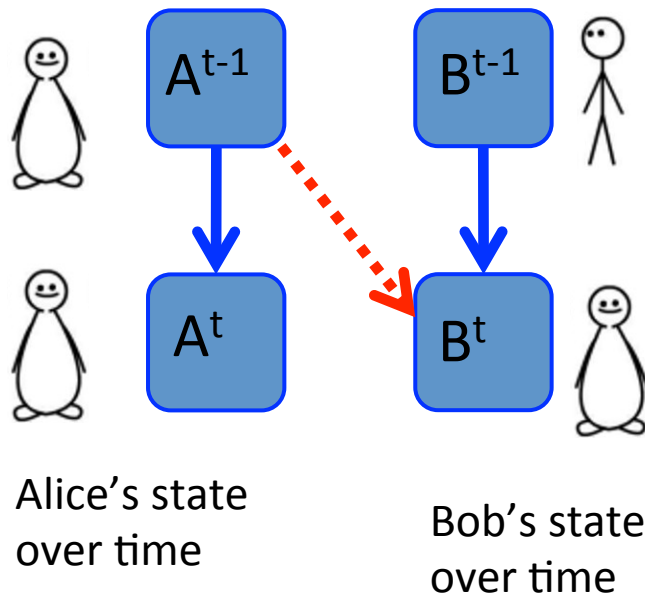


Context

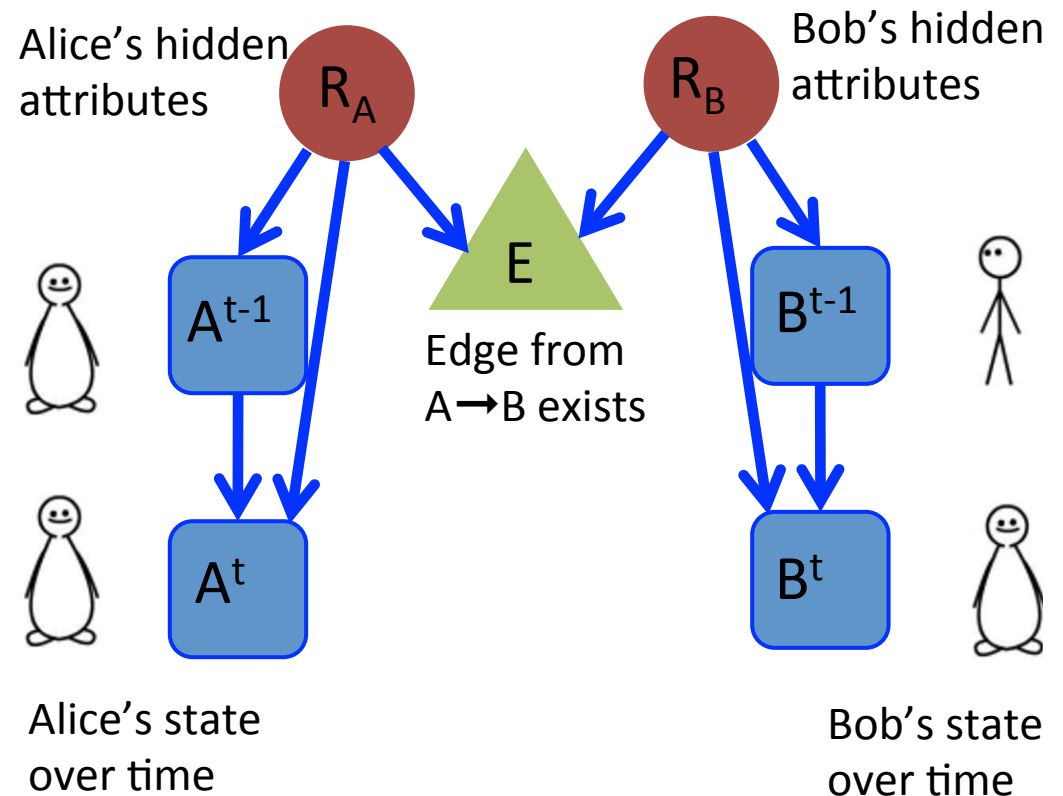




Influence or contagion explanation



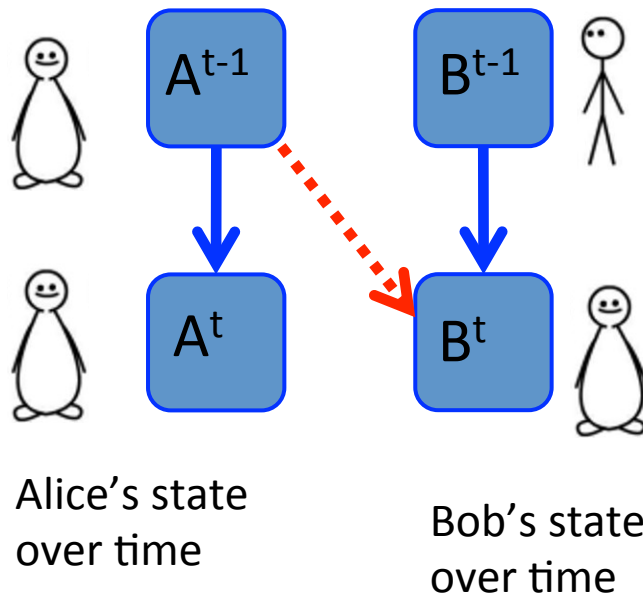
Selection/Latent homophily explanation



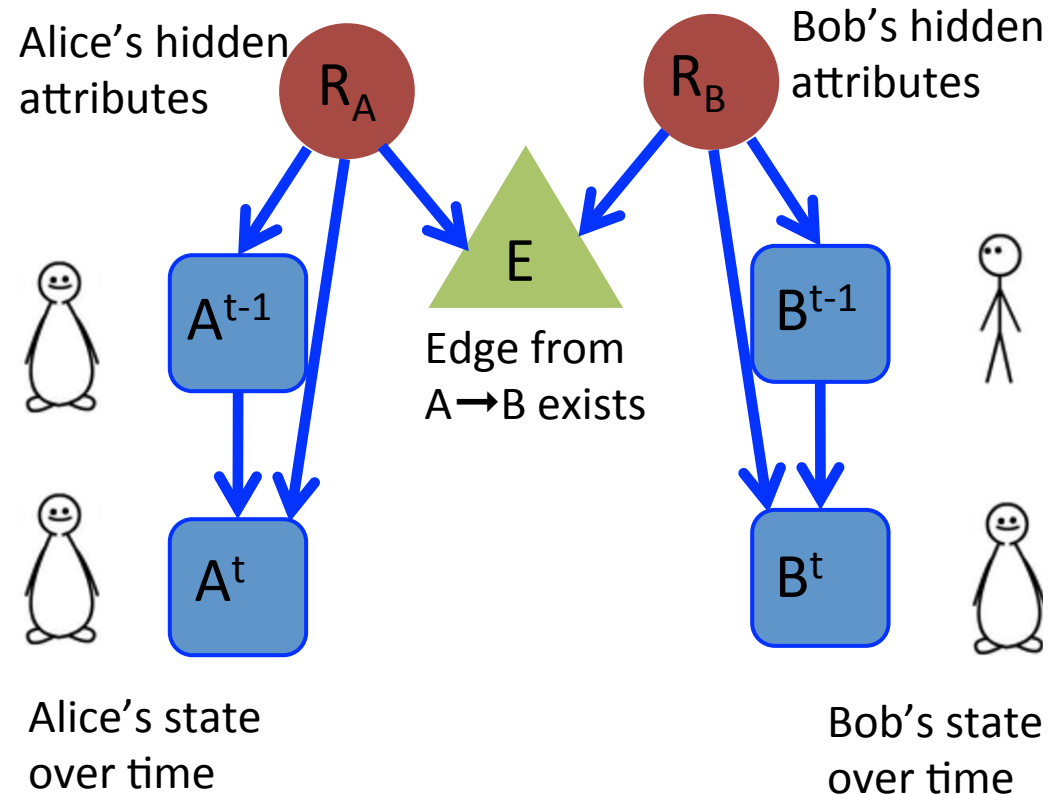
Is it easy to tell the difference?

NO: *Shalizi & Thomas, Soc. Methods & Research, 2011.

Influence or contagion explanation



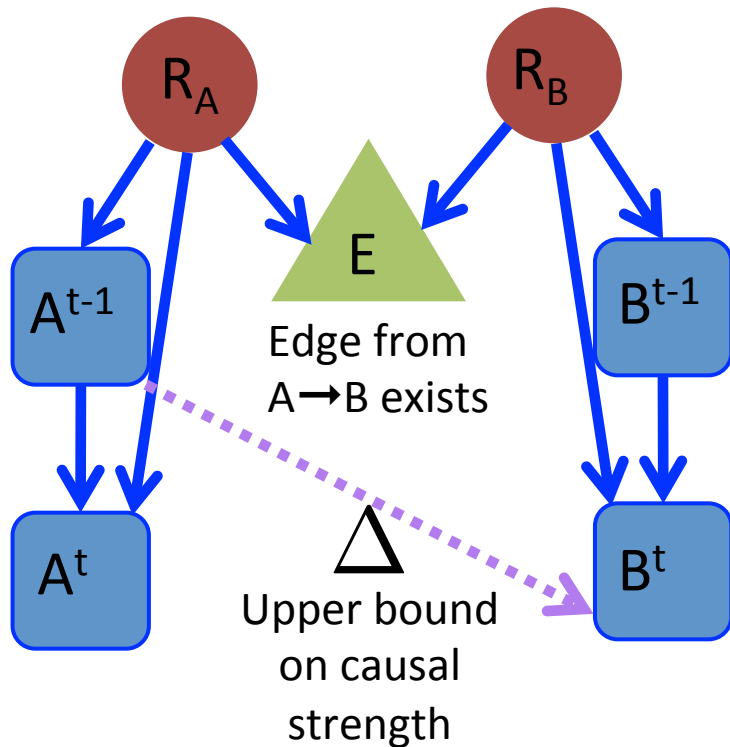
Selection/Latent homophily explanation



*Shalizi & Thomas, Soc. Methods & Research, 2011.

- Current tests for contagion (as in obesity study) can be fooled by latent homophily
- AND contagion can look like homophily

A Statistical Test



Find an observable, $c(A,B)$, so that:

$$\forall P \in \mathcal{P}_\Delta \quad \langle c(A, B) \rangle_P \leq 0$$

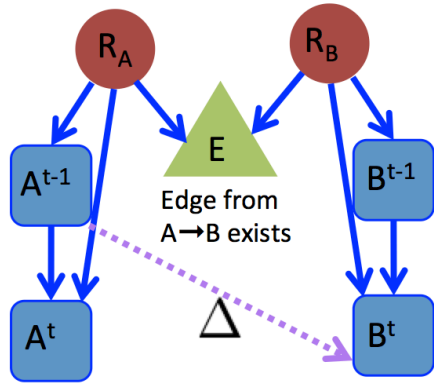
THEN, if an observed distribution $\hat{P}(A, B|E)$ violates this bound:

$$\langle c(A, B) \rangle_{\hat{P}} \neq 0$$

That implies causal strength $> \Delta$ is required to explain correlations. *

*With a caveat discussed in a few slides

How to Construct Tests



$$\forall P \in \mathcal{P}_\Delta \quad \langle c(A, B) \rangle_P \leq 1$$

Equivalent to:

$$P \in \mathcal{P}_\Delta \iff \exists P(A_t|A_{t-1}, R_A), P(R_A, R_B|E), P(B_t|B_{t-1}, A_{t-1}, R_B)$$

$$P(A_{1:T}, B_{1:T}|E=1) = \sum_{R_A, R_B} P(R_A, R_B|E) \prod_{t=1}^T P(A_t|A_{t-1}, R_A) P(B_t|B_{t-1}, A_{t-1}, R_B)$$

$$\delta_l \leq P(B_t = 1|B_{t-1}, A_{t-1} = 1, R_B) - P(B_t = 1|B_{t-1}, A_{t-1} = 0, R_B) \leq \delta_u \langle c(A, B) \rangle \leq 1$$

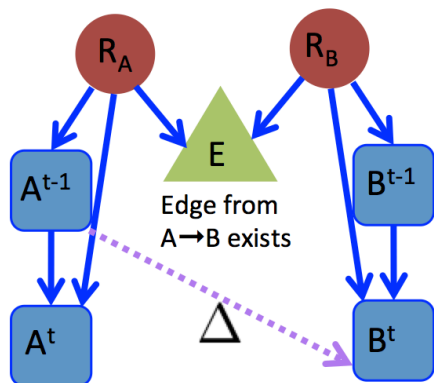
$f(\vec{x})$

x_1, x_2, \dots

Convex hull of an
algebraic surface

$$g_i(\vec{x}) \geq 0$$

How to Construct Tests



$$\forall P \in \mathcal{P}_\Delta \quad \langle c(A, B) \rangle_P \leq 1$$

Equivalent to:

$$1 - \vec{c} \cdot \vec{f}(\vec{x}) \geq 0$$

Polynomial

$$\forall \vec{x}, g_1(\vec{x}) \geq 0, \dots$$

(Linear) polynomial inequalities

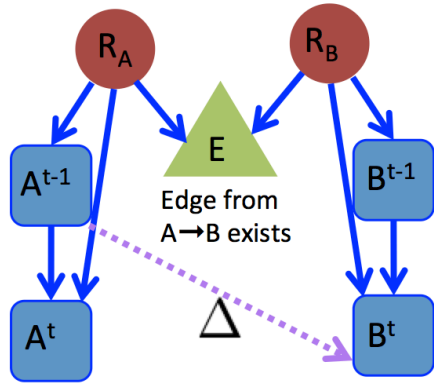
Handelman's representation

Theorem 0.1. *(Handelman's representation) Any polynomial, $\gamma - h(x)$, that is positive on a compact domain $\mathcal{K} = \{x : g_1(x) \geq 0, \dots, g_s(x) \geq 0\}$, where the $g_i(x)$ are linear, can be written in this form,*

$$\gamma - h(x) = \sum_{k \in \mathbb{N}^s} \lambda_k \prod_{i=1}^s g_i(x)^{k_i},$$

using non-negative λ 's, with k representing a vector of non-negative integers.

How to Construct Tests



$$\forall P \in \mathcal{P}_\Delta \quad \langle c(A, B) \rangle_P \leq 1$$

Equivalent to:

$$1 - \vec{c} \cdot \vec{f}(\vec{x}) \geq 0 \quad \forall \vec{x}, g_1(\vec{x}) \geq 0, \dots$$

Polynomial (Linear) polynomial inequalities

Search for a test, c , becomes a Linear Program

$$1 - \vec{c} \cdot \vec{f}(\vec{x}) = \sum_{k \in \mathbb{N}^s} \lambda_k \prod_{i=1} g_i(x)^{k_i} \quad \leftarrow \text{Handelman representation}$$

$$\vec{\lambda}_k \geq 0, \sum_i k_i \leq d_{max} \quad \leftarrow \text{Bounded degree relaxation}$$

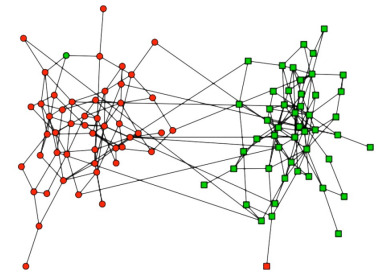
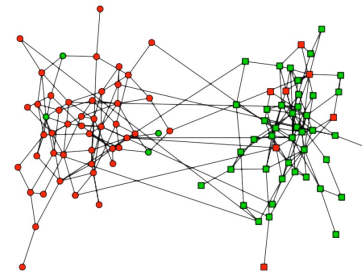
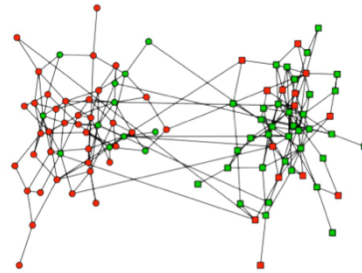
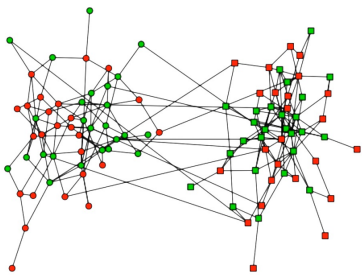
Spot the contagion!

t=1

t=2

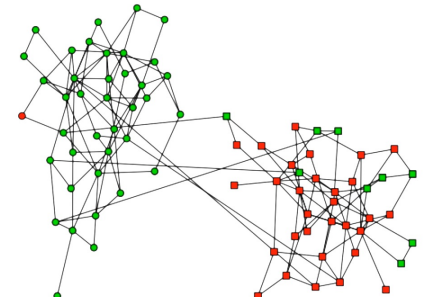
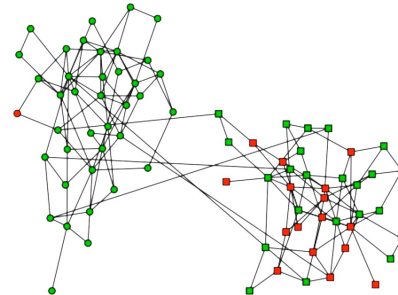
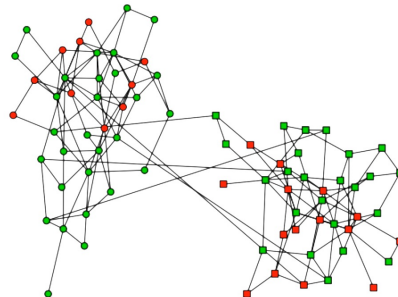
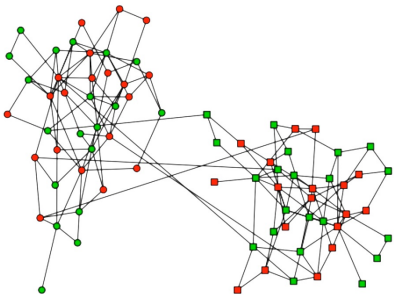
t=3

t=4



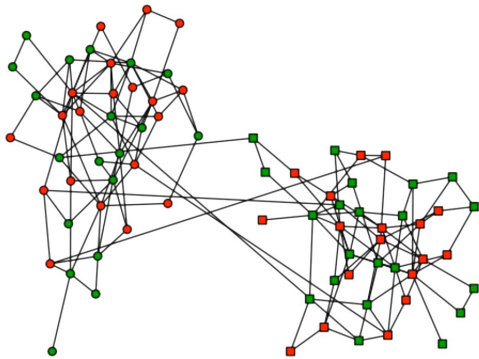
Model A

Model B

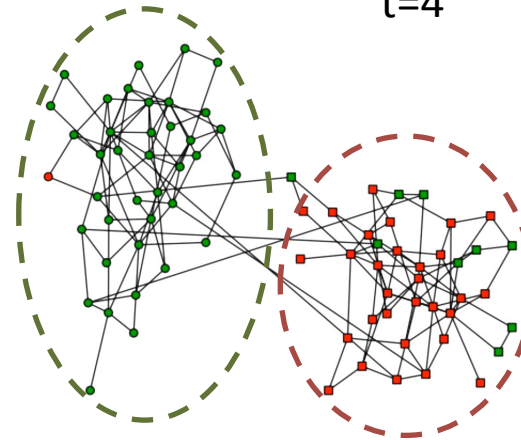


Two plausible explanations

t=1



t=4



Group 1:

- Prefer to befriend people in group 1 (homophily)
- Have a tendency to become green

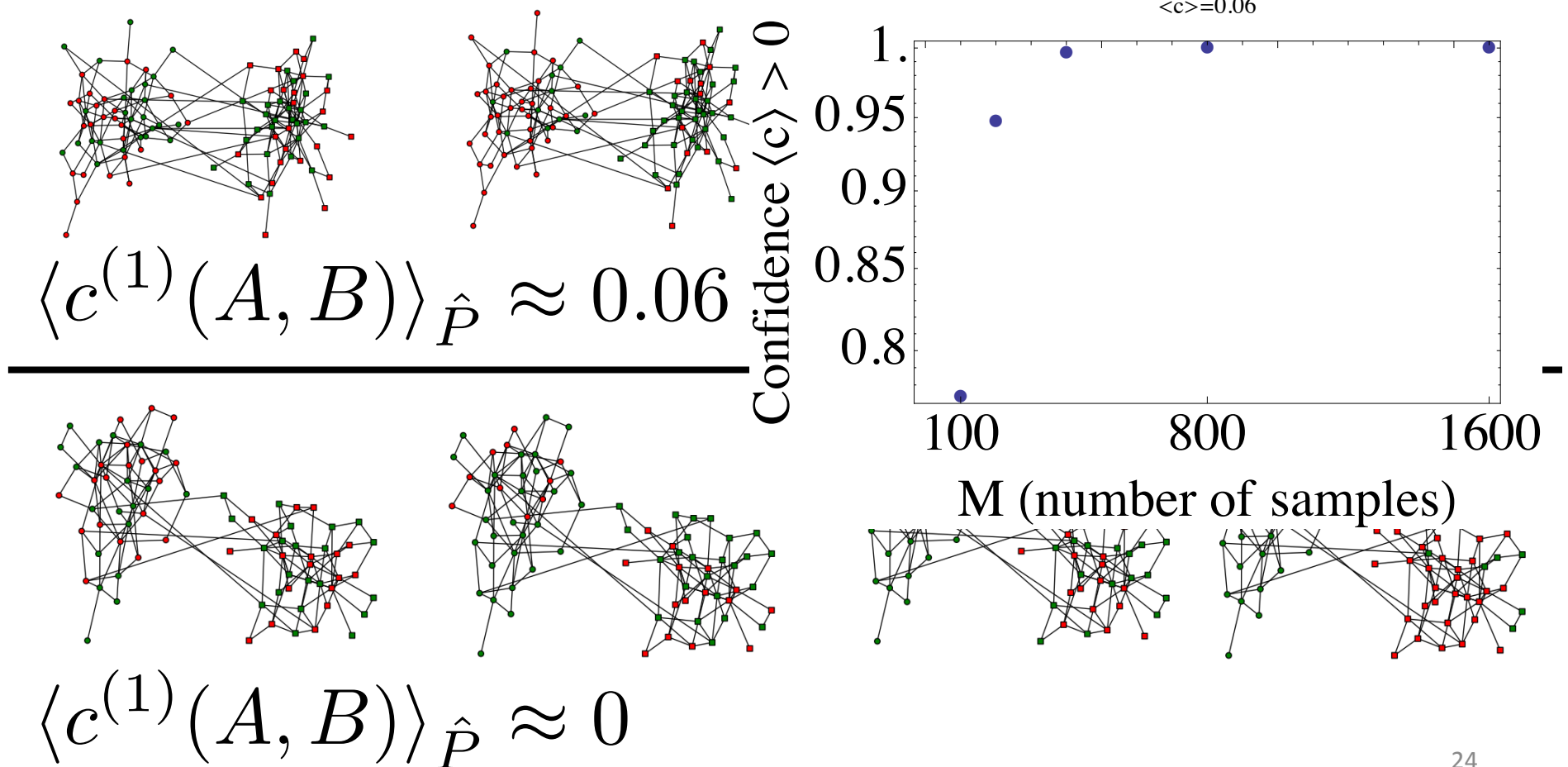
Group 2:

- Prefer to befriend people in group 2 (homophily)
- Have a tendency to become red

We construct a test, c , so that:

$$\text{No influence} \Rightarrow \langle c^{(1)}(A, B) \rangle_P = 0$$

$$\text{Estimate } \hat{P}(A_{1:4}, B_{1:4}|E) \Rightarrow \langle c^{(1)}(A, B) \rangle_{\hat{P}}$$



Framingham Heart Study Data

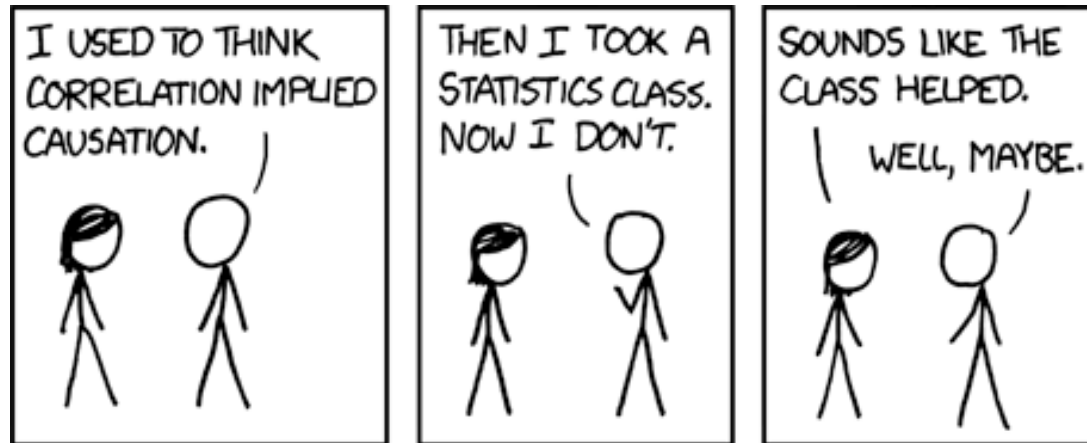
- 5124 subjects, decades long study
 - Extensive data, including: height, weight, marriage, social and familial ties, alcohol, smoking, cancer, heart disease, diabetes, psych exam...
- Edge: if B declared A as an unrelated friend
- Binary outcome: BMI increased more than median

$$\langle c^{(1)} \rangle_{\hat{P}_{obesity}} = 0.055 > 0$$

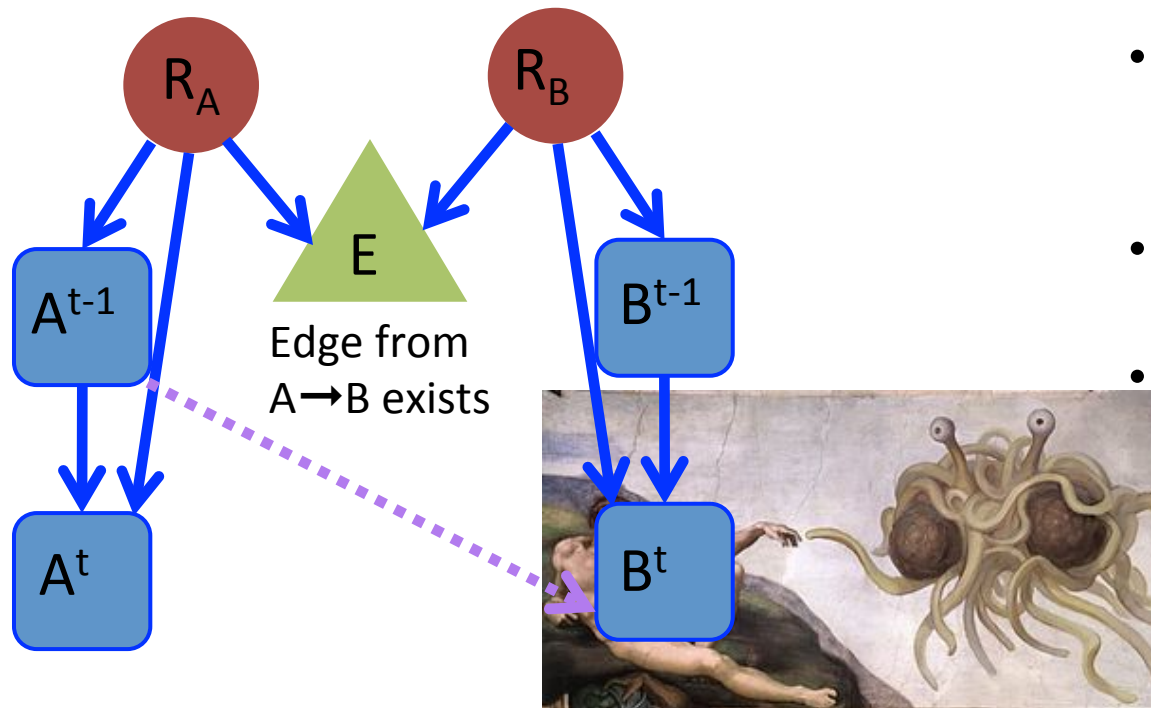
with confidence 99.8 ... % (Using 705 pairs)

Homophily alone does not explain the
spread of obesity

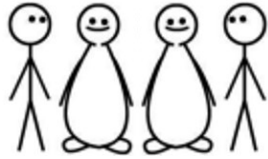
Causal Caveats



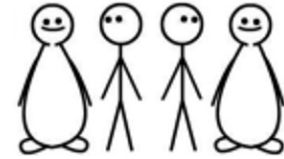
xkcd.com



- No model of this form can explain the spread of obesity, but...
- Adding this causal link is not the only alternative
- “external dynamics”



Conclusion



- Hidden Variables → Nontrivial constraints on observations
- Allows results like: Spread of obesity NOT explained by homophily (regardless of hidden attributes)
- Future work: Other HV models, better confidence bounds, relax stationary assumption, *analyze all FHS data* (smoking, marriage, depression...)

Pre-print: bit.ly/UwUct2

Slides: bit.ly/T3LBis

Contact: gregv@isi.edu

Thank you. Questions?