Applied Symbolic Dynamics

From 1D to 2D to ODE

Bailin HAO

Institute of Theoretical Physics, Academia Sinica, Beijing
T-Life Research Center, Fudan University, Shanghai
The Santa Fe Institute, New Mexico

http://www.itp.ac.cn/~hao/
http://tlife.fudan.edu.cn/

A semester-long course on Applied Symbolic Dynamics (ASD) will be condensed into this one-hour lecture. We shall describe the basic idea and show how ASD works on a few examples. No lecture notes will be given except for a few figures. Those interested in details may consult the following

References

1. Hao Bailin, *Elementary Symbolic Dynamics and Chaos in Dissipative Systems*, a Monograph, World Scientific, 1989 (Freely downloadable from Hao's website thanks to kind permission of the publisher).

2. Bailin Hao and Weimou Zheng, *Applied Symbolic Dynamics and Chaos*, World Scientific, 1998.

The Unimodal or Logistic Map

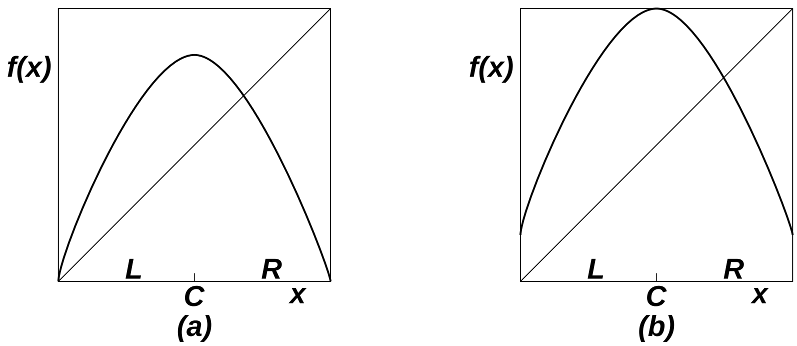


Figure 1: Examples of unimodal map. The “normalization” at the two end points of the interval I is not essential.

Doing Iteration on the Graph

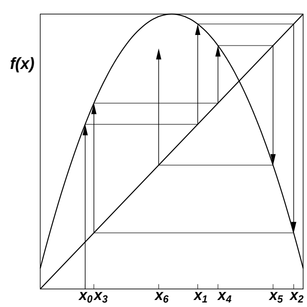
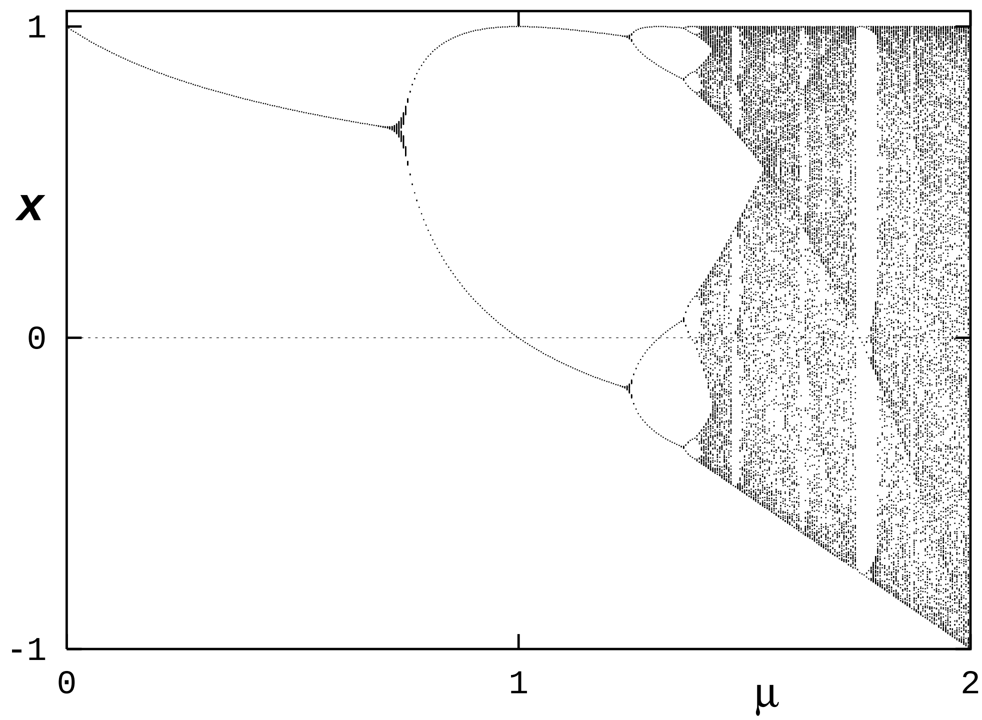


Figure 2: Graphic representation of iterations of a map.

$$
\begin{aligned}
&x_1 = f(x_0), \\
&x_2 = f(x_1), \\
&x_3 = f(x_2), \\
&\ldots \;\; \ldots \\
&x_n = f(x_{n-1}), \\
&\ldots \;\; \ldots
\end{aligned}
$$

A Bifurcation Diagram of the Unimodal Map



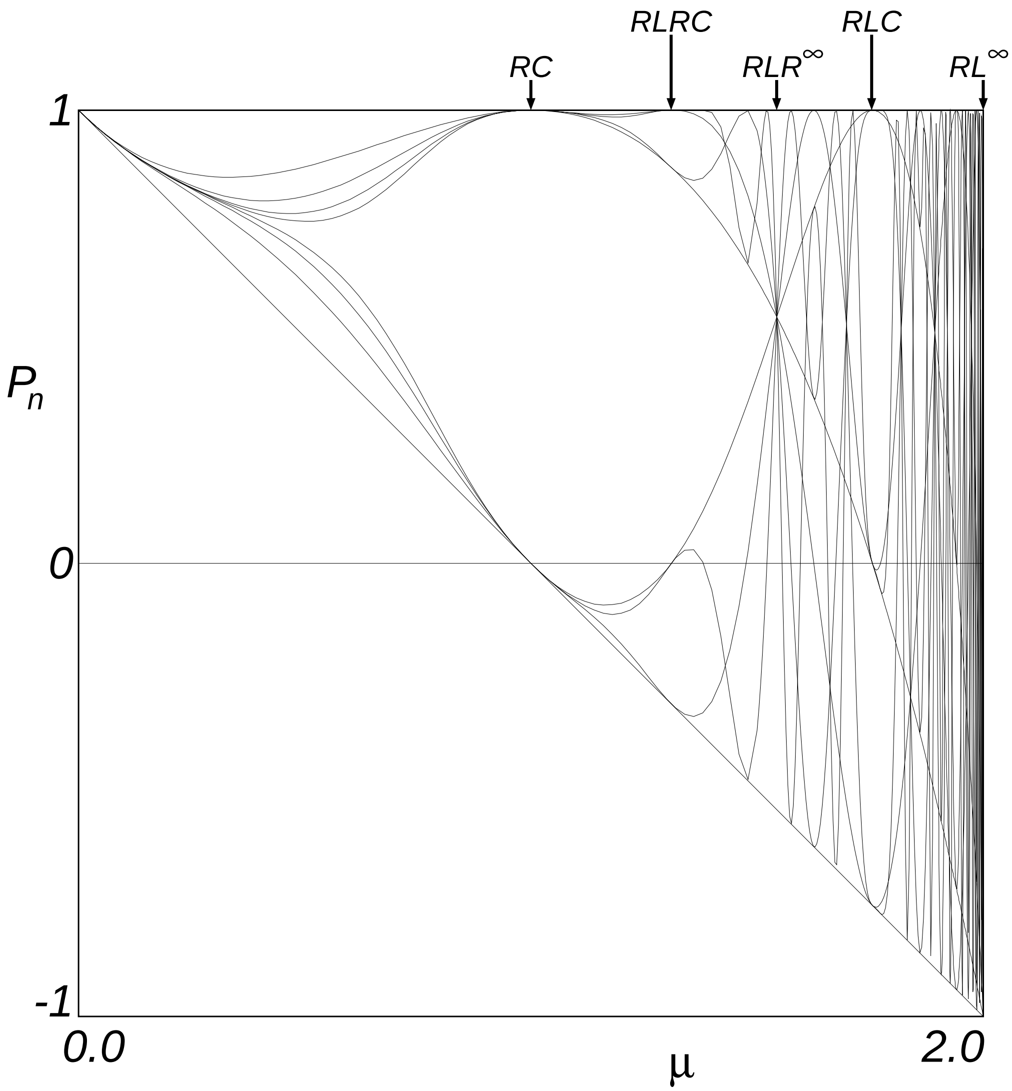
Looking at this diagram, how many questions one can ask?

Example: the thickening of lines near bifurcation points is related to critical slowing down.

See:

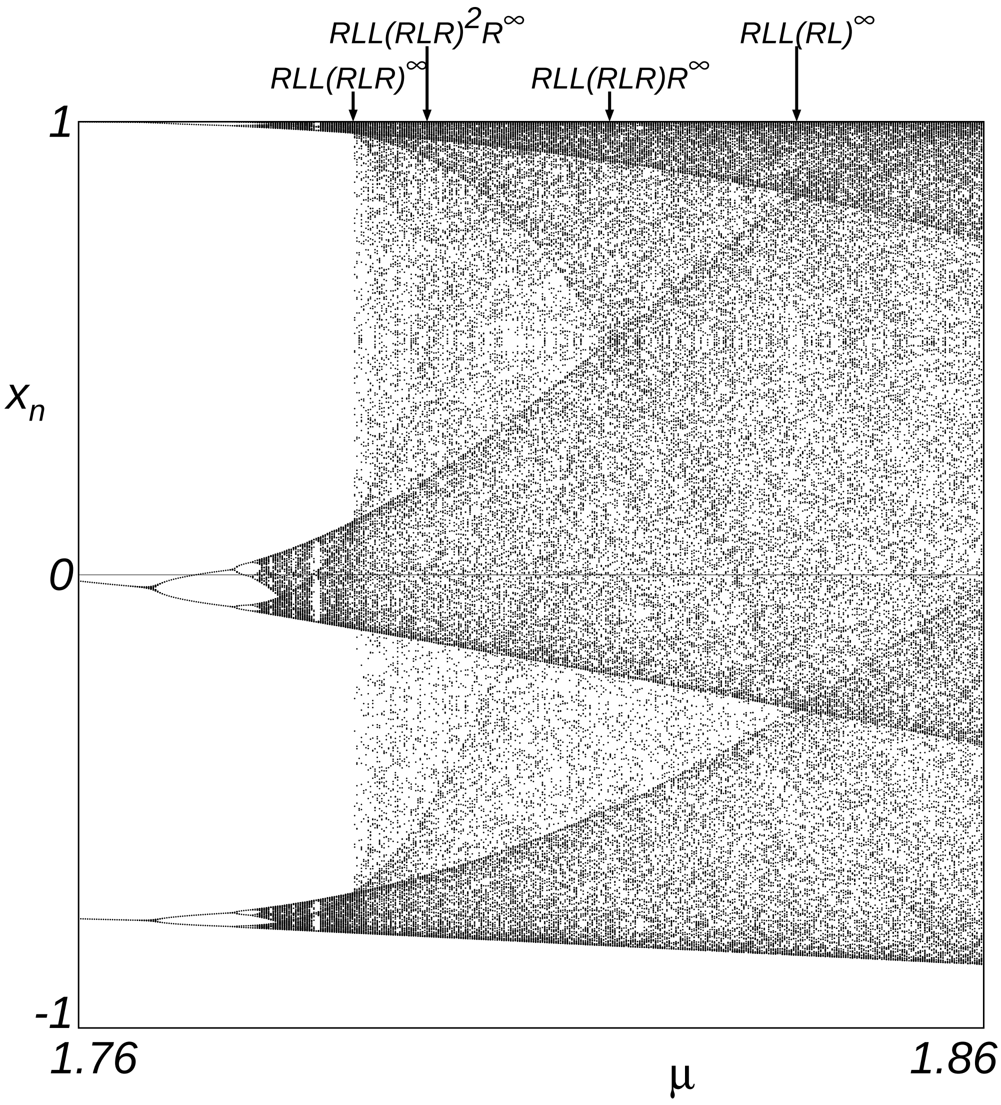
Bailin Hao, Critical slowing down in one-dimensional maps and beyond, *J. Stat. Phys.*, **121(5/6) (December 2005), 749 – 757.**

Dark Lines in the Bifurcation Diagram



Can we write down the equations for all these dark lines?

Bifurcation diagram near the period 3 window.



Dark Lines near the Period 3 Window

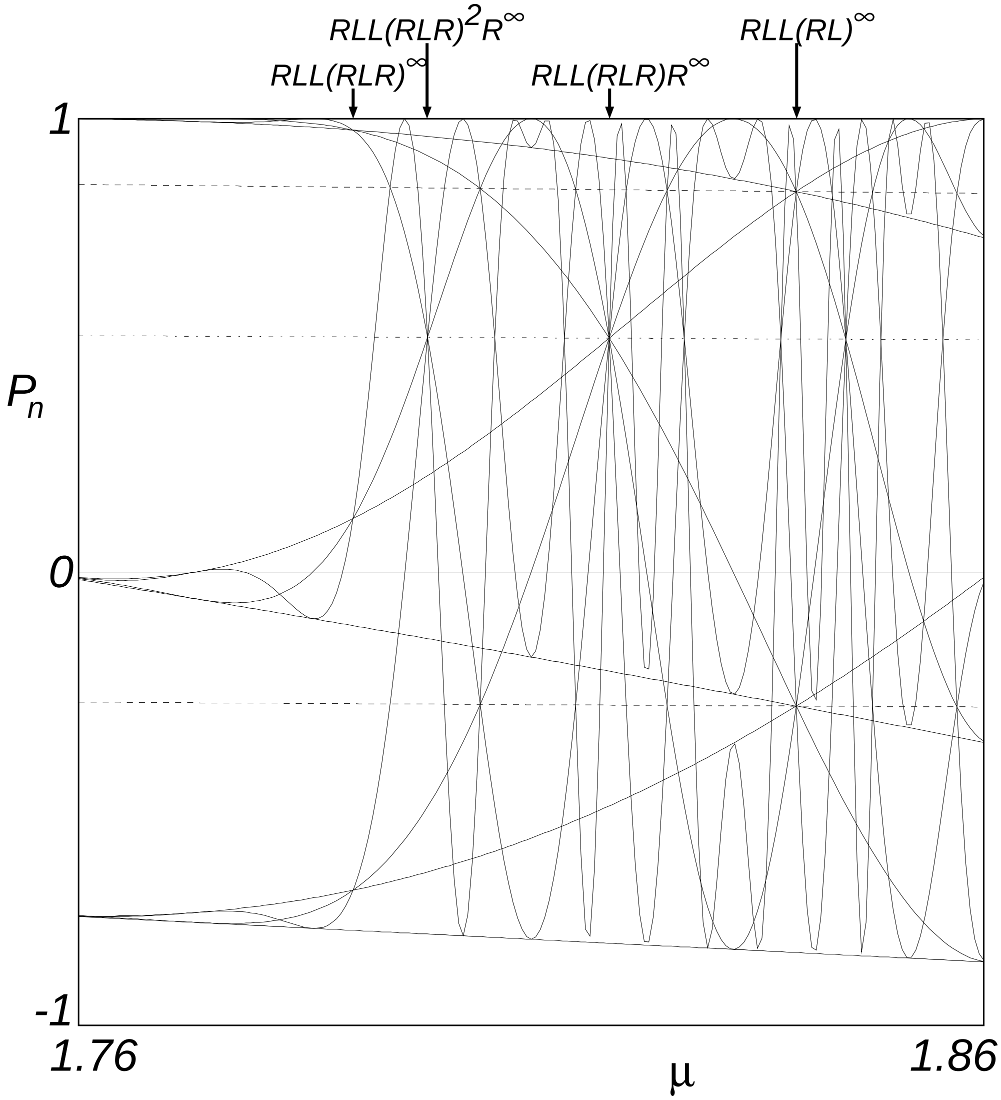


Figure 3: Approximately fro RLC to RL^2RC.

Dynamically invariant range $U = [f^2(C), f(C)]$

Kneading sequence: $K = f(C)$.

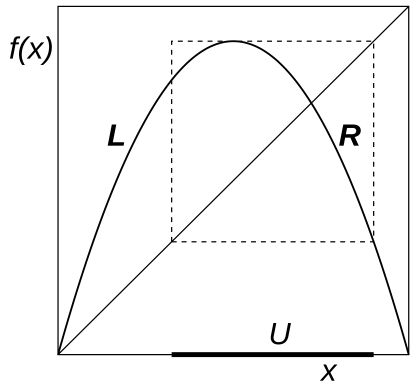


Figure 4: The dynamical invariant range $U = [f^2(C), f(C)]$. All points in U have symbolic sequences no larger than the kneading sequence $K = f(C)$.

Natural order $L < C < R$ **and ordering of all symbolic sequences**

All sequences starting from points within U **are no greater than the kneading sequence** $K = f(C)$**.**

Admissibility conditions