

# The Coevolution of Residential and Friendship Networks: An Extension of the Schelling Model\*

Bruno Abrahao<sup>†</sup>, Maria Pilar Opazo<sup>‡</sup>, Zhiyuan Song<sup>§</sup>, Bogdan State<sup>¶</sup>

September 1, 2010

## Abstract

During the past few decades social network analysis has produced a great deal of insight into the workings of social systems. While social scientists have put a lot of work into the investigation of residential, friendship, trust, exchange or discussion networks, scientific inquiry has typically limited itself to investigating the characteristics of networks of only one kind. This approach has produced plentiful insight on the structure and function of different kinds of social networks, but the interaction between the different kinds of social networks has received insufficient investigation so far. Our work, in which we examine the interaction of residential and social networks represents an attempt at advancing this field of inquiry. More specifically, we extend a classic model of residential segregation (Schelling, 1968) by incorporating a social network that constructs – and is influenced by – residential preferences. We use Agent-Based Modelling to examine how social network topology affects residential segregation in the Schelling model. Given its current popularity in social simulation, extending the Schelling model is an important task in its own right, but we seek to achieve something more fundamental than a mere rehashing of an old model. We deploy Schelling’s model as the basis for a way to understand multiplex networks, and seek to give a formal, methodologically practicable expression to Granovetter’s concept of embeddedness.

## 1 Literature Review

Our work speaks to several lines of social scientific inquiry. As a secondary goal of this paper we extend the admittedly large body of literature that has developed in the field of residential segregation research following Schelling’s foundational paper. Our work goes beyond the problem of residential networks,

---

\*This is an incomplete version of a working paper for SFI CSSS 2010. Please do not cite it

<sup>†</sup>Dept. of Computer Science, Cornell University

<sup>‡</sup>Dept. of Sociology, Columbia University

<sup>§</sup>Dept. of Biology, Stanford University

<sup>¶</sup>Dept. of Sociology, Stanford University

however. By bringing in another kind of social network, we also attempt to develop theoretical insights on the evolution of several networks. Given the scarce amount of work existing on the interaction of two or more networks, we believe this effort to be theoretically valuable in itself. By highlighting the interaction of two social networks, our work also speaks to the literature on embeddedness, a concept which has taken hold in Economic Sociology during the past twenty years. Moreover, because the discussion of network influence mechanisms is crucial to our model, we also draw heavily upon the social psychological literature on prejudice and attitude formation.

### 1.1 Embeddedness

Our work seeks to provide a quantitative interpretation of Granovetter's (1986) concept of embeddedness by extending Schelling's model. In his programmatic article, Granovetter sought to discredit both the atomistic, under-socialized, view based on rational-choice theory and common in economics, and the over-socialized, institutional view of economic action, typical of many sociological accounts. Instead, Granovetter proposes a middle-way, that does not deny the agency of social actors, at the same time accounting for the force of social context. Our work seeks to perform a first step in the direction of "embedding" the Schelling model, even though we recognize the limitations of our argument. Certainly, residential segregation does not arise only from individual preferences, individual preferences are not only shaped by friendship networks, and friendship networks do not evolve only as a result of propinquity. But small steps are necessary in order to understand the complex interweaving of networks and mechanisms that produces society. We thus believe our study represents a small, but significant, advance in the right direction of inquiry.

### 1.2 Residential Segregation

Residential segregation has been called the "missing link" of racial inequality (Charles, 2003). According to Massey and Denton (1996), the concentration of racial groups in different neighborhoods serves to mediate and exacerbate the effects of other mechanisms of inequality. Why segregation occurs is still an open question to social scientists, and this paper makes no pretense at answering this rather large question. Rather, we aim here to shed light on the relationship between residential segregation and the evolution of social networks.

Several kinds of explanations have been advanced with respect to the sources of segregation. Self-selection based on socio-economic status, as well as discrimination in the housing market (Charles, 2003) are two credible explanations for residential segregation that we have to overlook, in the interest of our model. Instead, the mechanism our model uses – as all other implementations of Schelling – is one of "place-based stratification" (Charles, 2003), where individuals become more willing to move if too many members of racial out-groups live in their current neighborhoods.

### 1.3 Formation of Segregation Preferences

Charles's 2003 review is suggestive of the fact that inter-group attitudes are far from being the sole mechanism underpinning residential segregation. Still, she concludes that "active racial prejudice is a critical component of preferences for integration, and therefore, the persistence of racially segregated communities." The question naturally arises as to what the source of these attitudes is. In the typical simulation study of residential segregation this question is left to the social-psychological literature to answer. The level of discriminatory or tolerant preferences is taken as a given variable, independent of the segregation situation.

This assumption appears relatively straight-forward if one considers the residential network in isolation. In the typical iteration of the Schelling model agents have fixed preferences, and they move about until these preferences are satisfied. Very few variables are exogenous in real social interaction, however, and racial attitudes are arguably no exception. In particular, the idea has been advanced long ago (Allport, 1954), in the form of the so-called "contact hypothesis," that meaningful inter-ethnic contact fosters positive inter-racial attitudes. The validation of this proposition has had an apparently mixed record in social-psychological research, with many researchers (i.e., Hewstone and Brown, 1986 [more refs]) challenging the very validity of this assertion, given that many studies failed to show an attenuation of prejudice resulting from inter-group contact.

The work of Thomas Pettigrew has brought necessary specificity into social scientists' understanding of the role of inter-group contact. Pettigrew (1998) emphasized the four conditions Allport cited as making contact effective at reducing prejudice: equal status, common goals, inter-group cooperation, and authority support. Not all conditions are absolutely necessary for the reduction of prejudice, but each one makes the contact mechanism more efficient at improving inter-group attitudes. Schelling-type models typically avoid introducing status and resource differential among agents (but see Benard and Willer, 2007 for a counter-example). We likewise assume status equality, and do not posit any differences in wealth that would complicate both inter-group contact and access to housing. We likewise assume Allport's fourth condition to hold: in our model there is at least enough support of desegregation from local authorities to prevent the adoption of any pro-segregation measures such as red-lining or neighborhood-level racial covenants. The middle two of Allport's conditions - common goals and inter-group cooperation - receive less emphasis in our model, as we are agnostic with respect to the existence of group-level goals, and the degree of inter-group cooperation possible in order to achieve them. Not all contacts between the members of different social groups are of equal importance in the reduction of prejudice, as Pettigrew (1998) points out. Meaningful, close, long-lasting social relationships are far more important for the lessening of negative views than are fleeting acquaintanceships. We similarly build a social network that emphasizes "strong" rather than "weak" ties (Granovetter, 1973), ties that would be relevant in the updating of racial preferences.

A meta-analysis completed by Pettigrew and Troop (2000) found that out of 515 studies, 94% showed results confirmatory of the contact hypothesis, leading

them to conclude that “generally, intergroup contact relates negatively and significantly with prejudice.” And while Allport’s conditions, discussed previously, do seem to facilitate the negative effect of intergroup contact on prejudice, Pettigrew and Troop find that significant reductions in prejudice occur even when the conditions are not met. Almost any kind of inter-group contact has the potential to impact prejudice negatively, and even though our model satisfies several of Allport’s conditions, and does not explicitly fail to satisfy any other conditions, we are confident that the same mechanism would operate even in the absence of status equality and a favorable legal environment.

#### 1.4 The Schelling Model

Thomas Schelling’s early work set off a very productive stream of research into social processes. Scholars of mathematics, natural and social sciences have used Schelling’s insights to develop the new method of agent-based modeling, to investigate systems that resist a reduction to equations. Schelling not only pioneered a new methodology, but also expanded knowledge of the dynamics of residential segregation. In particular, Schelling’s work has provided the view that relatively “reasonable” individual preferences, that would not normally be considered to be racist can result in nearly complete residential segregation. All it takes, it seems, for high levels of segregation to occur, is a desire to be surrounded by even a small number of one’s own kind in a neighborhood. This extremely powerful conclusion has revolutionized social scientists’ understanding of residential segregation.

Schelling’s model has not been sheltered scientific controversy. Massey and Denton (1996) have raised the question as to whether basing a model on residential preferences can provide a sufficient explanation for residential segregation, a phenomenon which is in the very least facilitated (if not enabled) by structures of inequality. Massey and Denton assert that the Schelling model assumes the existence of discriminatory mechanisms that keep minorities out of certain neighborhoods to which majority residents can choose to move: were this objection true, the Schelling model could in effect be reduced to a tautology. Fossett (2008) discusses these objections and offers several grounds to dismiss them. He points out that in the Schelling model whites (the majority) do not move to the most segregated neighborhood but simply seek out the first housing position that would satisfy their integration preferences. Fossett likewise argues against the idea that Schelling assumed discrimination to exist by default. All that is necessary for a Schelling-like process to occur, according to Fossett, is simply the existence of ethnocentric residential preferences.

Fossett (2008) also points out that the percentage mix matters. Even though minorities such as African-Americans have been found to have weak preferences towards their own group, the fact that these preferences nonetheless exist produces what Fossett terms the “paradox of weak minority preferences.” As perfect integration requires an even mixing of the entire city population, the smaller a minority, the lower their ethnocentric preferences have to be in order for system-wide integration to be achieved. Thus emerges a confusing

result. Even though, i.e., Blacks in the United States have relatively low ethnocentric residential preferences – which individually could well be judged as “integrationist” – these preferences ultimately end up promoting segregation. In a 10% Black, 90% White city, Blacks holding a preference for as little as 20% Black-neighbors would nonetheless be unsatisfied with the “maximum integration” situation where they would have about 10% Black neighbors. In a typical minority-majority ethnic mix, segregation may well emerge as a result of a perverse dynamic of relatively well-meaning preferences.

The effect of agent “vision” has similarly been used to criticize the Schelling model. In their study, Laurie and Jaggi (2003) found the pessimistic conclusion of the Schelling model – that segregation can emerge from relatively benevolent individual decisions – to be unwarranted, an artifact of previous implementation using a short range of “vision” for agents, which could only “see” the neighbors one square removed, in either a Moore or a von Neumann neighborhood. Laurie and Jaggi posited that increasing the range of vision produces a striking transition in the effect of preferences on residential segregation. Whereas segregation becomes more accentuated with highly ethnocentric preferences, higher ranges of vision yield, according to Laurie and Jaggi, far lower levels of segregation than they do in the typical Schelling model. Fossett and Waren (2006) examined Laurie and Jaggi’s findings, turning their attention to the 50%-50% ethnic mix used by Laurie and Jaggi, which Fossett and Waren found to be optimal for the promotion of integration as a stable state. For this reason it is important to examine the evolution of a Schelling-type model with respect to various ethnic mixes.

[Yinger (1995) – heterogeneity of preferences, Fossett (2006) rejects, see also Clark (2006)]

In addition to being the subject of many academic disputes, the Schelling model has likewise been extended in various directions. Benard and Willer (2007) provide an illustration of how status and resources can be integrated in Schelling’s framework. Macy and van de Rijt (2006) extend Fossett’s findings with respect to the primacy of individual choices over institutional factors, and show that segregation may arise even in situations where agents have a strong preference for multiculturalism, if their second preference is for ethnic homogeneity. Vinkovic and Kirman (2006) have gone as far as to create a physical analogue of the Schelling model, but as Clark and Fossett (2007) point out, it is crucial to consider the Schelling model in view of its embeddedness into a social context, and not remove it entirely from all other processes at work in a society. We seek to perform a first step in this direction by illustrating how a social network can evolve parallel to the residential network examined in canonical implementations of the Schelling model.

Schelling’s choice of a lattice for a residential grid, where actors live in self-centered Moore neighborhoods may seem misguided and unrealistic to many students of actual cities, which are larger and more irregular, and have more complicated neighborhood structures than Schelling’s stylized grid. Nonetheless Fossett and Dietrich (2008) present results of simulations which essentially confirm the validity of Schelling’s insights irrespective of city size, city shape, or

neighborhood structure. Indeed, the only element with a differential impact on the Schelling model seems to be vision, the size of the neighborhood the agent considers relevant when deciding whether to move or not (Fossett and Dietrich, 2008). Expanding vision leads to different stable-state configurations for the Schelling model, and while this parameter is not the main focus of our research, we analyze its influence on our outcome in the section devoted to robustness checks.

## 1.5 Formation of New Friendships

We contend that increased, meaningful inter-group contact at the neighborhood level will act to reduce prejudice. This assertion is supported by the findings of Ihlanfeldt and Scafidi (2001), who use data from the Multicity Study of Urban Inequality to show that neighborhood contact has a significant and negative effect of inter-ethnic prejudice. Notably, white prejudice is reduced only by contact with equal- or higher-status racial minorities, whereas any contact reduces black prejudice, although encounters with alters of higher-status has a greater negative effect. If we combine these insights with the demand placed by Pettigrew (1998) on the “contact hypothesis,” namely that only meaningful, long-term contact can reduce prejudice, friendship across group boundaries becomes an obvious mediating mechanism between contact and the reduction of prejudice.

Under conditions of inter-ethnic animosity, or, in the very least, homophily, making friends outside one’s salient identity groups is not a probable event. Indeed, as McPherson, Smith-Lovin and Cook (2001) note, “birds of a feather stick together,” and the principle of homophily ensures that most social bounds connect people similar in age, occupation, education, ethnicity, or any other number of characteristics. Homophily is a seemingly inexorable force against which any integrationist policies must contend. But homophily is not a universal law: making connections across social boundaries is possible if not always likely. When it comes to superseding homophily, propinquity is a strikingly influential factor: Nahemow and Lawton (1976) found that inter-racial friendships as well as friendships across age groups occurred almost without exception between people living close together. The kind of contact is to make an agent more tolerant of out-group neighbors seems to be overwhelmingly concentrated in one’s current neighborhood.

Examining the interaction of Schelling’s model with a social network brings to light a wholly new perspective on residential segregation. We no longer witness static, memory-less agents whose minds are made up in advance over what ethnic mix would make them happy. Rather, we see agents displaying a kind of cognition: meaningful contact developed at the neighborhood level translates into friendships, which ultimately translate into higher or lower levels of tolerance towards out-group neighbors. At any point in time, the friendship network is a reflection of all previous states of the residential network, and vice-versa.

The rule we stipulate for the formation of residential preferences is relatively

straight-forward: an actor will tolerate as high a percentage of out-group neighbors as they have out-group friends, weighted by their status. This measure may appear arbitrary, but one should not lose sight of the fact that it is merely designed to act as a parsimonious, mathematical measure of a far more complicated process. Each friendship triggers pleasant memories, and may make an actor more likely to identify with a larger group (i.e., New Yorkers or Americans) inclusive of both in- and out-groups, a process proposed by Gaertner et al. (1996) as mediating the influence of contact. Indeed, the influence of every friendship may not be the same and may not be constant over time. While we are cognizant of this limitation in our model, we decide to view all friendships alike, to avoid extending our model beyond the limit of relevant analysis.

One substantive issue with the mechanism we propose has to do with what kind of information is communicated through the social network. To the extent that preferences for or against residential segregation made the object of frequent discussion, one could argue that the social network acts as a conduit for information about each actor's threshold, which the actor broadcasts to their friends. This assumption would involve the possibility that intolerant preferences be freely transmissible in the network. There is ample literature that shows this not to be the case, however. Already in 1982, Pettigrew noted a steep decline in Americans' explicitly racist attitudes, and so it seems highly unlikely that Americans' would communicate their preferences for segregation explicitly to each other. Moreover, an entire literature has developed in social psychology on the effects of implicit, sub-conscious intergroup bias. [CITATIONS; I.e., Blair and Banaji, etc.]. Thus explicit communication seems too unlikely a process to be useful for our model. Instead we propose a mechanism whereby actors simply take stock of their friends' "color," weighted according to the normalization mechanism proposed above, and adjust their tolerance levels accordingly. Having many friends belonging to the other group should make one more tolerant - but as mentioned above, the effect should be lower if one's friends are low-status, and even lower when the actor is high-status.

## 1.6 A Stochastic Model

Our project will examine two mechanisms - propinquity and contact - through which the networks influence each other, but does not rely on these mechanisms alone. Tempting though the approach may be, to be relevant a social model must eschew determinism, and opt instead for a stochastic approach. Allowing for random factors to influence our agents' decisions forms part of our investigative strategy as well. This allows us to account for the existence of myriad other chance factors, independent of the propinquity and contact mechanisms that we believe influence the interaction of residential and friendship networks. A graphical representation of our conceptual framework is provided in Figure 1.6.

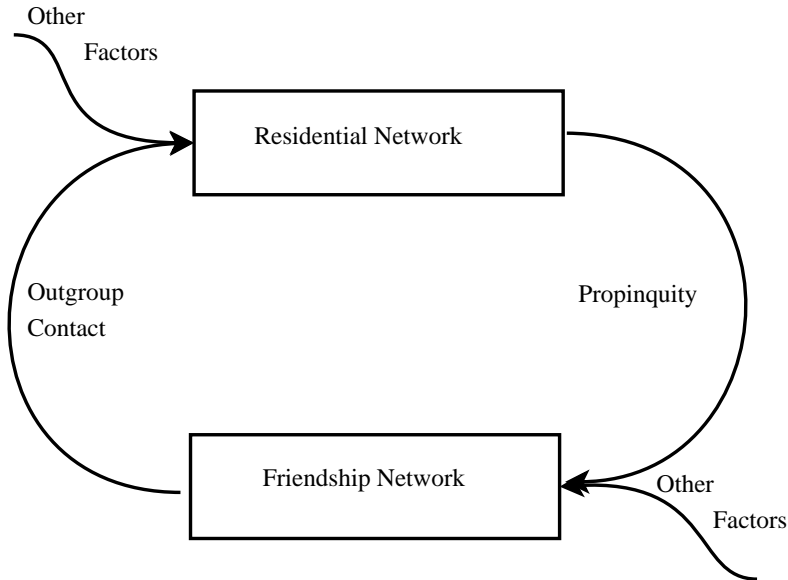


Figure 1: Conceptual Framework Used In Our Model

## 2 Model and Methodology

For simplicity and generality, we adopted the common settings of a two-dimensional segregation model. The residential area is a two-dimensional  $N \times N$  square lattice with periodic boundary conditions, i.e., a torus containing  $N^2$  cells. Each cell corresponds to a dwelling unit, all of equal quality. Some trivial trait divides the whole population in two groups, e.g., red and green, of households. Each location may thus be occupied by a red agent, a green agent, or may be vacant. Here, we chose the rule of Moore neighborhood (8 immediate neighboring cells surrounding the focal cell). Suppose an agent's neighborhood is composed of  $n_s$  similar agents,  $n_d$  dissimilar agents and  $n_v$  vacant cells. Since  $n_s + n_d + n_v = 8$ , one needs two independent parameters to describe the composition of the neighborhood of the agent. We define a state  $x$  as a  $N^2$ -vector, each element of this vector labeling a cell of the  $N \times N$  lattice. Each state  $x$  thus represents a specific configuration of the community. We note  $X$  the set of all possible configurations with a fixed population.

### 2.1 Utility function

The utility function is a measure of an individual's happiness according to his/her neighborhood composition. Here, we measure the happiness of an agent  $i$  as a function of the fraction  $s_i$  of similar neighbors, i.e.,  $s_i = n_s i / (n_s i + n_d i)$ . For simplicity, we adopted an indicator function with a threshold  $s_i^*$ , beyond



which the utility is 1 (happy) and otherwise 0 (unhappy).

$$\begin{aligned} u_i(s) &= 1 \text{ if } s_i \geq s_i^* \\ &= 0 \text{ otherwise.} \end{aligned} \tag{1}$$

In our model, agents could have heterogeneous neighborhood thresholds, and they update their thresholds through their experience with friends. Denote by  $r_i$  the fraction of friends from the same group in agent  $i$ 's friendship network, he updates his neighborhood threshold in proportion to the difference between the composition of his friendship network and his former neighborhood threshold, according to a delta rule with fixed weight  $\delta$ .

$$s_i^*(t+1) = s_i^*(t) + \delta (r_i(t) - s_i^*(t)). \tag{2}$$

## 2.2 Dynamic moving rule

A simplest moving rule is to allow unhappy agents to move randomly and globally to an open cell. However, for the model to be more realistic and interesting, the agents in our model can search and assess open cells first and then make decisions on whether to move, and the process of information acquisition and decision-making is not error-free. The probability to move decays with distance as an agent is less probable to do a remote search. Furthermore, we also allow happy agents to move occasionally due to some external reason like mood change. To avoid clash, agents try to move one by one, with a total number of  $m$  trials in each round.

To implement the moving rule, an agent and an open cell are chosen randomly and independently in each round. The agent makes a decision (by throwing a dice) whether to move to this cell according to the following rule, where  $d$  is the distance between his current location and the found new cell:

$$\begin{aligned} Pr \{move\} &= \frac{1 - \epsilon - \xi}{d}, \text{ if } \Delta u = 1 \\ &= \frac{\epsilon}{d}, \text{ if } \Delta u = 0 \\ &= \frac{\xi}{d}, \text{ if } \Delta u = -1. \end{aligned} \tag{3}$$

$\frac{\epsilon}{d}$  is the probability of a neutral move (from happy to happy, or from unhappy to unhappy), and  $\frac{\xi}{d}$  is the probability of a move from happy to unhappy, where  $\xi \leq \epsilon \ll 1$ .

By introducing positive probability to neutral and unfavorable move, we got an irreducible Markov chain. It is not only consistent to reality, but also theoretically important to allow occasional neutral and unfavorable move as we want to avoid initial condition dependence in simulation, which could be a problem for analyzing the outcome of simulations. We lay out more details on this issue in the discussion section.

Table 1: Parameters Used In The Model

$\delta$	Learning rate from friendship network
$m$	Number of move trials per round
$p$	New friend tie formation rate
$N_f$	Number of friend ties

### 2.3 Friendship updating rule

An agent could randomly make new friends and lose old friends through time. For simplicity, we assume that the probability to make new friends depends on proximity, and we start with an extreme case that agents can only make new friends with their immediate neighbors. A tie between old friends may break with probability  $p$  in each round, so the life expectancy of a tie is  $1/p$  rounds. The total number of friendship ties is fixed as  $N_f$  in the model so that we expect to observe on average  $pN_f$  ties to die and the same number of new ties to form in each round.

We devote a separate sub-section in our Robustness Checks section to the implementation of a more realistic propinquity mechanism that allows for friendships to be formed outside an immediate neighborhood.

### 2.4 Parameter combination

In this model, we tested various combinations of four important parameters (Table 1) by simulation. Instead of estimating the dynamic response to the variation of each parameter, here we investigate the relative rate of migration versus adaptation.  $m$  controls agent moving frequency, and  $\delta$  is the key parameter to control learning rate. On the one hand, if agents move frequently but update their neighborhood thresholds slowly, fast convergence to high-level segregation would be expected as in the case with fixed neighborhood preference. On the other hand, if agents only inertly move but got used to their neighborhood fast as they make friends with their neighbors, the community could freeze into any state, segregated or integrated, almost with equal probability.