Selected Annotated Bibliography

to accompany:

Some Foundations in Complex Systems: Tools and Concepts

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1 Introduction

This bibliography is intended to accompany my lectures and lecture notes on Foundations in Complex Systems, delivered at the 2005 Santa Fe Institute Complex Systems Summer School in Beijing, PRC, July 2005. My hope is that this will be a useful initial guide to the literature. I have not tried to be encyclopedic and list every reference possible. Rather, I've been been selective and somewhat choosy. I'm particularly interested in references that will help graduate students pursue these ideas further, gain a background sufficient so that they can read and review the research literature, and ultimately to contribute to the research and teaching of complex systems. For the most part this bibliography is not intended to be a review of the current research in any field. An exception to this is the sections on measures of complexity, entropy convergence, and computational mechanics. These sections are somewhat more thorough, and without being exhaustive, contain a survey of the research literature in these areas.

In a few years many of the summer school participants will have an appointment at a university and will perhaps have an opportunity to develop a class in complex systems or incorporate ideas from complex systems into existing classes. These people will then be faced with the task of finding texts and papers suitable for students at different levels. To that end, I've included here a number of textbooks, some of which are probably too basic for many of the summer school participants, but which might be appropriate for future students.

This is the first draft of this bibliography. As such, I'm certain there are some errors and omissions. If you have corrections or suggestions for additions, please send them to me at the address above. I can't promise that I'll add everything I get sent, but I'd nevertheless be grateful for input and feedback. This bibliography contains only works written in English. I would welcome suggestions for non-English works to add.

Needless to say, the opinions expressed below are solely those of the author and are not necessarily shared by the Santa Fe Institute. Likewise, the references listed below are not in any way officially endorsed by the Santa Fe Institute.

2 General Complex Systems

2.1 Textbooks

There isn't exactly a textbook on complex systems *per se*. However, there are now a number of books that cover various aspects of the topics I covered in my lectures, and also some that are broader in scope. Texts that are specific to a particular field, e.g. dynamical systems or computation theory, are listed under those fields and not here.

[1] R. Badii and A. Politi. *Complexity: Hierarchical structures and scaling in physics.* Cambridge University Press, Cambridge, 1997.

> An excellent book. Includes overview of dynamical systems, statistical mechanics, symbolic dynamics, probability. Discusses several different ways to characterize complexity. Many examples and almost 500 references. Recommended.

[2] G.W. Flake. *The Computational Beauty of Nature: Computer Explorations of Fractals, Chaos, Complex Systems, and Adaptation.* MIT Press, 1999.

> This is an excellent, broad, and widely accessible text. Covers an impressive range of topics, including chaos, game theory, fractals, genetic algorithms, neural networks, and

cellular automata. Flake doesn't go into too much depth in any one topic, but it's a great place to go to learn about a topic. Assumes some calculus, but not much else mathematics. Well written and sophisticated, despite the lack of advanced math. Highly recommended.

[3] N. Boccara. *Modeling Complex Systems.* Springer, 2004.

Although I've only skimmed it, this book looks great. As the title suggests, the emphasis is on modeling. Chapters on differential equations, chaos, cellular automata, networks, power-law distributions, and some general discussion of modeling, including agent-based modeling. Highly recommended.

2.2 Popular Books

[1] T.C. Schelling. *Micromotives and Macrobehavior.* W.W. Norton & Company, 1978.

An excellent book written in an engaging and entertaining style. Schelling is interested in how the behaviors and motives of individuals relate to large scale or societal outcomes. Most of the examples are from economics. Highly recommended.

[2] H.A. Simon. *The Sciences of the Artificial.* MIT Press, 1982.

[3] M.M. Waldrop. *Complexity: The Emerging Science at the Edge of Order and Chaos.* Simon and Schuster, 1992.

I read this book around ten years ago, and I recall liking it a lot. It's a little sensational, but I think it's well written and a decent, albeit now somewhat dated, overview of some areas of complex systems.

[4] J.H. Holland. *Emergence: From Chaos to Order.* Perseus Books, 1999.

[5] M. Gladwell. *The Tipping Point: How Little Things Can Make a Big Difference.* Back Bay Books, 2002.

I read this a few years ago and found it quite interesting. Not exactly complex systems *per se*. Gladwell looks at how ideas and innovation spread through societies and businesses, and examines the conditions underwhich a small change may cause a system to switch suddenly from one type of behavior to another.

2.3 Overviews, Reviews, and Commentary

[1] C.R. Shalizi. Methods and techniques of complex systems science: An overview. In T.S. Deisboeck, J.Ya. Kresh, and T.B. Kepler, editors, *Complex Systems Science in Biomedicine.* Kluwer, 2004. `http://arxiv.org/abs/nlin.AO/0307015`.

An excellent, thorough review. Covers many tools: statistical learning and model selection; time series analysis; cellular automata; agent-based models; the evaluation of complex-systems models; information theory; and ways of measuring complexity. Contains 265 references. Highly recommended.

[2] John Horgan. From complexity to perplexity. *Scientific American*, June 1995. 104-109.

I don't agree with the premise of this opinion piece at all, but it makes interesting reading.

[3] S.A. Levin. Complex adaptive systems: Exploring the known, the unknown, and the unknowable. *Bulletin of the American Mathematical Society*, 40:3–19, 2002. `http://www.princeton.edu/~seasplan/lifesciences/Bulletin%20Am%20Math%20%Soc.pdf`.

> I've only skimmed this. Appears to be a nice review of complex adaptive systems in the context of evolutionary ecology.

[4] J.B. Rosser. On the complexities of complex economic dynamics. *Journal of Economic Perspectives*, 13:169–192, 1999. `http://cob.jmu.edu/rosserjb/GENERIC.CPX.doc`.

> A thorough, even-handed review of the applications of "complexity theory" to economics. Presents a good spectrum of views, from those who think complexity is mostly hype, to those who believe it has contributed significant new understandings. Contains around 125 references.

[5] M. Christen and L.R. Franklin. The concept of emergence in complexity science: Finding coherence between theory and practice. In *Proceedings of the Santa Fe Institute Complex Systems Summer School*, 2002. `http://www.ini.unizh.ch/~markus/articles/Emergence_def.pdf`.

> This looks to be an interesting and thoughful essay on different notions of emergence.

2.4 Mathematical Modeling and Computational Methods

Mathematical modeling isn't synonymous with complex systems. However, I think that facility with mathematical modeling is an important skill for many working in the areas of complex systems. Math modeling courses are being added to the curricula of many math departments, and there are some excellent textbooks being used. Most of these books have a discussion of general approaches to modeling, including dimensional analysis and scaling, estimation techniques, model verification, and choices and trade offs to consider when forming a model. I think modeling courses are a good idea. Usually, math classes focus on only one model or technique, e.g., Markov chains or differential equations. A class (or book) on modeling forces the student to think more critically about different models and to construct her or his own models, rather than just learning how to solve one particular type of model. Also, these books grapple with interesting "real-world" problems (as opposed to "toy models") in ways that most traditional textbooks don't.

Also in included in this section is a book on numerical methods and a book on Monte Carlo simulations.

[1] N. Gershenfeld. *The Nature of Mathematical Modeling*. Cambridge University Press, 1999.

> The strength of this book is the wide range of topics it covers: ordinary and partial differential equations, variational principles, probability, finite differences, finite elements, cellular automata, optimization and search methods, filtering and state estimation, and many more. Since this is a survey, none of these topics are covered in much depth. However, each topic is introduced in a clear and concise way, and there are references for those wishing to go further.

[2] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge University Press, second edition, 1995. `http://www.nr.com/`.

An excellent resource; anyone who does even a little scientific programing will want to have a copy handy. Even more valualbe than the sample code are the very, very clear (and funny) explanations of different numerical methods. There are versions in Fortran 77, Fortran 90, and c++, too. All except the c++ edition are available online.

[3] C.L. Dym. *Principles of Mathematical Modeling.* Elsevier, second edition, 2004.

Assumes background in calculus and trigonmetry. Many references and problems. Examples drawn from many fields.

[4] E. Beltrami. *Mathematical Models for Society and Biology.* Academic Press, 2002.

For junior-level undergraduates, assumes multivariable calculus and linear algebra. A wide array of applications.

[5] B. Barnes and G.R. Fulford. *Mathematical Modelling with Case Studies: A Differential Equation Approach Using Maple.* Taylor & Francis, 2002.

This book covers differential equations almost exclusively, so it isn't really a general modeling book. Nevertheless, it does emphasize modeling in a much more direct way than most differential equations textbooks. Makes extensive use of Maple, a computer algebra system.

[6] A.C. Fowler. *Mathematical Models in the Applied Sciences.* Cambridge University Press, 1997.

Unlike most of the other books in this section, this text is pitched to graduate students and not undergrads.

[7] M.M. Meerschaert. *Mathematical Modeling.* Academic Press, second edition, 1999.

[8] E.A. Bender. *An Introduction to Mathematical Modeling.* Dover Publications, 1978.

[9] M. E. J. Newman and G. T. Barkema. *Monte Carlo Methods in Statistical Physics.* Oxford University Press, 2000.

Monte Carlo simulations are used frequently in complex systems. This book is an excellent introduction to Monte Carlos. The writing is extremely clear, and the book contains much practical advice on how to efficiently code different algorithms. Very highly recommended.

3 Chaos and Dynamical Systems

The fields of chaos and dynamical systems have been around long enough that there are by now a number of excellent textbooks in this area. Below are some books and a few articles that I think are particularly good.

[1] R. May. Simple mathematical models with very complicated dynamics. *Nature*, 261:459, 1976.

May's highly influential "evangelical" paper on chaos in the logistic equation. Still current and interesting today.

[2] R. Shaw. *The Dripping Faucet as a Model Chaotic System.* Aerial Press, Santa Cruz, California, 1984.

This is an in depth analysis of a dripping faucet, a simple chaotic system. Includes much on information theory and some thoughts on complexity.

[3] P. Cvitanović. *Universality in Chaos*. World Scientific, 1989.

Comprehensive collection of reprints. Very handy. Cvitanović's introduction is excellent.

[4] Hao Bai-lin. *Elementary Symbolic Dynamics and Chaos in Dissipative Systems*. World Scientific, 1989.

[5] R.L. Devaney. *An Introduction to Chaotic Dynamical Systems*. Perseus Publishing, 1989.

A mathematics text at the advanced undergraduate level on chaotic dynamics. Very clear, but definitely a book for those with a math background. Recommended.

[6] C. Beck and F. Schlögl. *Thermodynamics of Chaotic Systems*. Cambridge University Press, 1993.

This is a great book. Very clear discussion of the thermodynamic formalism (aka multifractals), relationships between statistical mechanics, thermodynamics, and chaotic dynamical systems. Highly recommended. Fairly advanced, some experience with dynamics or statistical mechanics will be helpful.

[7] E. Ott. *Chaos in Dynamical Systems*. Cambridge University Press, 1993.

A good text on chaotic dynamical systems. For advanced undergraduates and graduate students in math, physics, or related fields.

[8] D. Kaplan and L. Glass. *Understanding Nonlinear Dynamics*. Springer-Verlag, 1995.

This is an excellent textbook on nonlinear dynamics. It assumes a knowledge of calculus, but little else in the way of a mathematical background. There's an emphasis on applications to biology. Includes some very good sections on cellular automata and boolean networks—topics which aren't covered in most nonlinear dynamics books. Highly recommended.

[9] L.S. Liebovitch. *Fractals and Chaos: Simplified for the Life Sciences*. Oxford University Press, 1998.

This isn't quite a book. Every other page is designed to be used as an overhead transparency, and the facing pages provide explanation for the overhead. The level is somewhat elementary—no calculus is used—but the discussion is quite clear. There are many helpful illustrations and a good number of biological examples. Recommended.

[10] S. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*. Perseus Books Group, 2001.

A standard text. Used in many advanced undergraduate and gratuate physics and math courses. Recommended.

[11] P. Blanchard, R.L. Devaney, and G.R. Hall. *Differential Equations*. Brooks/Cole, second edition, 2002.

This is a differential equations text; it's not a book on chaos. However, it's modern text, in that it treats nonlinear systems alongside linear ones and emphases numerical methods and a global approach to differential equations. It also has a chapter on discrete dynamical systems and the logistic map. If you find yourself one day teaching a differential equations course and wanting to choose a text that will help students someday work in complex systems, this would be an excellent choice.

[12] R.C. Hilborn. Sea gulls, butterflies, and grasshoppers: A brief history of the butterfly effect in nonlinear dynamics. *American Journal of Physics*, 72(4):425–427, 2004.

A short, fascinating article about the history of the term *the butterly effect*. Recommended.

[13] M. Frame, B. Mandelbrot, and N. Neger. Fractal geometry. `http://classes.yale.edu/fractals/`.

An online textbook. Includes a very good discussion of chaos, with many illustrations and small simulations. Assumes very little technical background. Highly recommended.

[14] D.P. Feldman. Exploring the logistic map. `http://hornacek.coa.edu/dave/Chaos`, accessed 25 Feb., 2005.

This is the website I used to make many of the figures in the section on chaotic dynamics. Here you will find programs that will let you make time series plots, bifurcation diagrams, and compare the orbits of two different initial conditions.

4 Time Series Analysis

I'm not familiar with this field at all. Here are a two books and a review article that should be a good places to start.

[1] H. Kantz and T. Schreiber. *Nonlinear Time Series Analysis*. Cambridge University Press, 1997.

[2] H.D.I. Abarbanel. *Analysis of Observed Chaotic Data*. Springer, 1997.

[3] E. Bradley. Time-series analysis. In M. Berthold and D. Hand, editors, *Intelligent Data Analysis: An Introduction*. Springer, 1999. `http://www.cs.colorado.edu/~lizb/papers/ida-chapter.html`.

5 Information Theory

[1] C. E. Shannon and W. Weaver. *The Mathematical Theory of Communication*. University of Illinois Press, Champaign-Urbana, 1962. `http://cm.bell-labs.com/cm/ms/what/shannonday/paper.html`.

Shannon's original information theory paper and a non-technical essay by Weaver. Both are highly readable and highly recommended.

[2] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. John Wiley & Sons, Inc., 1991.

This is, in my opinion, the best book on information theory. The text is very clear, the scope is broad, and the balance between rigor and intuition is just right. Highly recommended.

[3] A. Greven, G. Keller, and G. Warnecke, editors. *Entropy.* Princeton University Press, 2003.

An excellent edited volume. Has review chapters on entropy and thermodynamics (equilibrium and non-equilibrium), communications, data compression, chaos, etc. Written at a fairly mathematically advanced level.

[4] R. M. Gray. *Entropy and Information Theory.* Springer-Verlag, New York, 1990.

This is a good textbook, but it is at a much higher mathematical level than Cover and Thomas. I find this book too mathy, but those with a stronger math background will probably like this book a lot.

[5] D.P. Feldman. Information theory, excess entropy and statistical complexity: Discovering and quantifying statistical structure, 2002. `http://hornacek.coa.edu/dave/Tutorial/`.

A set of lecture notes on information theory that I wrote a few years ago. Covers similar material to that which I discussed in my lectures.

[6] D.J.C. MacKay. *Information Theory, Inference & Learning Algorithms.* Cambridge University Press, 2002. `http://wol.ra.phy.cam.ac.uk/mackay/itprnn/book.html`.

A large book on information theory applied to inference and learning. Looks very good. The entire book is available online.

[7] Entropy and information theory on the web. Maintained by R.Gunesch. `http://www.math.uni-hamburg.de/home/gunesch/entropy.html`.

A large list of links. Information theory applied to many different disciplines. Lots of good stuff here.

[8] T. Schneider. Information theory primer. `http://www-lmmb.ncifcrf.gov/~toms/paper/primer/`.

A very elementary review of basic information theory. Includes a review of logarithms.

6 Computation Theory

[1] J. G. Brookshear. *Theory of Computation: Formal Languages, Automata, and Complexity.* Benjamin/Cummings, 1989.

I like this book and found it much more readable than Hopcroft and Ullman.

[2] J. E. Hopcroft and J. D. Ullman. *Introduction to Automata Theory, Languages, and Computation.* Addison-Wesley, Reading, 1979.

This is a standard reference in automata theory. It's certainly very good. However, I find it difficult to read; for me, the formalism gets in the way of some of the ideas. Good for details, but for an introduction I'd recommend Brookshear.

7 Excess Entropy and Entropy Convergence

[1] P. Szépfalusy and G. Györgyi. Entropy decay as a measure of stochasticity in chaotic systems. *Phys. Rev. A*, 33(4):2852–2855, 1986.

[2] P. Szepfalusy. Characterization of chaos and complexity by properties of dynamical entropies. *Physica Scripta*, T25:226–229, 1989.

[3] A. Csordás and P. Szépfalusy. Singularities in Rényi information as phase transitions in chaotic states. *Phys. Rev. A.*, 39(9):4767–4777, 1989.

[4] P. Gaspard and X.-.J Wang. Noise, chaos, and (ϵ, τ)-entropy per unit time. *Physics Reports*, 235:291–343, 1993.

> This is a review of an approach to entropy convergence that is different than the one I took in my lectures. The authors consider a system that is continuous, and then examine how the entropy varies as different discretations of that continuous variable are taken.

[5] M. Feixas, E. del Acebo, P. Bekaert, and M. Sbert. An information theory framework for the analysis of scene complexity. *Eurographics*, 1999. `http://ima.udg.es/~feixas/publications/Feixas99B.ps.gz`.

[6] J.P. Crutchfield and D.P. Feldman. Regularities unseen, randomness observed: Levels of entropy convergence. *Chaos*, 15(25-54), 2003.

> A thorough treatment of entropy and entropy convergence, including detailed discussion of the excess entropy and transient information. Many examples.

[7] J. P. Crutchfield and N. H. Packard. Symbolic dynamics of noisy chaos. *Physica D*, 7:201–223, 1983.

> This is the paper that introduced the idea of entropy convergence as a measure of complexity.

[8] D.P. Feldman and J.P. Crutchfield. Structural information in two-dimensional patterns: Entropy convergence and excess entropy. *Physical Review E*, 67:051104, 2003.

> Extends excess entropy to two dimensions and applies it to configuration generated by a two-dimensional Ising model with nearest- and next-nearest-neighbor interations. Includes many references on two-dimensional measures of complexity.

[9] W. Ebeling, L. Molgedey, J. Kurths, and U. Schwarz. Entropy, complexity, predictability and data analysis of time series and letter sequences. In H.-J. Schellnhuber A. Bunde, J. Kropp, editor, *The science of disaster: Climate disruptions, heart attacks, and market crashes*. Springer Berlin/Heidelberg, 2002.

> A nice review of entropy and entropy convergence. Has many references to earlier work by Ebeling, co-workers, and others. Ebeling has done a lot of work on the entropy of large texts, e.g., *Moby Dick*, financial time series, and biosequences. Includes many references. Also has a brief discussion of measures of complexity.

[10] T. Schürmann. Scaling behaviour of entropy estimates. *Journal of Physics A*, 35:1589, 2002. `http://arxiv.org/abs/cond-mat/0203409`.

Determines how the Lempel-Ziv estimates for the entropy scale for the critical circle map and the period-doubling accumulation point of the logistic map.

[11] J. P. Crutchfield and D. P. Feldman. Synchronizing to the environment: Information theoretic constraints on agent learning. *Advances in Complex Systems*, 4:251–264, 2001. http://hornacek.coa.edu/dave/Publications/STTE.html.

Entropy convergence and transient information applied to agent learning.

[12] D.P. Feldman and J.P. Crutchfield. Synchronizing to a periodic signal: The transient information and synchronization time of periodic sequences. *Advances in Complex Systems*, 7:329–355, 2004. http://hornacek.coa.edu/dave/Publications/TIPS.html.

An exhaustive exploration of the transient information of periodic sequences.

7.1 Numerically Estimating Entropies

The easiest way to estimate the entropy of a random variable is to estimate the probability of outcomes by observing the frequencies with with outcomes occur, normalizing, and then using Shannon's formula to get the entropy. In my experience this works quite well. However, this approach leads to a biased estimate. A fair amount has been written about ways to improve upon naive entropy estimates. The papers listed below should be a good place to start.

[1] T. Poschek, W. Ebeling, and H. Rose. Guessing probability distributions from small samples. *Journal of Statistical Physics*, 1995.

[2] T. Schürmann and P. Grassberger. Entropy estimation of symbol sequences. *Chaos*, 6:414–427, 1996. http://arxiv.org/abs/cond-mat/0203436.

[3] T. Dudok de Wit. When do finite sample effects significantly affect entropy estimates? *European Physical Journal B*, 11:513–16, 1999.

[4] T. Schürmann. Bias analysis in entropy estimation. *Journal of Physics A*, 37:L295, 2004. http://arxiv.org/abs/cond-mat/0403192.

[5] I. Nemenman. Inference of entropies of discrete random variables with unknown cardinalities. NSF-ITP-02-52, KITP, UCSB, 2002. http://arXiv/physics/0207009.

8 Measures of Complexity

In general, this section is more technical than others. This section does not include computational mechanics and its applications and extensions, or excess entropy and entropy convergence as these topics get a sections of their own.

8.1 Early Foundational Papers

These are some papers that I think are particularly good and/or have been particularly influential. Those who want to get a thorough sense of the "field" of complexity measures will, in my opinion, want to check out most of these papers and probably also the reviews and surveys listed in the subsequent section.

[1] P. Grassberger. Toward a quantitative theory of self-generated complexity. *Intl. J. Theo. Phys.*, 25(9):907–938, 1986.

Introduces several measures of complexity and includes a very nice general discussion of what complexity is and different ways it can be measured. Highly influential.

[2] K. Lindgren and M. G. Norhdal. Complexity measures and cellular automata. *Complex Systems*, 2(4):409–440, 1988.

An excellent paper on different measures of complexity applied to cellular automata. In my opinion, this paper deserves to be much better known than it is. Highly recommended.

[3] D. Zambella and P. Grassberger. Complexity of forecasting in a class of simple models. *Complex Systems*, 1988.

Puts forth a measure of complexity that captures how difficult it is to optimally predict a sequences. Calculates this quantity for one-dimensional elementary cellular automata.

[4] K. Lindgren. Entropy and correlations in discrete dynamical systems. In J. L. Casti and A. Karlqvist, editors, *Beyond Belief: Randomness, Prediction and Explanation in Science*, pages 88–109. CRC Press, Boca Raton, Florida, 1991.

A nice paper applying information theoretic measures of complexity to lattice systems, mostly cellular automata. Includes extensions to more than one dimension.

[5] K. Lindgren, C. Moore, and M. Nordahl. Complexity of two-dimensional patterns. *J. Stat. Phys.*, 91:909–951, 1998.

Extensions of formal computation theory to two-dimensional patterns. I'm surprised that more people haven't followed up this work.

8.2 Reviews, Surveys, and Commentary

[1] R. Landauer. A simple measure of complexity. *Nature*, 336(6197):306–307, 1988.

[2] B. Wackerbauer, A. Witt, H. Atmanspacher, J. Kurths, and H. Scheingraber. A comparative classification of complexity measures. *Chaos, Solitons & Fractals*, 4(1):133–173, 1994.

A nice review and discussion of a number of different complexity measures.

[3] B. Edmonds. Hypertext bibliography of measures of complexity. `http://bruce.edmonds.name/combib/`.

A large bibliography of different measures of complexity. Useful, even though it was last updated in 1997.

[4] D. P. Feldman and J. P. Crutchfield. Measures of statistical complexity: Why? *Physics Letters A*, 238:244–252, 1998. `http://hornacek.coa.edu/dave/Publications/MSCW.html`.

This paper is a critique of a measure of complexity that was proposed a few years earlier. It includes a brief review and general discussion of complexity measures, including thoughts on criteria for a complexity measure.

[5] D.P. Feldman and J.P. Crutchfield. Discovering non-critical organization: Statistical mechanical, information theoretic, and computational views of patterns in simple one-dimensional spin systems. Santa Fe Institute Working Paper 98-04-026, `http://hornacek.coa.edu/dave/Publications/DNCO.html`.

This long paper compares statistical mechanical measures of structure with the excess entropy and statistical complexity. To do so, these quantities are calculated exactly for one-dimensional Ising models. These results are then used to draw comparisons between these different approaches to structure. Especially recommended for readers with a background in statistical mechanics who wish to learn about excess entropy and computational mechanics.

[6] C. H. Bennett. How to define complexity in physics, and why. In W. H. Zurek, editor, *Complexity, Entropy, and the Physics of Information*, volume VIII of *Santa Fe Institute Studies in the Sciences of Complexity*, pages 137–148. Addison-Wesley, 1990.

Some interesting remarks on possible criteria for a definition of complexity.

[7] W. Bialek, I. Nemenman, and N. Tishby. Predictability, complexity, and learning. *Neural Computation*, 13:2409–2463, 2001. `http://arxiv.org/abs/physics/0007070`.

Some very interesting results relating entropy convergence to the difficulty of learning (in the sense of machine learning) a process. Contains a nice discussion of other measures of complexity.

[8] I. Nemenman w. Bialeck and N. Tishby. Complexity through nonextensivity. *Physica A*, 302:89–99, 2001. `http://arxiv.org/abs/physics/0103076`.

8.3 Other Complexity Measures

These are papers discussing some of the other complexity measures I mentioned in my lectures. In my opinion, these papers are less essential or have been less influential than those listed above. But, you can read these and decide for yourself.

[1] S. Wolfram. Universality and complexity in cellular automata. *Physica*, 10D:1–35, 1984.

An early paper examining complexity in the context of cellular automata. Quite influential.

[2] C. H. Bennett. On the nature and origin of complexity in discrete, homogeneous locally-interacting systems. *Found. Phys.*, 16:585–592, 1986.

[3] M. Koppel. Complexity, depth, and sophistication. *Complex Systems*, 1:1087–1091, 1987.

[4] S. Lloyd and H. Pagels. Complexity as thermodynamic depth. *Annals of Physics*, 188:186–213, 1988.

[5] J. Rissanen. *Stochastic Complexity in Statistical Inquiry.* World Scientific Publisher, Singapore, 1989.

[6] W. Li. On the relationship between complexity and entropy for Markov chains and regular languages. *Complex Systems*, 5(4):381–399, 1991.

[7] M. Gell-Mann and S. Lloyd. Information measures, effective complexity, and total information. *Complexity*, 2(1):44–52, 1996.

Another UTM-based complexity measure.

9 Computational Mechanics

9.1 Foundational Papers and Reviews

[1] J. P. Crutchfield and K. Young. Inferring statistical complexity. *Phys. Rev. Lett.*, 63:105–108, 1989. `http://cse.ucdavis.edu/~cmg/compmech/papers/ISC.pdf`.

This is the paper that introduces statistical complexity and computational mechanics. Includes calculations of the statistical complexity for the logistic map.

[2] J. P. Crutchfield and K. Young. Computation at the onset of chaos. In W. H. Zurek, editor, *Complexity, Entropy and the Physics of Information*, volume VIII of *Santa Fe Institute Studies in the Sciences of Compexity*, pages 223–269. Addison-Wesley, 1990. `http://cse.ucdavis.edu/~cmg/compmech/papers/CompOnset.pdf`.

A detailed and clear application of computational mechanics to the order-disorder transitions in the logistic equation. Highly recommended.

[3] J. P. Crutchfield. The calculi of emergence: Computation, dynamics, and induction. *Physica D*, 75:11–54, 1994. `http://cse.ucdavis.edu/~cmg/compmech/pubs/CalcEmergTitlePage.htm`.

A thorough treatment of computational mechanics. Includes a largly non-techical introduction. Many examples.

[4] C. R. Shalizi and J. P. Crutchfield. Computational mechanics: Pattern and prediction, structure and simplicity. *Journal Statistical Physics*, 104:819–881, 2001.

Mathematical foundations of causal states and computational mechanics. Careful proofs of optimality and minimality.

9.2 Applications and Extensions

[1] J. E. Hanson and J. P. Crutchfield. Computational mechanics of cellular automata: An example. *Physica D*, 103(1-4):169–189, 1997.

Computational Mechanics applied to cellular automata.

[2] J. P. Crutchfield and D. P. Feldman. Statistical complexity of simple one-dimensional spin systems. *Phys. Rev. E*, 55(2):1239R–1243R, 1997. `http://hornacek.coa.edu/dave/Publications/SCS1DSS.html`.

Computational Mechanics Applied to One-dimensional Ising systems.

[3] D. R. Upper. *Theory and Algorithms for Hidden Markov Models and Generalized Hidden Markov Models*. PhD thesis, University of California, Berkeley, 1997. `http://cse.ucdavis.edu/~cmg/papers/TAHMMGHMM.pdf.gz`.

Computational mechanics applied to hidden Markov models. This is quite mathematically rigorous; it is a dissertation written for a mathematics Ph.D.

[4] J. Delgado and R. V. Solé. Collective-induced computation. *Phys. Rev. E*, 55(3):2338–2344, 1997. `complex.upf.es/~ricard/PREcollective.pdf`.

Computational mechanics applied to coupled map lattices

[5] W. M. Goncalves, R. D. Pinto, J. C. Sartorelli, and M. J. de Oliveira. Inferring statistical complexity in the dripping faucet experiment. *Physica A*, 257:385–389, 1998.

Computational Mechanics applied to a chaotic dripping faucet.

[6] D.P. Feldman and J.P. Crutchfield. Discovering non-critical organization: Statistical mechanical, information theoretic, and computational views of patterns in simple one-dimensional spin systems. Santa Fe Institute Working Paper 98-04-026, `http://hornacek.coa.edu/dave/Publications/DNCO.html`.

This long paper compares statistical mechanical measures of structure with the excess entropy and statistical complexity. To do so, these quantities are calculated exactly for one-dimensional Ising models. These results are then used to draw comparisons between these different approaches to structure. Especially recommended for readers with a background in statistical mechanics who wish to learn about excess entropy and computational mechanics.

[7] A.J. Palmer, C.W. Fairall, and W.A. Brewer. Complexity in the atmosphere. *IEEE Transactions on Geoscience and Remote Sensing*, 2000.

Statistical complexity and mutual information applied to atmospheric data.

[8] A.J. Palmer, T.L. Schneider, and L.A. Benjamin. Inference versus imprint in climate modeling. *Advances in Complex Systems*, pages 73–89, 2002.

The authors inferr ϵ-machines from observational data.

[9] C.R. Shalizi, K.L. Shalizi, and J.P. Crutchfield. An algorithm for pattern discovery in time series. Technical Report 2002-10-60, Santa Fe Institute, 2002. `http://arxiv.org/abs/cs.LG/0210025`.

[10] D. P. Varn, G. S. Canright, and J. P. Crutchfield. Discovering planar disorder in close-packed structures from X-ray diffraction: Beyond the fault model. *Phys. Rev. B*, 66(17):156, 2002.

Computational Mechanics applied to layered solids known as polytypes.

[11] R.W. Clarke, M.P. Freeman, and N.W. Watkins. Application of computational mechanics to the analysis of natural data: An example in geomagnetism. *Physical Review E*, 67:016203, 2003.

[12] C.R. Shalizi. Optimal nonlinear prediction of random fields on networks. In *Discrete Mathematics and Theoretical Computer Science, AB(DMCS)*. 2003. `http://arxiv.org/abs/math.PR/0305160`.

Computational mechanics extended to dynamical systems on networks.

[13] J.P. Crutchfield and O. Görnerup. `http://arxiv.org/abs/nlin.AO/0406058`.

Computational mechanics appled to self-assembling evolutionary systems.

[14] C.R. Shalizi and K.L. Shalizi. Blind construction of optimal nonlinear recursive predictors for discrete sequences. In M. Chickering and J. Halpern, editors, *Uncertainty in Artificial Intelligence: Proceedings of the Twentieth Conference [UAI 2004]*, pages 504–511. AUAI Press, 2004. `http://arxiv.org/abs/cs.LG/0406011`.

This paper and the previous one present an algorithm, known as CSSR, for estimating causal states from time series data. I.e., it is an algorithm for building recursive hidden Markov models from discrete-valued time series and other discrete sequential data. Source code and documentation are available at `http://www.cscs.umich.edu/~crshalizi/CSSR/`.

[15] C.R. Shalizi, K.L. Shalizi, and R. Haslinger. Quantifying self-organization with optimal predictors. *Physical Review Letters*, 93:118701, 2004. `http://arxiv.org/abs/nlin.AO/0409024`.

Extension of computational mechanics and causal states to more than one dimension, and application to cellular automata.

[16] K. Young, U. Chen, J. Kornak, G.B. Matson, and N. Schuff. Summarizing complexity in high dimensions. *Physical Review Letters*, 94:098701, 2005.

Extends computational mechanics to more than one dimension. Applies this to two-dimensional representations of the brain, focusing on the progression of Alzheimer's disease.

10 Power Laws

There is, of course, a great deal that has been written about power laws. The few references below aren't the last word, by any means, but they should help you be critical and careful consumers (and users) of power laws.

[1] D. Sornette. Multiplicative processes and power laws. *Physical Review E*, 57:4811, 1998. `http://arxiv.org/abs/cond-mat/9708231`.

[2] W.J. Reed and B.D. Hughes. From gene families and genera to incomes and internet file sizes: Why power laws are so common in nature. *Physical Review E*, 66:067103, 2002.

Shows that exponentially sampling an exponential yields a power law. Recommended.

[3] M.E.J. Newman. Power laws, pareto distributions and zipf's law. *Contemporary Physics*, In press. `http://arxiv.org/abs/cond-mat/0412004`.

A thorough, clear review of the many different physical and mathematical processes that generate quantities that are distributed according to a power. Also a discussion of how to recognize a power law empirically, with many examples. Very highly recommended.

[4] M.L. Goldstein, S.A. Morris, and G.G. Yen. Problems with fitting to the power-law distribution. *The European Physical Journal B*, page 255, 2004. `http://arxiv.org/abs/cond-mat/0402322`.

[5] C.R. Shalizi. Speaking truth to power about weblogs, or, how not to draw a straight line. `http://cscs.umich.edu/~crshalizi/weblog/232.html`.

A weblog entry containing an excellent discussion of the perils of fiting a line to a log-log plot and proclaiming the discovery of a power law.

11 Miscellaneous

Finally, there are a few books that aren't complex systems related but which I nevertheless thought might be of interest to some CSSS students. For the most part, these are books about the social, economic, political, and philosophical aspects of science. I think it is very valuable for scientists to have done some reading and thinking in these areas.

[1] T.S. Kuhn. *The Structure of Scientific Revolutions.* The University of Chicago Press, 1996.

> Originally published in 1962. This is a tremendously influential short book about the nature of scientific knowledge, how science is done, and in particular how scientific knowledge changes. The phrase "paradigm shift" was coined by Kuhn. Not everyone will agree with all Kuhn says, and those who disagree will do so for very different reasons. An excellent, thought-provoking book. Highly recommended.

[2] R.C. Lewontin. *Biology As Ideology: The Doctrine of DNA.* Perennial, 1993.

> Even more readers will likely disagree with Lewontin than Kuhn. This short book is a highly readable discussion of science as a social, economic, and political institution, focusing on biology and DNA. Highly recommended.

[3] G. Farmelo, editor. *It Must be Beautiful: Great Equations of Modern Science.* Granta Books, 2002.

> Usually books with titles like this make me cringe. But this book is actually great. It's a very interesting set of essays, mostly by historians of science, about various scientific paradigm shifts. Topics covered include quantum mechanics, general relativity, information theory, mathematical evolution, and chaos. Highly recommended. These essays are, for the most part, very fun and interesting.