Power grids statistical modeling

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Motivation

- Power grids have grown organically over the past century (naturally random)
  - More balancing options: economic benefits + safety
- Our initial quest → a systematic approach to design the cyber support for the power grid
- Design and analysis of power grids has been based on reference samples and case studies
  - Does not help establishing macroscopic trends
- Can we capture in a model key features of the ensemble?

Background

What should we model?

- System of systems

Generators,Loads

Transmission Lines

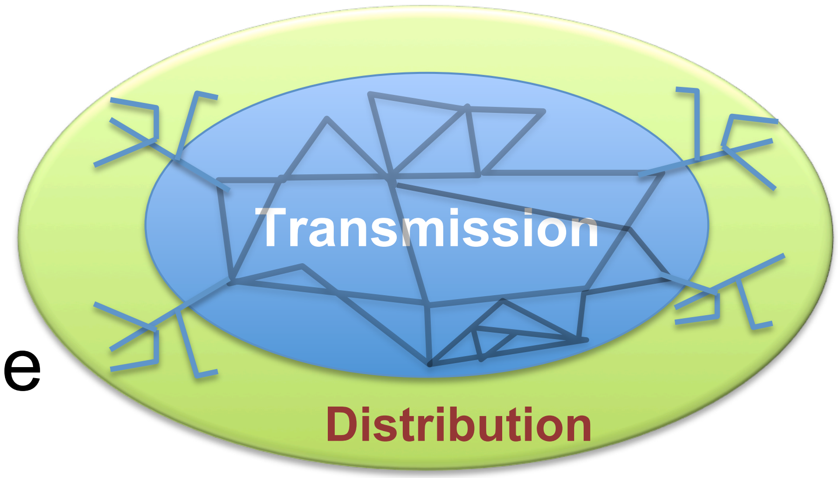
Power systems gear:
Switches,
Relays,Transformers...

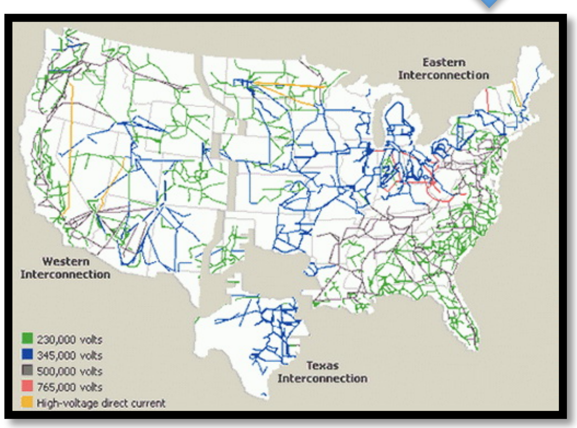
Computers and Sensors
(Substations, PLC,
Supervisory control)

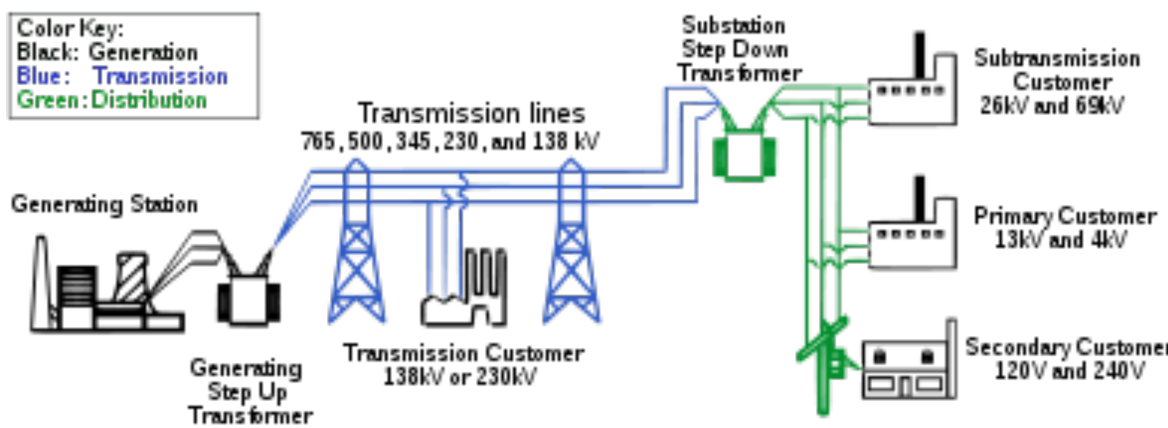
Market players (supply and
demand)

The grid transmission lines

- 3 sections:
  - High, Medium and Low voltage sections
- High and medium voltage networks wide areas

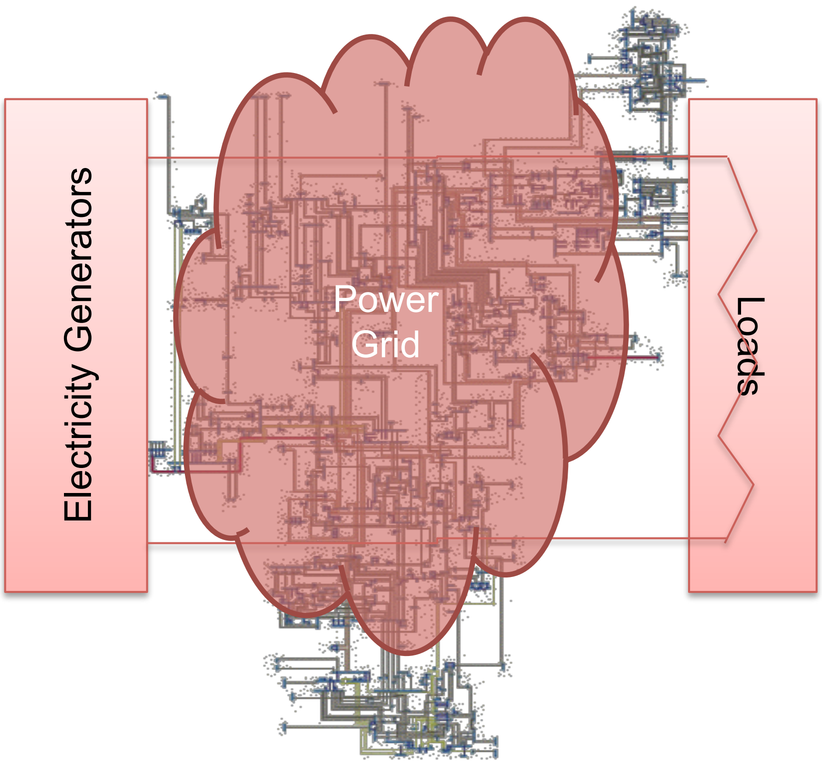






What we model

- Topological and electrical characteristics of the transmission grid
- As captured by the statistical properties of the grid admittance matrix
- Also one data point for Medium Voltage Distribution
- Leave out the distribution network



Admittance matrix and the graph topology

- Line-Node Incidence Matrix (M x N):

Line i connected to node $j \rightarrow A_{i,j} = 1,\ A_{j,i} = -1$

else $A_{i,j} = A_{j,i} = 0$

- Admittance matrix

$$Y = A^T diag(y_1, \ldots, y_M) A$$

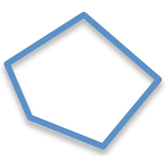
- Observation: Y is a weighted graph Laplacian
  - complex weights given by the admittances of the lines

The laws for the grid

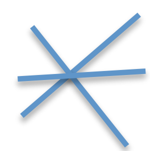
- Voltage, Currents, Powers → narrow spectrum

AC ~ 60-50 Hz

- Electrical transient dynamics → unimportant
  - Circuit laws replaced with algebraic equations (frequency) relating "*phasors*" (complex numbers whose phase and amplitude match the AC signal V and I)
- Kirchhoff's Voltage/Current laws (KVL-KCL)



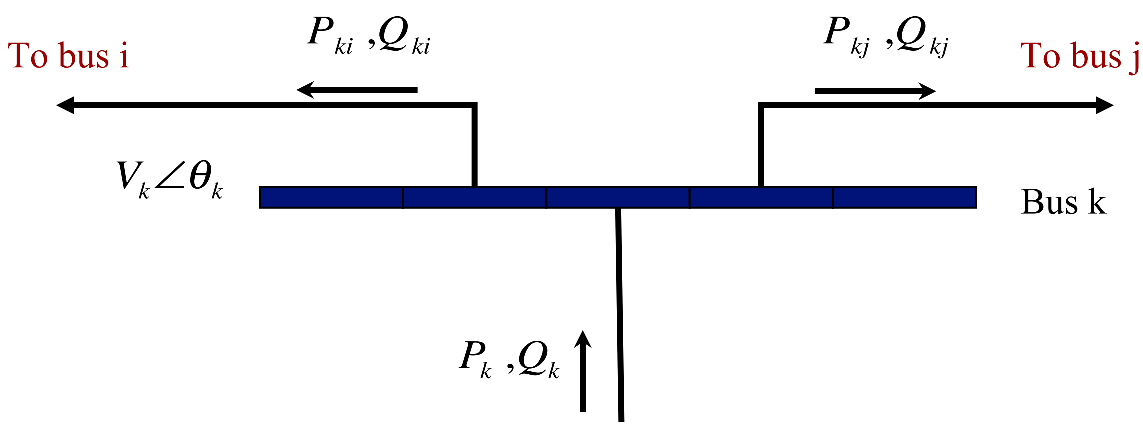
$$\sum_{i\in \text{Circuit}} V_i = 0, \qquad \sum_{i\in \text{Node}} I_i = 0$$



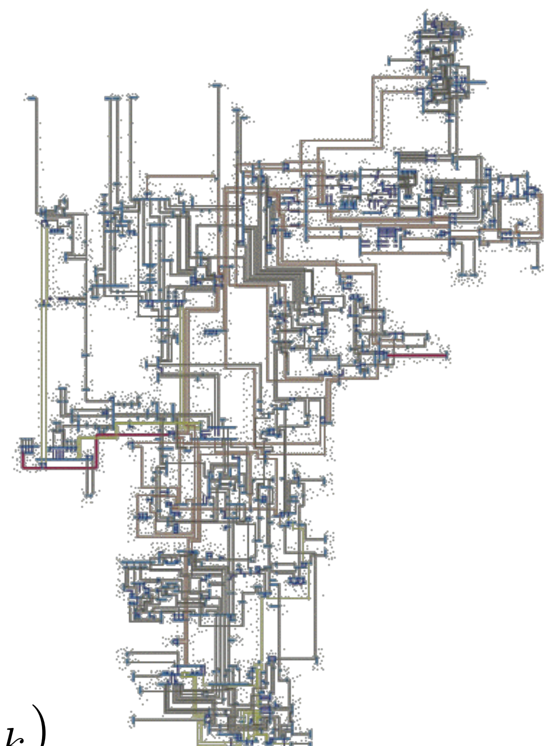
- Ohm's law

$$V_i = Z_i I_i$$

Relationship with power: The balance equations



Power Injection = Losses Admittance matrix



$$P_i + jQ_i = V_i I_i^* = V_i \sum_{j \in E} Y_{ij}^* V_j^*$$

AC Power Flow Model

$$P_i = \sum_{k=1}^{n} |V_i V_k| G_{ik} \cos(\theta_{ik}) + B_{ik} \sin(\theta_{ik})$$

$$Q_i = \sum_{k=1}^{n} |V_i V_k| G_{ik} \sin(\theta_{ik}) - B_{ik} \cos(\theta_{ik})$$

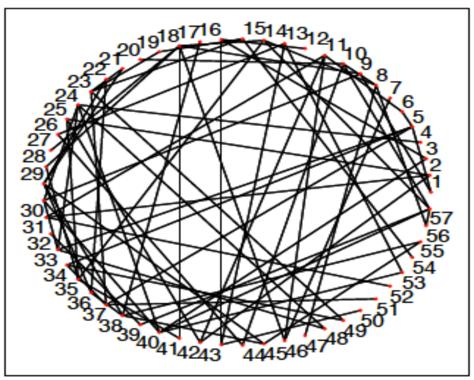
Precursors of our study

- Most of the literature has used real grids or reference models for testing
  - IEEE 30 57, 118 and 300 bus systems
  - Power systems test case archive
    - http://www.ee.washington.edu/research/pstca/
- Scalable models to grasp macroscopic trends
  - [Parashar and Thorp ’04] ring topology + “continuum model”
  - [Rosas-Casals, Valverde, Solé ’07] tree topology

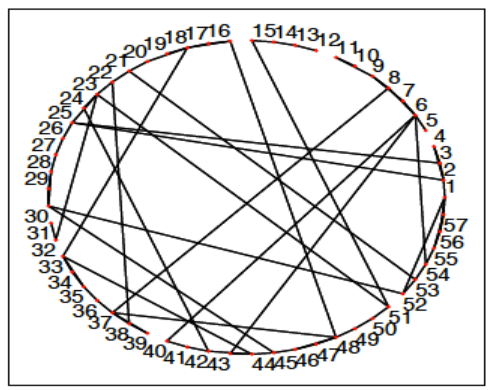
Random topology models

- '98 Watts and Strogatz, *Nature*
  - *Conjecture: Power Grids are small world networks*

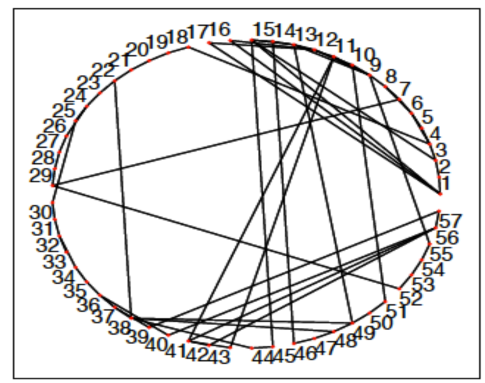
Erdos Reny



Small World



Power network



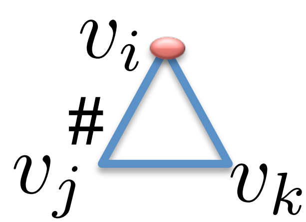
- Topological studies: [Newman '03], [Whitney & Alderson'06] [Wang, Rong,'09]
- Degree distribution: [Albert et al. '04],[Rosas-Casals et al. '07]

Small world → high clustering coefficient

- high average clustering coefficient of the sample power grid network examined

Definition of clustering coefficient

$$C_i = \frac{2|\{e_{jk}\}|}{k_i(k_i - 1)}$$



$$\forall \ (v_j, v_k) \in N_i, e_{jk} \in E$$

$(N_i = \text{ neighbors of } v_i)$

$(E = \text{edges})$

$(k_i = \text{degree of } i)$

Erdos Renyi

Grid	$C(G)$	$C(R)$
IEEE-30	0.2348	0.094253
IEEE-57	0.1222	0.048872
IEEE-118	0.1651	0.025931
IEEE-300	0.0856	0.009119
NYISO-2935	0.2134	0.001525
WSCC-4941	0.0801	0.000540

Our analysis

Degree distribution

- [Albert et al. ’04,Rosas-Casals’07] Geometric PDF
- Way to highlight:

  Probability Generating Function (PGF)

  - For a mixture model

$$G_k(z) = G_{k_1}(z) \ldots G_{k_p}(z)$$

Our analysis result

1. The degree distribution is a mixture of a truncated exponential and finite support random variable
2. The average degree vs. N is O(1)

Why the PGF?

- A finite support Probability Mass Function (PMF) is a finite order polynomial
  - We should see ‘zeros’ in the PGF

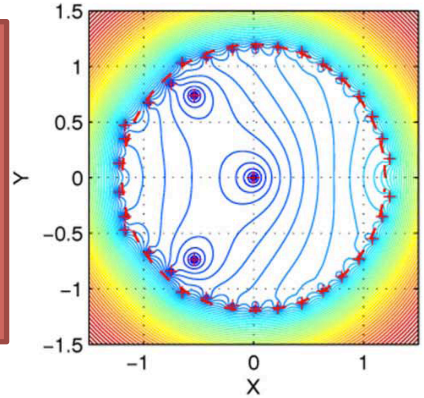
$$G_{\mathcal{D}}(z) = p_0 + p_1 z + \ldots + p_{k_t} z^{k_t}$$

- A purely geometric random variable is the reciprocal of a first order polynomial → ‘pole’
  - Impossible to observe, in practice a ‘clipped’ version

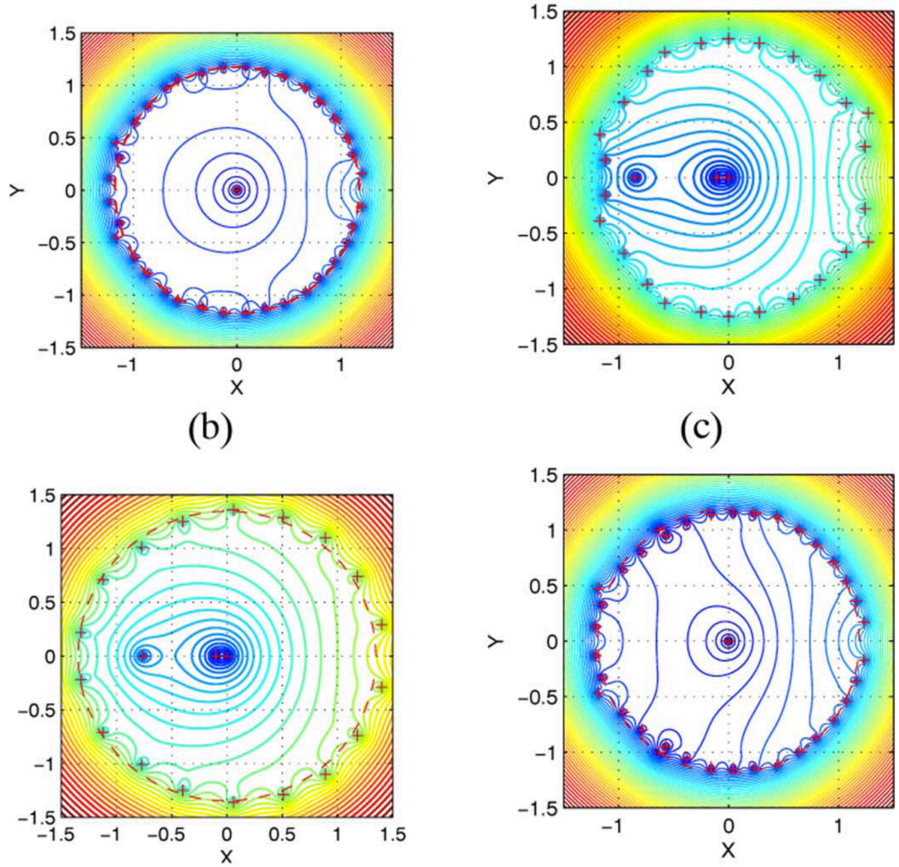
$$G_{\mathcal{G}}(z) \propto \frac{1 - [z(1-p)]^{k_{\max}+1}}{1-(1-p)z}$$

PGF NYSO data

(a) All buses
(b) Gen buses
(c) Load buses.
(d) Connection buses.
(e) Gen+Load buses.
The zeros are red '+'

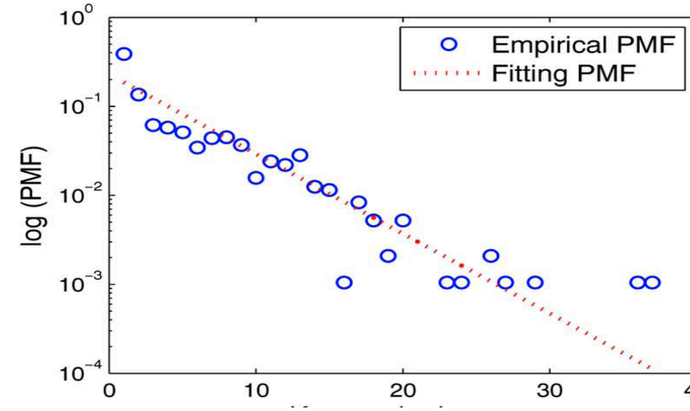


(a)

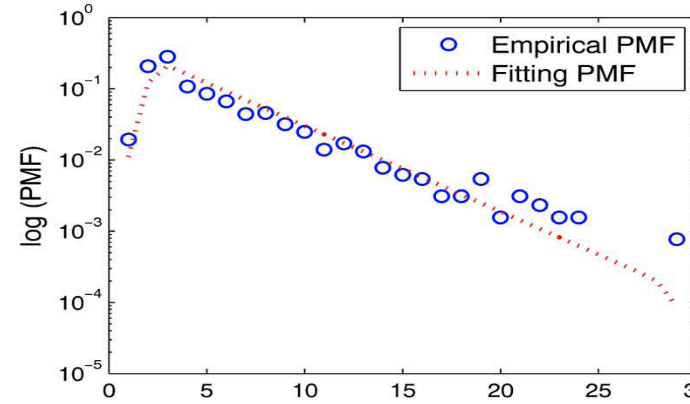


(b) (c)

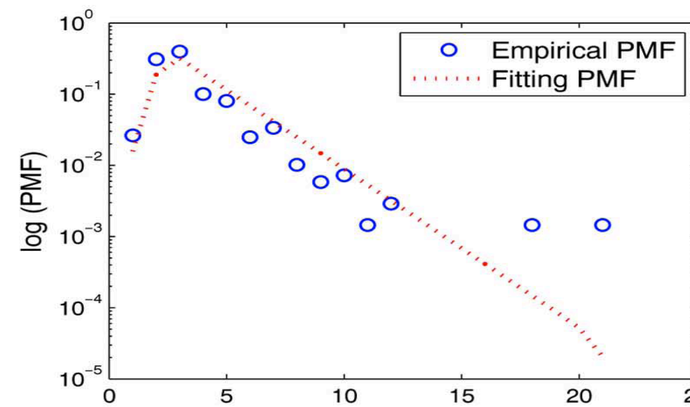
(d) (e)



Degree of Generator buses



Degree of Load buses



Degree of Connection buses

WSCC versus NYSO degree distribution

$$G_k(z) = G_{\mathcal{D}}(z) G_{\mathcal{G}}(z)$$

ESTIMATE COEFFICIENTS OF THE TRUNCATED GEOMETRIC AND THE IRREGULAR DISCRETE FOR THE NODE DEGREES IN THE NYISO AND WSCC SYSTEM

node groups	max($\underline{k}$)	p	k_{max}	k_t	$\{p_1, p_2, \cdots, p_{k_t}\}$
All	37	0.2269	34	3	0.4875, 0.2700, 0.2425
Gen	37	0.1863	36	1	1.000
Load	29	0.2423	26	3	0.0455, 0.4675, 0.4870
Conn	21	0.4006	18	3	0.0393, 0.4442, 0.5165
Gen+Load	37	0.2227	34	3	0.4645, 0.3385, 0.1970
All-WSCC	19	0.4084	16	3	0.3545, 0.4499, 0.1956

Vulnerability studies

- Fraction of nodes removal before breakdown
  - [Rosas-Casals et. al '07] $r = 1$ [Wang et al. '09]

$$f_c^{rand} = \left(1 - \ln \frac{f_c^{sel}}{r}\right) \quad f_c^{sel} = 1 - \frac{1}{<k> - 1}$$

The Theoretical versus the Empirical Critical Breakdown Thresholds

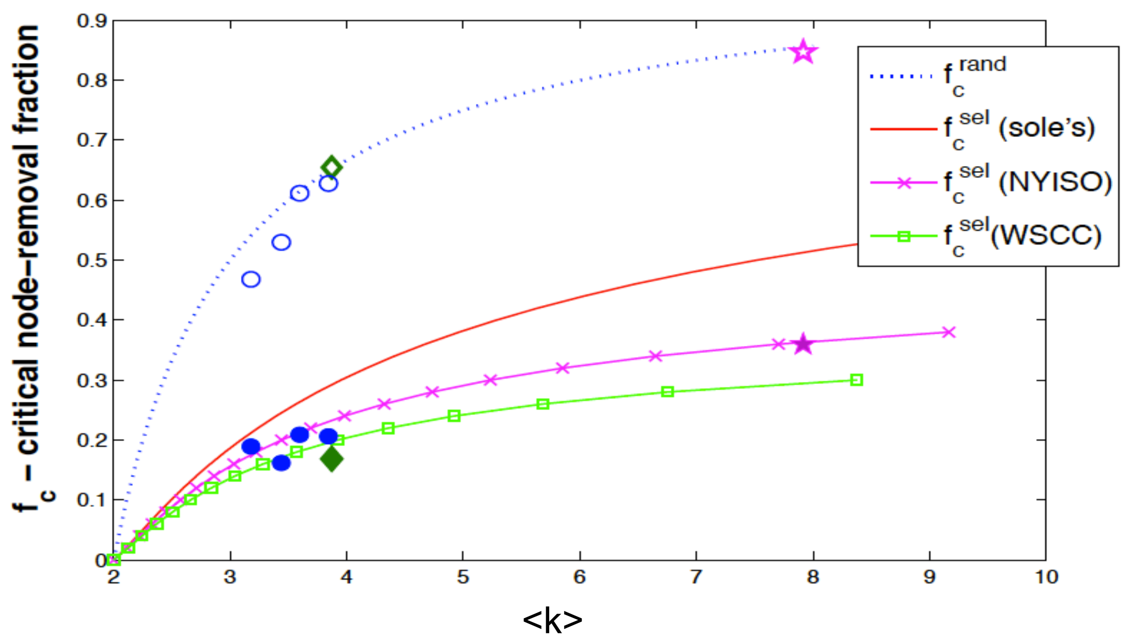
$r_{NYISO} = 1.4074$

$r_{WSCC} = 1.9690$

IEEE (circles), WSCC **(diamond), NYISO (star)**

Hollow - f_c^{rand}

Filled - f_c^{sel}



Small World conjecture

- Some evidence contradicting it
  - For a SW network with N nodes, to guarantee with high probability a connected network (no isolated component) the scaling laws for the average degree *<k> >> log N*
  - The average degree in power grids is ~ constant (3-4)

N: Number of nodes
m: number of lines
-
<k> Average Degree
<l> Average shortest path length
ρ Pearson Coefficient
r{k>k} Ratio of nodes with largest nodal degree

TOPOLOGICAL CHARACTERISTICS OF REAL-WORLD POWER NETWORKS

	(N, m)	$\langle l \rangle$	$\langle k \rangle$	ρ	$r\{k > \bar{k}\}$
IEEE-30	(30,41)	3.31	2.73	-0.0868	0.2333
IEEE-57	(57,78)	4.95	2.74	0.2432	0.2105
IEEE-118	(118,179)	6.31	3.03	-0.1526	0.3051
IEEE-300	(300, 409)	9.94	2.73	-0.2206	0.2367
NYISO	(2935,6567)	16.43	4.47	0.4593	0.1428
WSCC	(4941, 6594)	18.70	2.67	0.0035	0.2022

Average shortest path

- Observation: $< l > \approx 3 \log_{10}(N)$

N: Number of nodes
m: number of lines
-
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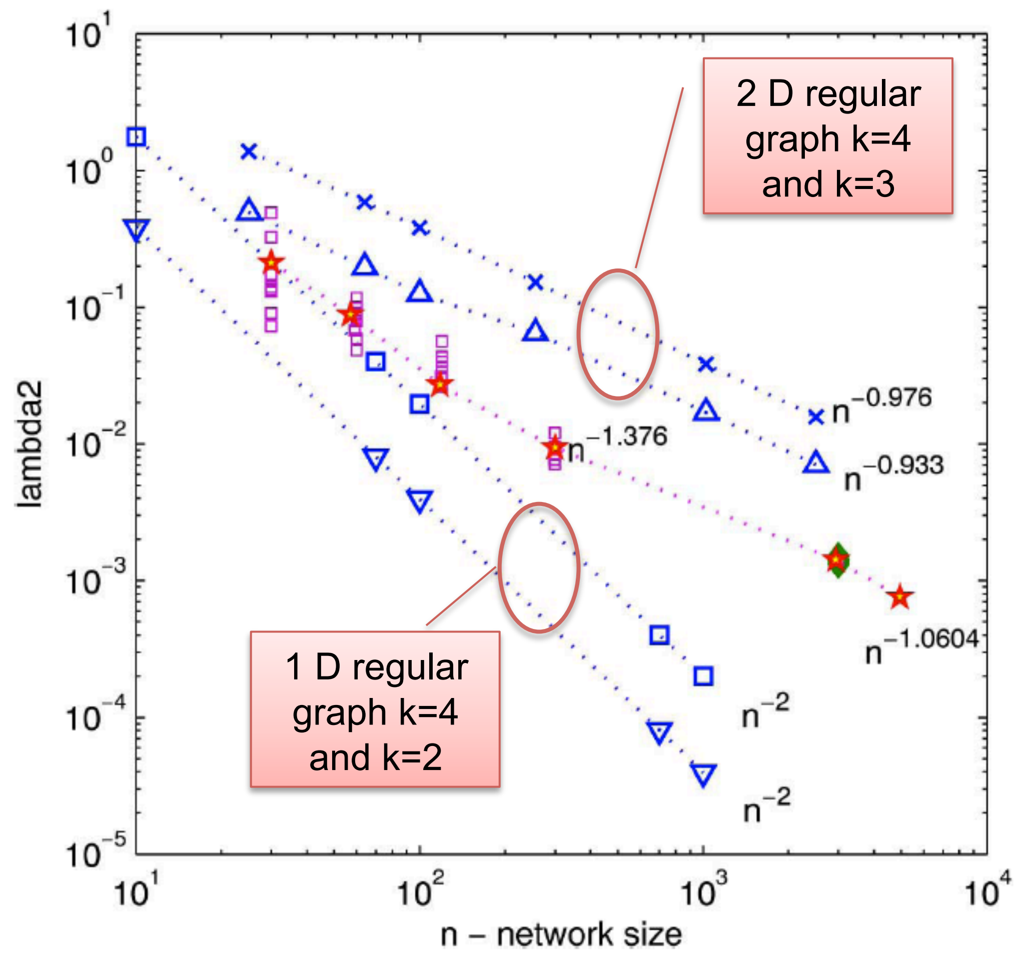
- Not bad to overlay communications with the lines – relatively short distance

Algebraic connectivity

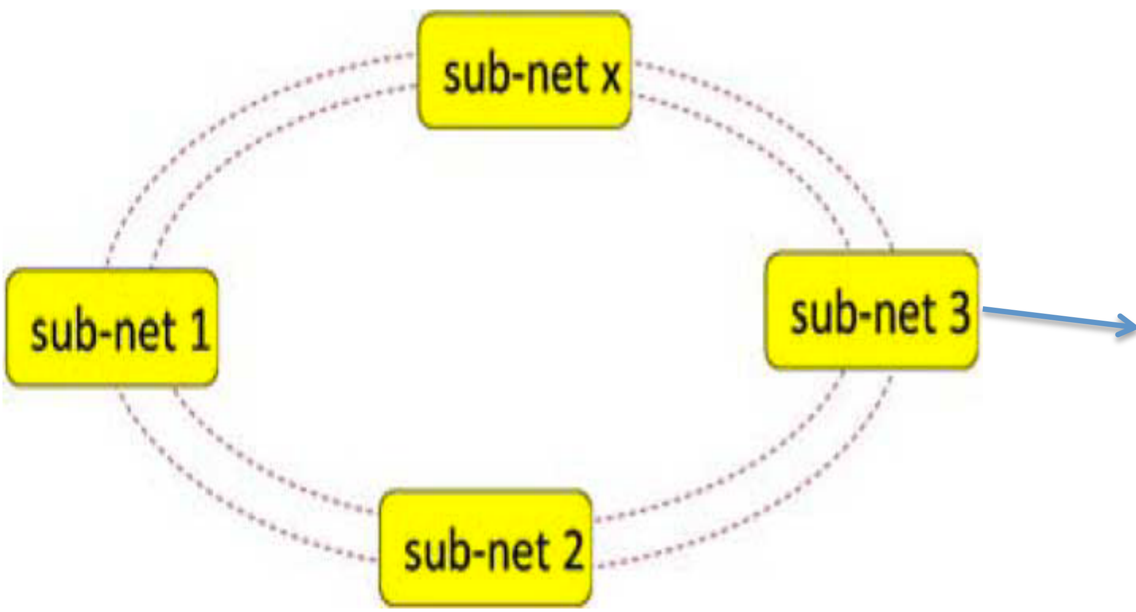
- Graph Laplacian second smallest eigenvalue
- Values shown in ☆

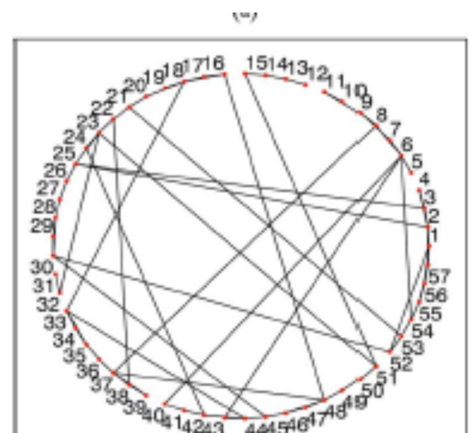
	$\lambda_2(L)$
IEEE-30	0.21213
IEEE-57	0.088223
IEEE-118	0.027132
IEEE-300	0.0093838
NYISO-2935	0.0014215
WSCC-4941	0.00075921



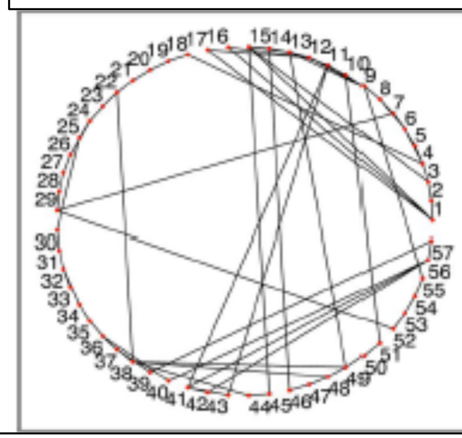
Plausible topology

- The model that matches this trend is what we call Nested-Small-world graph
  - IEEE → SW subnet 30; NYSO & WSCC → SW sub-net 300





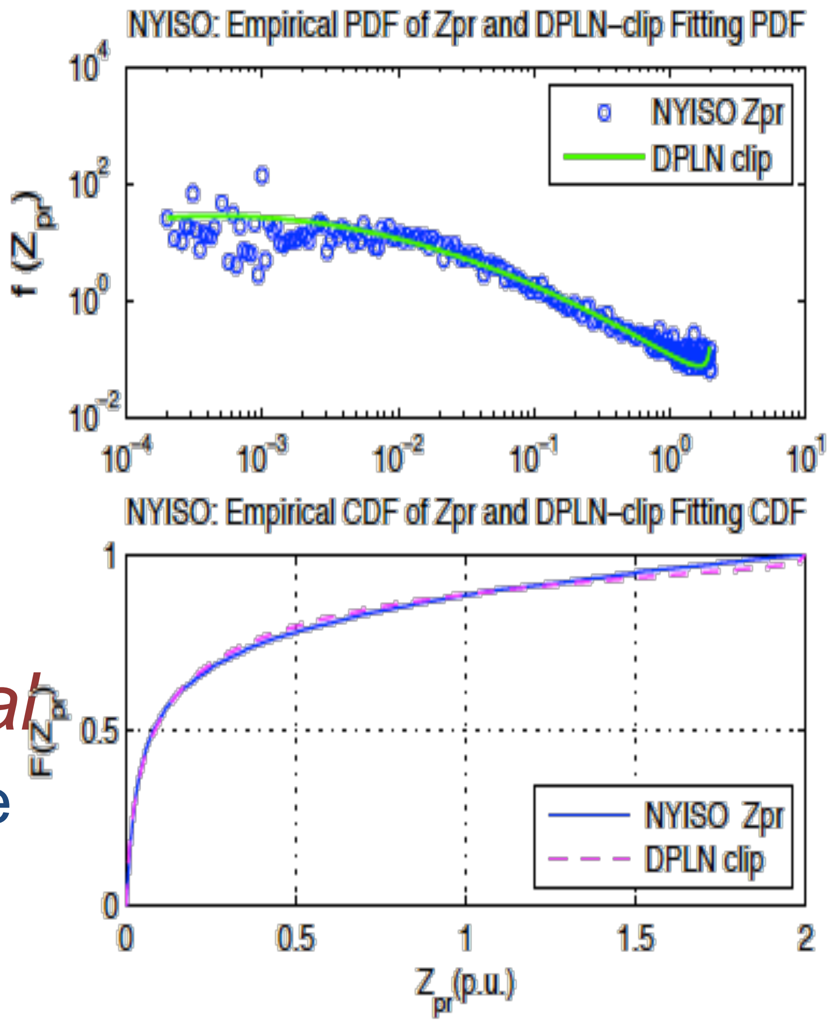
SW: independent rewiring



IEEE 300: Correlated rewiring

Impedance distribution

- Absolute values of the impedances
  $Z_{pr} = R + jX \approx jX$
- Prevailingly <u>*heavy tailed distributions*</u>
- NYSO best fit → *clipped Double Pareto Log-normal*
  - Did not pass KS test but was the closest to pass it



Distributions comparison

DISTRIBUTION FITTING FOR LINE IMPEDANCES

System	Fitting Distribution	ML Parameter Estimates (*alph*=0.05)
IEEE-30	$\Gamma(x\|a,b)$	$a = 2.14687$ $b = 0.10191$
IEEE-118	$\Gamma(x\|a,b)$	$a = 1.88734$ $b = 0.05856$
IEEE-57	$gp(x\|k,\sigma,\theta)$	$k = 0.33941$ $\sigma = 0.16963$ $\theta = 0.16963,$
IEEE-300	$gp(x\|k,\sigma,\theta)$	$k = 0.45019$ $\sigma = 0.07486$ $\theta = 0.00046,$
NYISO-2935	$logn_{clip}(x\|\mu,\sigma,Z_{\max})$	$\mu = -2.37419$ $\sigma = 2.08285$ $Z_{max} = 1.9977$
	$dPlN_{clip}(x\|\alpha,\beta,\mu,\sigma,Z_{\max})$	$\alpha = 44.25000$ $\beta = 44.30000$ $\mu = -2.37420$ $\sigma = 2.082600$ $Z_{max} = 1.9977$

Gamma:

$$\Gamma(x \mid a, b) = \frac{1}{b^a \Gamma(a)} x^{a-1} e^{x/b},$$

Generalized Pareto (GP):

$$gp(x \mid k, \sigma, \theta) = \left(\frac{1}{\sigma}\right)\left(1 + k\frac{(x-\theta)}{\sigma}\right)^{-1-(1/k)}.$$

Lognormal:

$$\log n(x \mid \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(\log x - u)^2}{2\sigma^2}}.$$

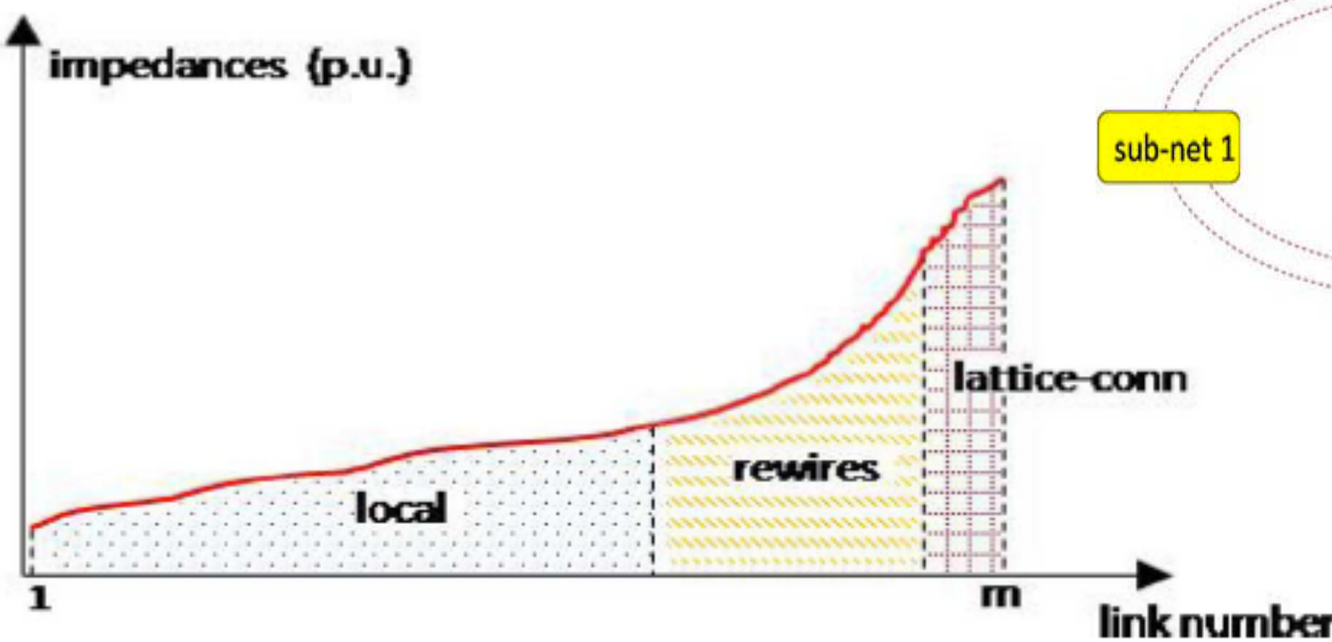
DPLN:

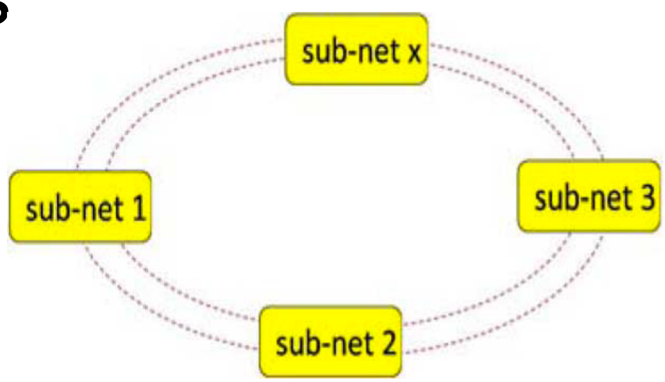
$$\mathrm{DPLN}(x|\alpha,\beta,\mu,\sigma) = \frac{\alpha\beta}{\alpha+\beta}\Bigg[A(\alpha,\mu,\sigma)x^{-\alpha-1}\Phi\left(\frac{\log x - \mu - \alpha\sigma^2}{\sigma}\right) + A(-\beta,\mu,\sigma)x^{\beta-1}\Phi^c\left(\frac{\log x - \mu + \beta\sigma^2}{\sigma}\right)\Bigg].$$

where $A(\theta, \mu, \sigma) = e^{(\theta\mu + \theta^2\sigma^2/2)}$.

Impedance attribution

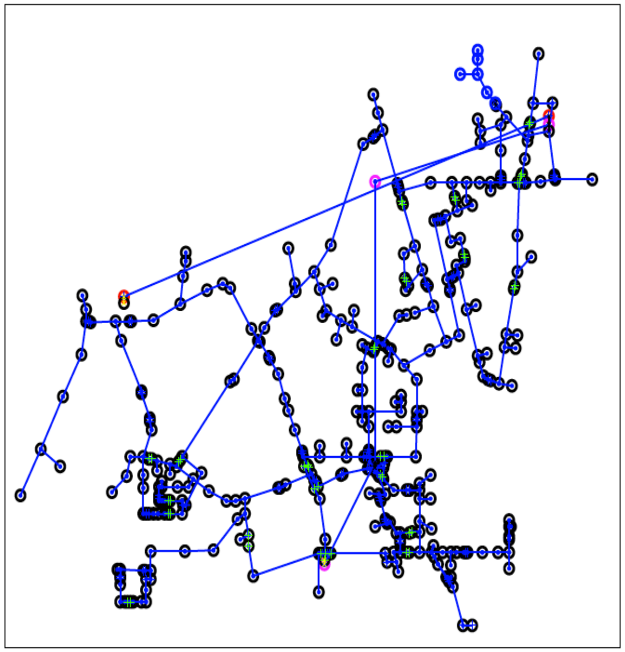
- Impedance grows with distance
- Conjecture: local → short; rewires → medium; lattice connections → long lines





396-node Medium Voltage distribution network

- US distribution utility
  - The power supply from the 115 kV-34.5 kV step-down substation.
  - Most nodes or buses in the network are 12.47 kV (>95%), and only a small number of them are 34.5 kV or 4.8 kV.



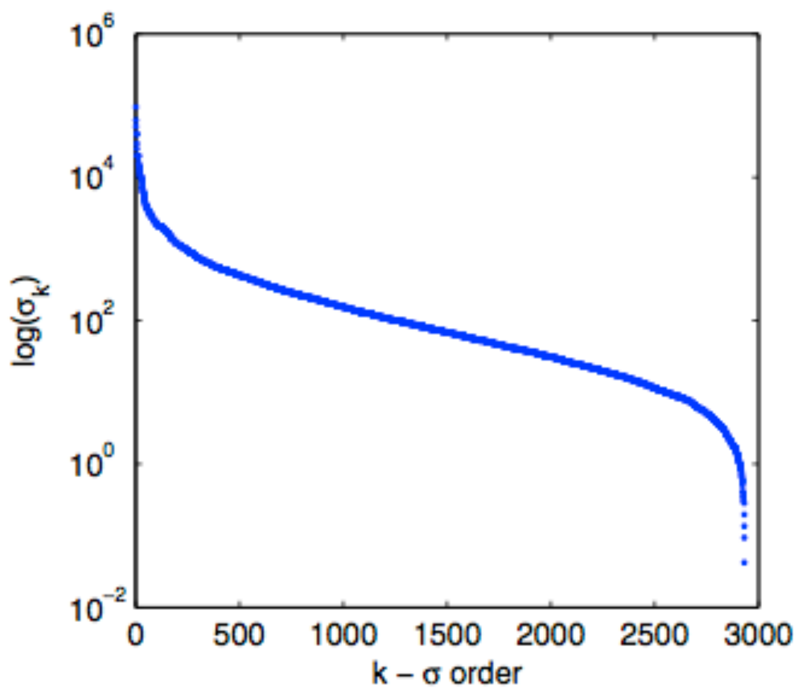
	(N, m)	$\langle k \rangle$	$\langle l \rangle$	ρ	$\lambda_2(L)$	$C(G)$
IEEE-300	(300, 409)	2.73	9.94	-0.2206	0.0094	0.0856
WSCC	(4941, 6594)	2.67	18.70	0.0035	0.00076	0.0801
396-node MV-Distr	(396, 420)	2.12	21.10	-0.2257	0.00030	0

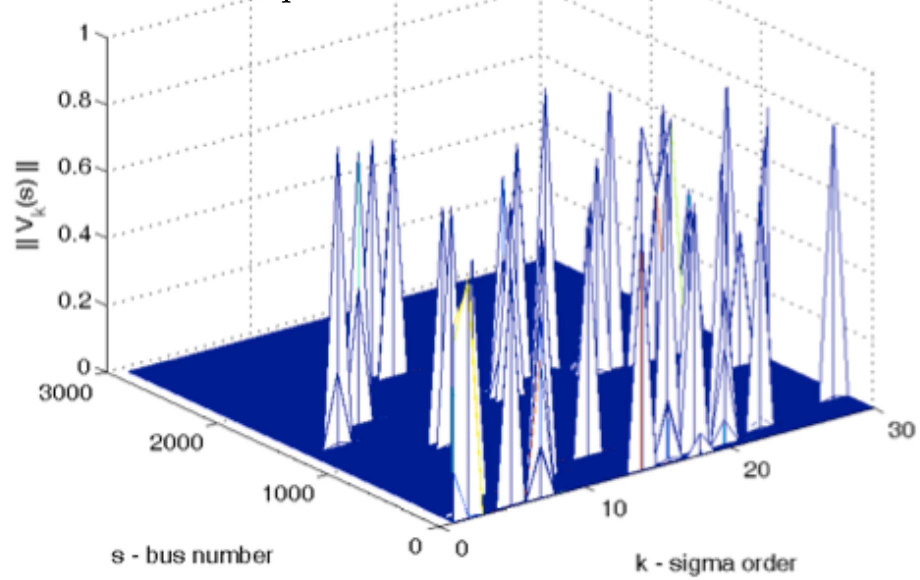
Interesting facts relating to CPS

Sparse principal eigenvectors

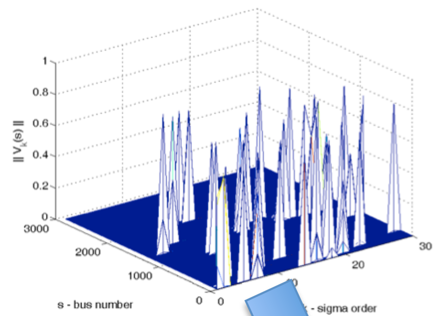
- We have found that the $\mathbf{Y} = \mathbf{U \Lambda U^H}$ has sparse eigenvalues with sparse principal components

$$\mathbf{U} = [\underbrace{\mathbf{u}_1, \cdots, \mathbf{u}_K}_{\triangleq \text{Principal}}, \underbrace{\mathbf{u}_{K+1}, \cdots, \mathbf{u}_N}_{\triangleq \text{Minor}}]$$

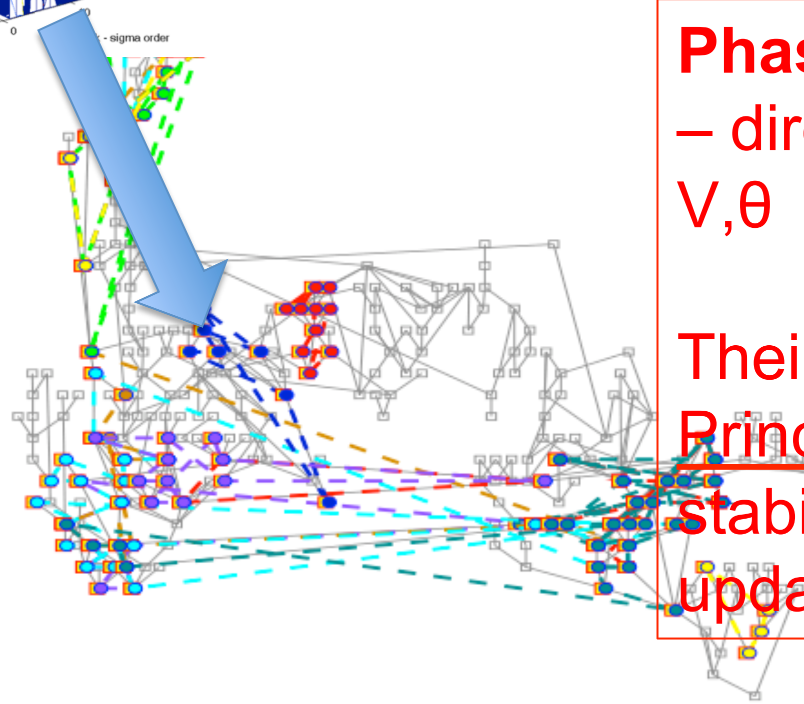




Sensor placement for robust state estimation



$$\text{Clique}_k(\gamma) = \left\{ i : [\mathbf{u}_k] \geq \frac{\gamma}{\sqrt{N}} \right\}$$



Phasor Measurement Units – directly measure the state V,θ

Their placement on the K Principal Cliques best stabilizes the Gauss-Newton update for State Estimation

Geometrical insights from AC to DC Power flow

- Admittance matrix $\mathbf{Y} = \mathbf{G} + \mathrm{i}\mathbf{B}'$

conductance | susceptance

- Susceptance >> Conductance
- Small angle difference $|\theta_i - \theta_j|, V_i \approx 1$
- DC Power Flow Model approximaion

$$\mathbf{P} = \mathbf{B}\boldsymbol{\theta}, \quad \mathbf{B} = -\mathbf{B}' \text{ with shunt removed}$$

Power injection | Phase angle

Impact on Power Injections

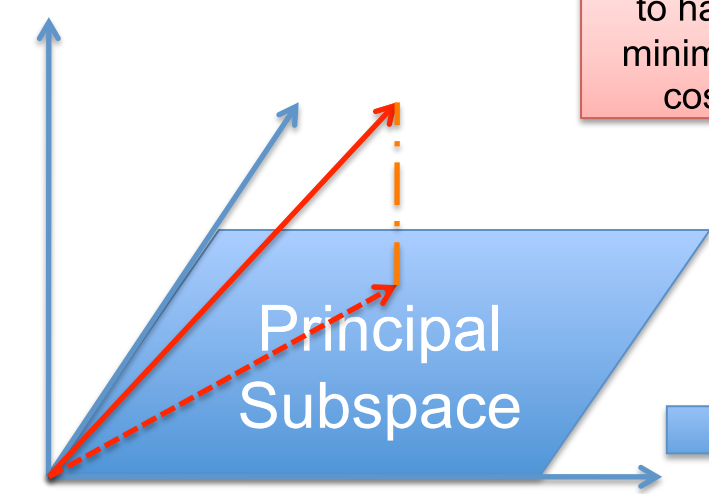
- Low rank approximation $\mathbf{B} = \sum_{k=1}^{K} \lambda_k \mathbf{u}_k \mathbf{u}_k^T + \mathcal{O}(\lambda_{K+1})$

$$\mathbf{B}\boldsymbol{\theta} = \mathbf{P}_G - \mathbf{P}_L$$

$$\sum_{k=1}^{K} \rho_k \lambda_k \mathbf{u}_k \approx \mathbf{P}_G - \mathbf{P}_L, \quad \rho_k = \mathbf{u}_k^T \boldsymbol{\theta}.$$

Dispatched to have minimum cost

The balance constraint in the Optimal Power Flow Economic dispatch will tend to line up the injection with the principal subspace

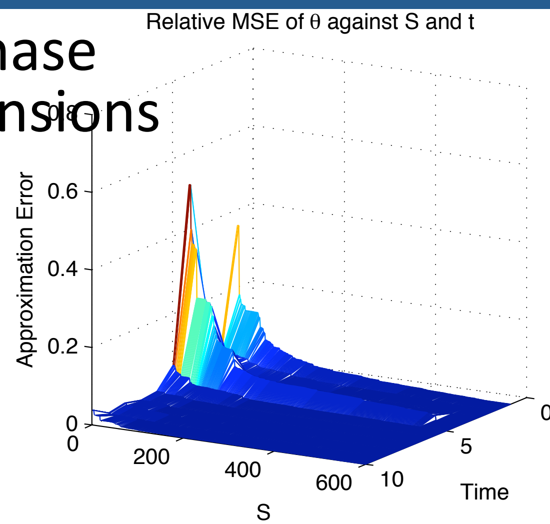


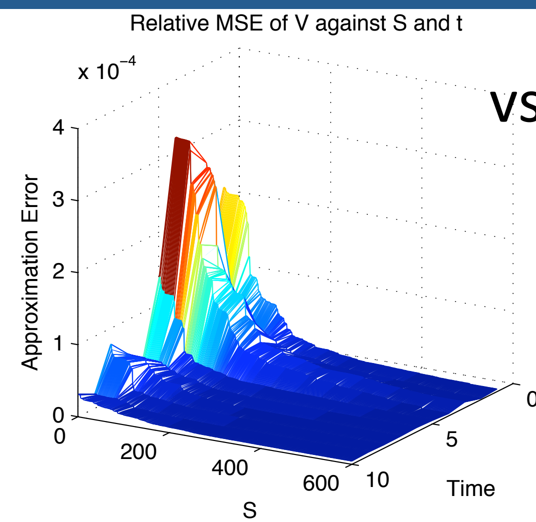
Load fluctuation → OPF generation adjustment

The sensitivity analysis suggests that greatest variations are in the least significant subspace component

Low Dimensional Representation

MSE of phase
vs # of dimensions

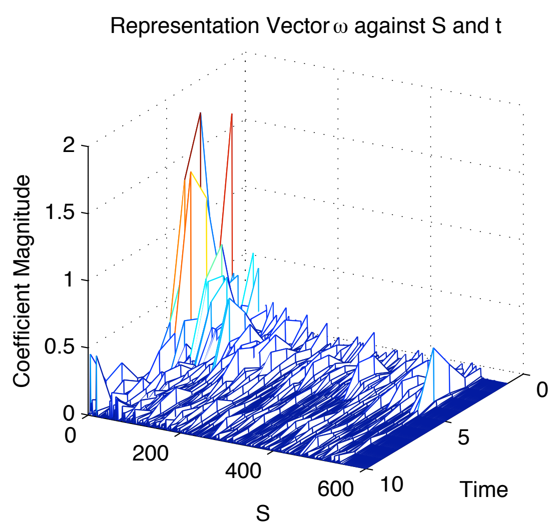


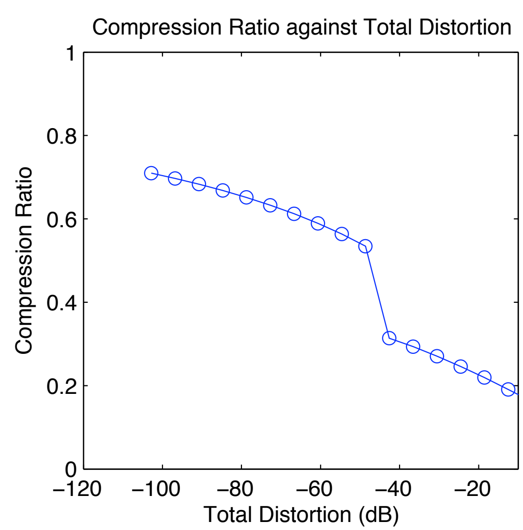


MSE of voltage
vs # of dimensions

IEEE-300 bus system

10 snapshots





Conclusions

- The admittance matrix of power grids has peculiar features that follow clear statistical trends
- The analysis can help grasping macroscopic phenomena
- It is critical to understand how to design it better
- It is critical incorporate flexibility and adaptability via monitoring and control

References

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