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# An agent-based model of forest use with social learning

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**Summary**

We present an agent based model for deforestation under social learning. We assume a forest is composed of a number of land parcels. Each parcel is in either a forested or a deforested state, and the state transition is described via a two-state Markov chain. Landowners decide whether or not to cut trees based on the relative expected utility of forested and deforested parcels. The expected utility is altered by information input about actual utility, in particular the average experience in a society. We study the global landscape dynamics that result from individual decisions. We observed stationary-forested, stationary-deforested, and cyclic-forested landscapes that are induced by collective deforestation. We investigated feedback from global landscape dynamics to individual decision. We found that if the value of forested land increased with the decline of forest density, collective deforestation was more likely to occur. Extinction of forest ecosystems was predicted if the forest recovery rate was diminished by an intensive deforestation. We discuss the implications for forest policy and management.

## Introduction

Forest resources are one of most important natural resources upon which people depend but have been overexploited (Moran and Ostrom, 2005; Millennium Ecosystem Assessment, 2005). Deforestation has been studied in terms of land-use models, in which natural processes such as ecological successions and physical disturbances and human decision making are combined (Veldkamp and Fresco, 1996; Parker et al. 2003; Walker, 1999, 2003; Walker et al. 2004). In such models, each landowners are assumed to make decision that maximizes their utility. However, human understanding of ecological and social dynamics is clouded by uncertainty, and therefore it may be difficult to know the true value about utility of forest use. To cope with the uncertainty, landowners may develop their expectation about the utilities by learning from experience.

In this project, we develop a decision model about forest use under social learning. We investigate following questions: (1) how global resource-use patterns result from the decisions of individual landowners under social learning?, (2) how global landscape dynamics feedback to the individual decision? Based on results, we will discuss the implications for forest policy and management.

## The model

We assume that a forest is composed of *N* land parcels. For simplicities sake, we assume that a landowner owns only one parcel. This landowner agent may represent a single person, a household, or a group of people. Let $S_i(t)$ be the state variable at *i*th land parcel that is given as follows:

$$S_i(t) = \begin{cases} 1 & \text{if parcel is in forested} \\ 0 & \text{if parcel is in deforested} \end{cases} \quad (1)$$

A forested parcel becomes deforested following the landowner's decision to cut trees (called "deforestation decision") (Fig. 1a). The deforestation decision is made with rate $r(t)$ every year, that is controlled by the net gain of deforestation. We will explain how to calculate the net gain of deforestation and the deforestation rate in the next section. Once the forested land is deforested, the resultant deforested land may develop secondary vegetation. Such reforestation helps to restore nutrient and water cycling, and leads to the development of a forest with species composition and biomass accumulation rates similar to the original forest. Therefore we assume that deforested land reverts back to forested land at rate $\mu$ and the expected number of years needed for forest recovery is $1/\mu$ (Fig. 1a).

Landowners receive utility depending on the state of their land. Let $u_i(t)$ bet the actual utility received by landowner *i* at time *t*. $u_i(t)$ is given as follows:

$$u_i(t) = \begin{cases} b & \text{if } S_i(t) = 1 \\ c & \text{if } S_i(t) = 0 \text{ and } t_D = 0, \\ 0 & \text{if } S_i(t) = 0 \text{ and } t_D > 0 \end{cases} \quad (2)$$

where $t_D$ is the time when deforestation occurs. $b$ in the above equation is a positive constant that indicates the utility caused by ecosystem services when the parcel is in a forested state. $c$ in equation (2) is the utility of deforestation, which is the total revenue received (e.g. monetary benefits by timber sales) minus the cost incurred (e.g. harvesting/transportation cost) when the landowner is engaged in deforestation. We assume that landowners receive the utility $c$ just after deforestation (i.e. $t = t_D$), but afterwards (i.e. $t \geq t_D + 1$) the utility of deforested land declines to a level of 0 that is even lower than that of forested land, because a bare land does not produce the same range of ecosystem services (Fig. 1b).

*2.2 Individual decision about deforestation*

We assume two options about forest use, deforestation and forest conservation. The landowner makes his decision about which option to choose every year. The decision of a landowner regarding whether or not to choose deforestation option is influenced by the net gain of deforestation. The net gain of deforestation is defined as the expected utility received by deforestation minus the expected utility loosed by deforestation as given by the following,

$$[\text{Net gain of deforestation}] = V_D(t) - V_F(t), \qquad (3)$$

where $V_D(t)$ and $V_F(t)$ are the expected utilities of deforestation and that of forest conservation at time $t$.

We assume that the landowner chooses deforestation option more frequently if it results in a larger net gain:

$$r(t) = \frac{1}{1 + e^{-\beta(V_D(t) - V_F(t))}}, \qquad (4)$$

where $\beta$ is a positive constant. Equation (2) represents a probabilistic decision called logitdynamicsingametheory(HofbauerandSigmond,2003),andhasbeenused to describe individual land-use or water-use decisions (Satake and Iwasa, 2006). $\beta$ is a parameter controlling the degree of stochasticityin decision making. If $\beta$ is close to 0, parameter controlling the degree of stochasticity in decision making. If $\beta$ is close to 0, the landowner's behavior resembles a deterministic decision, i.e. he chooses the forest-use option that represents the larger value among the two with a certainty and the inferior option is selected with zero probability. Otherwise the choices becomes probabilistic - the option producing larger utility is more likely to be selected than the inferior option. The probabilistic choice models were introduced by mathematical psychologists (e.g. Luce, 1959), and McKelvey and Palfrey (1995) provided a general framework to extend the probabilistic choice approach to the case of multiple decision-makers (called the Quantal Response Equilibrium). We adopt probabilistic decision rather than deterministic decision because it is likely that the landowner's valuation system about utility includes some degree of error.

*2.3 Social learning: overcome environmental uncertainty*

Satake and Iwasa (2006) analyzed the decision dynamics in which landowners have full access to information to estimate the true utility values of forest use. However, it is likely that landowner's knowledge about utilities is incomplete. In such a case, the landowner must learn utility values from experience.

The individual's expected utility about forest conservation is updated according to the following dynamics:

$$V_F(t+1) = (1-\alpha)V_F(t) + \alpha\pi_F(t), \quad (5a)$$

where $\alpha$ is the parameter that defines a memory time ranging from 0 to 1 and $\pi_F(t)$ is the information input from experience about the utility of forested parcel at time *t*. As $\alpha$

increases, the memory size is reduced. The shortest memory corresponds to $\alpha = 1$ where landowner's expected utility is determined based only on the current information input. The expected utility is determined over a period $T \sim 1/|\log(1-\alpha)|$. Similarly, the individual's expected utility from a plot of deforested land changes according to the following:

$$V_D(t+1) = (1-\alpha)V_D(t) + \alpha\pi_D(t), \tag{5b}$$

where $\pi_D(t)$ is the information input about the utility of forested parcel at time *t*. $\pi_F(t)$ and $\pi_D(t)$ are given by:

$$\pi_F(t) = \sum_{i=1}^{N} S_i(t)u_i(t) / \sum_{i=1}^{N} S_i(t), \tag{6a}$$

$$\pi_D(t) = \sum_{i=1}^{N} (1 - S_i(t))u_i(t) / \sum_{i=1}^{N} (1 - S_i(t)), \tag{6b}$$

where $S_i(t)$ and $u_i(t)$ are the state variable (equation (1)) and the actual utility received by landowner *i* in year *t* (equation (2)). The learning dynamics in equations (5) and (6) assume that the agents all share their information about utility with each other, so that the actions and resultant experiences of one agent provide information about the choices on land-use of others, which is called "social learning" (Sobel, 2000). We acknowledge this is a simplified implementation of social learning. In future work we may explore the consequences of various variations of knowledge exchange via different social network structures.

$\alpha$ in equation (5) is called learning rate according to the model of reinforcement learning (Bradtke and Duff, 1994) because a large $\alpha$ indicates a greater application of present information in developing expectations. $\alpha$ is also called a "forgetting" rate according to the learning dynamics in game theory (Erev and Roth, 1998; Roth and Erev, 1995) because a larger $\alpha$ implies a smaller memory about past experience. Although there are multiple different ways how to call the parameter $\alpha$, we call it memory time after Giardina and Bouchaud (2003) who model updating dynamics of the performance of agent's strategies in financial market. Our formulation is also equivalent to the learning model in Rapoport and Chammah (1970) who studied Prisoner's Dilemma game under learning dynamics.

*2.5 Linking individual decision with global landscape dynamics*

In order to investigate macroscopic landscape patterns emerged from individual decisions, we developed a global landscape dynamics about coupled human and forest ecosystem under the assumption that a society is composed of a number of landowners (i.e. $N$ is sufficiently large). Let $x(t)$ be the density of forested land, and $y(t)$ be the density of just harvested parcels. $1-x(t)-y(t)$ represents the density of parcels that were deforested more than a year ago, and have not recovered to forest yet.

The global landscape dynamics is given as follows:

$$x(t+1)=\mu(1-x(t))+\left(1-\frac{1}{1+\exp\left[-\beta\left(V_D(t)-V_F(t)\right)\right]}\right)x(t), \tag{7a}$$

$$y(t+1)=\frac{1}{1+\exp\left[-\beta\left(V_D(t)-V_F(t)\right)\right]}x(t), \tag{7b}$$

$$V_F(t+1)=(1-\alpha)V_F(t)+\alpha b, \tag{7c}$$

and

$$V_D(t+1) = (1-\alpha)V_D(t) + \alpha \frac{cy(t+1)}{1-x(t+1)}. \quad \text{(7d)}$$

It is easily expected that the value of forest conservation ($V_F(t)$) converges to $b$ according to equation (7c). Therefore dynamics are reduced to three variables: $x(t)$, $y(t)$, and $V_D(t)$.

We also performed computer simulation about the individual decisions by assuming a finite number of agents ($N$=10000). As for initial state, we gave random numbers ranging from 0 to 1 for expected utility about forest conservation ($V_F(0)$) and deforestation ($V_D(0)$) for each landowner, and then updated these values based on the dynamics under social learning (equations (5) and (6)).

## 3. Result

### *3.1 Equilibrium state in a forested landscape*

The global landscape dynamics in equation (7) have a single positive equilibrium state:

$$(x^*, y^*, V_F{}^*, V_D{}^*) = \left( \frac{1+\exp[-\beta(\mu c - b)]}{1+\exp[-\beta(\mu c - b)] + 1/\mu}, \frac{1}{1+\exp[-\beta(\mu c - b)] + 1/\mu}, b, \mu c \right). \quad \text{(8)}$$

The equilibrium forest density ($x^*$) decreases as the equilibrium net gain of deforestation (i.e. $V^*_D - V^*_F = \mu c - b$) increase. On the contrary, the equilibrium density of just harvested parcels ($y^*$) increases as $\mu c - b$ increase. The equilibrium state in equation (8) is independent of the memory time ($\alpha$). However, $\alpha$ greatly influences the stability of the equilibrium state as we explain later.

We illustrated several examples of the temporal dynamics of expected utility

($V_F(t)$ and $V_D(t)$), forest density ($x(t)$), and the density of just harvested forest ($y(t)$) generated from computer simulation (Fig. 2). $V_F(t)$ converges to a constant level of $b$ regardless of the memory time ($\alpha$) (Figs. 2a, 2b, and 2c). However the dynamics of $V_D(t)$ strongly depend on $\alpha$. When the memory time is small (i.e. $\alpha = 0.05$), the social learning leaded to a constant value of $V_D(t)$ (Fig. 2a) that is equivalent to $V_D^*$ given in equation (8). As a consequence, the forest density and the density of just harvested parcels converge to constant levels (Fig. 2d). As the memory time becomes large (i.e. $\alpha = 0.2$ and $\alpha = 0.9$), $V_D(t)$ starts to fluctuate across time (Figs. 2b and 2c). Consequently the forest density drastically declines and shows recovery afterwards, which is repeated quasi-periodically (Figs. 2e and 2f).

*3.3 When collective deforestation occurs?*

In this section, we investigate when collective deforestation occurs. When the equilibrium state (equation (8)) is stable, the system converges to a stationary state in which forest density is constant across time due to balanced fraction of forest recovery and deforestation. In contrast, if the equilibrium is unstable, forest density varies across time. Here we define collective deforestation as the unstable dynamics for global landscape. We analyzed the stability of the dynamics (equation (7)) by calculating eigenvalues of the Jacobi Matrix (Appendix A).

Collective deforestation was expected when the memory time ($\alpha$) is relatively large (Fig. 3). It also is influenced by the magnitude of forest recovery rate ($\mu$) and the degree of stochasticity in decision making that is controlled by a parameter $\beta$ (Fig. 3). Collective deforestation was observed when $\mu$ is of an intermediate

Collective deforestation was observed when $\mu$ is of an intermediate magnitude. The dynamics is stable either when $\mu$ is too small or too large. The degree of stochasticity in decision making negatively influenced the occurrence of collective deforestation – as stochastic deforestation becomes more frequently (i.e. as $\beta$ decreases), collective deforestation is restricted to occur in a smaller parameter region. This is because, when deforestation decision occurs very frequently and therefore when the forest density stays at low level, the decisions of deforestation by small fraction of individuals are not pronounced enough to alter expectation of others.

*3.4 Feedback from global landscape dynamics to individual decision*

In this section, we analyze the feedback from global landscape dynamics to individual decision. We especially focus on three situations; (1) when forest value is negatively correlated with the global forest density, (2) when forest recovery rate is positively correlated with the global forest density, and (3) when timber price decreases as timber supply increases.

<u>(1) When forest value is negatively correlated with the global forest density</u>

If a large area is deforested, people may identify much more value of remaining forest. We model this situation by simply considering that the actual utility of forest is given as a decreasing function of forest density;

$$b(x) = \hat{b} + a(1 - x), \qquad (9)$$

where $\hat{b}$ is the actual utility of forest when the entire land is covered with forest (i.e.

$x$=1) . $a$ in equation (9) is a positive constant ranging from 0 to 1. The actual utility, therefore, linearly decreases as the forest density ($x$) increases.

We developed a global landscape dynamics for this situation, and obtained the condition when collective deforestation occurs (Appendix B). Results showed that if forest value is negatively correlated with the global forest density, the system becomes more unstable comparing to the situation under constant forest value (Fig. 4).

(2) When forest recovery rate is positively correlated with the global forest density

If a large area is deforested, forest regrowth may be suppressed because of a loss of seed supply, degradation of soil nutrient, and water shortage. Therefore we consider that the forest recovery rate is given as a linearly increasing function of the forest density;

$$\mu(x) = hx, \tag{10}$$

where $h$ is a positive constant ranging from 0 to 1.

We found that the forested ecosystem with density dependent forest recovery has two equilibrium points, one of which is the extinction of forest ecosystem, i.e. $\left(x^*, y^*, V_D{}^*\right) = (0,0,0)$, and another is positive equilibrium for all the three variables (Appendix B). The former equilibrium is stable if:

$$h < \frac{1}{1+\exp^{\beta b}}. \tag{11}$$

The above equation means that either as the decision approaches deterministic (i.e. as $\beta$ increases) and as the actual utility of forested land ($b$) increases, the extinction of forest ecosystem becomes unlikely. In particular, if the decision is close to deterministic (i.e. $\beta \to \infty$), extinction never happen.

As for the latter equilibrium, we investigated the stability, and illustrated the boundary that separates stable and unstable region in Fig. 5. The equilibrium is unstable in large parameter region, and the stable forested state is realized only when the memory time ($\alpha$) is small.

(3) When timber price decreases as timber supply increases

We consider the third situation where timber price is determined by the amount of timber supplied. This represents that a society does not have any external market, and timber provided at the society is consumed in that society. We simply model this situation by considering that the actual utility of deforestation decreases with a density of landowners who just harvested their forest:

$$c(y) = \hat{c} - dy, \tag{12}$$

where $\hat{c}$ is a basic utility of deforestation when nobody provides timber supply, and $d$ is a positive constant ($0 \leq d \leq \hat{c}$). $y$ is a density of landowners who just harvested trees and sell timbers. We here simply consider that the landowner sells timber immediately after harvesting although this assumption is relaxed to allow time delay effect .

We similarly investigated the condition when collective deforestation occurs (Appendix B), and found that the unstable parameter region was not slightly different from the situation where constant timber price was assumed (Fig. 6).

## 4. Discussion

This paper presents a simple decision model of forest use under social learning and investigates the global landscape pattern emerged from the individual landowner's decisions. Existing models about forest use include human decision making based on statistical models of past observations (Veldkamp and Fresco, 1996) or increasingly comprehensive agent-based models (Parker et al. 2003). Notwithstanding an obvious benefit of reality, a drawback of comprehensive agent-based model is the lack of rigorous understanding analysis that can be performed to understand the underlying model dynamics. The approach focus of this paper is a simple agent-based model, extending Markov chain models, for which we can analyze rigorously the consequences of different behavioral assumptions of the agents.

Our analysis shows that exchange of information about forest-use value among landowners can trigger harvesting of resources at a larger scale and results in a global resource shortage. This collective deforestation occurs when our resource users quickly forget past experience and use public available information only on short term benefits.

Regeneration of forest resources is slow. Because of this slow recovery process, it is difficult to directly observe how decisions made by previous generation affect the state and shape of forest in the present time. This implies that social learning within the same generation is not efficient to learn from the events that occurred long time ago and that have been experienced by previous generation. To promote intergenerational information exchange, a society needs to develop a system with which cumulative body of knowledge about forest value is handed down through generations by cultural transmission (as discussed in Berkes and Folke 1998). These traditional value will allow future human generations to learn to sacrifice short-term gains to obtain long-term benefits. It will prevent reemergence of overexploitation of forest resources.

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**Appendix A**

*Stability analysis*

The Jacobi matrix $\mathbf{J}$ of the dynamics in equation (7) in the text is given by:

$$\mathbf{J} = \begin{vmatrix} -\mu + 1 - \dfrac{1}{1+e^{-\beta\left(V^*{}_D - V^*{}_F\right)}} & 0 & \dfrac{-\beta e^{-\beta\left(V^*{}_D - V^*{}_F\right)}}{\left(1+e^{-\beta\left(V^*{}_D - V^*{}_F\right)}\right)^2} x^* \\ \dfrac{1}{1+e^{-\beta\left(V^*{}_D - V^*{}_F\right)}} & 0 & \dfrac{\beta e^{-\beta\left(V^*{}_D - V^*{}_F\right)}}{\left(1+e^{-\beta\left(V^*{}_D - V^*{}_F\right)}\right)^2} x^* \\ \dfrac{\alpha c y^*}{\left(1-x^*\right)^2} & \dfrac{\alpha c}{1-x^*} & 1-\alpha \end{vmatrix}, \quad \text{(A1)}$$

where $x^*$, $y^*$, $V_F{}^*$, and $V_D{}^*$ are given in equation (8) in the text. We obtained eigenvalues of the Jacobi matrix $\mathbf{J}$ numerically because characteristic equation is not tractable analytically. If absolute values of all of the eigenvalues obtained numerically are less than 1, the system is stable.

**Appendix B**

*(1) When the value of forest is a decreasing function of forest density*

We develop a global landscape dynamics emerged from individual decision when the actual utility of forest is given as a decreasing function of the forest density as in equation (9) in the text, and investigated the stability of equilibrium state. The global landscape dynamics are the same as in equation (8) in the text except that the dynamics for the expected value of forested land is given by;

$$x(t+1) = \mu(1-x(t)) + \left(1 - \frac{1}{1+\exp\left[-\beta\left(V_D(t) - V_F(t)\right)\right]}\right)x(t) \quad \text{(B1)}$$

$$y(t+1) = \frac{1}{1+\exp\left[-\beta\left(V_D(t) - V_F(t)\right)\right]} x(t) \tag{B2}$$

$$V_F(t+1) = (1-\alpha)V_F(t) + \alpha\left[\hat{b} + a(1-x(t)\right] \tag{B3}$$

$$V_D(t+1) = (1-\alpha)V_D(t) + \alpha \frac{cy(t)}{1-x(t)} \tag{B4}$$

From equations (B3) and (B4), we have $(V_F^*, V_D^*) = (\hat{b} + a(1-x^*), \mu c)$. From equations (B1) and (B2), we have;

$$\left(1 - \frac{1}{1+\exp\left[-\beta\left(V_D^* - V_F^*\right)\right]}\right) x^* = \mu(1-x^*), \tag{B5}$$

$$y^* = \frac{1}{1+\exp\left[\beta\left(V_D^* - V_F^*\right)\right]} x^*. \tag{B6}$$

By instituting $(V_F^*, V_D^*) = (\hat{b} + a(1-x^*), \mu c)$ into equation (B5), we have the forest density at equilibrium state, $x^*$. By instituting $x^*$ into equation (B6), we have $y^*$.

The Jacobi matrix of the above dynamics is given by;

$$\mathbf{J} = \begin{vmatrix} -\mu + 1 - \frac{1}{1+e^{-\beta\left(V^*_D - V^*_F\right)}} & 0 & A & -A \\ \frac{1}{1+e^{-\beta\left(V^*_D - V^*_F\right)}} & 0 & -A & A \\ -\alpha a & 0 & 1-\alpha & 0 \\ \frac{\alpha c y}{(1-x)^2} & \frac{\alpha c}{1-x} & 0 & 1-\alpha \end{vmatrix}, \tag{B7}$$

where $A = \beta e^{-\beta\left(V^*_D - V^*_F\right)} x^* / \left(1 + e^{-\beta\left(V^*_D - V^*_F\right)}\right)^2$. We calculated the eigen values of the jacobi matrix numerically, and investigated whether or not the absolute values of all eigen values are less than 1.

*(2) When forest recovery rate is a decreasing function of forest density*

If forest recovery rate is given as a decreasing function of forest density as in equation (10) in the text, global landscape dynamics become;

$$x(t+1) = (1 - r(t))x(t) + hx(t)(1 - x(t)), \tag{B8}$$

$$y(t+1) = r(t)x(t), \tag{B9}$$

$$V_D(t+1) = (1-\alpha)V_D(t) + \alpha \frac{cy(t)}{1 - x(t)}, \tag{B10}$$

where

$$r(t) = \frac{1}{1 + \exp^{-\beta\left(V_D(t) - V_F(t)\right)}}. \tag{B11}$$

There are two equilibrium points, one of which is $\left(x^*, y^*, V_D{}^*\right) = (0,0,0)$, and another is positive equilibrium. As for the latter equilibrium, we have $V_D{}^* = chx^*$. And equilibrium forest density and that of just harvested parcel, $x^*$ and $y^*$, are obtained from the following equations;

$$hx^* + \frac{1}{1 + \exp^{-\beta\left(chx^* - b\right)}} - h = 0, \tag{B12}$$

$$y^* = r^* x^*. \tag{B13}$$

The Jacobi matrix of the above system is given by;

$$\mathbf{J} = \begin{vmatrix} -2hx^* + h + 1 - r^* & 0 & -A \\ r^* & 0 & A \\ \dfrac{\alpha c y^*}{\left(1 - x^*\right)^2} & \dfrac{\alpha c}{1 - x^*} & 1 - \alpha \end{vmatrix}, \tag{B14}$$

where $A$ is given as the same formula in a previous section. As for the first equilibrium, $(x^*, y^*, V_D{}^*) = (0,0,0)$, we have three eigen values, $\lambda_1 = 0$, $\lambda_2 = 1 - \alpha$, and $\lambda_3 = h + 1 - 1/(1 + \exp^{\beta b})$. By noting that $\alpha$ is a positive constant ranging from 0 to 1, the system is stable if

$$h < \frac{1}{1+\exp^{\beta b}}. \tag{B15}$$

For the positive equilibrium, the characteristic equation is not mathematically tractable, and therefore we solved the characteristic equation numerically.

*(3) When timber price is a decreasing function of the density of harvesting*

If timber price is given as a decreasing function of the density of harvesting as in equation (12) in the text, we have the following global landscape dynamics;

$$x(t+1) = (1-r(t))x(t) + \mu(1-x(t)), \tag{B16}$$

$$y(t+1) = r(t)y(t), \tag{B17}$$

$$V_D(t+1) = (1-\alpha)V_D(t) + \alpha\frac{\left(\hat{c}-dy(t)\right)y(t)}{1-x(t)}. \tag{B18}$$

At equilibrium, we have $V_D{}^* = \mu\left[\hat{c}-d\mu\left(1-x^*\right)\right]$. Equilibrium forest density and that of just harvested parcel, $x^*$ and $y^*$, are obtained from following equations;

$$r^*x^* = \mu\left(1-x^*\right), \tag{B19}$$

$$y^* = r^*x^*. \tag{B20}$$

The Jacobi matrix of the above dynamics is given by:

$$\mathbf{J} = \begin{vmatrix} 1-r^*-\mu & 0 & -A \\ r^* & 0 & A \\ \frac{\alpha\left(\hat{c}-dy^*\right)y^*}{\left(1-x^*\right)^2} & \frac{\alpha\left(\hat{c}-2dy^*\right)}{1-x} & 1-\alpha \end{vmatrix}, \tag{B21}$$

where $A$ is given in a section (1) in Appendix B. We investigated the stability numerically by calculating eigen value of the Jacobi matrix $\mathbf{J}$.

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**Figure captions**

Figure 1 (a) Diagram of two-state Markov chain at a single land parcel. The parcel is in either forested (*F*) or deforested (*D*) state. $r$ and $\mu$ are the deforetation and forest recovery rate respectively. (b) The change of actual utility at single land parcel. The landowner will receive utility value of *b* if he manages forested land. He will obtain utility values of *c* when he cuts trees, but utility values of deforested land stays at low level 0 after deforestation until the parcel reverts back to forested state.

Figure 2 (a), (b), and (c): The expected utility of forest conservation ($V_F$: dashed lines) and that of deforestation ($V_D$: solid lines). (d), (e), (f): Forest density ($x(t)$: dashed lines) and frequency of deforestation ($y(t)$: solid lines). Other parameters are $b = 0.5$, $c = 2.0$, $\mu = 0.1$, and $\beta = 10$.

Figure 3 Phase diagram of the behavior of the model. Solid lines indicate the boundary that separate stable and unstable regions. The numbers attached these lines are magnitude of $\beta$. Other parameters are $b = 0.5$ and $c = 2.0$.

Figure 4 Phase diagram. Solid lines indicate the boundary that separate stable and unstable regions. The numbers attached these lines are magnitude of *a*. Other parameters are $b = 0.5$, $c = 2.0$, and $\beta = 10$.

Figure 5 Phase diagram. Solid lines indicate the boundary that separate stable and unstable regions. Other parameters are $b = 0.5$, $c = 2.0$, and $\beta = 10$.

Figure 6 Phase diagram. Solid lines indicate the boundary that separate stable and unstable regions. The numbers attached these lines are magnitude of *d*. Other parameters are $b = 0.5$, $c = 2.0$, and $\beta = 10$.

Fig. 1
Satake et al.

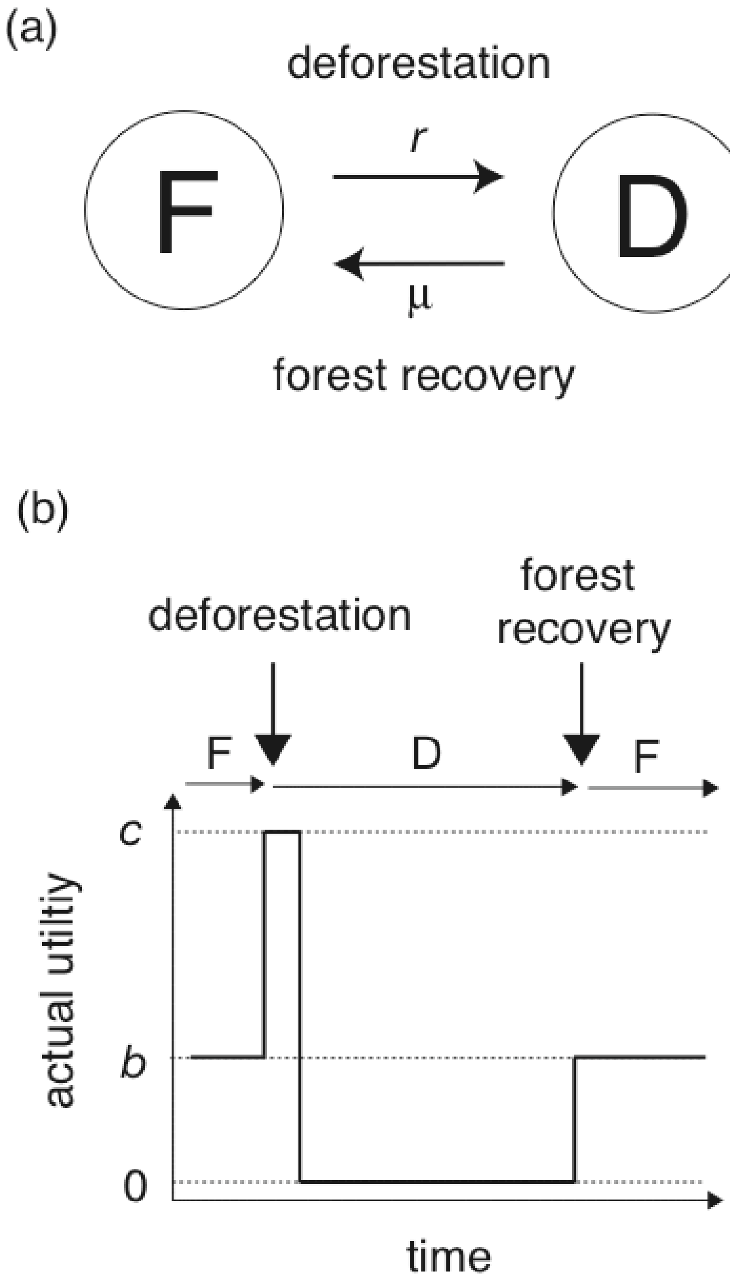

Fig. 2
Satake et al.

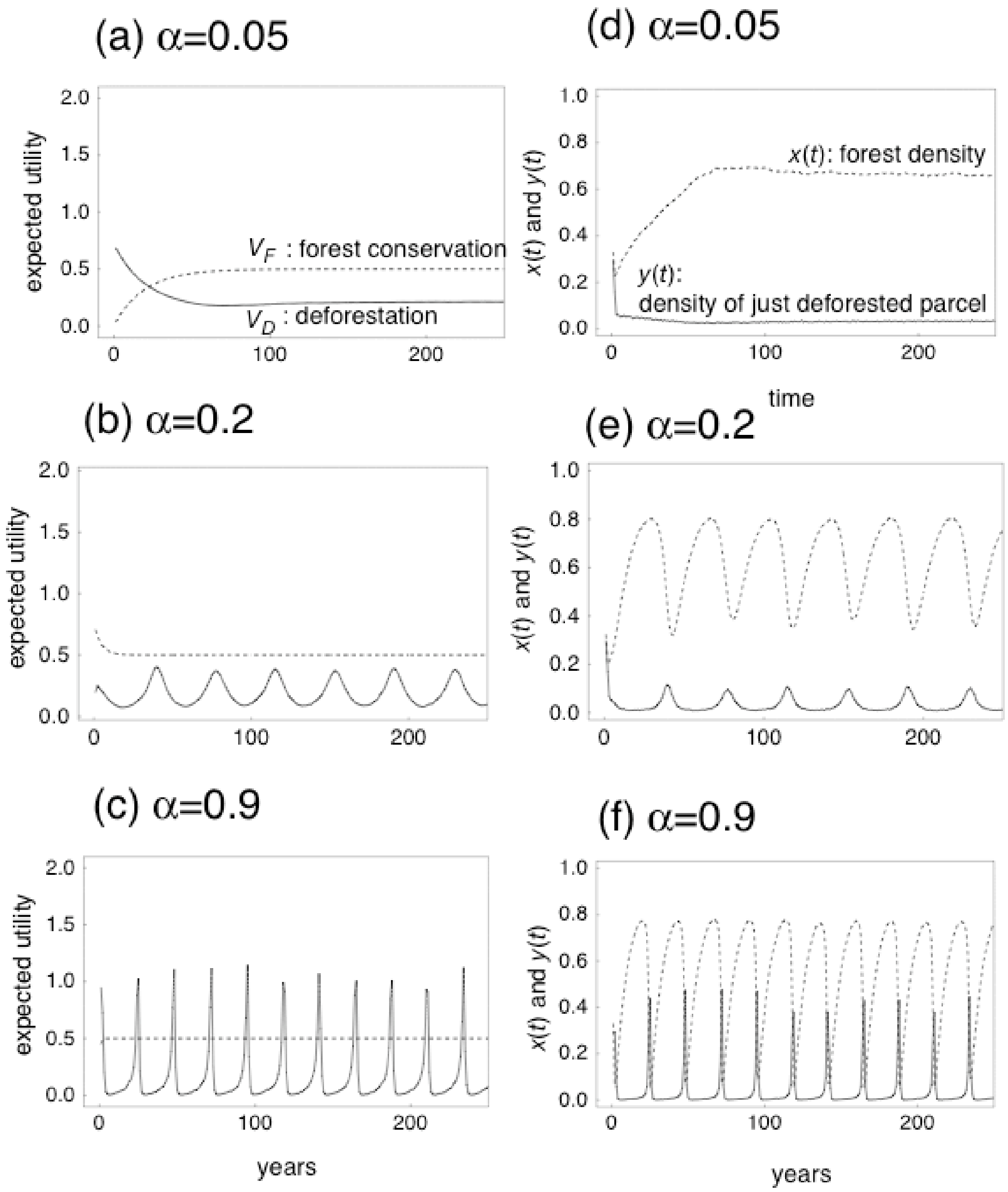

Fig. 3
Satake et al.

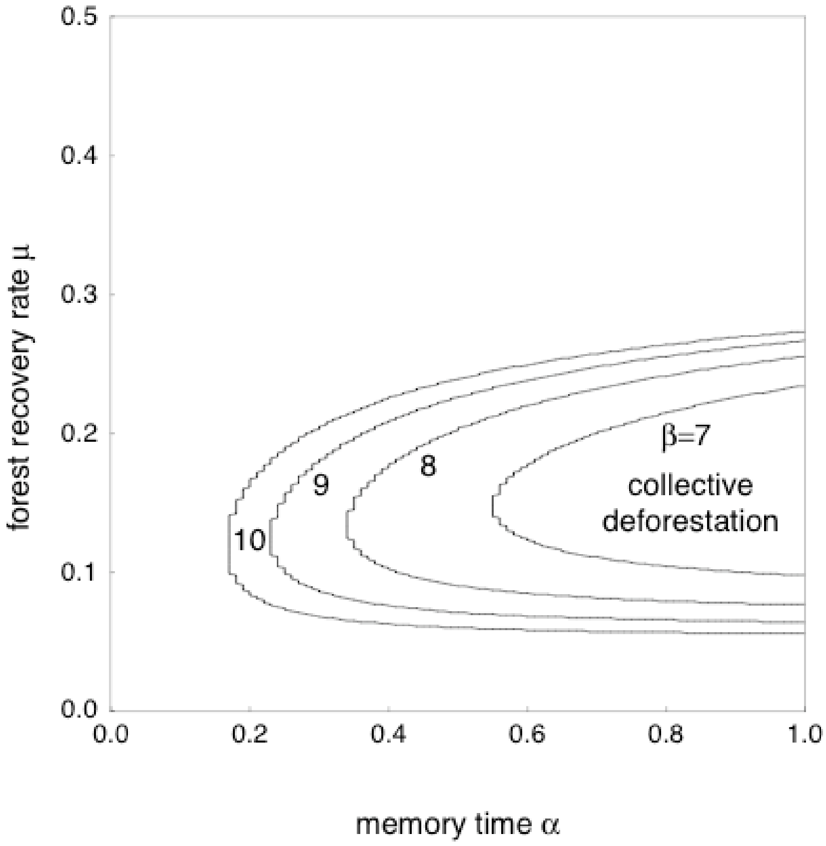

Fig. 4
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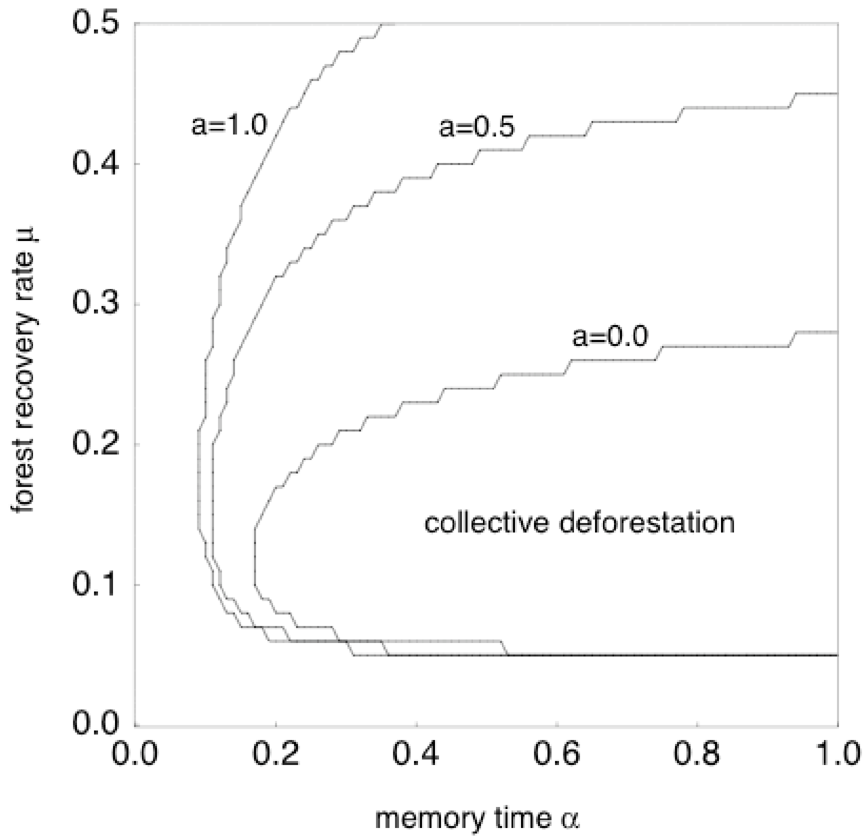

Fig. 5
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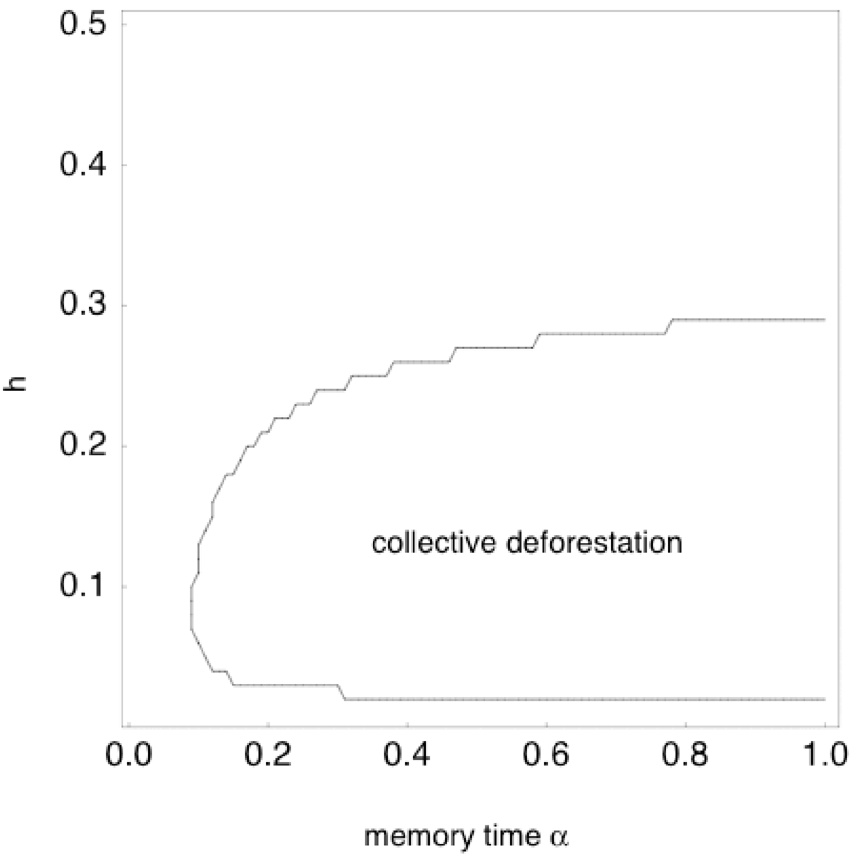

Fig. 6
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