Length Scales in Complex Time Series: Markov and Cryptic Orders

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Block-State Entropy Curve

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- Combine both observed information and state information
- $H[X_{0:0}, \mathcal{S}_0] = C_\mu$
- This curve gives us the crypticity
- Limits to the same asymptote as the block entropy

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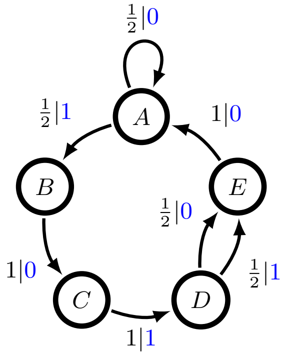
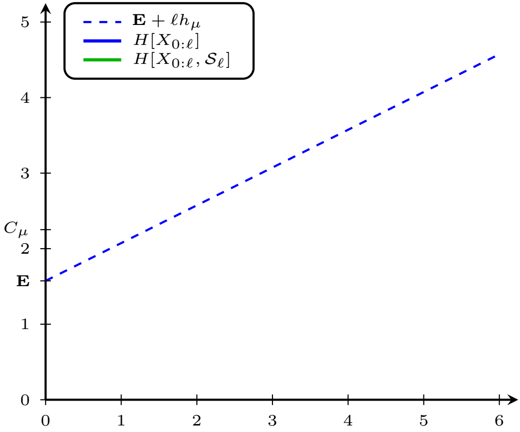
Block Entropy Curves Revisited





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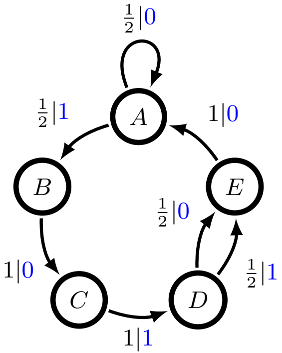

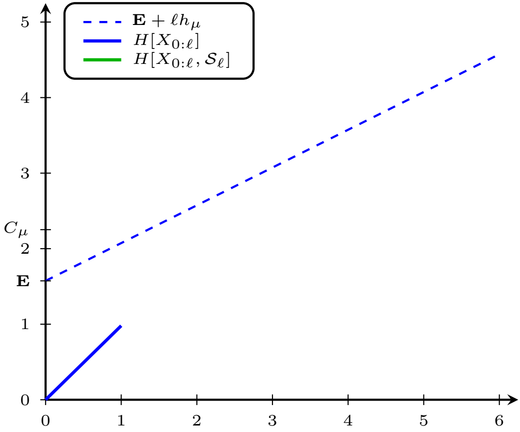

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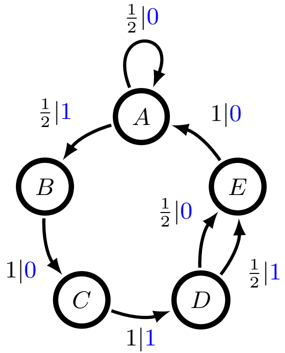

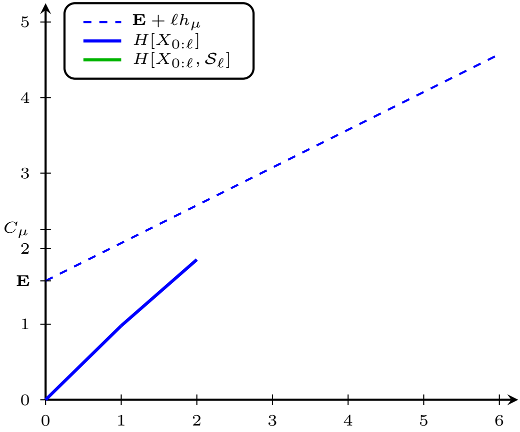

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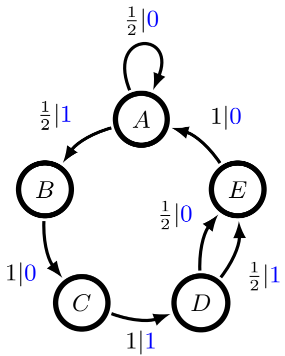

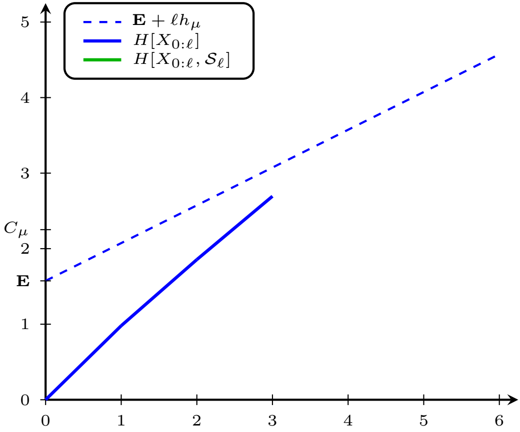

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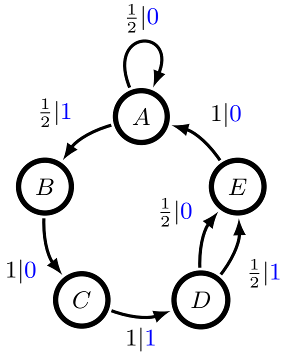

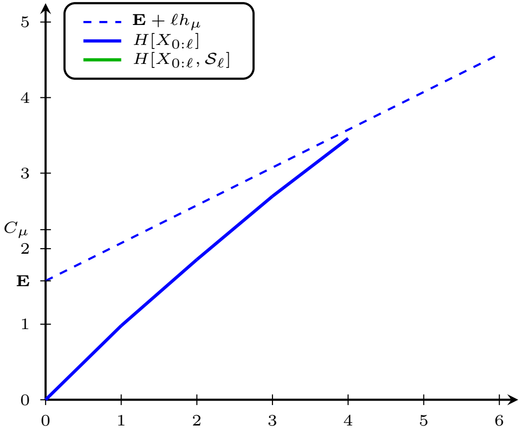

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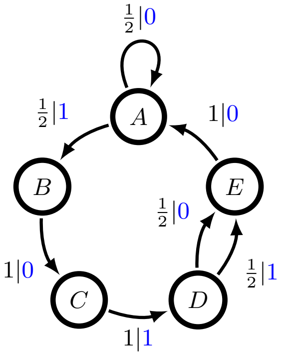

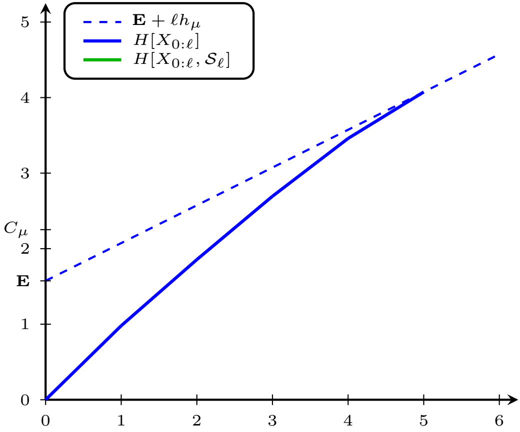

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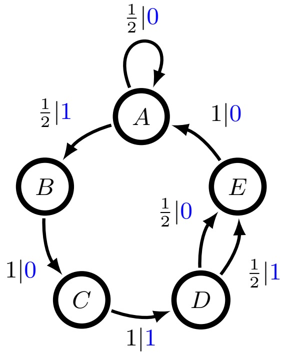

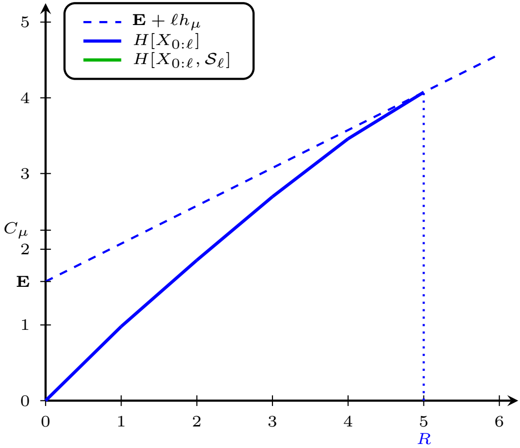

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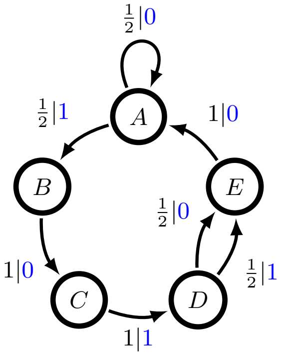

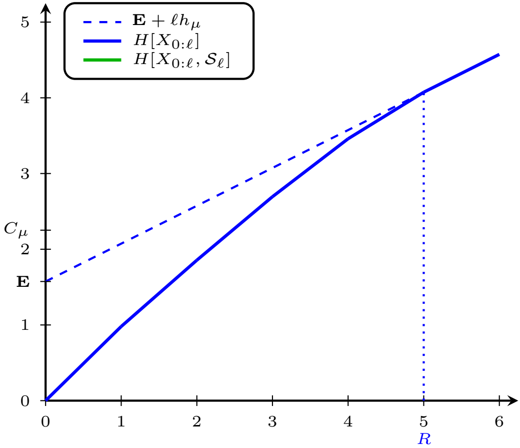

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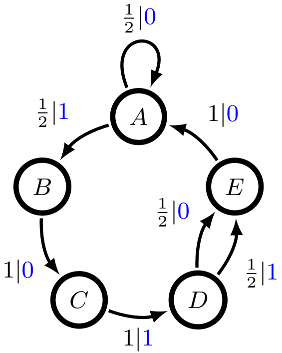

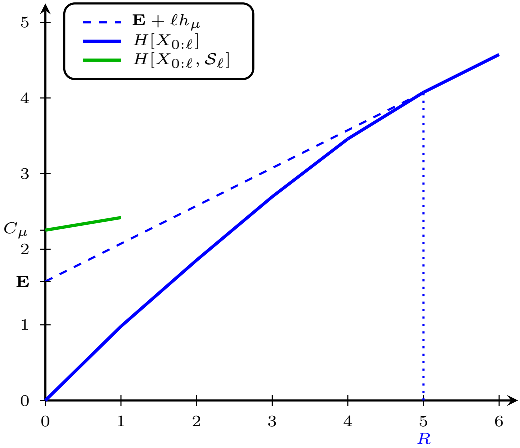

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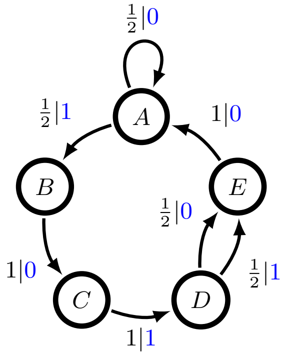

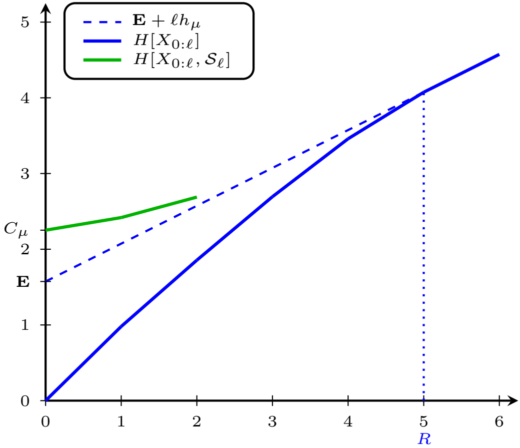

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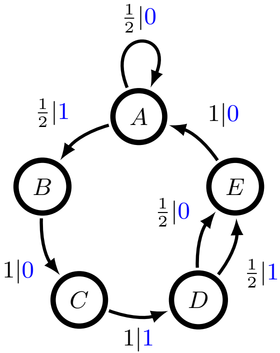

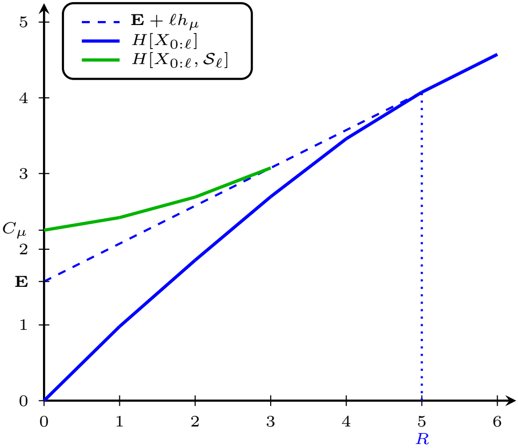

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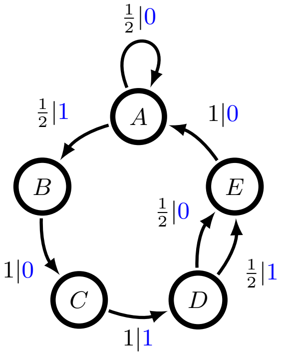

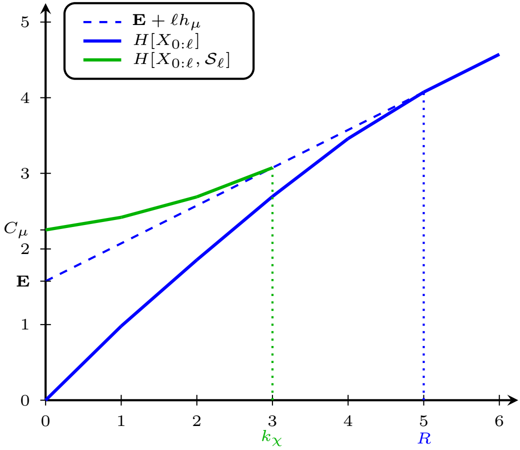

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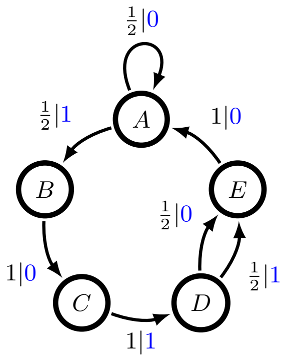

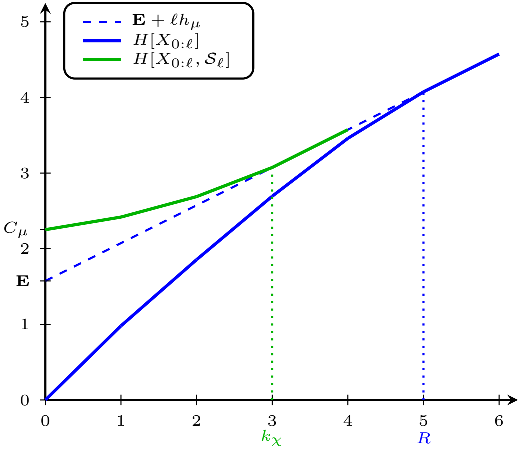

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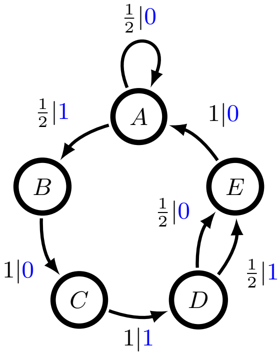

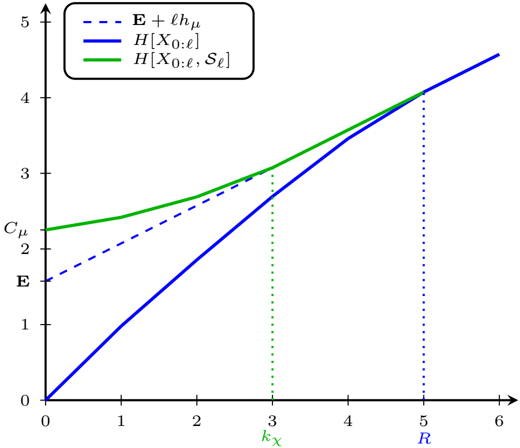

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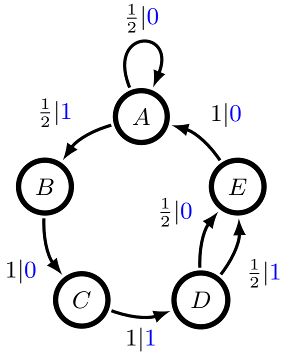

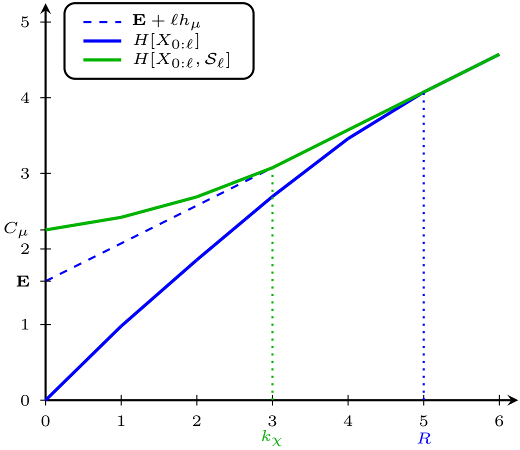

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- Knew $\mathbf{E}$ exactly
- Knew h_μ exactly
- Could differentiate *exactly on* the asymptote from *less than machine precision away from* the asymptote
- Could "guess" when R or k_χ were infinite, else we'd be computing block entropies indefinitely

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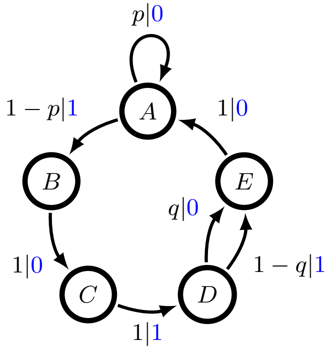
A Hint of a Solution



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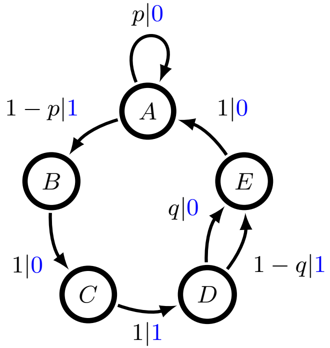

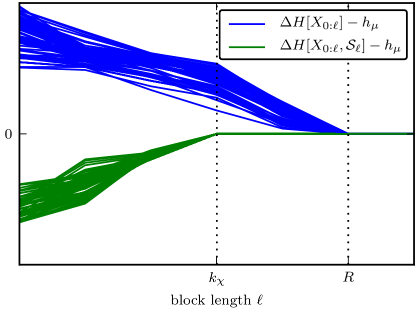

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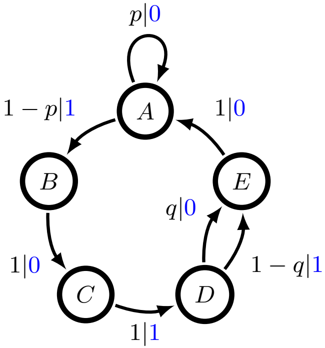

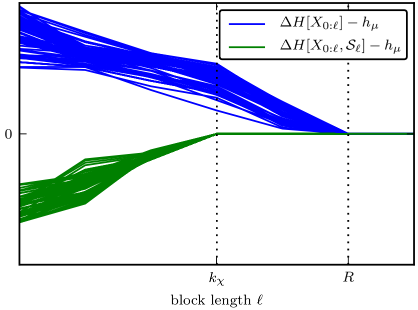

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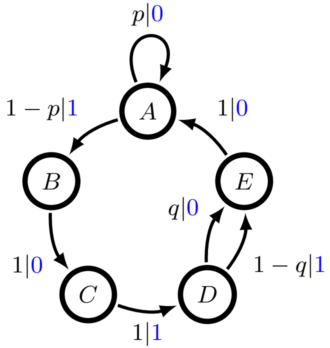
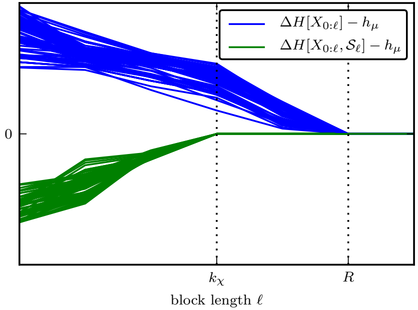
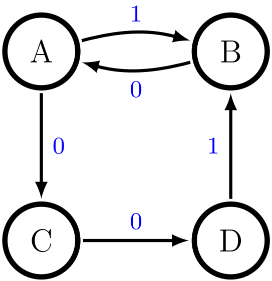




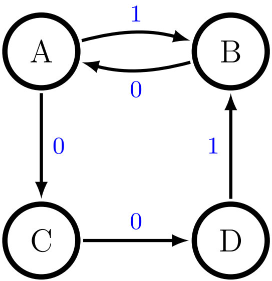
Markov and cryptic orders are *independent* of the probabilities!
They depend only on the *topology* of the ϵ-machine!

Walking Paths



Walking Paths

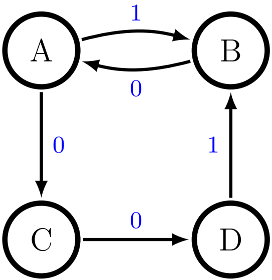
A
B
C
D



Walking Paths

A
B
C
D

0



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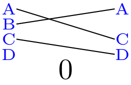

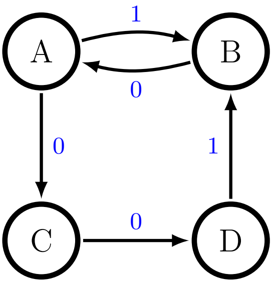

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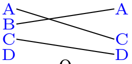

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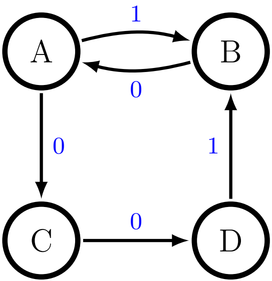

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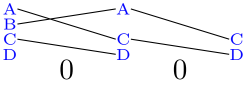

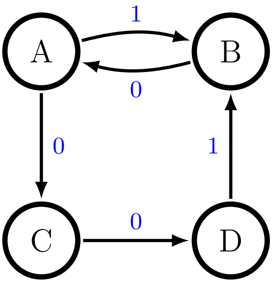

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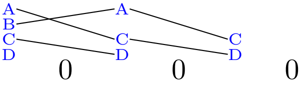

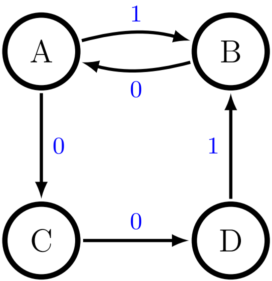

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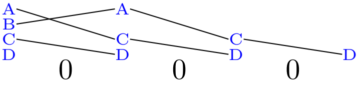

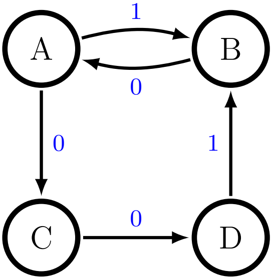

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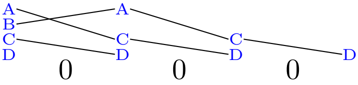

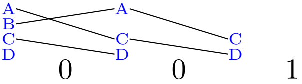

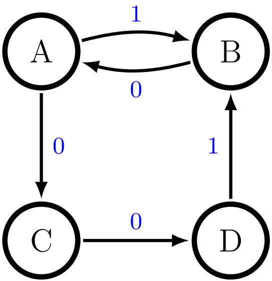

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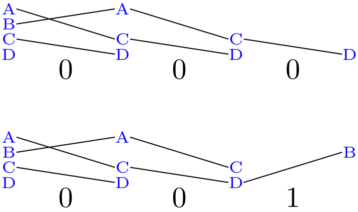

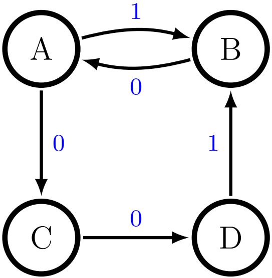

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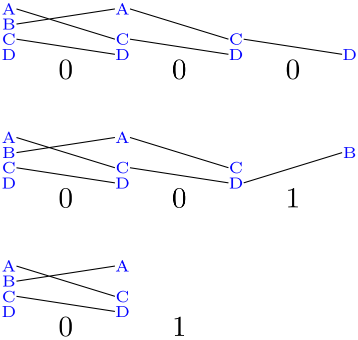

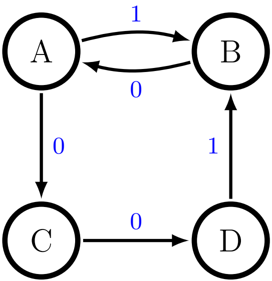

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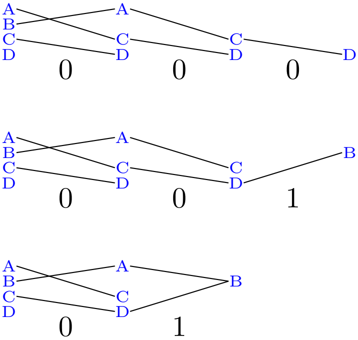

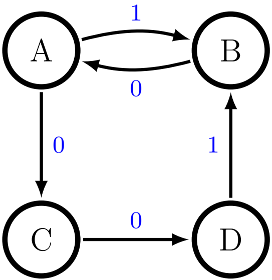

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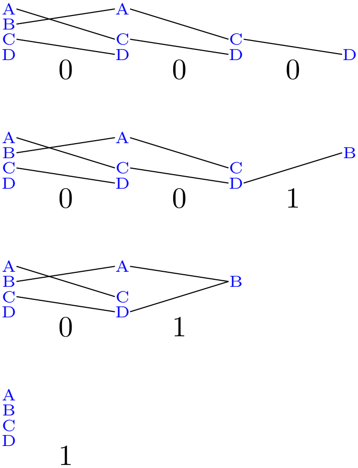

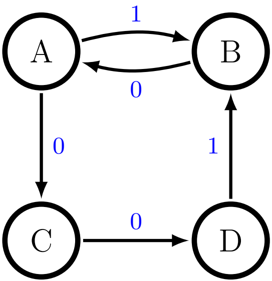

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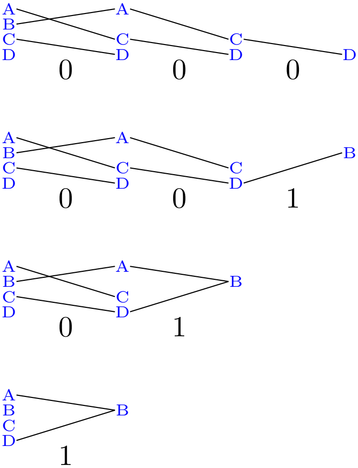

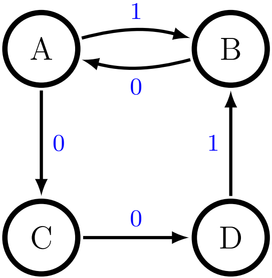

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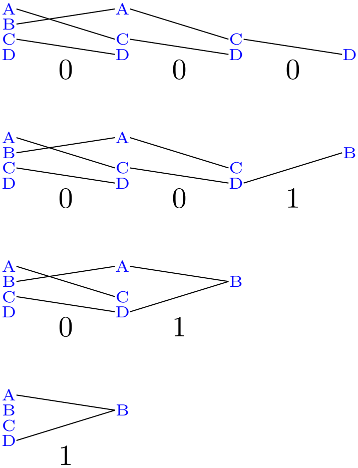
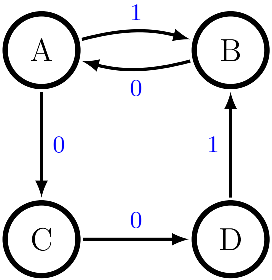
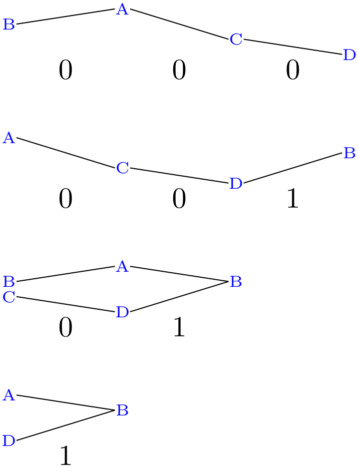


$R = 3$



Walking Paths



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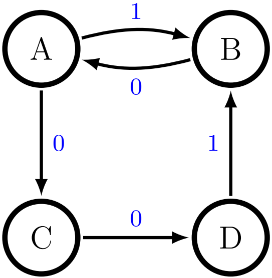

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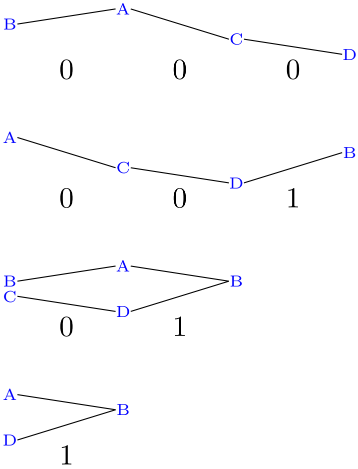

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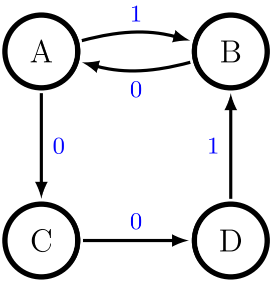

$k_\chi = 2$

Improvements

In what ways have we improved over the original method of computing R and k_χ?

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- But ... There could be an arbitrary number of synchronizing words, each of arbitrary length

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Further Improvements

Can we do even better than this method?

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- Yes!
- We can check all synchronizing words in *parallel* by analyzing and manipulating the graph structure of the ϵ-machine
- For details on that, see Ryan G. James, John R. Mahoney, Christopher J. Ellison, James P. Crutchfield: *Many Roads to Synchrony: Natural Time Scales and Their Algorithms*

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