Interacting Particles vs Interacting Agents

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Is there a physics of society?

- Most social scientists would be skeptical:
  - Where is behavior/decision-making in physics-type models?
    - Most social scientists really mean 'Where is the rationality?'
  - Many examples of this critique, e.g., of econophysics
  - Remark: many important social scientists studied physics/ chemistry/engineering (from Pareto to Engle)
- Related research programs:
  - *Systems Dynamics* (J. Forrester, Sloan School at MIT)
    - Models social processes as stocks and flows
  - Older traditions:
    - Early game theorists asked 'Is there a mathematics of society?'
    - *Polytechnicians* (Comte and others, critiqued by von Hayek)

My Focus

- People are not particles:
  - How to model behavior? People are purposive
  - Rational behavior as a standard (albeit unrealistic)
  - Behavioral experiments with people clearly support 'bounded rationality'
- Are there social environments in which behavior is unimportant?
  - Random search ('zero-intelligence') models
  - Micro-macro (multi-level) models

Outline

- *Historical remarks:* Mirowski on mathematical economics
- *Example* of a model with purposive behavior at the agent level while the macro-level seems explicable via ‘particles’
- *Argue* for the generality of the example: macroeconomics

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Mirowski:
Is Math Econ Archaic Physics?

- In *More Heat Than Light* (CUP, 1989), Phil Mirowski traces the historical development of mathematical economics
  - Subtitle: *Economics as Social Physics, Physics as Nature's Economics*
- 19*th* Century economists (Jevons, Edgeworth, Menger, Walras) were keenly aware of progress in (classical) physics and their work makes reference (sometimes direct, more often indirect) to physics and physicists of their day
- *'Subjective revolution'*: Scalar-valued utility is introduced as a function of goods consumed; behavior derives from it
- The formalism *for each agent* looks much like classical thermodynamics (e.g., conditions on $\partial U / \partial x_i$ for equilibrium)

Mirowski: continued

- Important difference: utility is *not* like energy as it is not interpersonally comparable
- Leaves us with a partial equilibrium story
- In the 1930-40s, Hicks and Samuelson extend received theory to general equilibrium:
  - Samuelson's main mentor a natural scientist (Wilson)
  - Papers in the *J. Math Econ*. develop inter-relations between classical thermodynamics and utility theory (e.g., L Hurwicz)
- Much of 20*th* C economics looks like 19*th* C physics

Mirowski: Concluded

- Mirowski argued that this was *not* good science, for many reasons, including there was no experimental basis for the formalism:
  - It was logical-deductive, not empirical
  - The mathematics not even very interesting
- Mirowski's critique both attacked and neglected by theorists
- 1990s saw the initial systematization of behavioral/ experimental economics
- IMHO, Mirowski vindicated by developments of past 20 years

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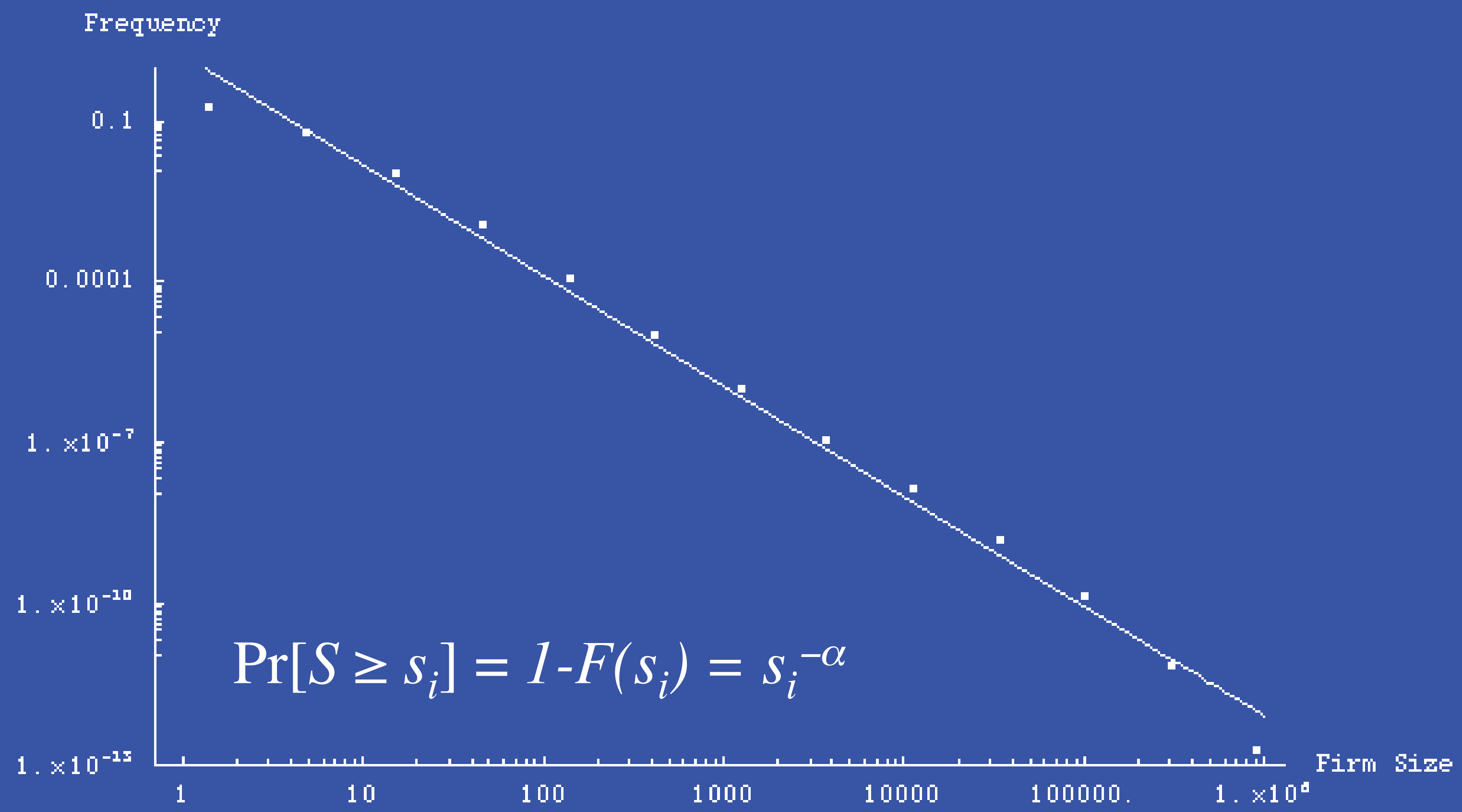
Multi-Agent Firms

- Goal: Build a microeconomic model that is capable of reproducing what we know empirically about firms
- Empirics: new micro-data available

FIRM SIZE DATA
(SIZE = EMPLOYEES)



"U.S. Firm Sizes are Zipf Distributed," RL Axtell, *Science*, *293* (Sept 7, 2001), pp. 1818-20

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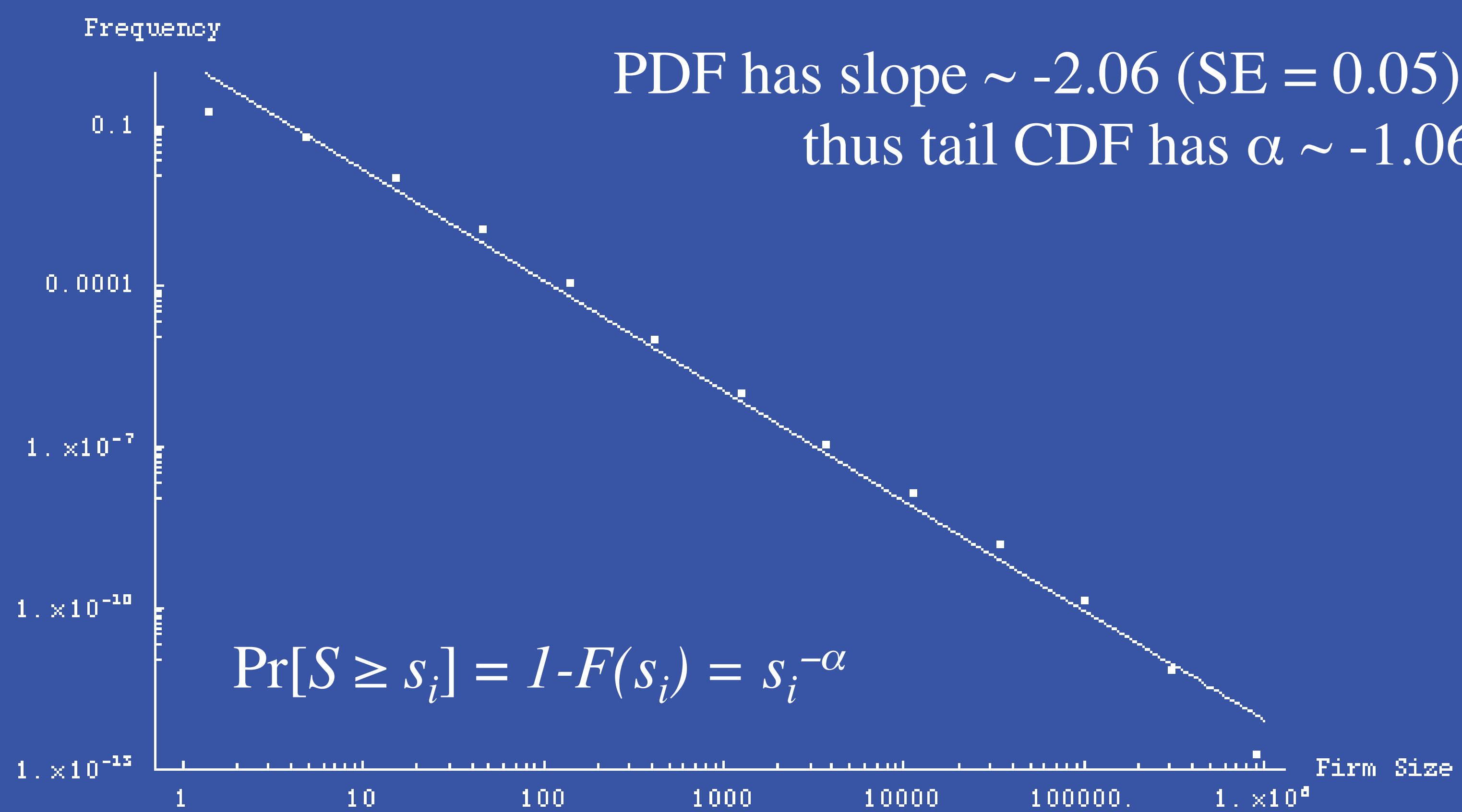

PDF has slope ~ -2.06 (SE = 0.05),
thus tail CDF has α ~ -1.06

$$\Pr[S \geq s_i] = 1\text{-}F(s_i) = s_i^{-\alpha}$$

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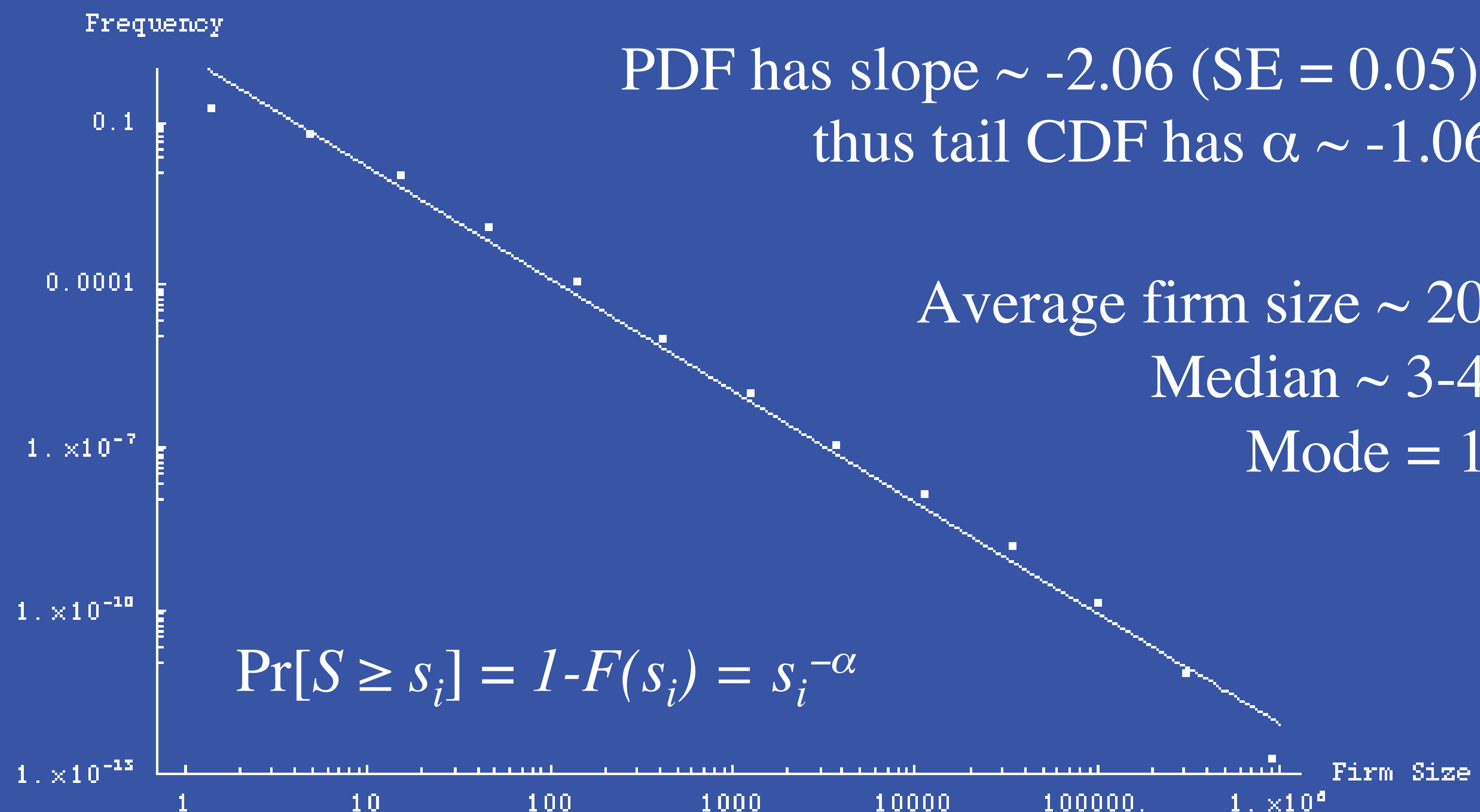

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Average firm size ~ 20
Median ~ 3-4
Mode = 1

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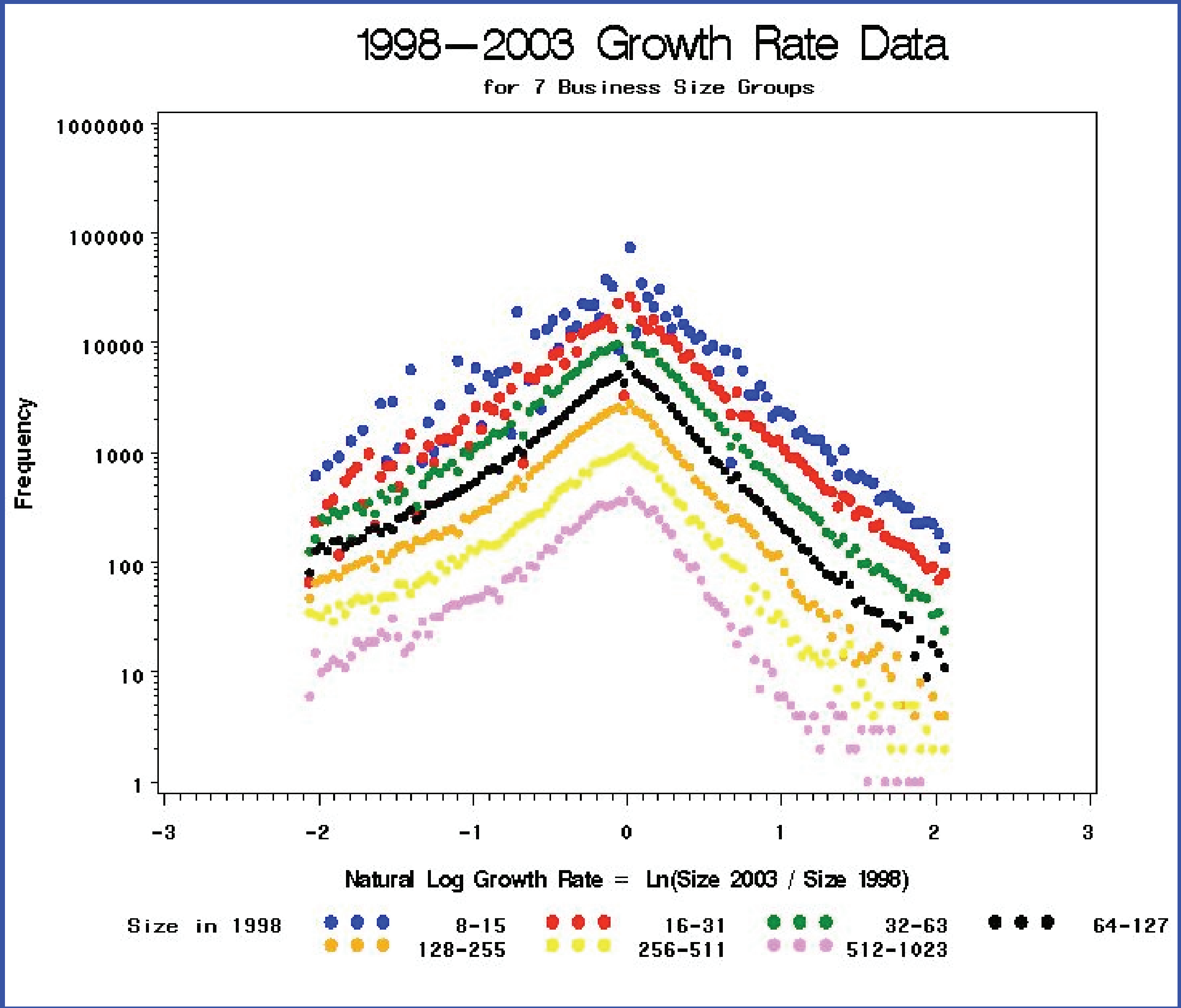
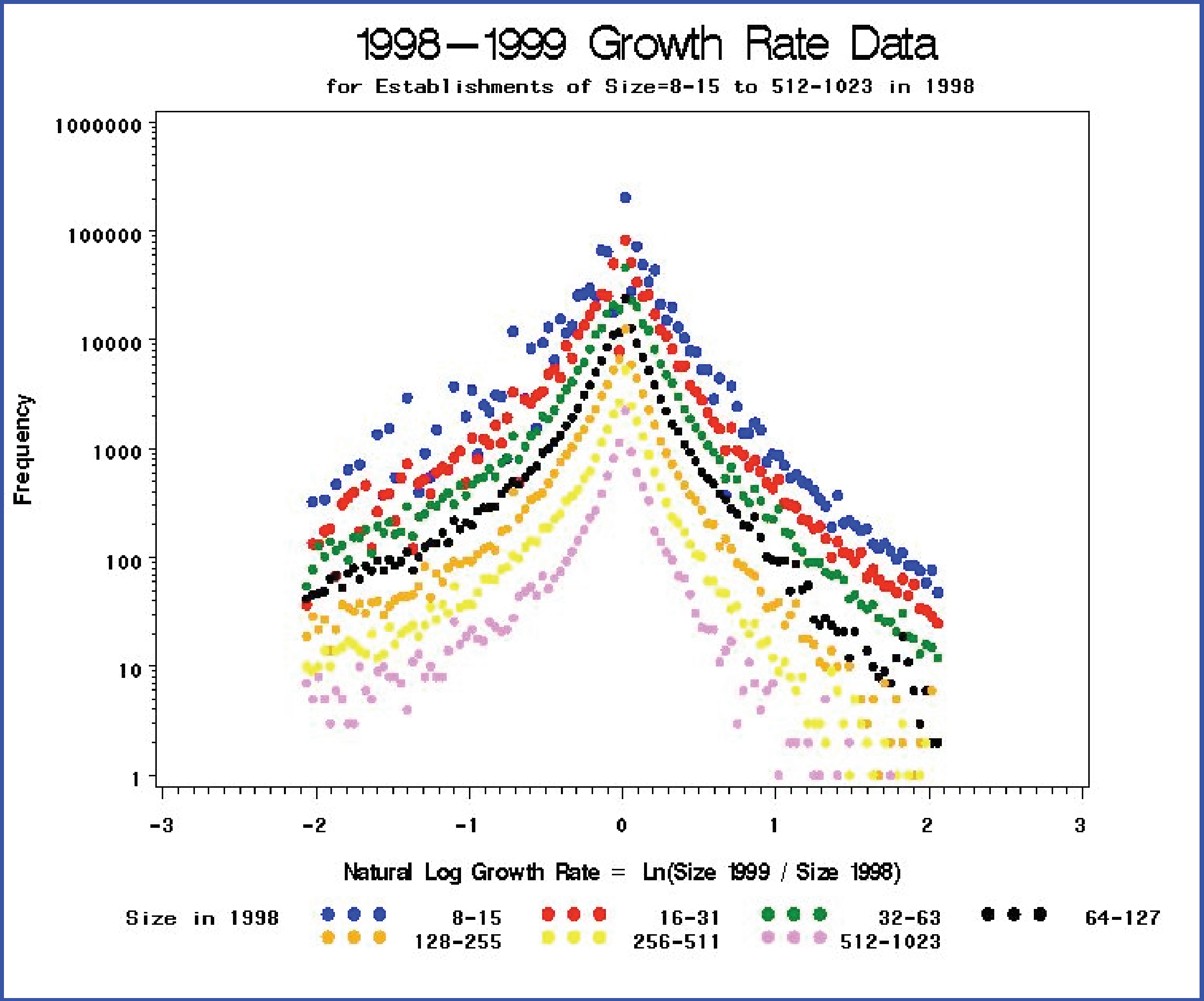
Source: Census 1997

Firm Growth Rates are Laplace Distributed over Time



Source: Axtell *et al.*, SBA-sponsored project, US Census data (Economic Census), based on IRS data

Firm Growth Rates are Subbotin Distributed



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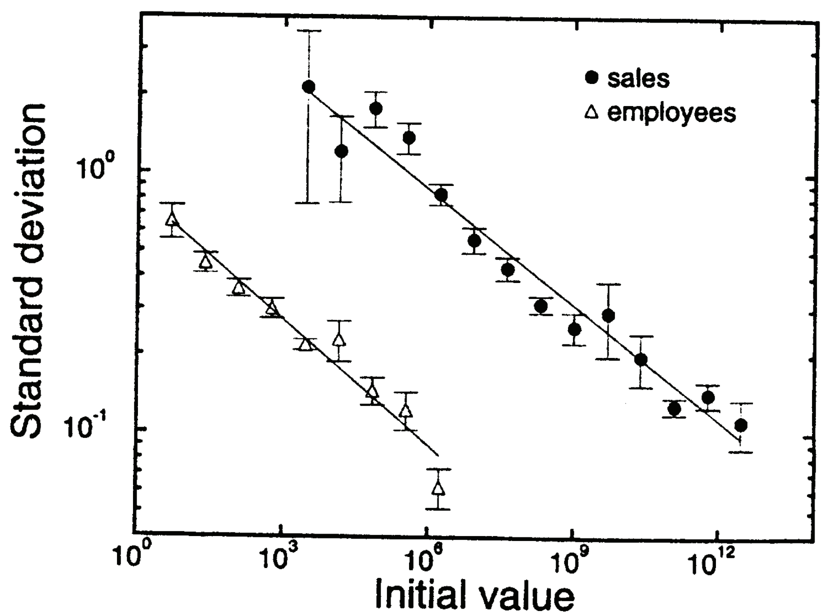
Dependence of Firm Growth Rate Variance on Size



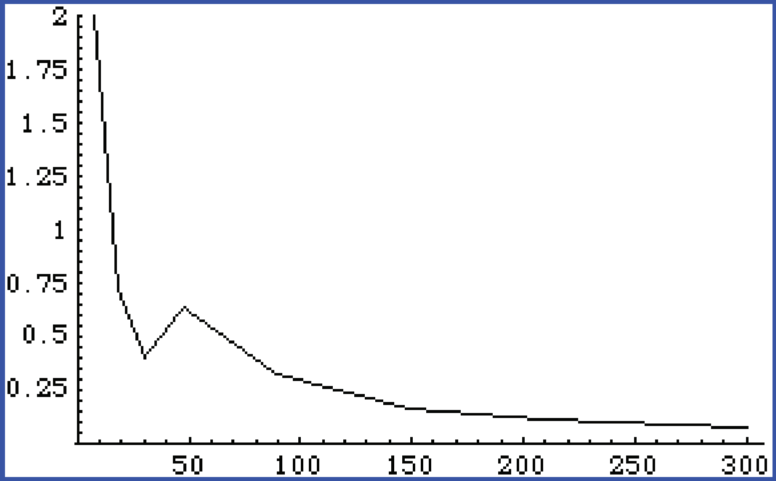
$S \sim r_0^{-\beta}$

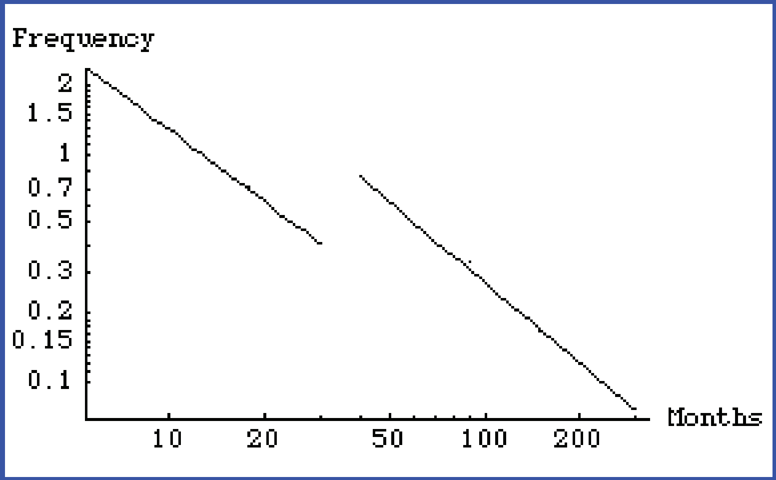
$\beta \approx 0.15 \pm 0.03$ (sales)

$\beta \approx 0.16 \pm 0.03$ (employees)

Stanley, Amaral, Buldyrev, Havlin, Leschhorn, Maass,, Salinger and Stanley, *Nature*, *379* (1996): 804-6

Job Tenure Distribution





More Firm Facts

- Wage rates increase with firm size (Brown and Medoff [1989]):
  - Log(wages) α Log(size)
  - Effect is opposite of neoclassical expectations (compensating differentials)
- Constant returns to scale at aggregate level (Basu [1997])
- Firm growth rates decline with age
- Firm exit probability declines with age

Requirements of an Empirically Accurate Theory of the Firm

- Produces a power law distribution of firm sizes
- Generates Laplace (double exponential) distribution of growth rates
- Yields variance in growth rates that decreases with size according to a power law
- Wage-size effect obtains
- Constant returns to scale
- Exponential distribution of job tenure
- Methodologically individualist (i.e., written at the agent level)
- No *microeconomic/game theoretic* explanation for any of these

Firm Formation Model

- *Heterogeneous* population of agents
- Situated in an environment of increasing returns (*team production*)
- Agents are *boundedly rational* (locally purposive not hyper-rational)
- Rules for dividing team output (*compensation* systems)
- Agents have *social networks* from which they learn about job opportunities

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Set-Up:

- Consider a group of *N* agents, each of whom supplies input ('effort') $e_i \in [0,1]$

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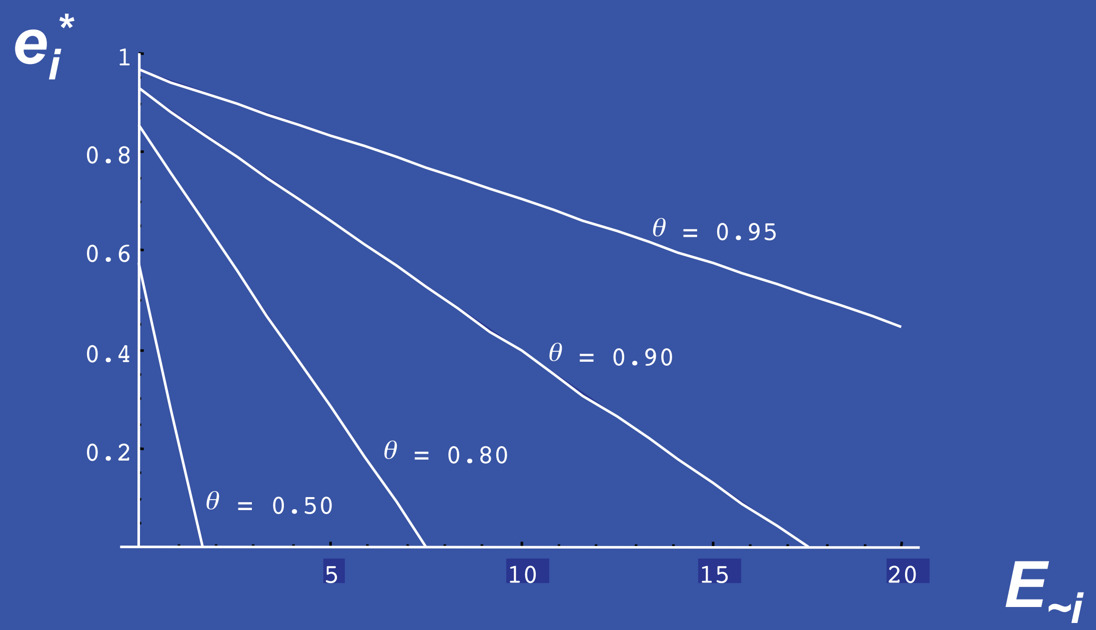
- Agents have Cobb-Douglas preferences for income (output shares) and leisure,

$$U^i(e_i) = S(e_i, E_{\sim i})^{\theta_i} (1-e_i)^{1-\theta_i}$$

NASH EQUILIBRIUM

Proposition 1: In any team, Nash equilibrium exists and is unique

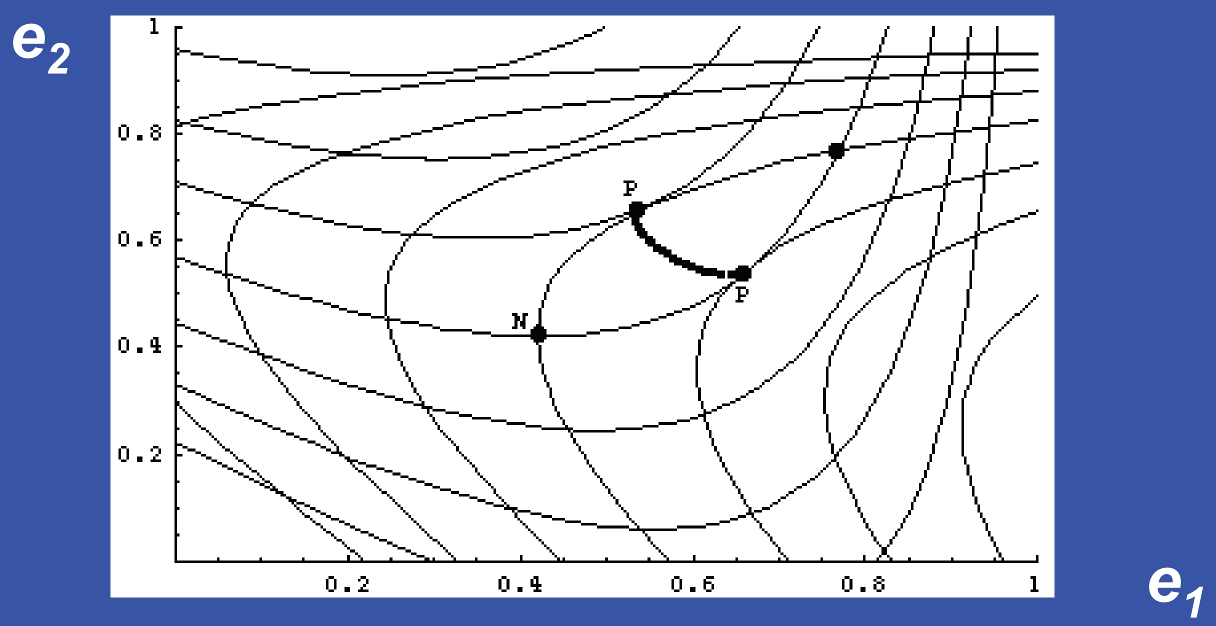
$$e_i^*(\theta_i, E_{\sim i}) = \max\left[0, \frac{-a - 2b(E_{\sim i} - \theta_i) + \sqrt{a^2 + 4ab\theta_i^2(1 + E_{\sim i}) + 4b^2\theta_i^2(1 + E_{\sim i})^2}}{2b(1+\theta_i)}\right]$$



TEAM PRODUCTION

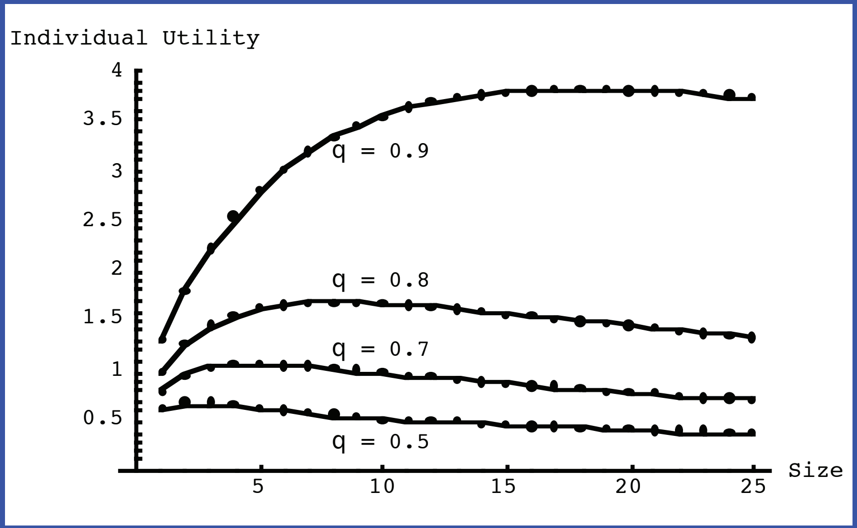
<u>Proposition 2</u>: Agents *under-supply* input at Nash equilibrium

Consider a 2 agent team:



Homogeneous Teams

Utility as a function of team size and agent type



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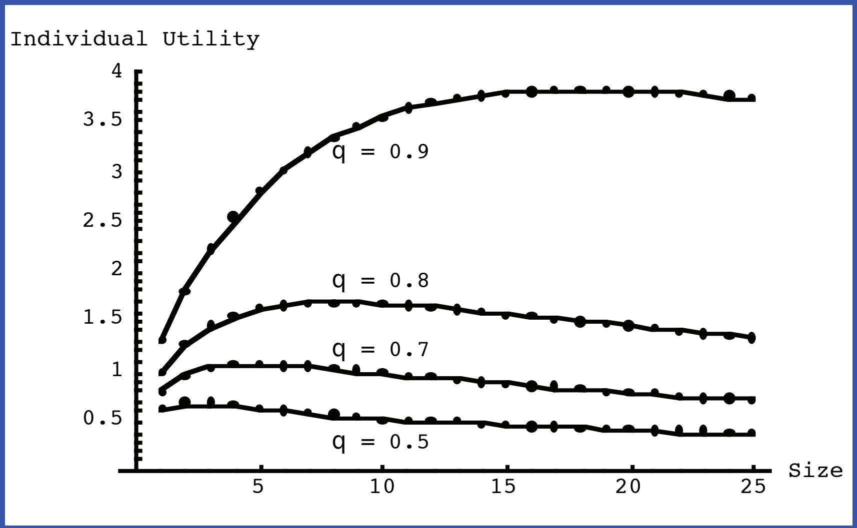
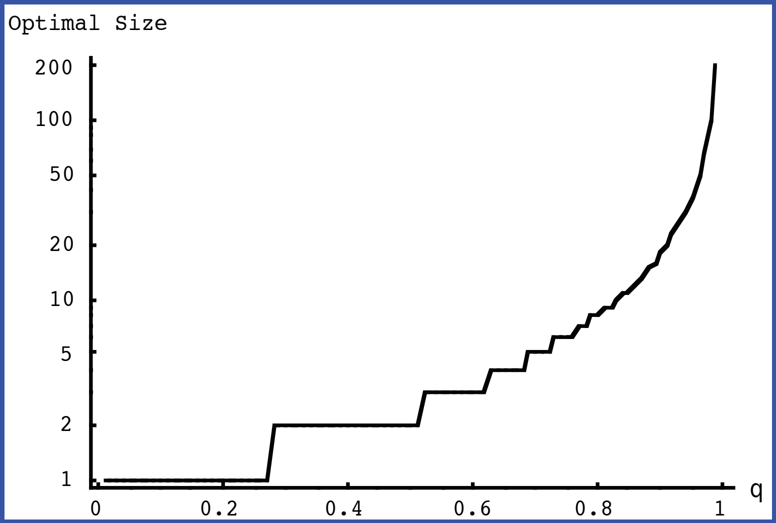


Optimal team size as a function of agent type



STABILITY OF NASH, I

Effort adjustment: Agents 'best reply' to last period's output

$$e_i(t+1) = \max\left[0, \frac{-a - 2b\left(E_{\sim i}(t) - \theta_i\right) + \sqrt{a^2 + 4ab\theta_i^2\left(1 + E_{\sim i}(t)\right) + 4b^2\theta_i^2\left(1 + E_{\sim i}(t)\right)^2}}{2b\left(1 + \theta_i\right)}\right]$$

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For *a* « *b* or $E_{\sim i}$:

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Proposition 3: There is an upper bound on stable group size

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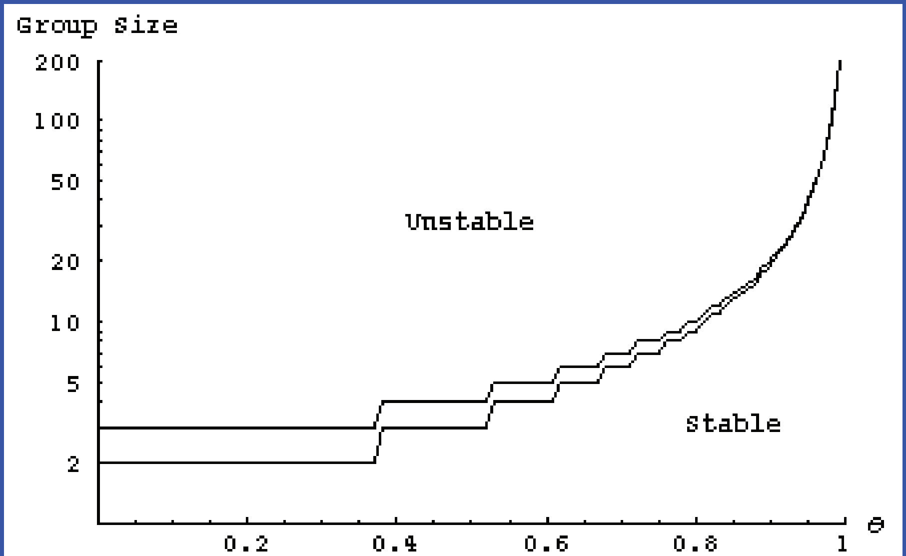
STABILITY OF NASH, III

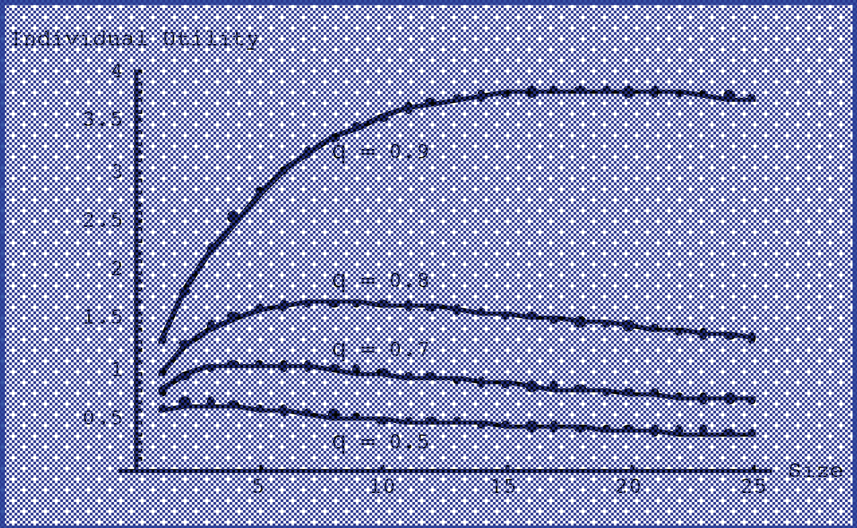
For homogeneous groups:

Stability boundary is close to size at which individual and group utilities are maximized

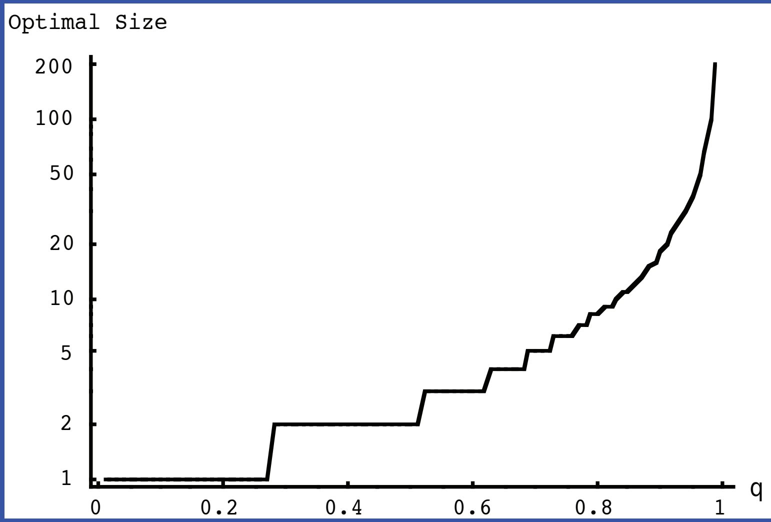


HOMOGENEOUS TEAMS

Utility as a function of team size and agent type



Optimal team size as a function of agent type



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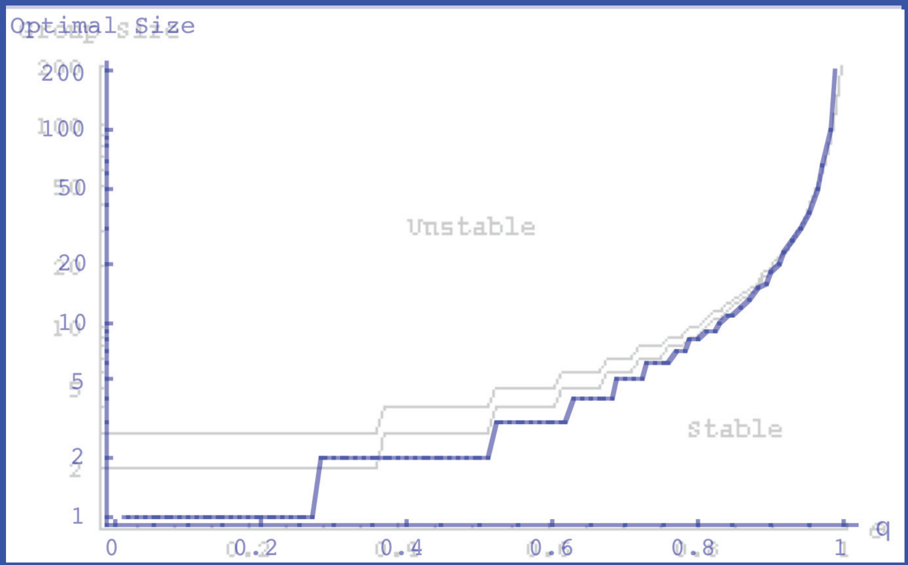

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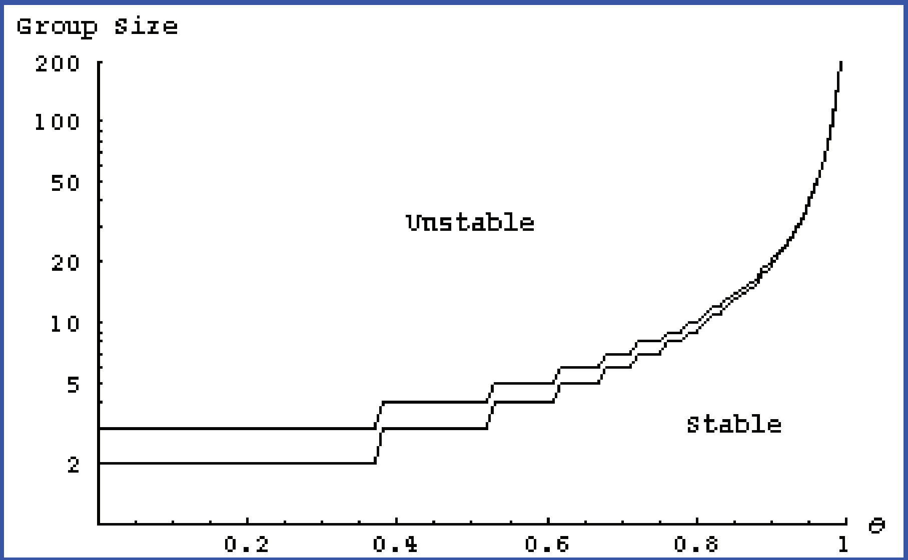

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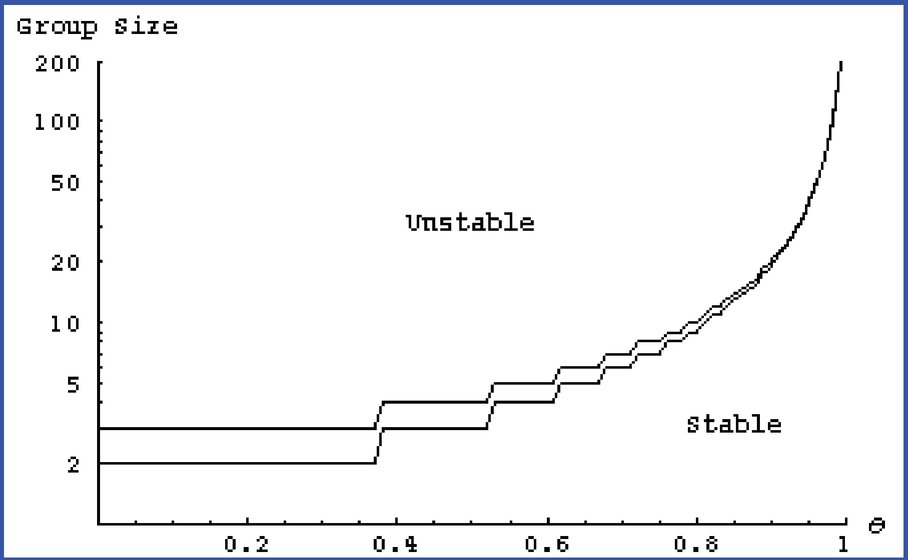

For heterogeneous groups:

$$N^{\max} = \frac{2}{1 - \max_i \theta_i}$$

Agent with largest preference for income determines maximum stable firm size

MOTIVATIONS FOR A COMPUTATIONAL MODEL

Deficiencies of the analytical model:

- Representative agent/representative group formulation
- Exclusive focus on equilibria, which provide *no information* since they are unstable
  - Unstable equilibria not explosive
  - Analogy with financial markets, *turbulence*
- Perfectly-informed, rational agents
- Synchronous updating of model with equations

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⇒Agent-based computing perfectly suited to by-pass these problems

Features of Agent Computing

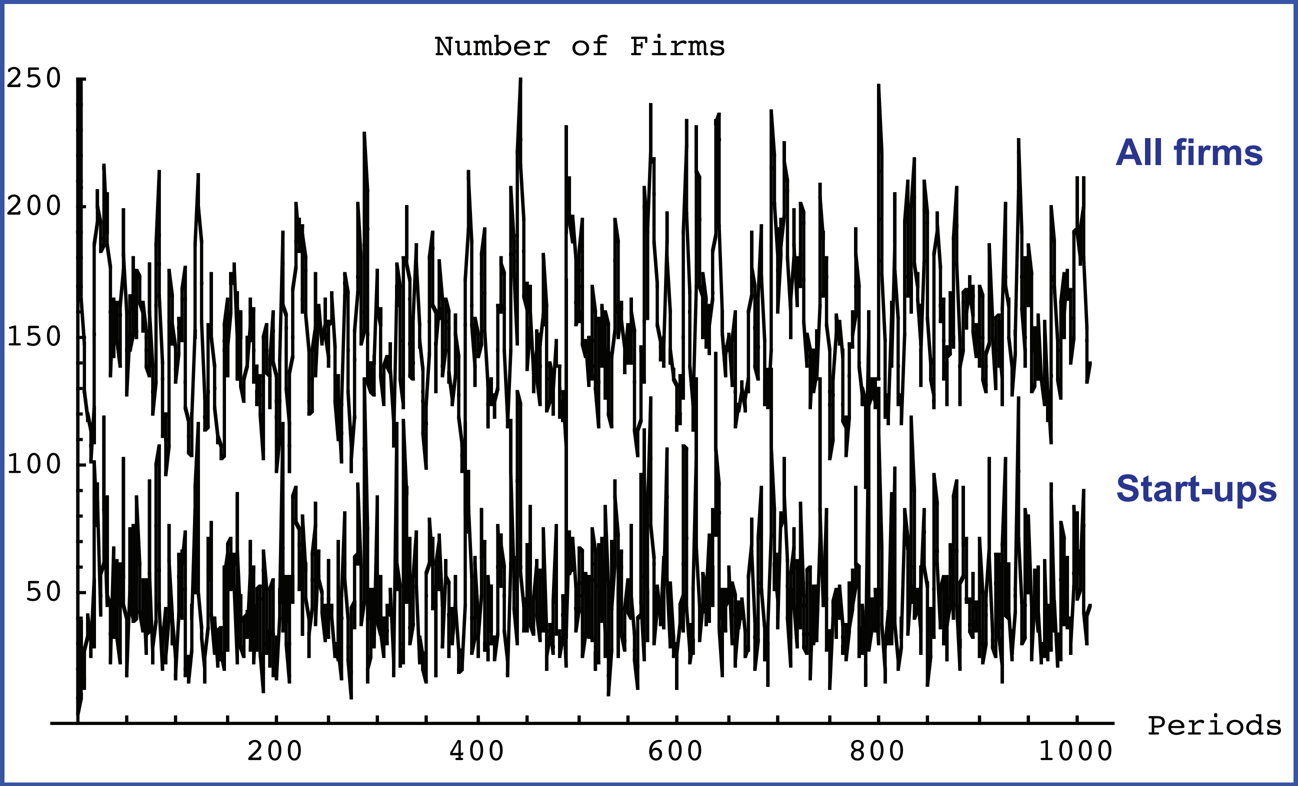
- *Heterogeneous agents*: replace representative agent, focus on distribution of behavior instead of average behavior; endogenous heterogeneity
- *Bounded rationality*: essentially impossible to give agents full rationality in non-trivial environments
- *Local/social interactions*: agent-agent interactions on some topology (e.g., graph, social network, space)
- Focus on *dynamics*: no assumption of equilibrium; paths to equilibrium and non-equilibrium adjustments
- Each realization a *sufficiency theorem*

THE AGENT MODEL

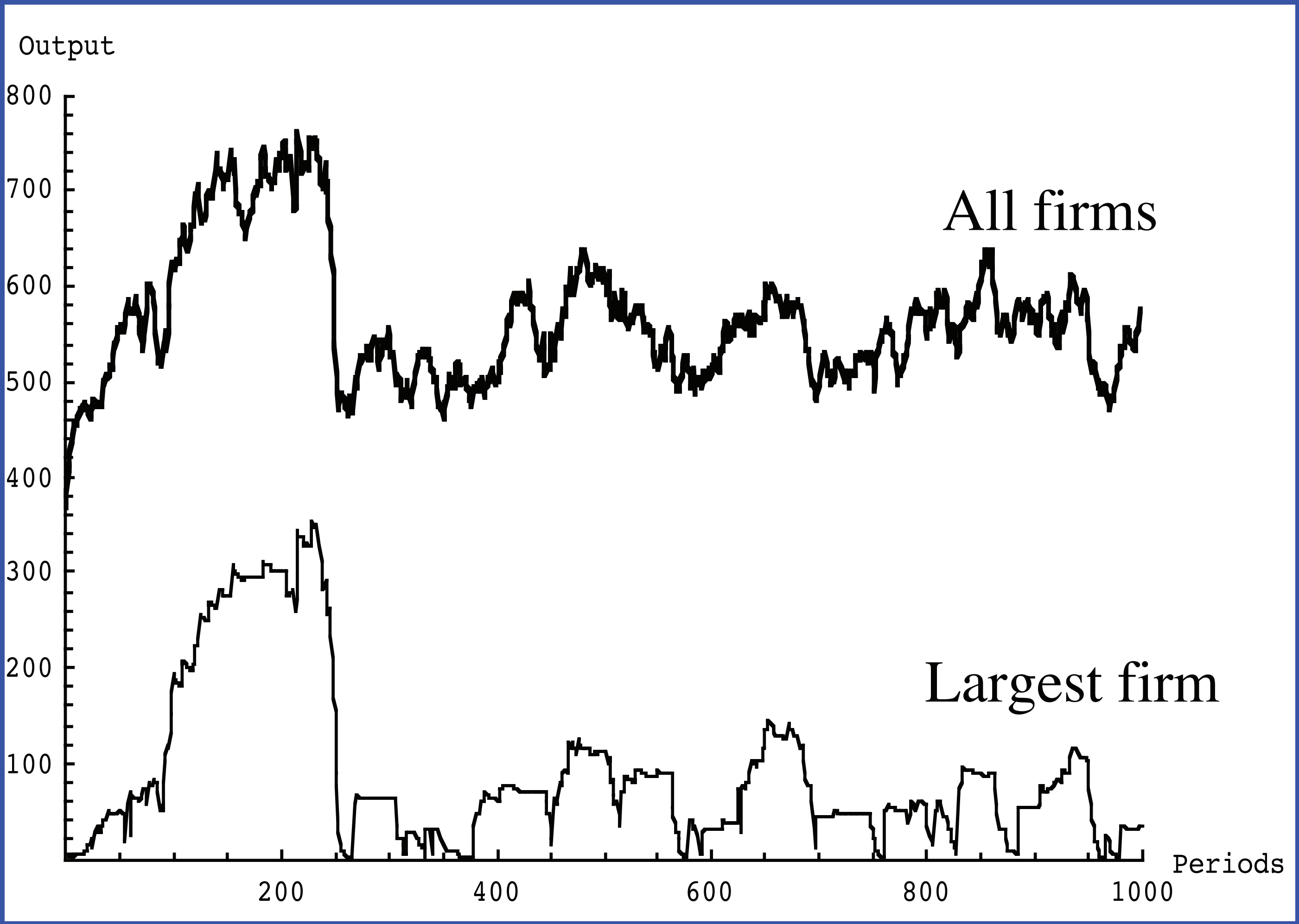
- Preference parameter, θ , distributed uniformly on (0,1)
- Firm output: $O(E) = E + E^{\beta}$, $\beta \geq 1$
- Each time period an agent is selected at random and computes its optimal effort level, e*, for:
  - staying a member of its present firm;
  - moving to a neighboring firm;
  - starting a new firm;
- The option that yields the greatest utility is selected

<Run Firms code>

NUMBER OF FIRMS TIME SERIES

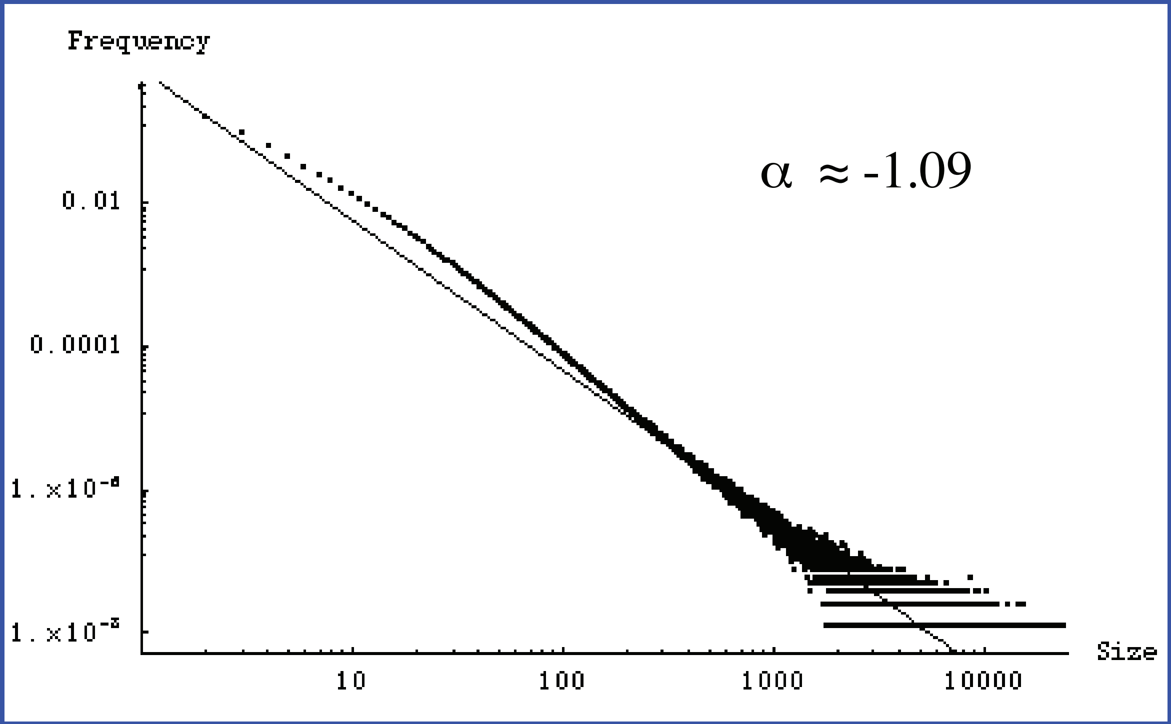


Output Fluctuations

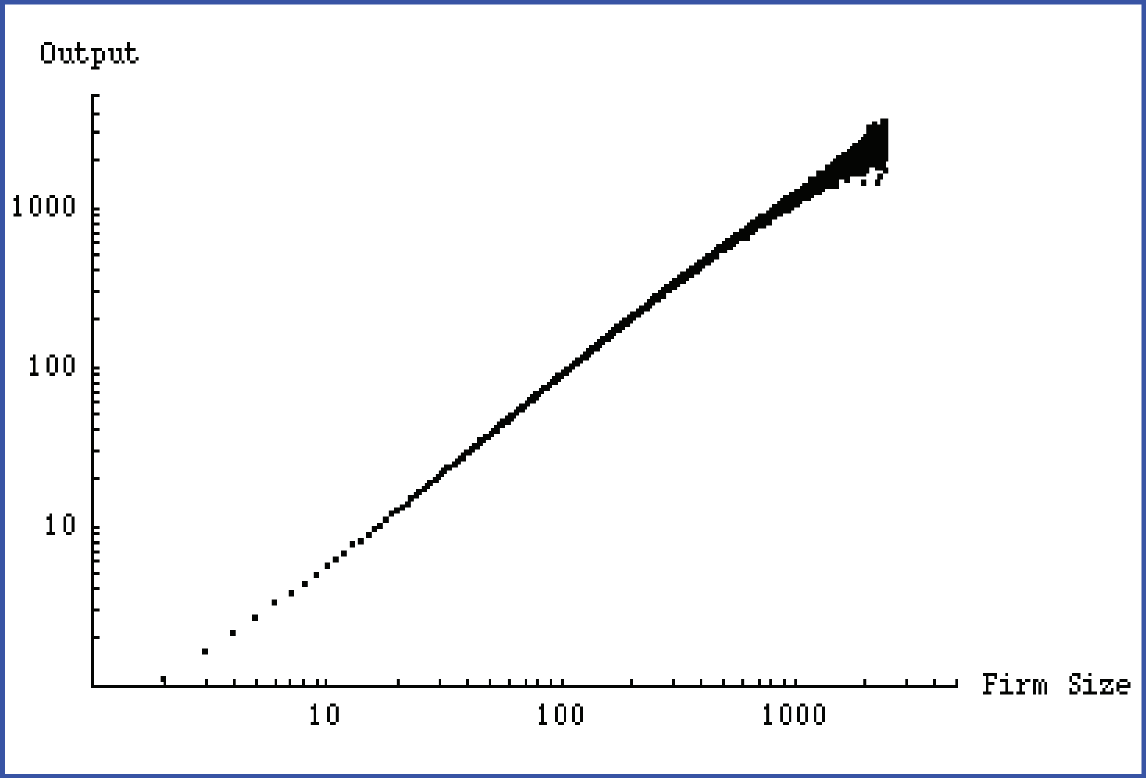


Firm Size Distribution

Firm size distribution follows a power law, f α $n^{-(1+\alpha)}$

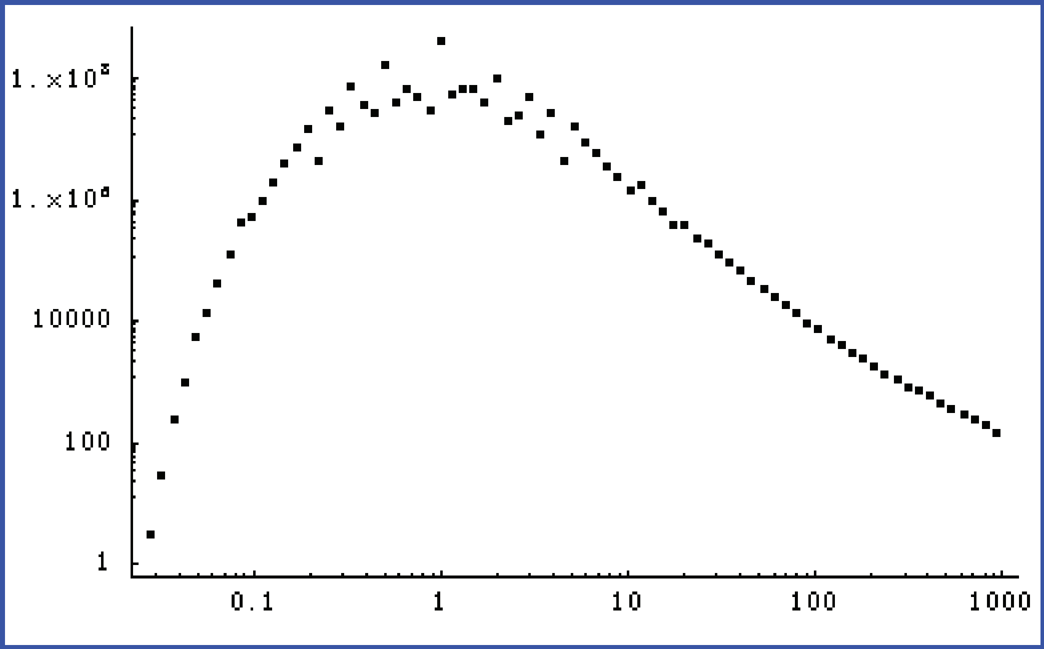


PRODUCTIVITY: OUTPUT VS. SIZE



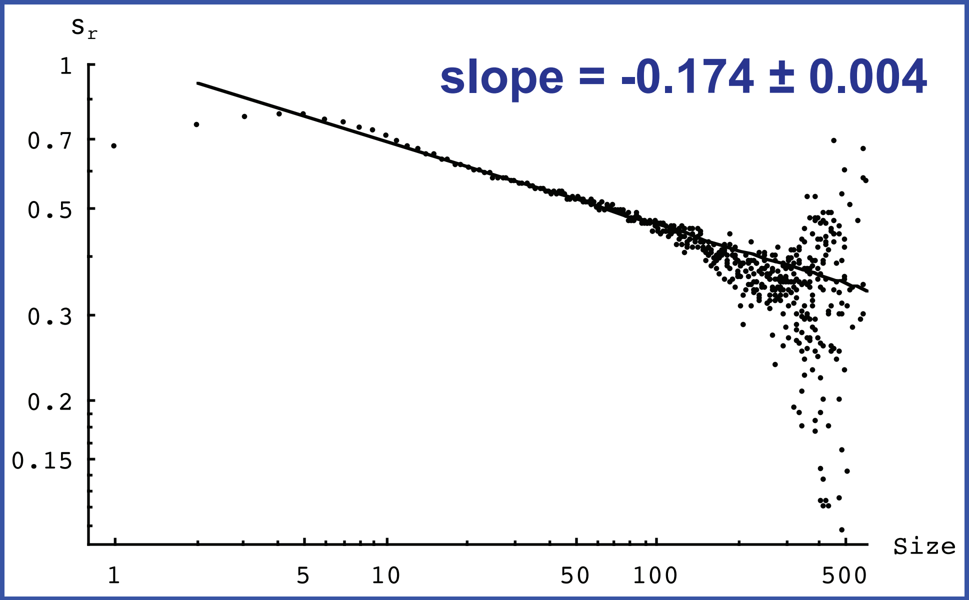
Near constant returns **at the aggregate level despite *increasing returns* at the local level**

Firm Growth Rate Distribution



Growth rates are heavy-tailed and asymmetric (more variance on firm growth than decline)

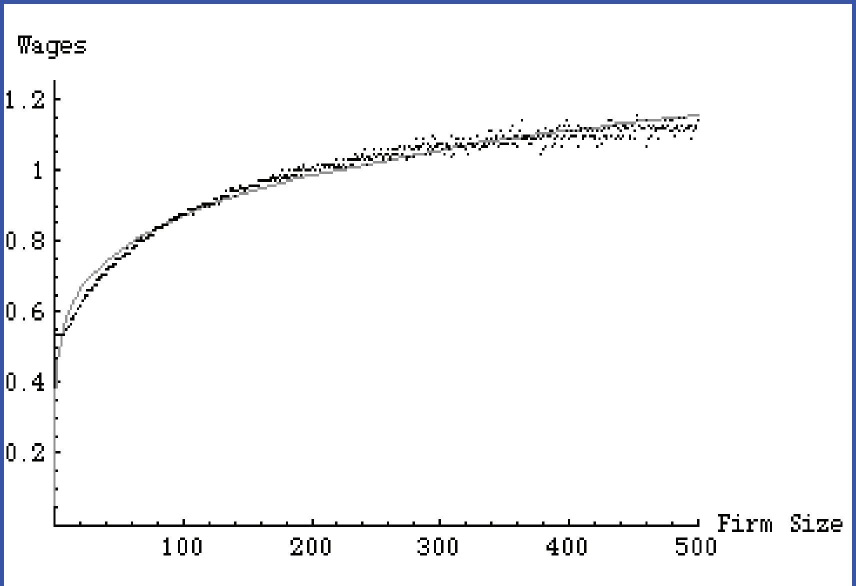
VARIANCE IN GROWTH RATES AS A FUNCTION OF FIRM SIZE

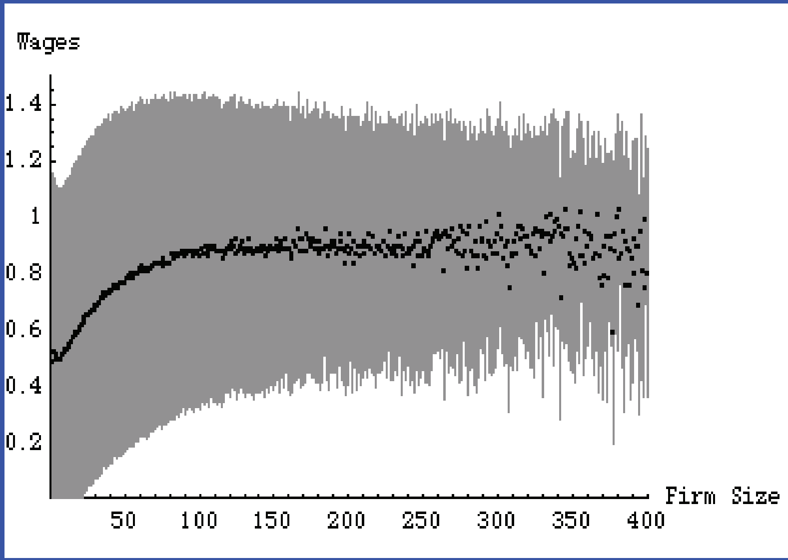


Stanley *et al.* [1996]: Slope ≈ -0.16 ± 0.03 (dubbed 1/6 law)

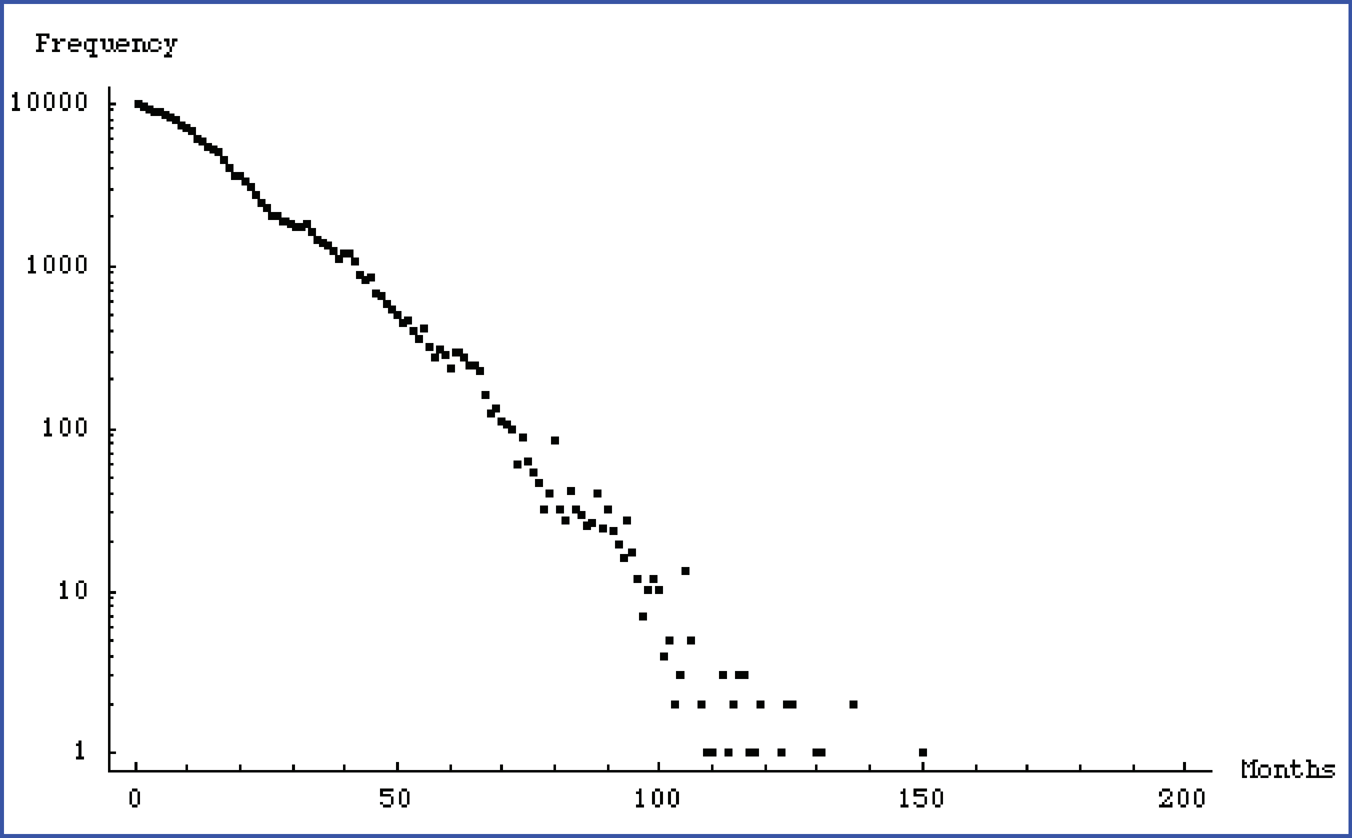
Wages as a Function of Firm Size

Brown and Medoff [1992]: wages α size $^{0.10}$

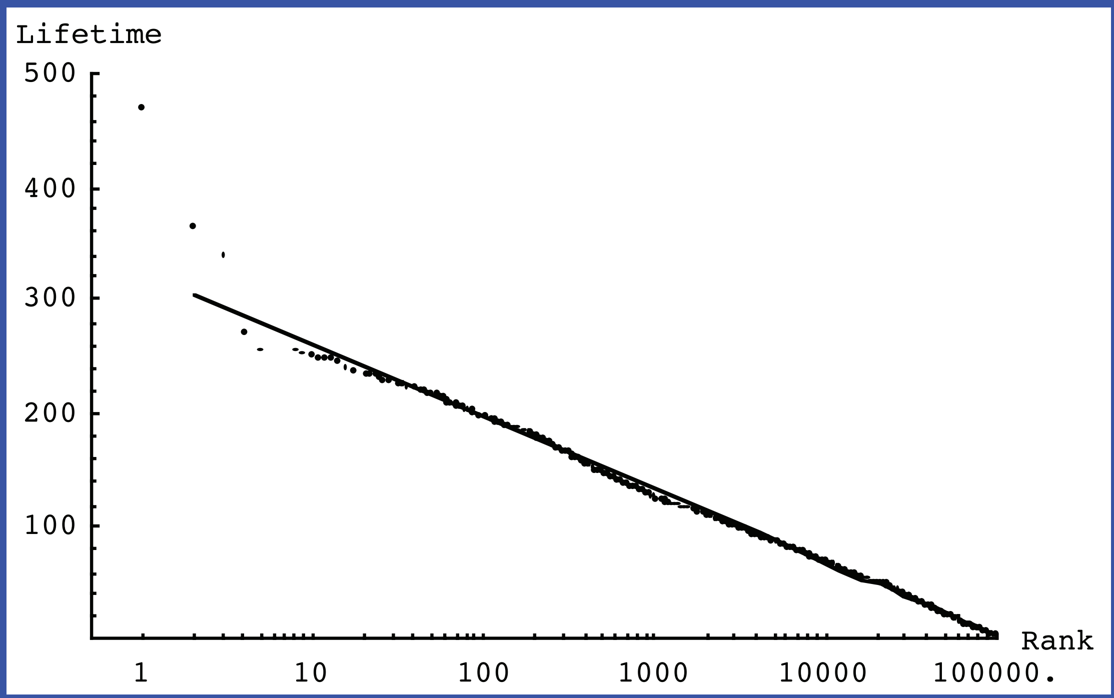




JOB TENURE DISTRIBUTION



Firm Lifetime Distribution



Data on firm lifetimes is complicated by effects of mergers, acquisitions, bankruptcies, buy-outs, and so on

Over the past 25 years, ~10% of 5000 largest firms disappear each year

Effect of Model Parametrization

- Importance of (locally) purposive behavior
- Vary *a*, *b*, and β: Greater increasing returns means larger firms
- Alternative specifications of preferences
- Role of social networks
- Agent 'loyalty' is a stabilizing force in large firms
- Bounded rationality: groping for better effort levels
- Alternative compensation schemes
- Firm founder sets hiring standards, filters new hires
- Firm founder acts as residual claimant

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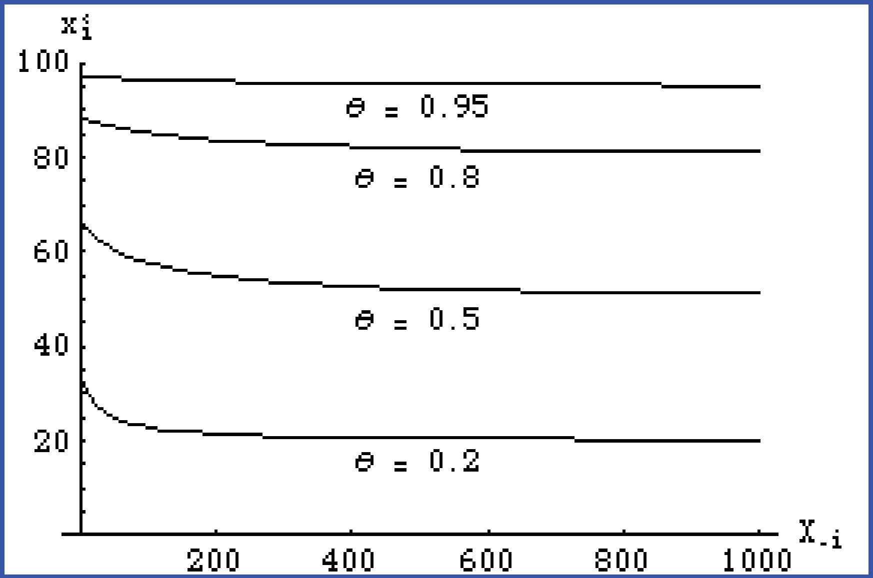
Sensitivity to Compensation System

Compensation proportional to input: $S_i(e_i, E) = \frac{e_i}{E} O(E)$

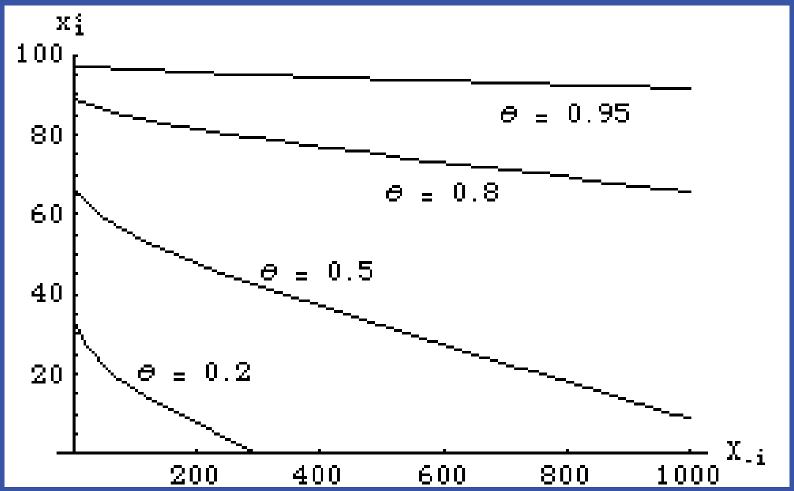


All firms now stable!

Mixed Compensation

Linear combination of compensation policies:

$$S_i(e_i,E) = \left(\delta \frac{e_i}{E} + (1-\delta)\frac{1}{N}\right) O(E)$$



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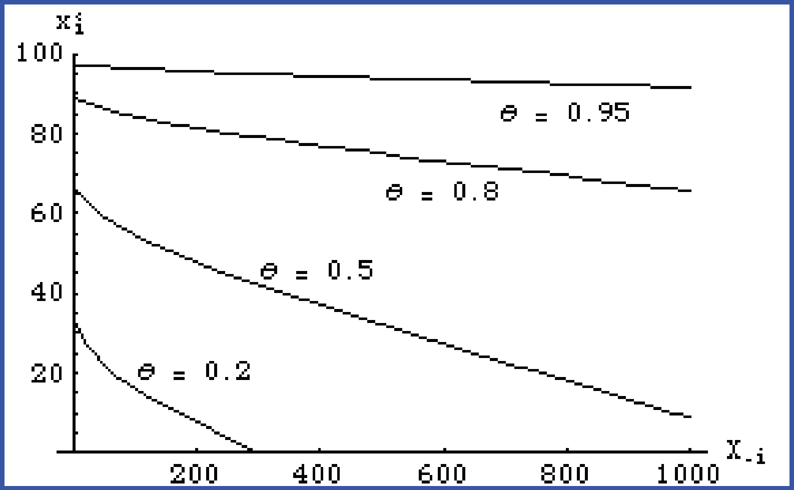

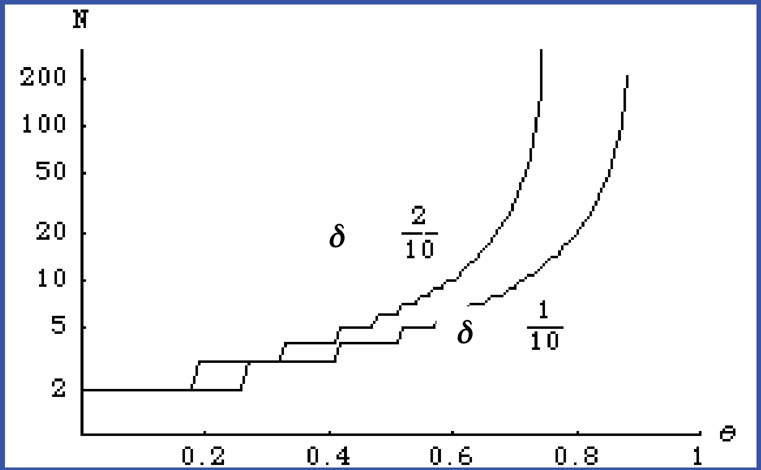

Now firms are again unstable

Summary of Agent Model

- Heterogeneous agents who 'best reply' locally and out-of-equilibrium in an economic environment of *increasing returns* with free agent entry and exit are *sufficient to generate* firms
- Highly non-stationary (turbulent) micro-data, *stationary* macro-data
- *Constant returns* at the aggregate level
- A *microeconomic* explanation of the empirical data
- Successful firms are those that can attract and maintain high productivity workers; profit maximization, to the extent it exists, is a by-product
- Analytically difficult model *tractable* with agents

SO FAR...

Microeconomic model
(purposive agents,
unstable Nash
equilibria)

SO FAR...

Computational model: micro to macro

Microeconomic model (purposive agents, unstable Nash equilibria)

So far...

Computational model: micro to macro

Microeconomic model (purposive agents, unstable Nash equilibria)

Derive macro behavior assuming agents are 'particles'

Becker-Doring Dynamics

$$1 + s \rightleftharpoons s+1$$

BECKER-DORING DYNAMICS

$$\frac{dn_s(t)}{dt} = J_{s-1,s} - J_{s,s+1}$$

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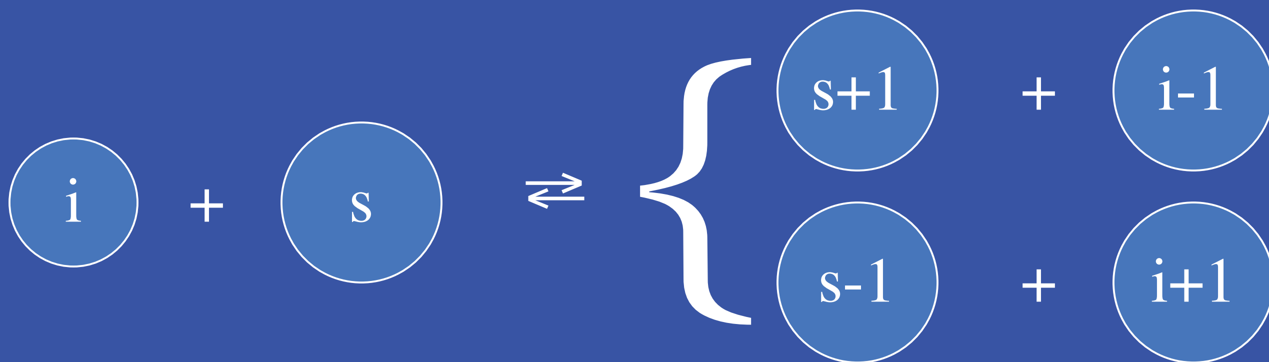

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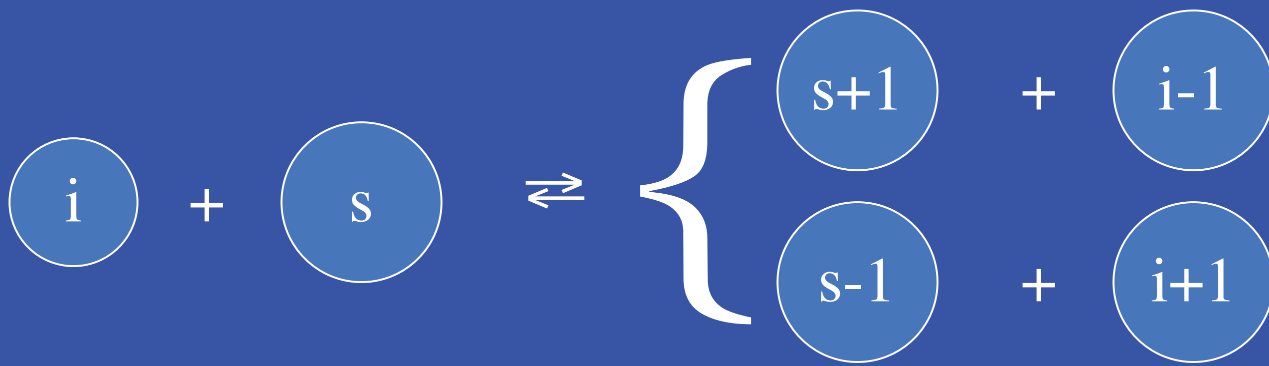

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MODIFIED BECKER-DORING
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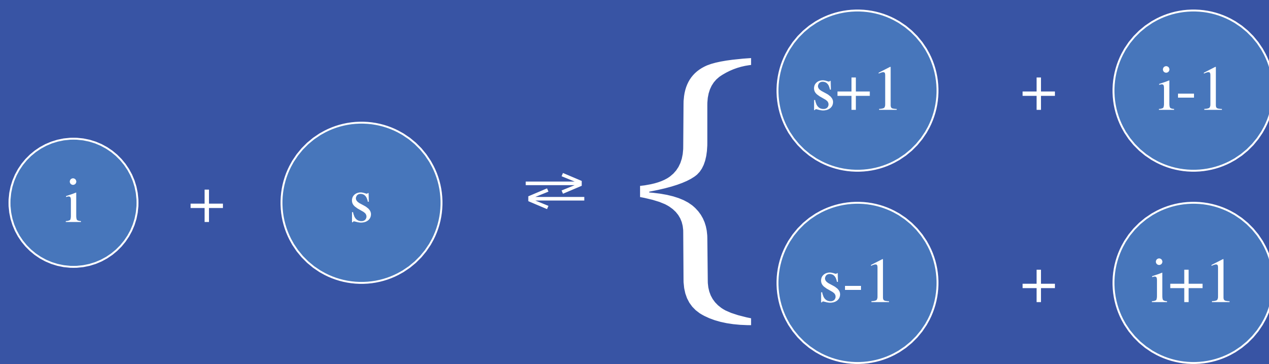

$$\frac{dn_s(t)}{dt} = J_{s-1,s} - J_{s,s+1}$$

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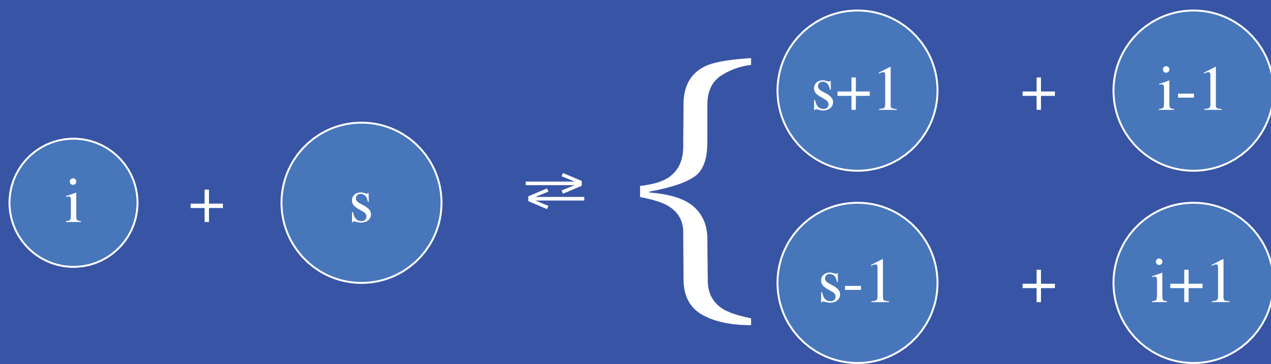

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$$\frac{dn_s(t)}{dt} = 0 \Rightarrow J_{s-1,s} = J_{s,s+1} = 0$$

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$$k_{i,s} = kis \Rightarrow csn_s = (s-1)n_{s-1}$$

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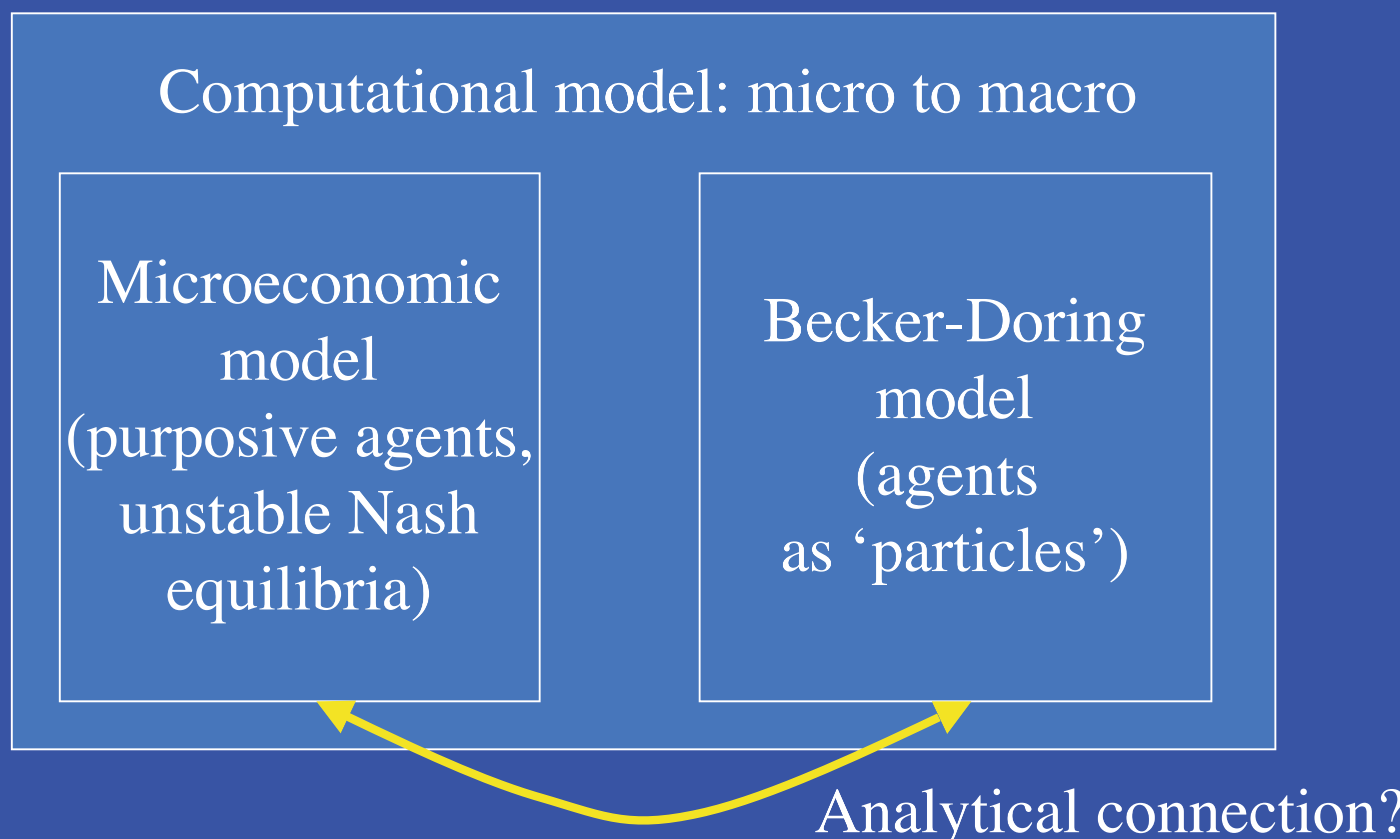
$n \sim \text{Pareto}(\alpha = 1)$

Summary...


Computational model: micro to macro
Microeconomic model (purposive agents, unstable Nash equilibria)
Becker-Doring model (agents as ‘particles’)
Analytical connection?

Agents and Game Theory, I

"We repeat most emphatically, that our theory is thoroughly static. A dynamic theory would unquestionably be more complete and therefore preferable…A static theory deals with equilibria…For real dynamics, which investigates the precise motions, usually far away from equilibria, a much deeper knowledge…is required."

von Neumann and Morgenstern

Games and Economic Behavior [1944]

Agents and Game Theory, II

"It is to be expected that mathematical discoveries of a stature comparable to that of the calculus will be needed in order to produce decisive progress in [game theory]…it is unlikely that a mere repetition of the tricks which served us so well in physics will do for social phenomena."

von Neumann and Morgenstern

Games and Economic Behavior [1944]

Outline

- *Historical remarks:* Mirowski on mathematical economics
- *Example* of a model with purposive behavior at the agent level while the macro-level seems explicable via 'particles'
- *Argue* for the generality of the example: macroeconomics

Macroeconomics

- Early macroeconomics had no *microfoundations* (Keynesian macro)
- Conventional practice today is to give micro-economic foundations to macroeconomics
  - Specifically, *representative agent* formulation of macroeconomics (1 consumer, 1 firm)
- Possibly guilty of the *fallacy of composition* (or division, depending on whether you view macro or micro having greater existential value)

MACRO IN PRACTICE

Why America must have a fiscal stimulus

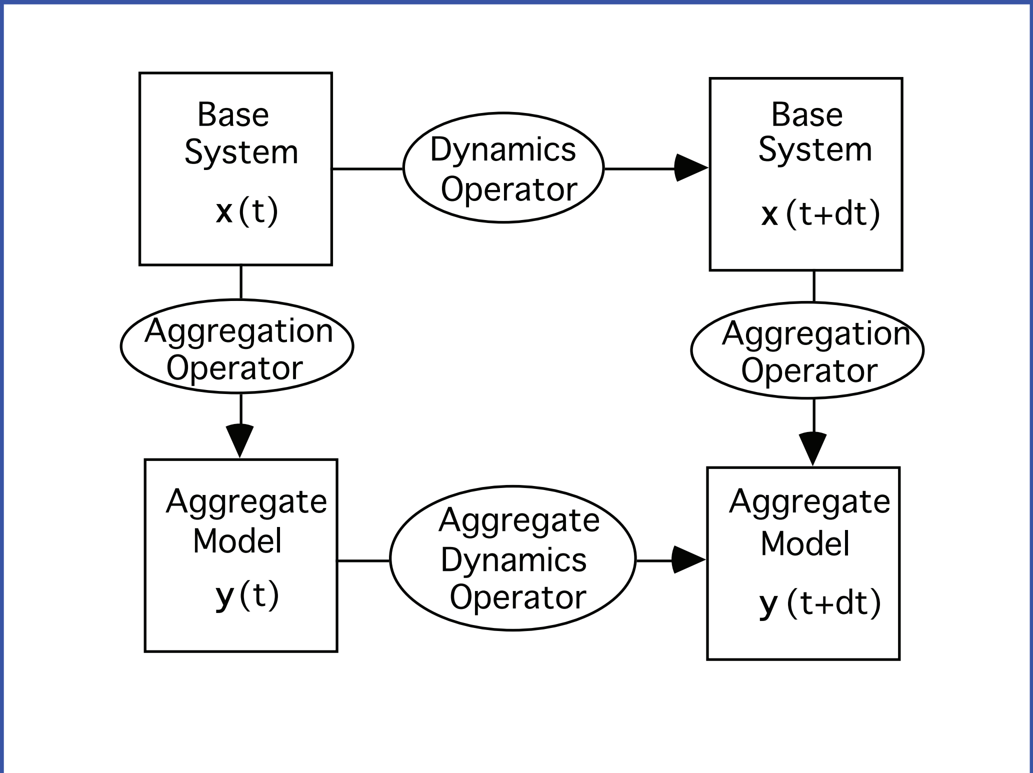
By Lawrence Summers
Published in the *Financial Times*: January 6 2008 18:27

The odds of a 2008 US recession have surely increased after a very poor employment report, growing evidence of weak holiday spending, further increases in oil prices, more dismal housing data and further writedowns in the financial sector. Six weeks ago my judgment in this newspaper that recession was likely seemed extreme; it is now conventional opinion and many fear that there will be a serious recession. Markets now predict the Federal Reserve will provide further stimulus to the economy by cutting rates by an additional 125 basis points on top of the 100 basis points by which they have already been cut so that rates fall to the 3 per cent range…

SUMMARY

- Desirability of rich behavioral specification of the agent level means *there is no physics of the micro-level!*
- However, under certain conditions there seems to be a *physics of the macro-level*.
- The general way this works is unclear.
- Significant implications for economics

WHEN DO MICRO AND MACRO LEVELS RESEMBLE EACH OTHER?

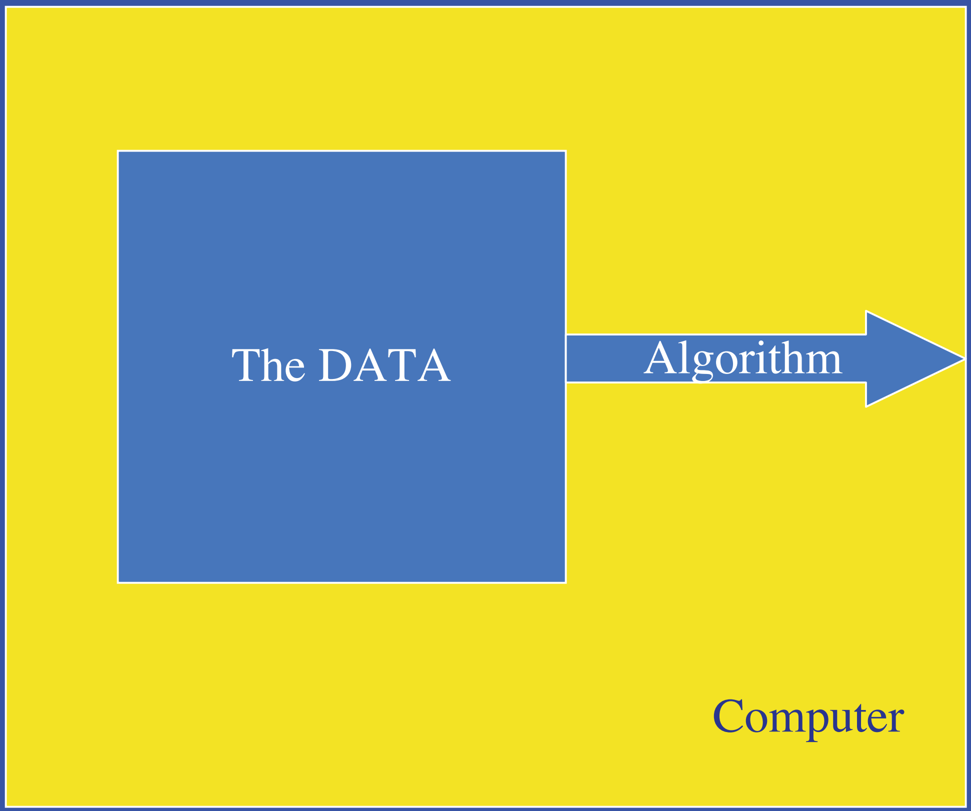


Older View of Science

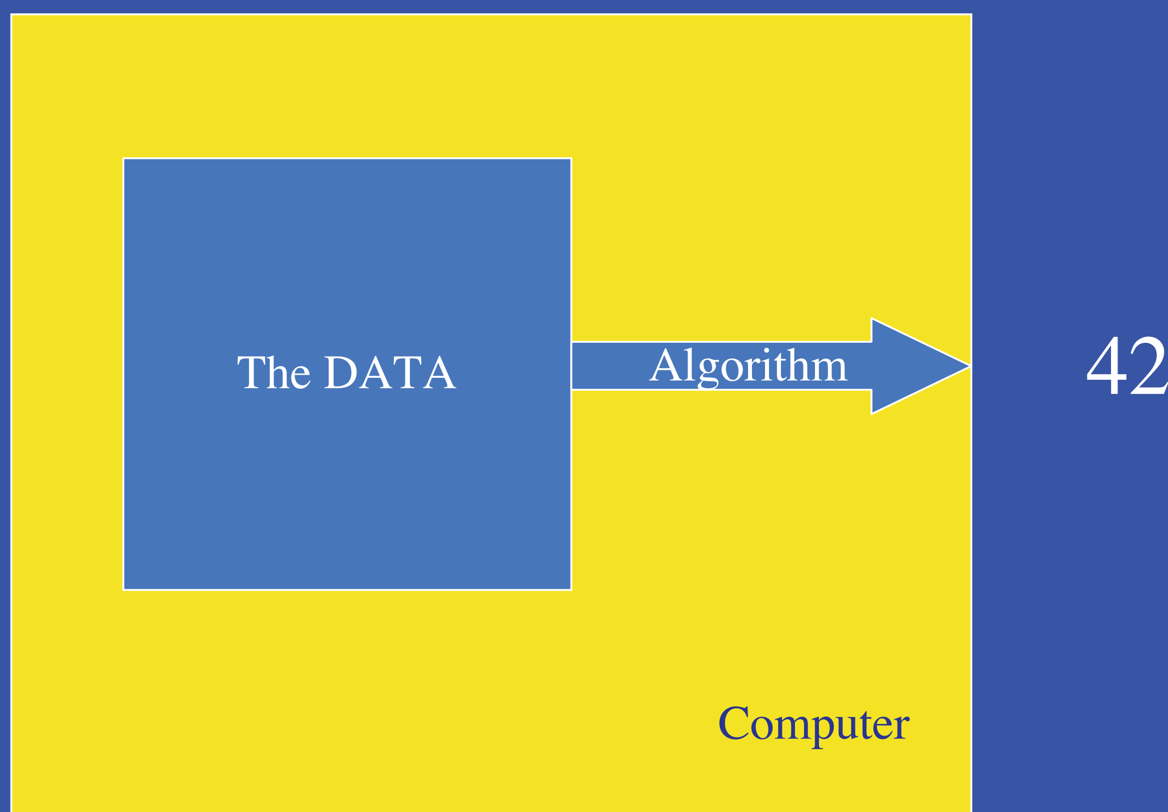
The DATA

OLDER VIEW OF SCIENCE

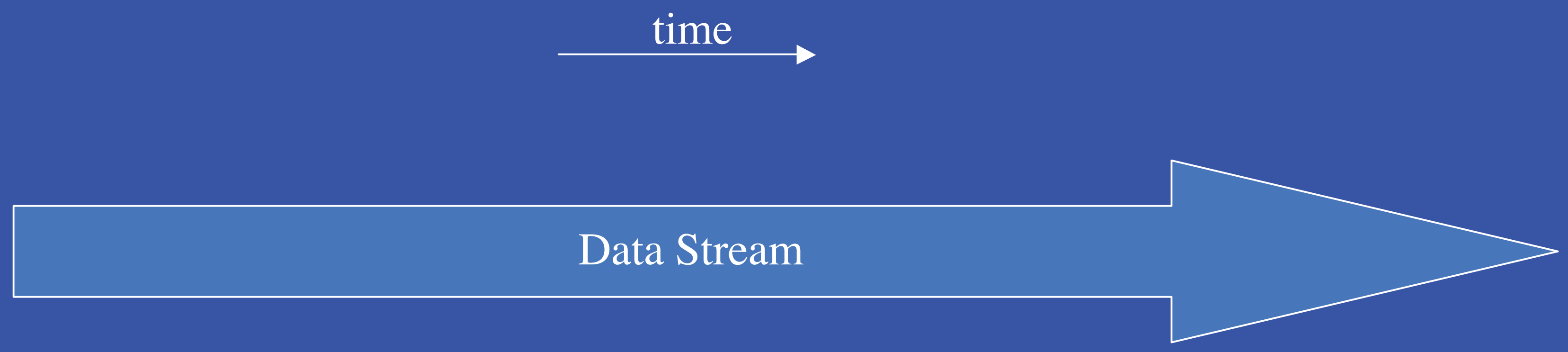
OLDER VIEW OF SCIENCE



OLDER VIEW OF SCIENCE



INTERACTIVE COMPUTING



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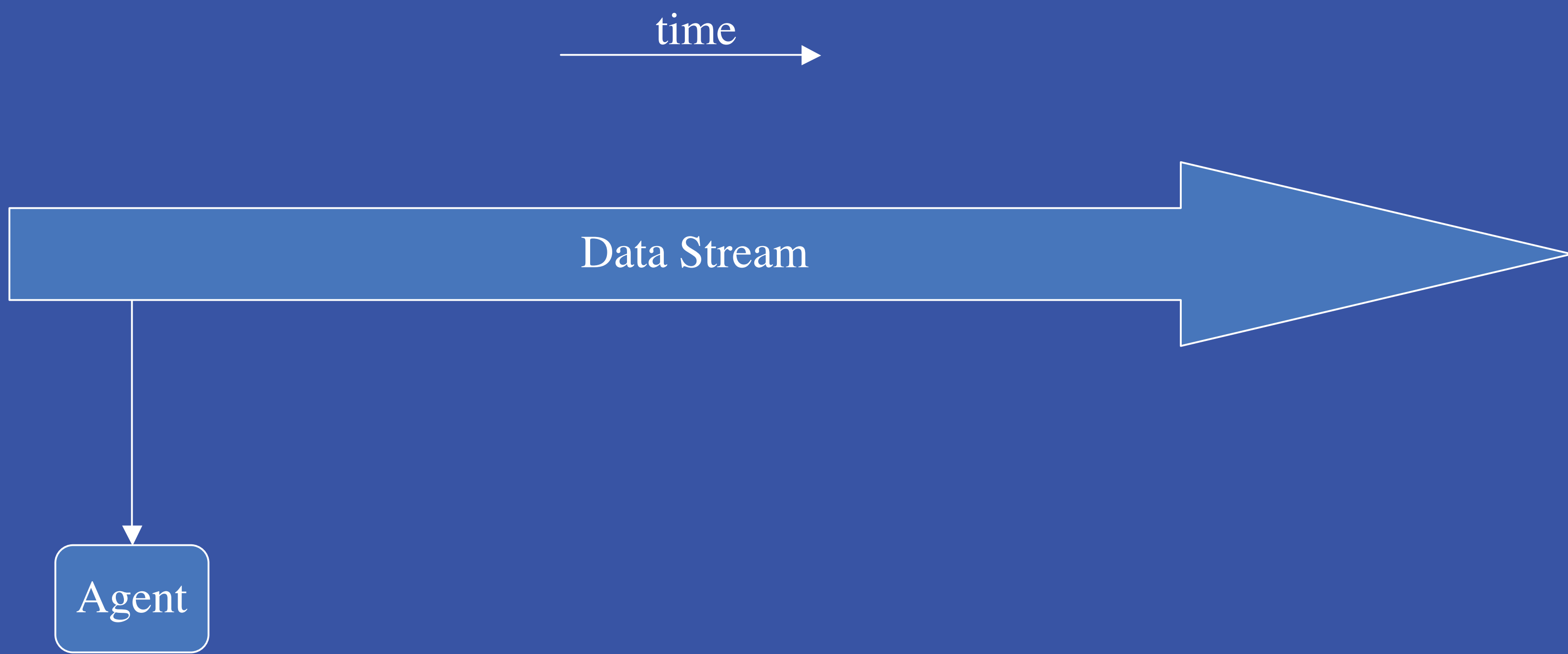

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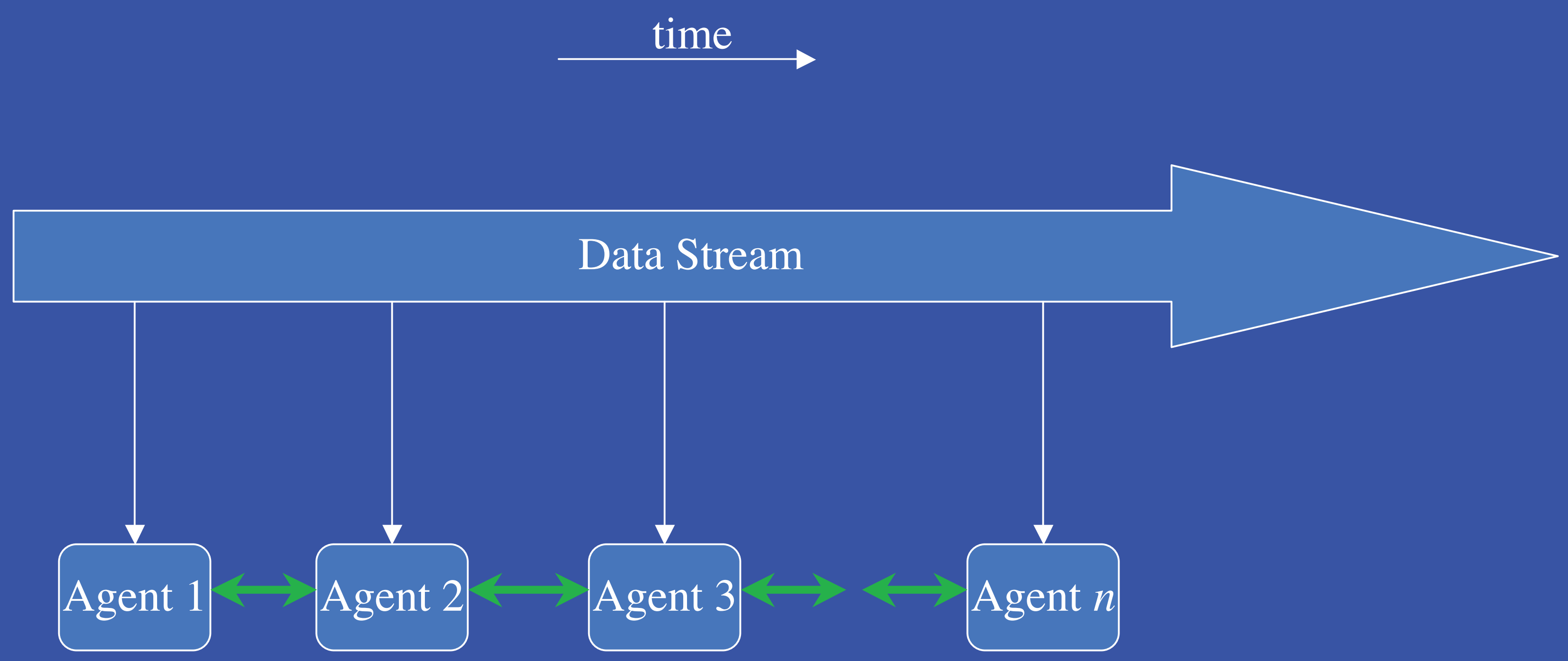

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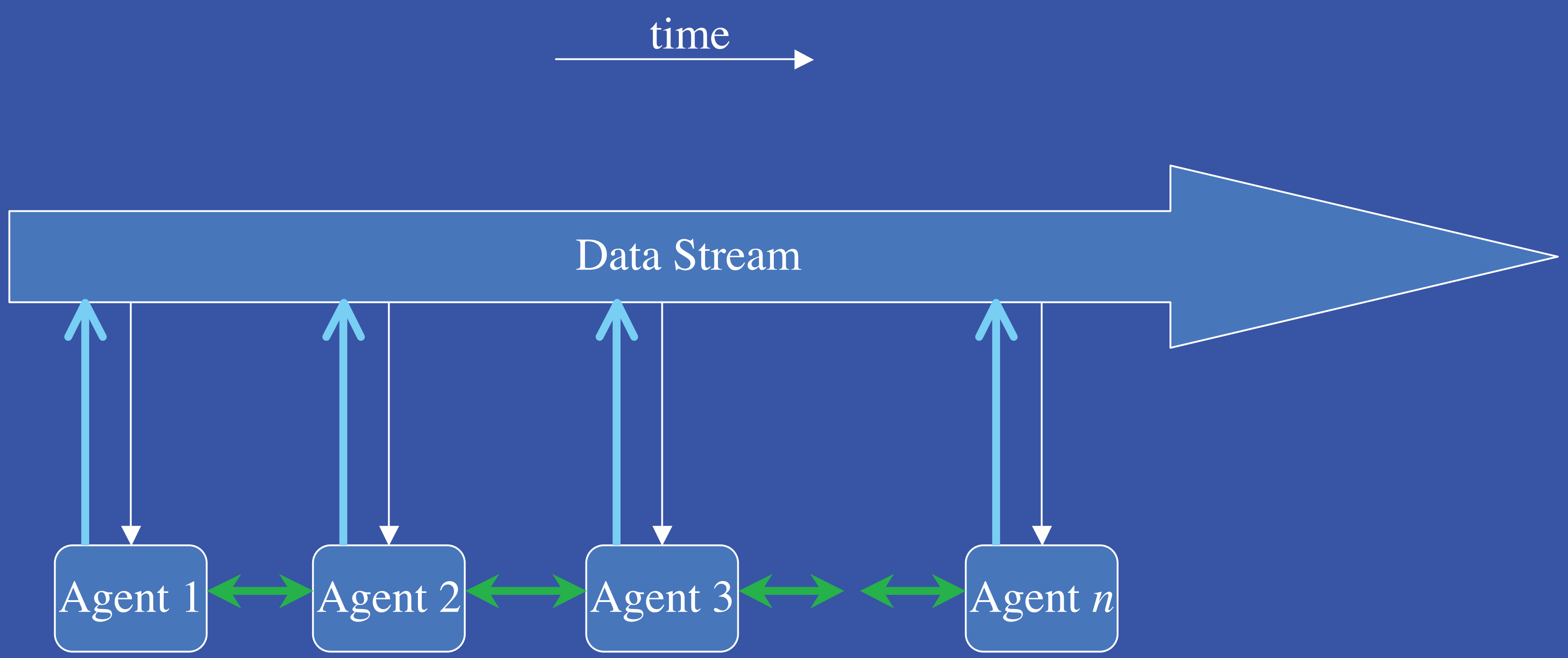

INTERACTIVE COMPUTING