Renormalization Group for Quantum Walks

Stefan Boettcher

EMORY

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Collaborators:

- Stefan Falkner (Emory U)
- Renato Portugal (LNCC, Brazil)

Support:

- NSF-Materials Research

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Quantum Algorithms

- Grover's Abstract Algorithm for Quantum Search
- Quantum Walks (QW) on the 1d-Line

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- Grover's Abstract Algorithm for Quantum Search
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The Renormalization Group (RG)

- Motivation: What is Universality in QW?
- RG for 1d-Walks, classical and quantum
- Results for Mean-Square Displacement
- Problems and Challenges
- QW on the Dual Sierpinski Gasket (DSG)

QW on the Dual Sierpinski Gasket (DSG)

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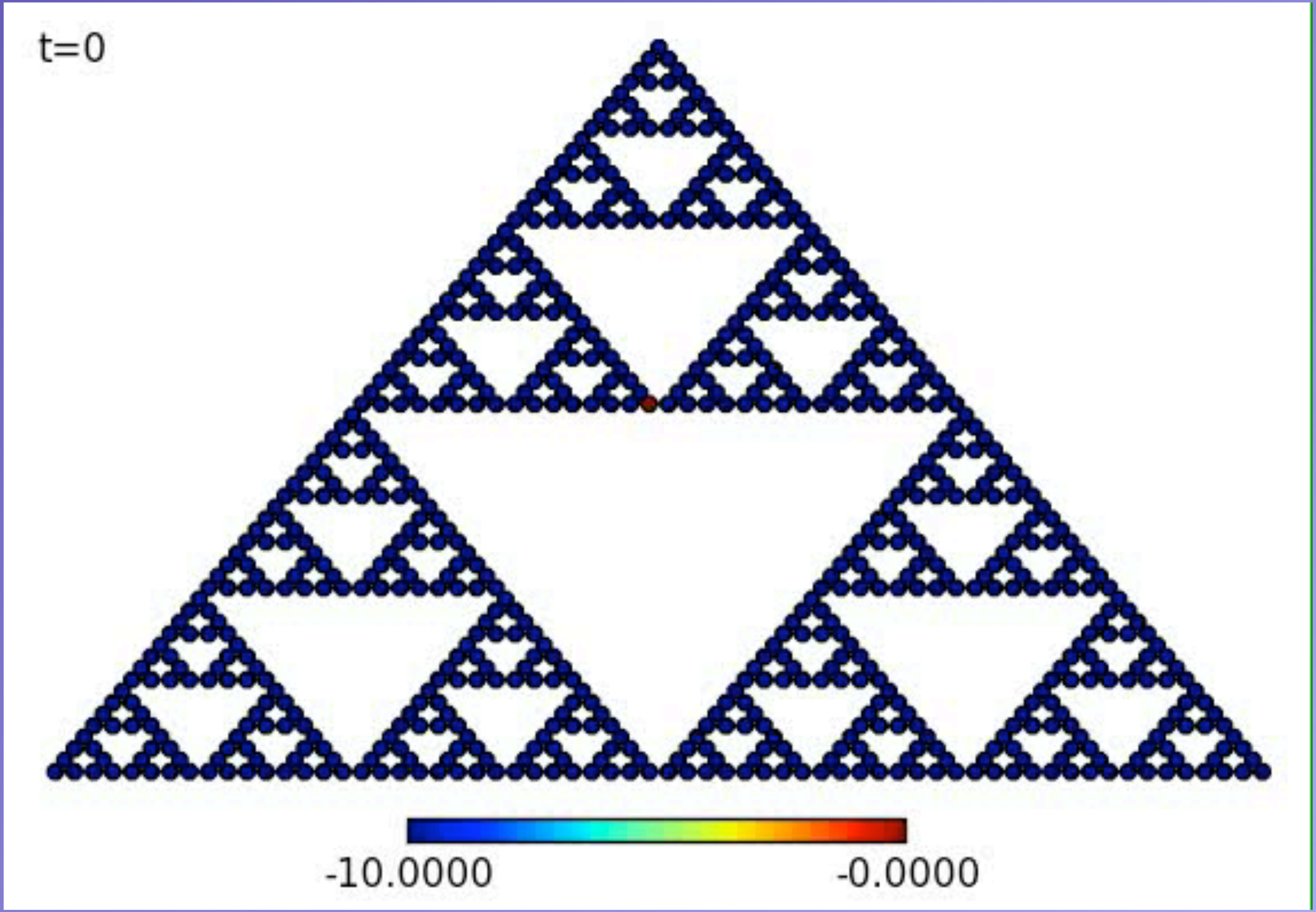
QW on the Dual Sierpinski Gasket (DSG)



Classical Search:

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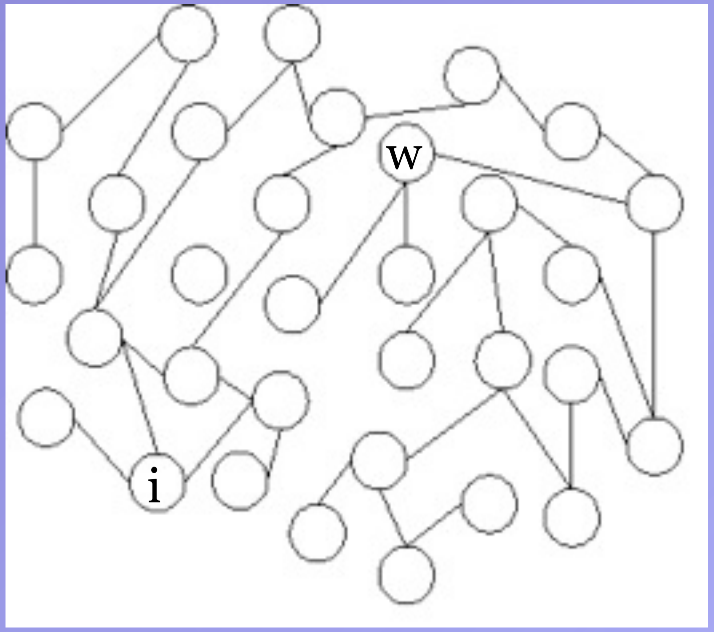
Problem ("First-Passage Time"):

Within any Structure of N sites,
Design an algorithm,
To find an arbitrary site w,
From any initial site i.



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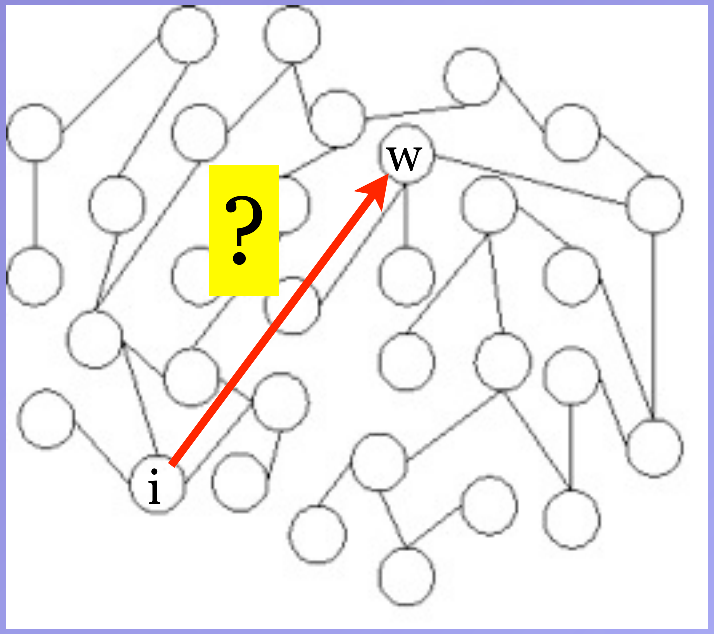

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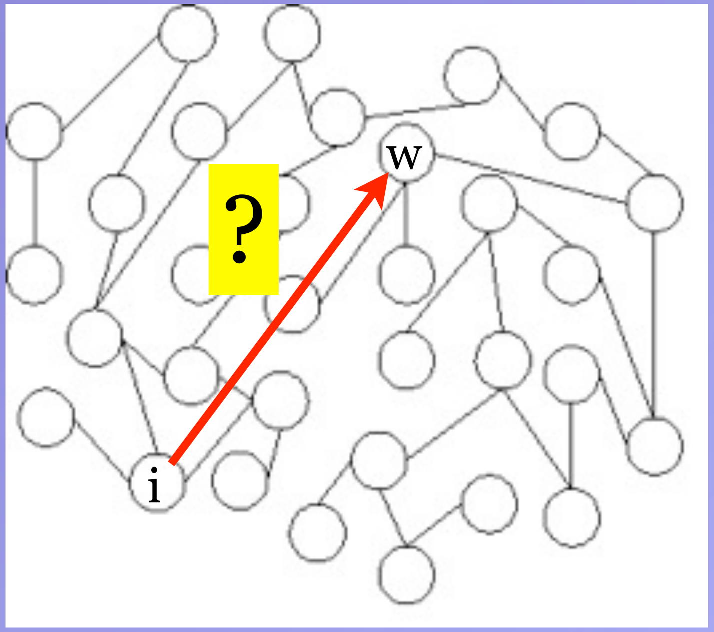
In general: $$T_{min} \sim O(N)$$



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Quantum Search:

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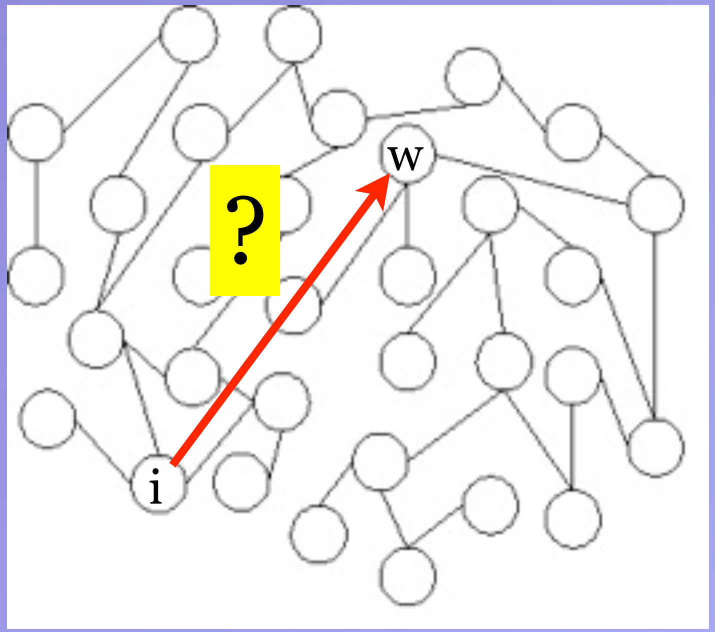
Within any N-dim. Hilbert Space (site-basis $\{|i\rangle\}$),
Design a propagator U,
To concentrate on an arbitrary site-state $|w\rangle$,
From a uniform initial state $|\Psi(0)\rangle = |s\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |i\rangle$.



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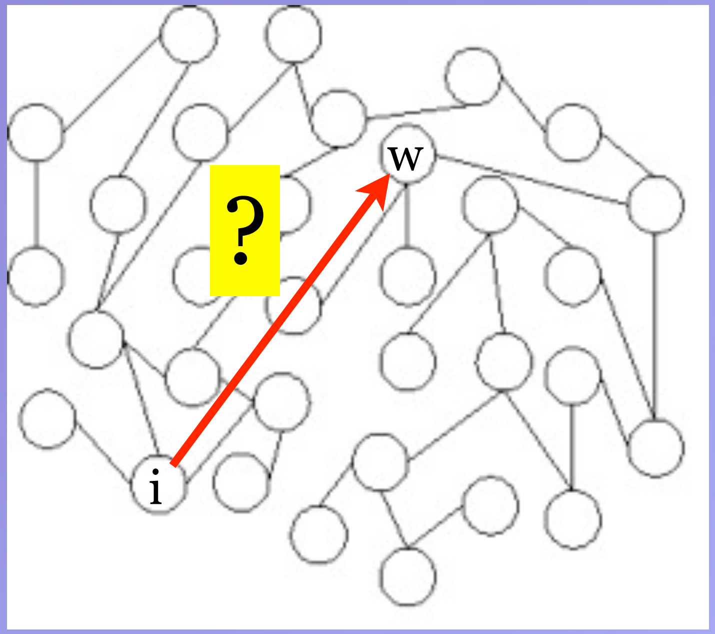
Grover (PRL1997):

$$T_{min} \sim O(\sqrt{N})$$



Grover's Algorithm:

$$|\Psi(0)\rangle = |s\rangle = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|i\rangle, \quad \text{goal:} \quad T_{min} = \min_t\left\{|\langle\Psi(t)|w\rangle| \sim 1\right\}$$

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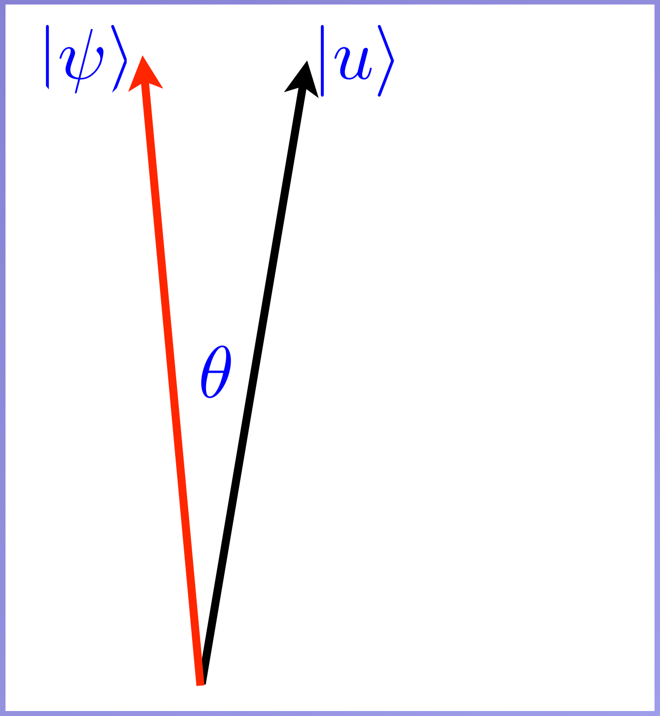

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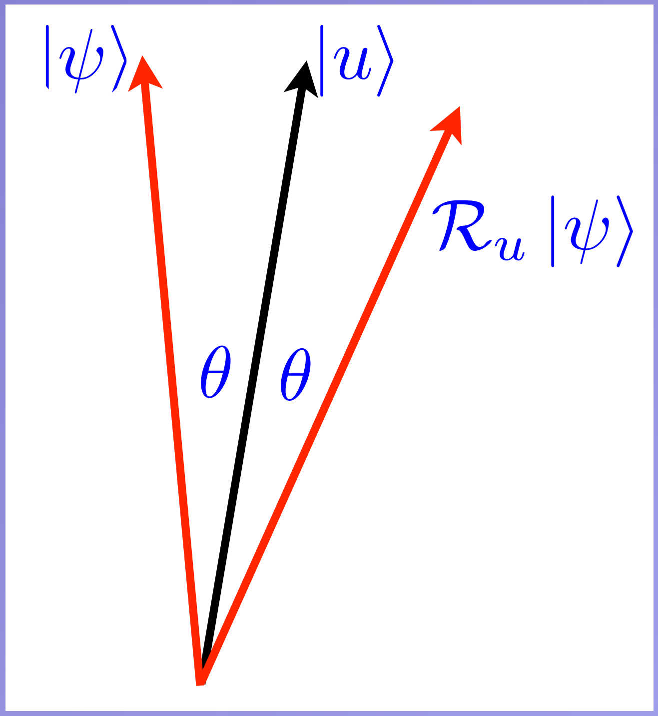

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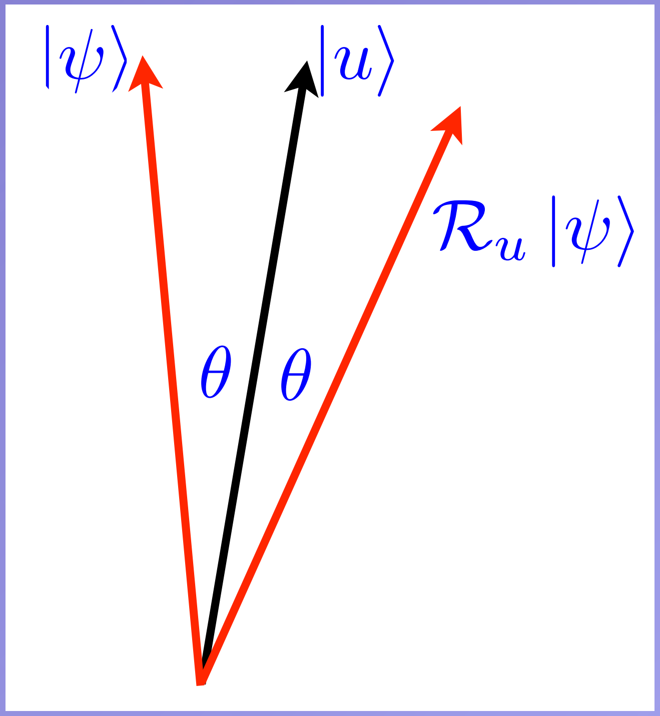

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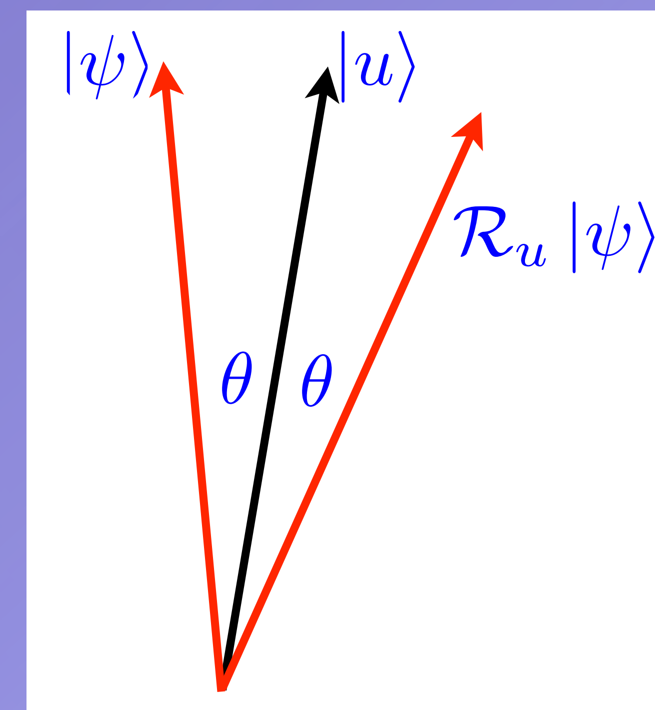

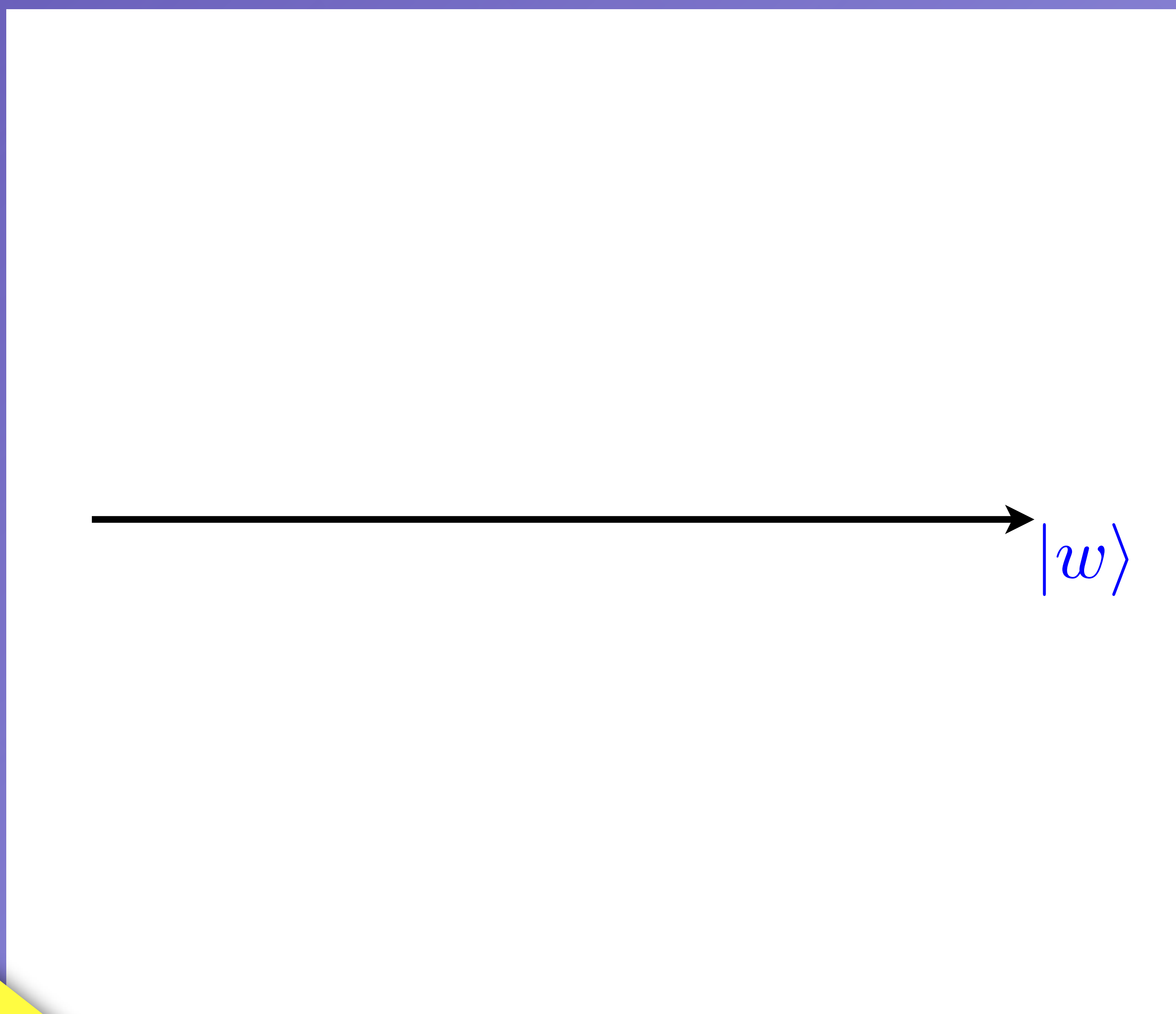

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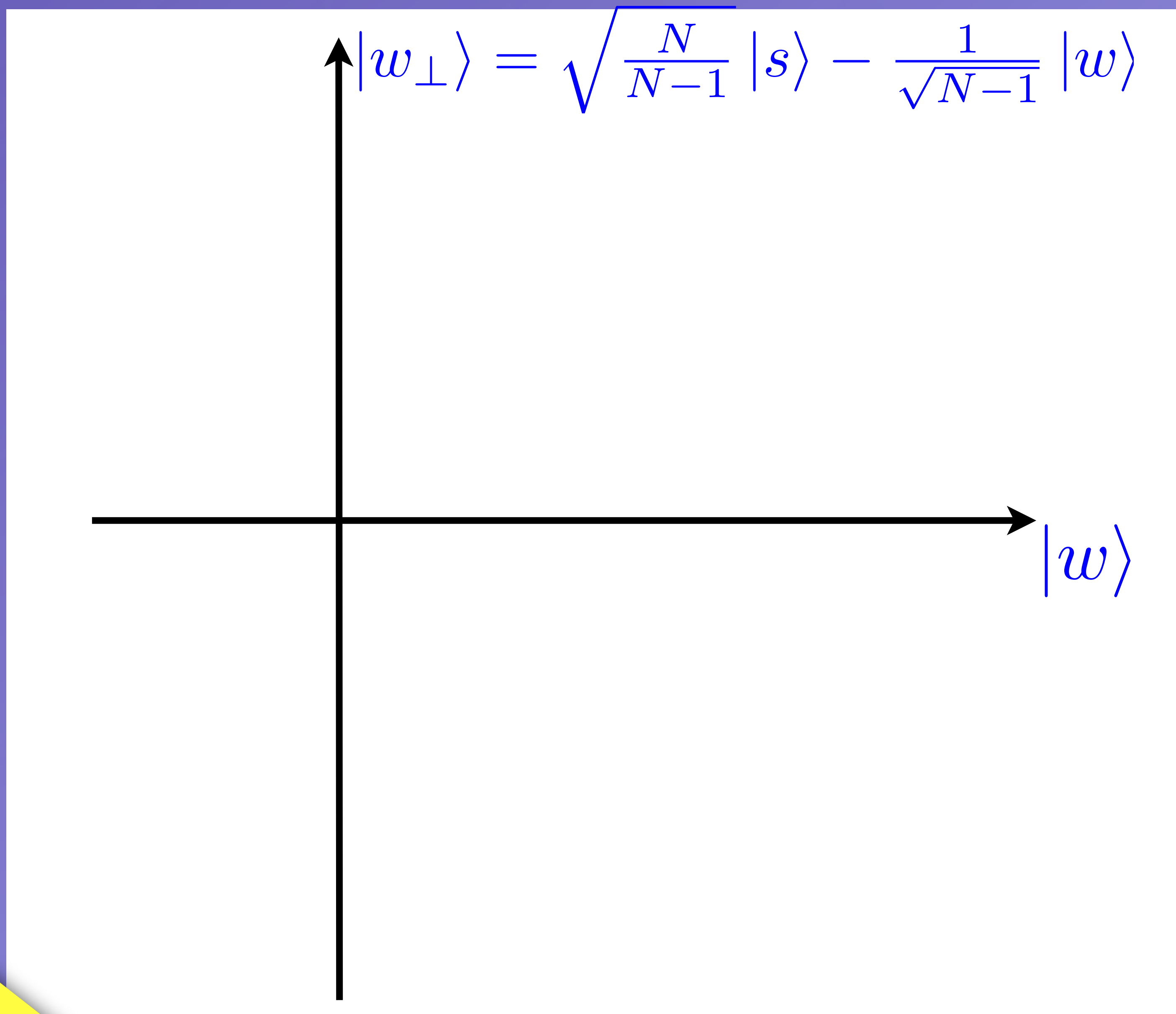

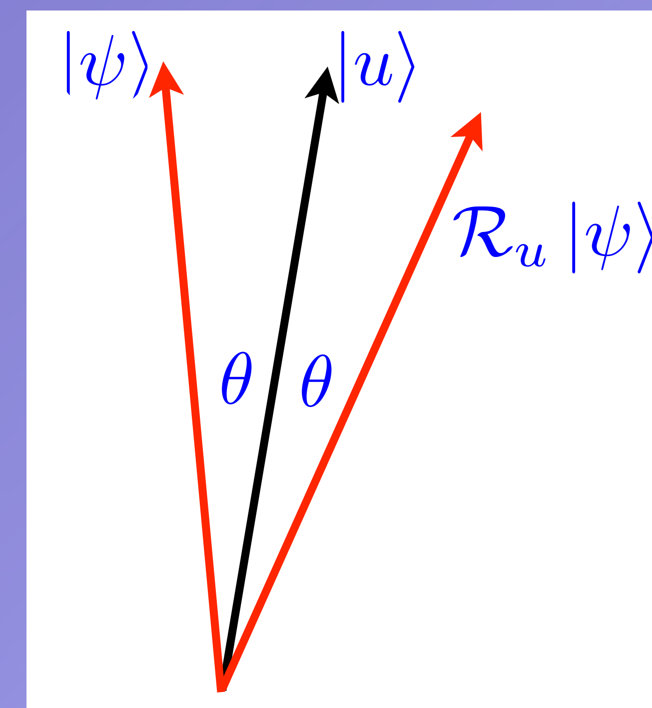

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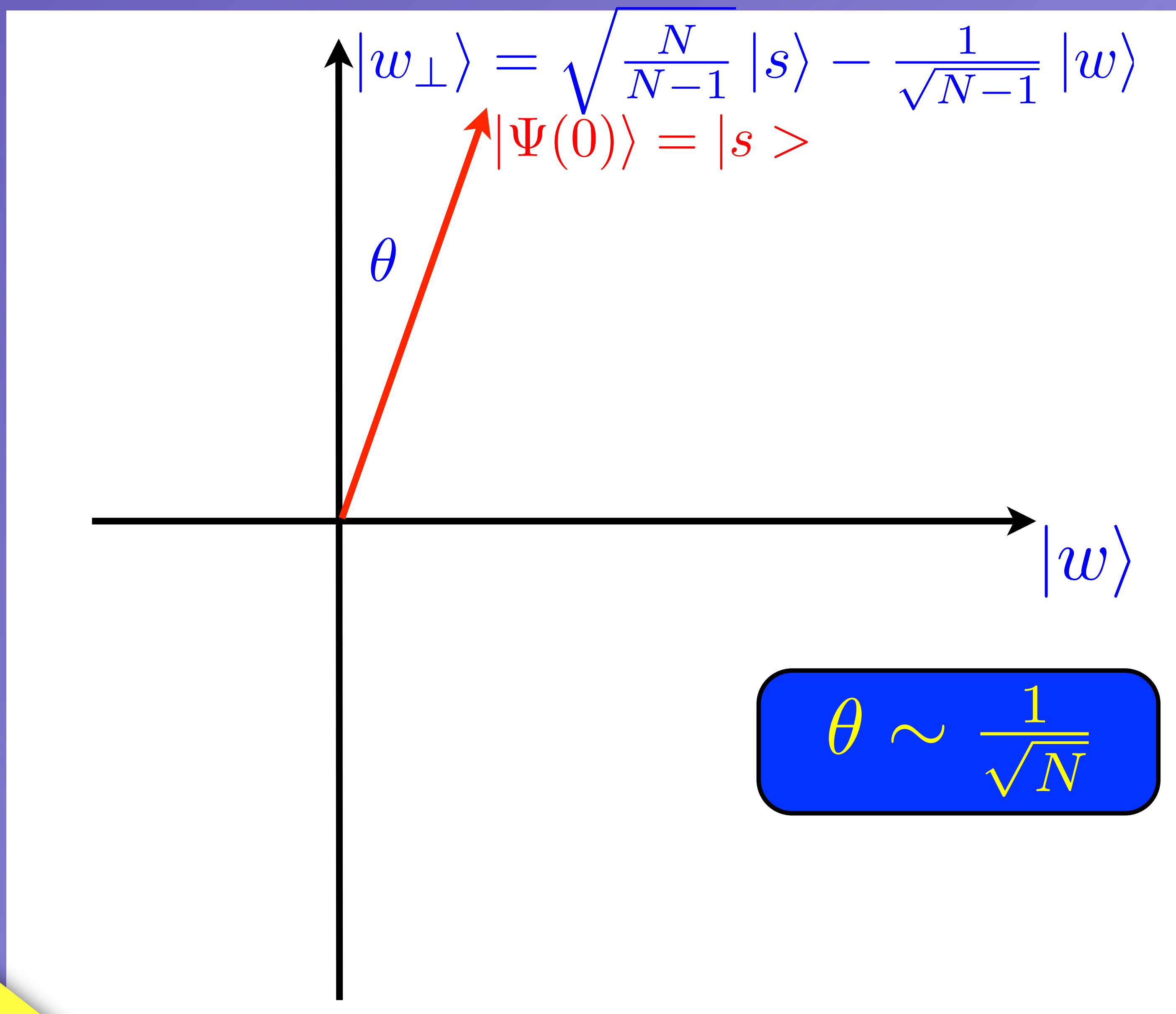

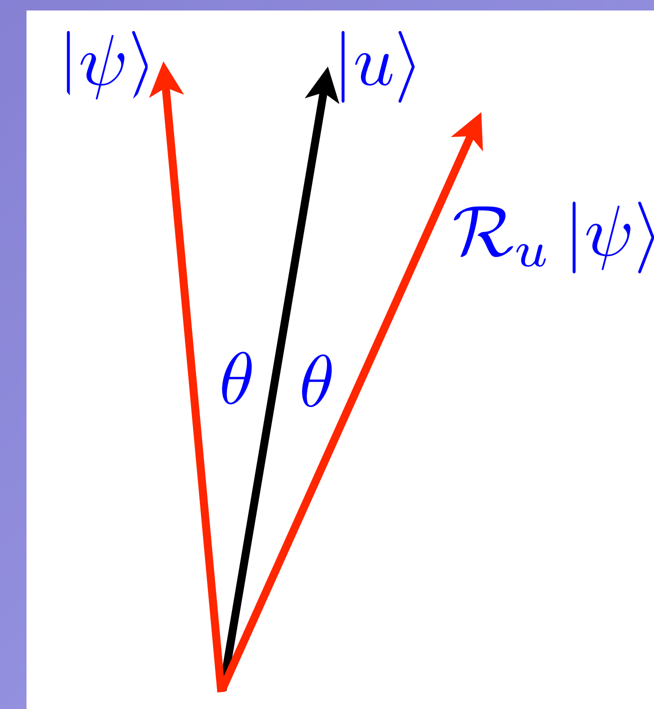

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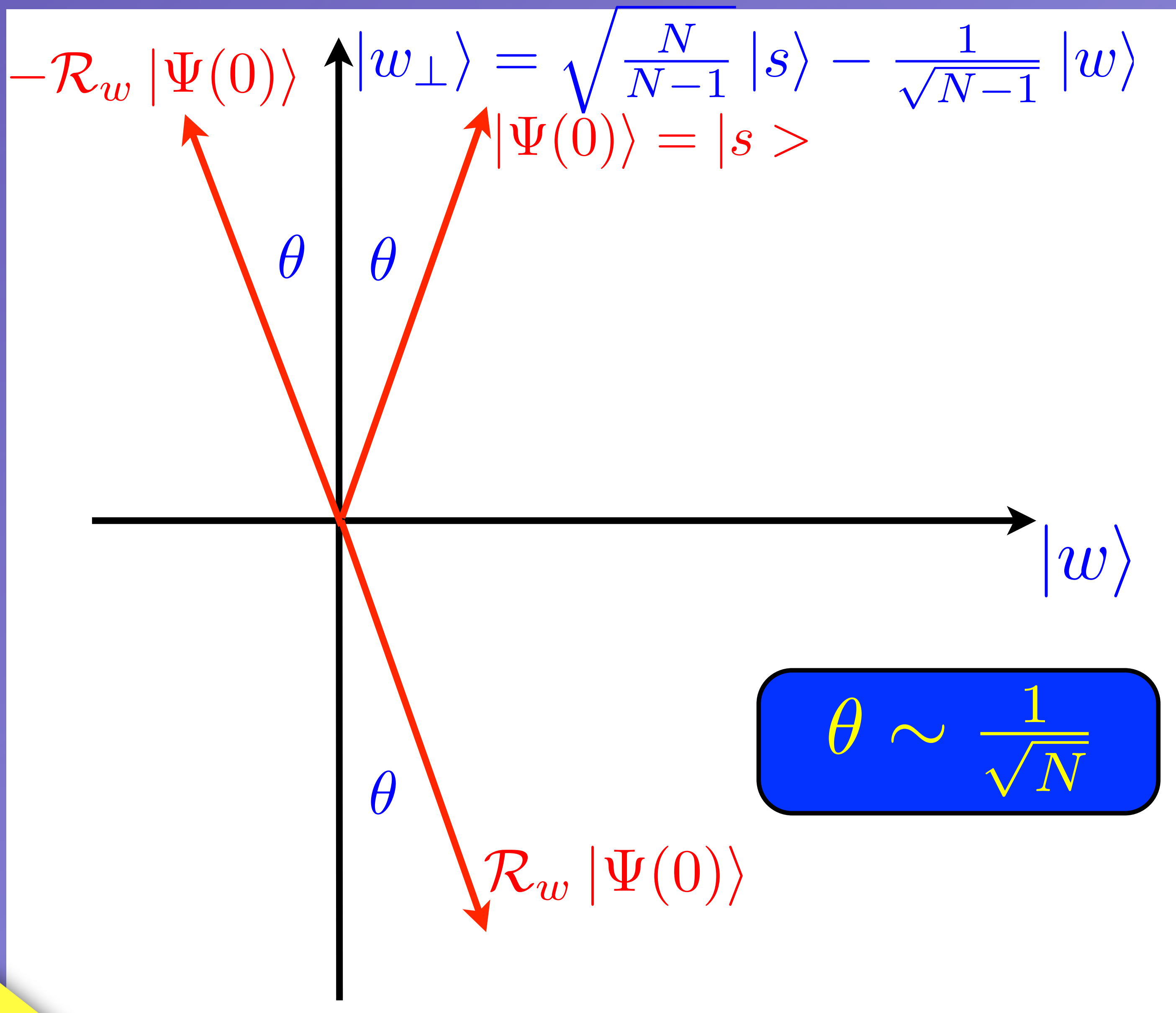

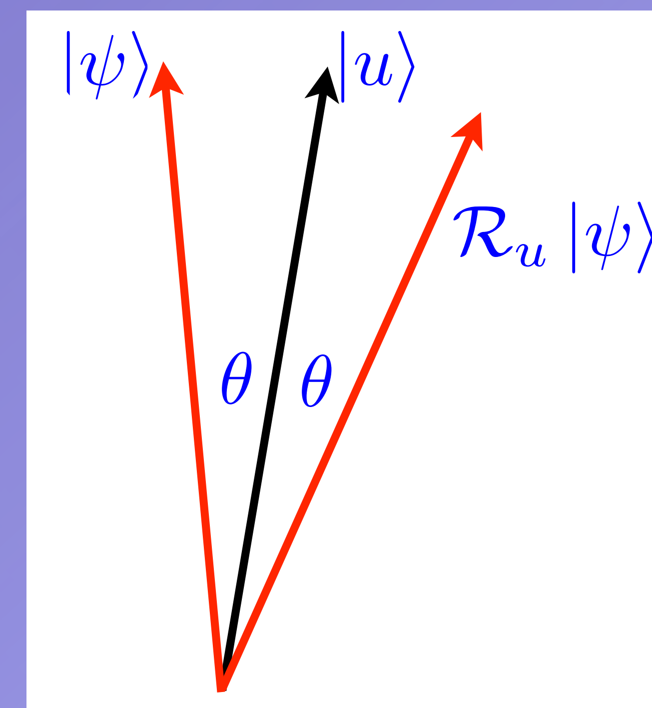

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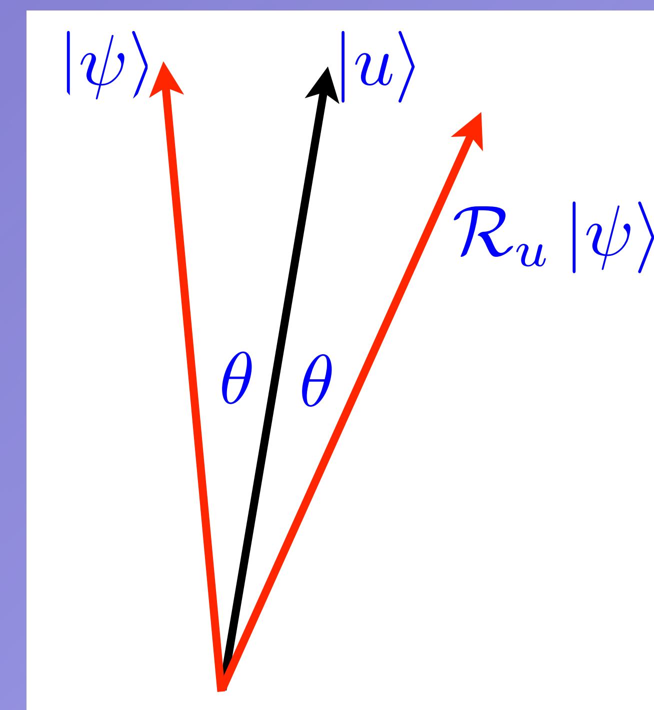

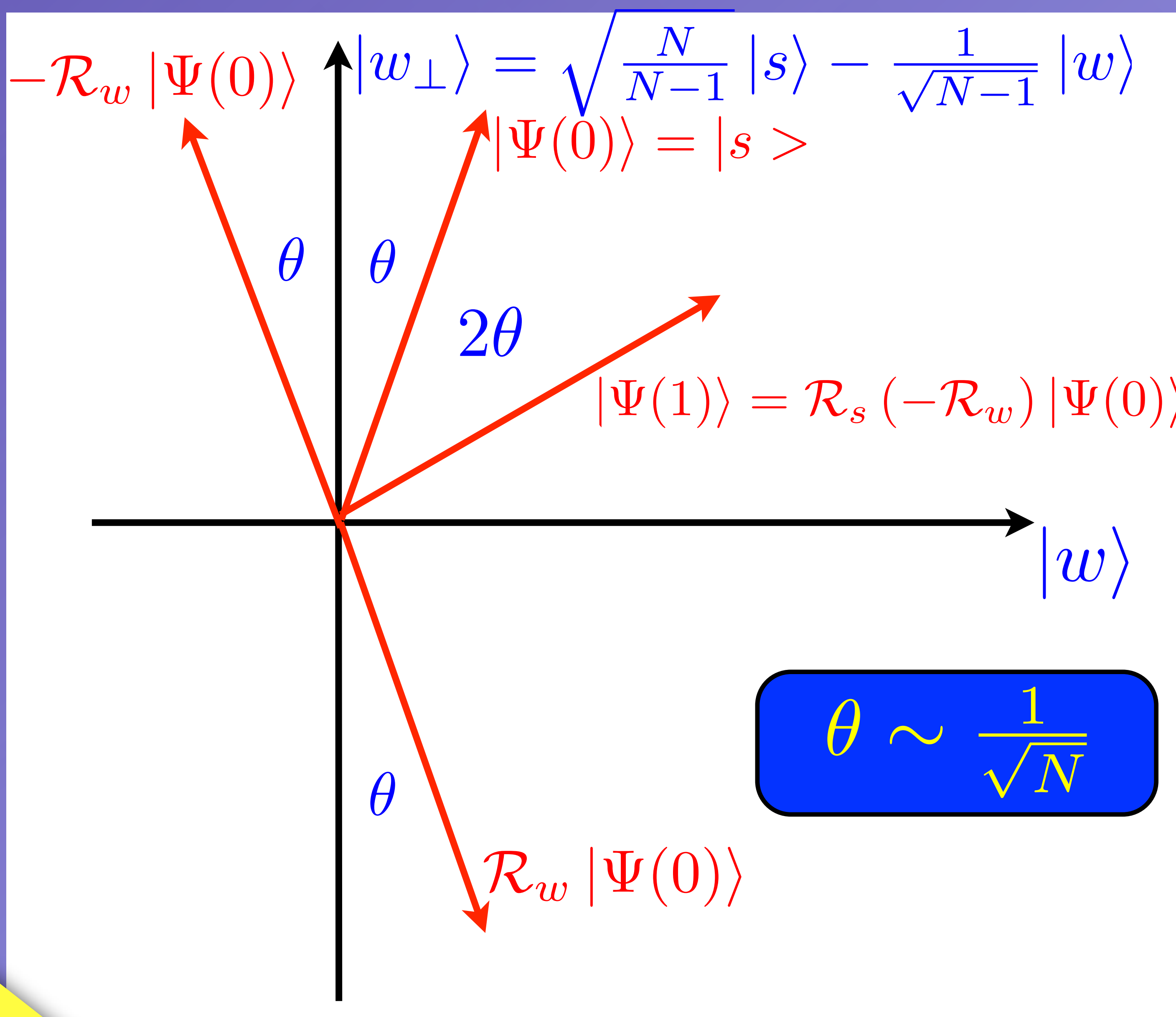

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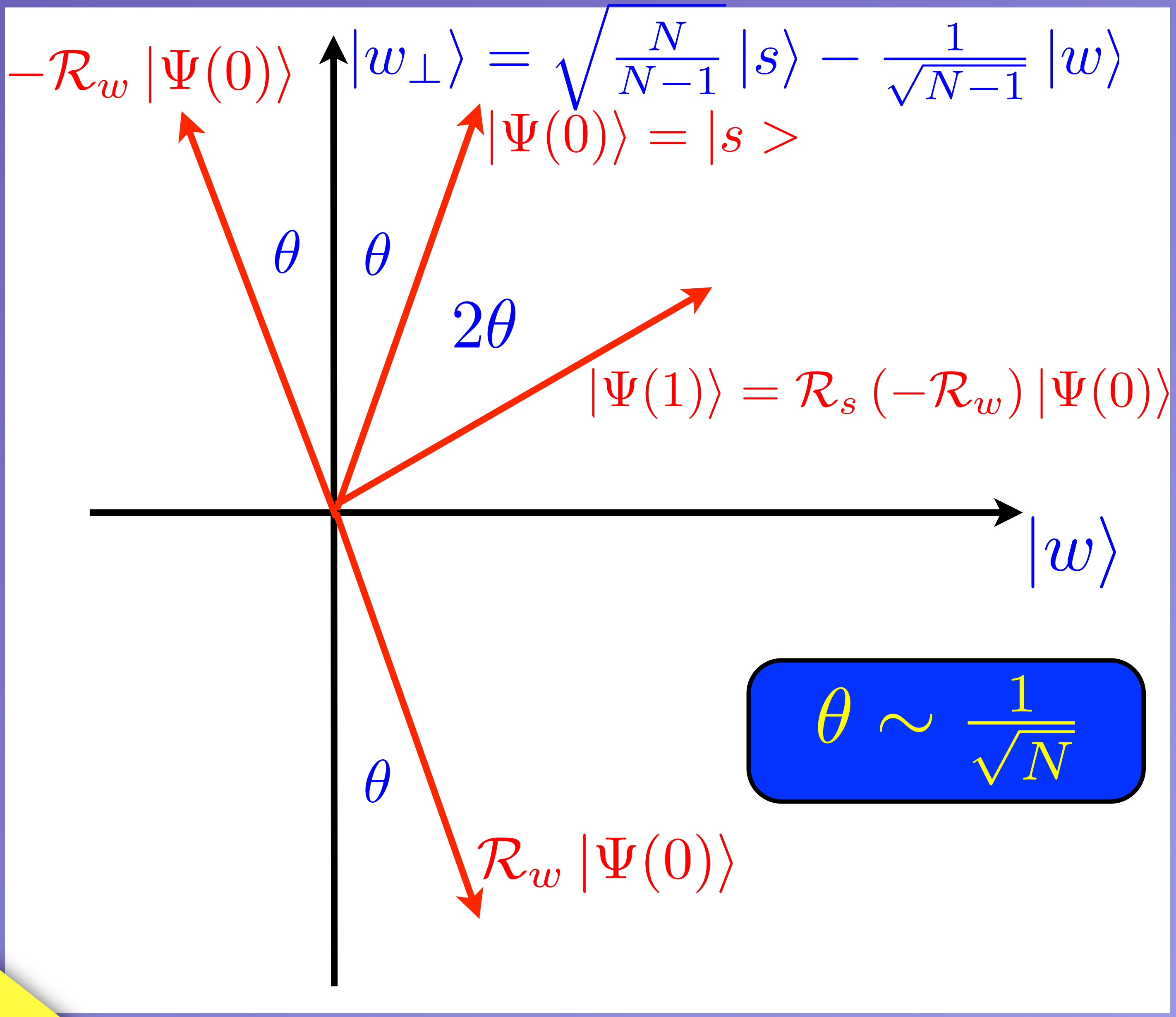

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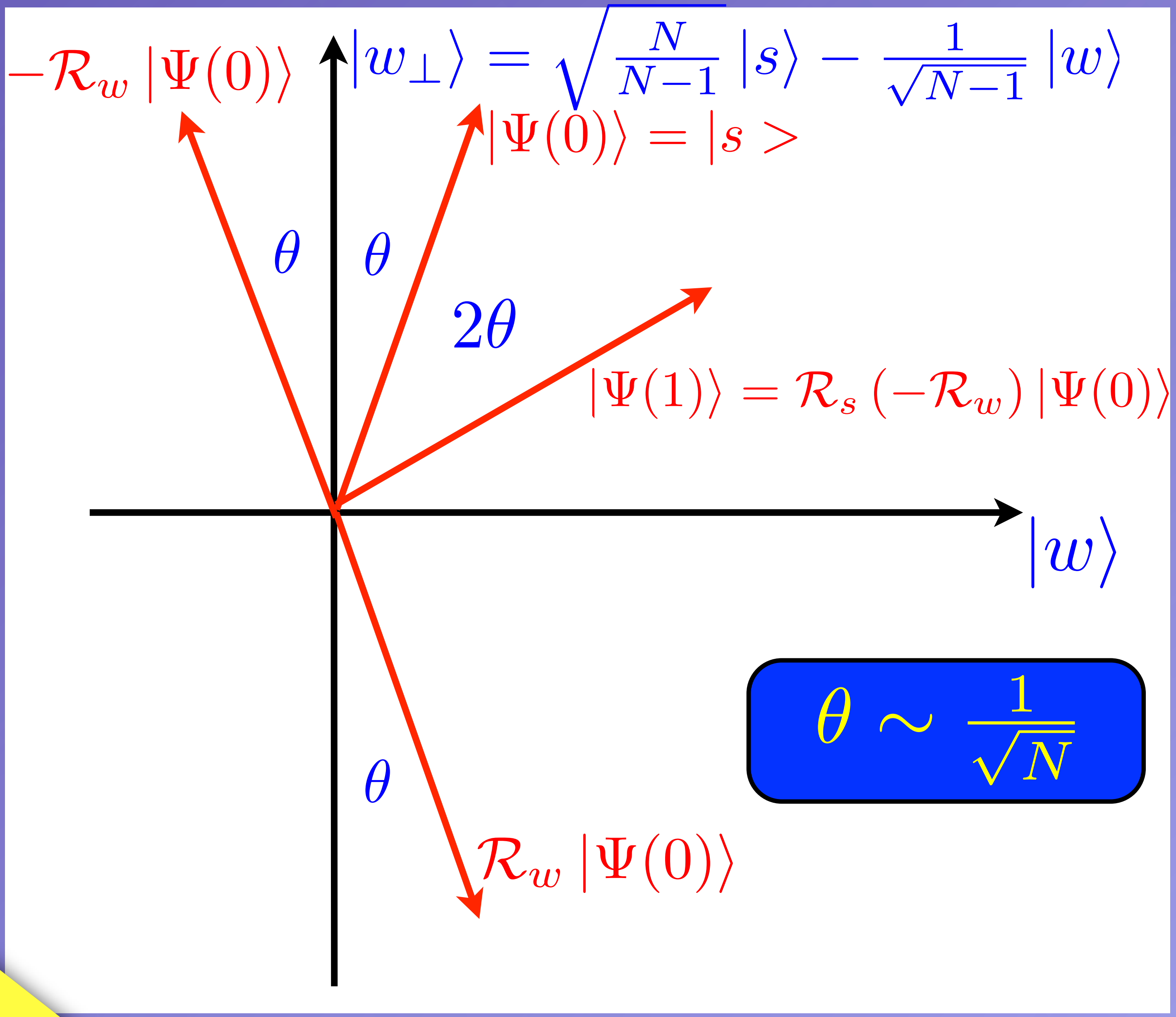

$$\text{Set:} \quad U = \mathcal{R}_s(-\mathcal{R}_w)$$

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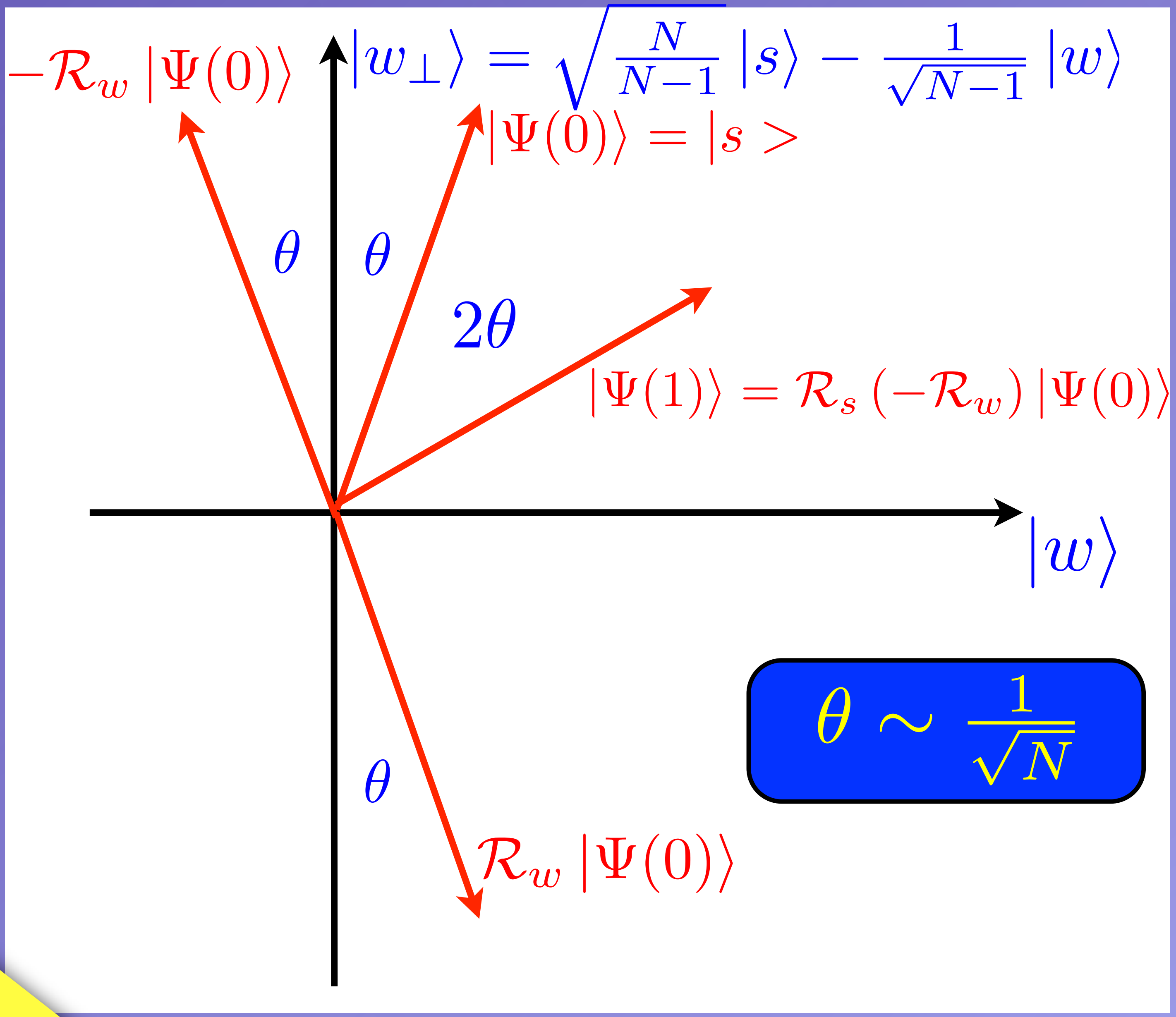

Set: $U = \mathcal{R}_s(-\mathcal{R}_w)$

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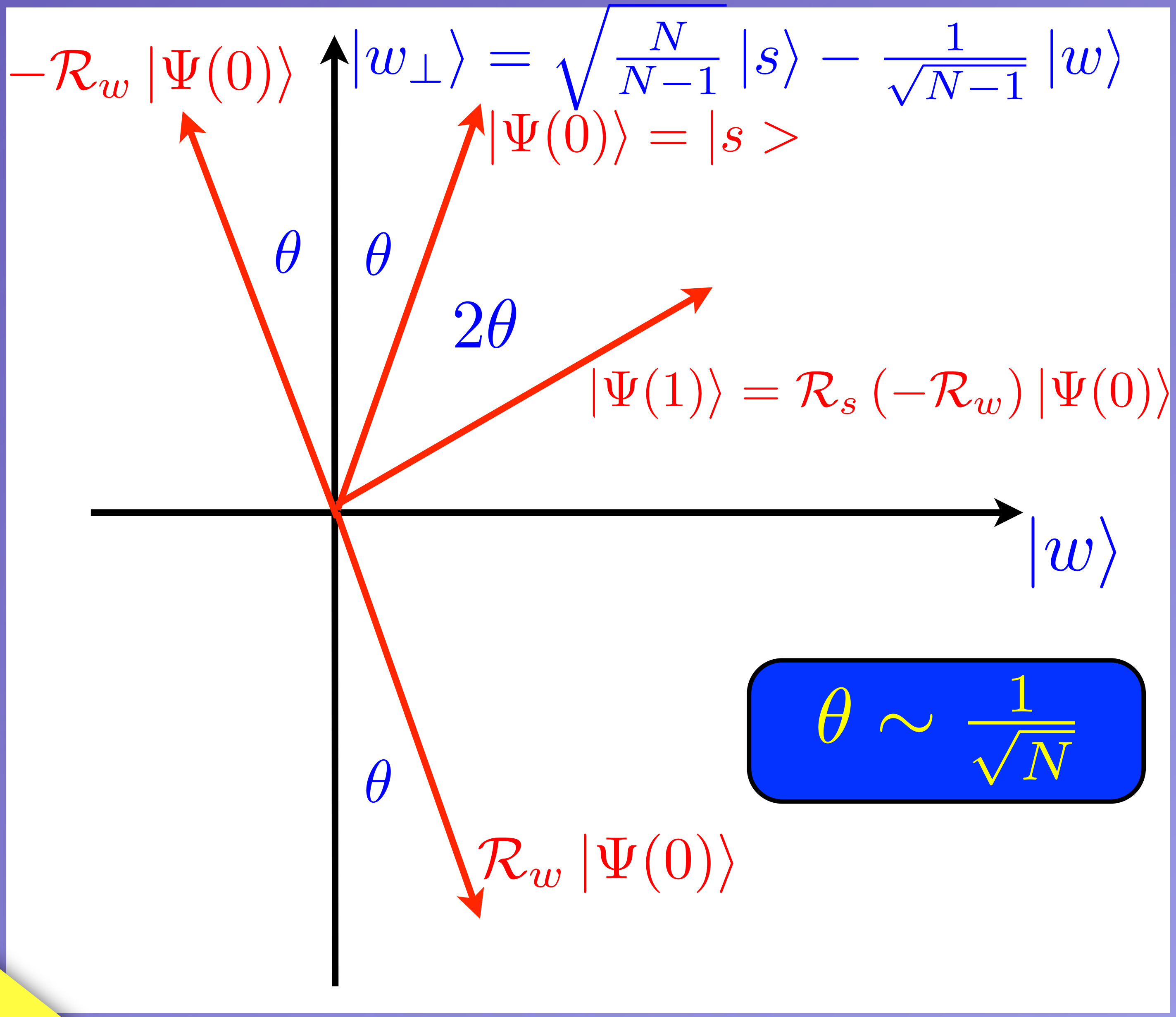

$$\text{Set:} \quad U = \mathcal{R}_s(-\mathcal{R}_w)$$

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$$\text{Need:} \quad 2\theta t \sim \frac{\pi}{2}$$

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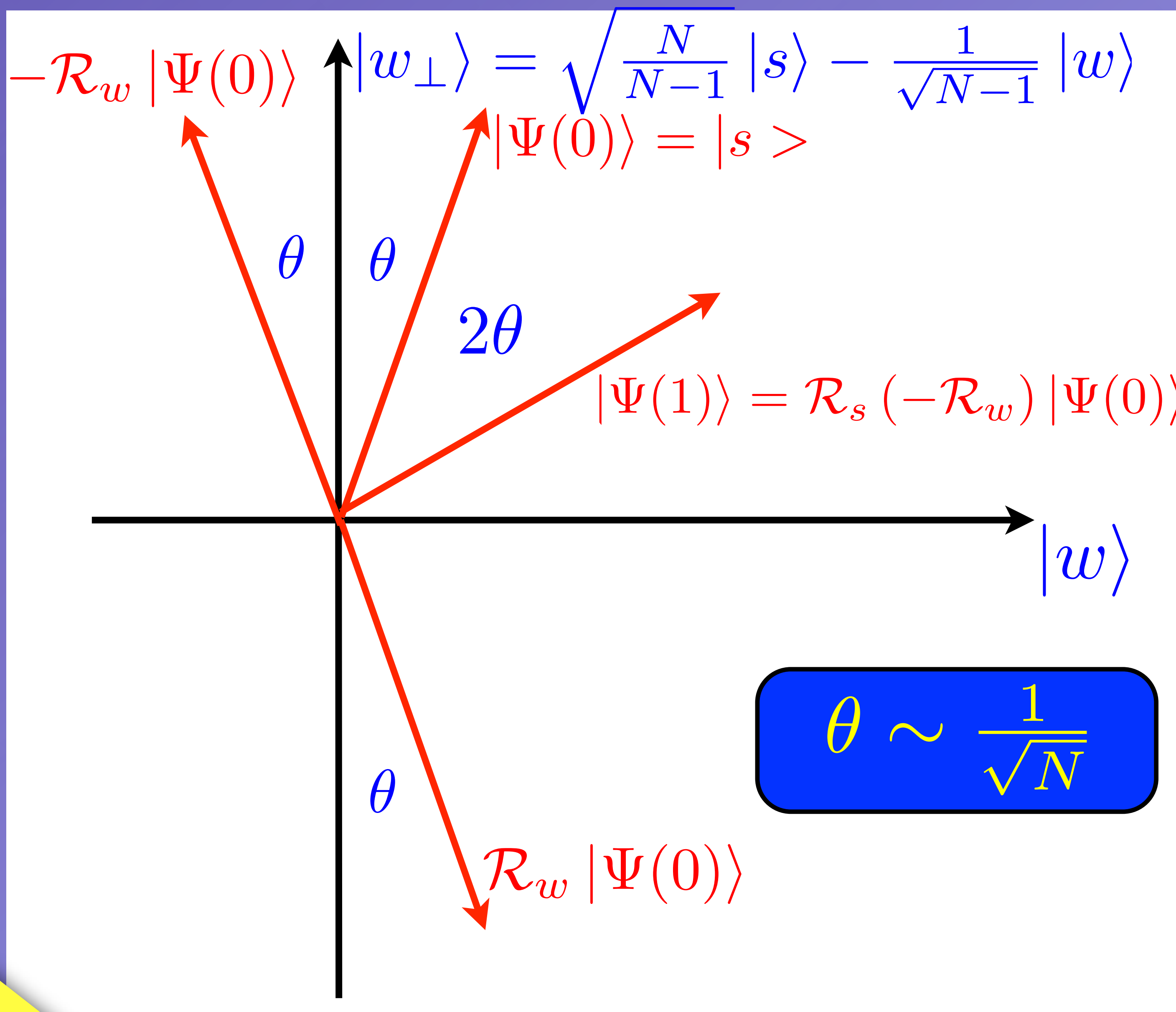

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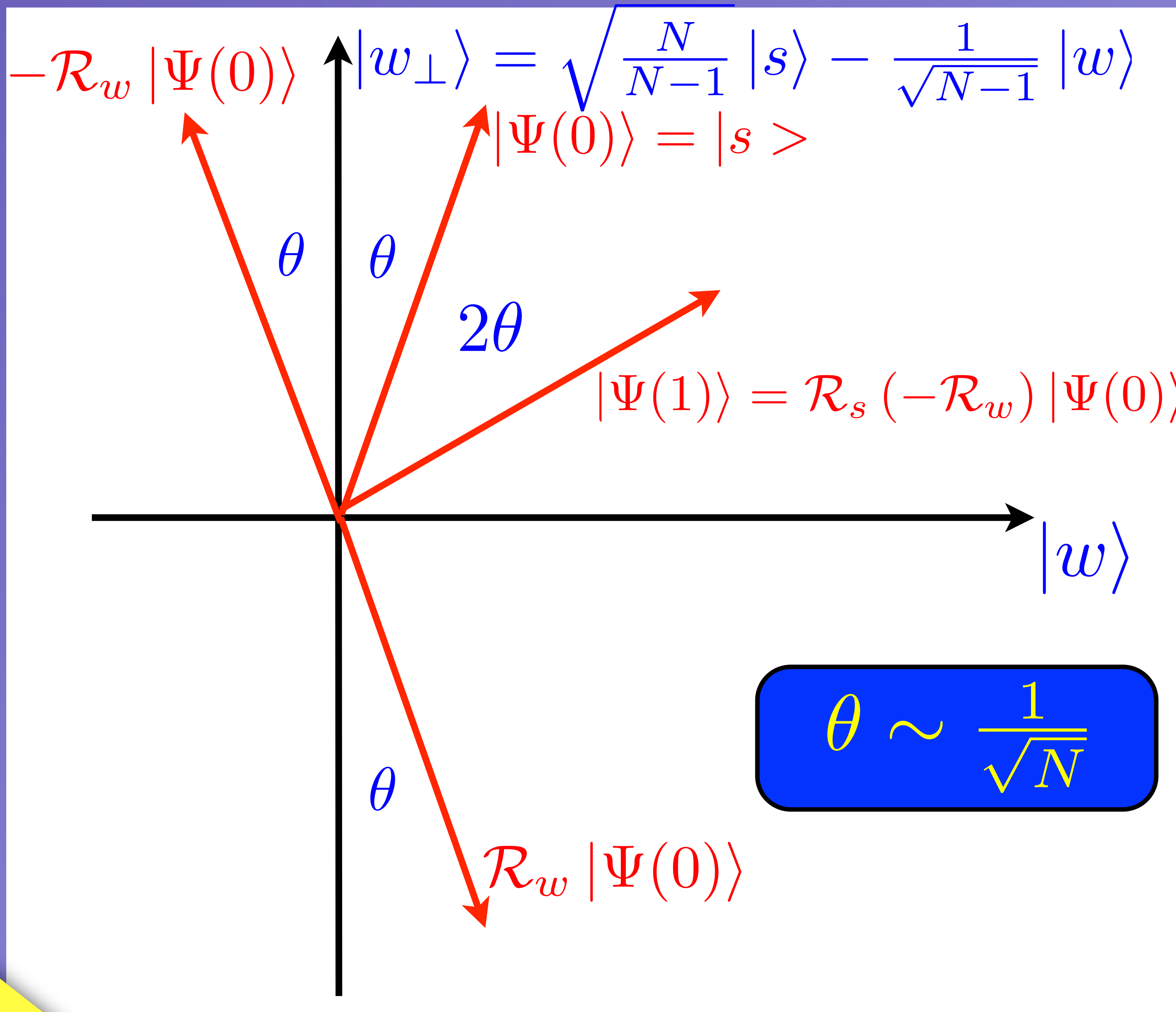

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$$\mathcal{R}_s = \frac{2}{N}\begin{bmatrix} 1 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 1 \end{bmatrix} - \mathbb{I}$$

Discrete-Time Q-Walk in a Geometry:

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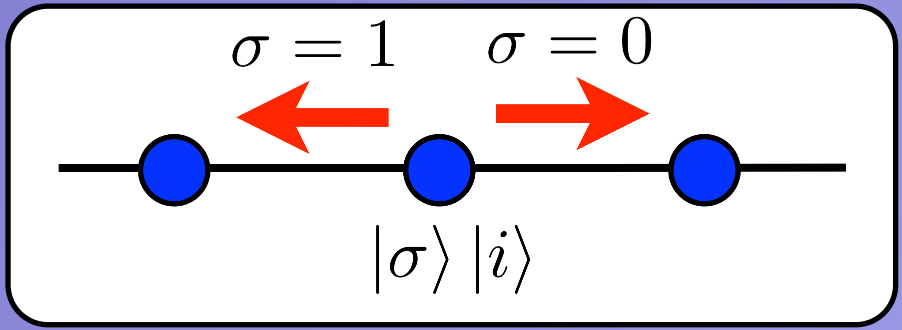

[Ambainis et al ('01)]

Discrete-Time Q-Walk in a Geometry:

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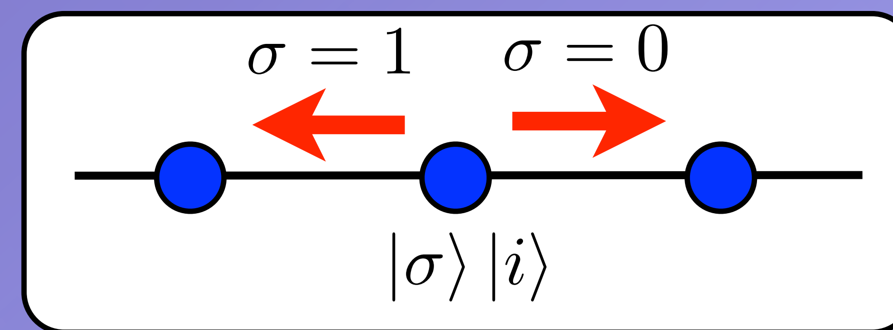
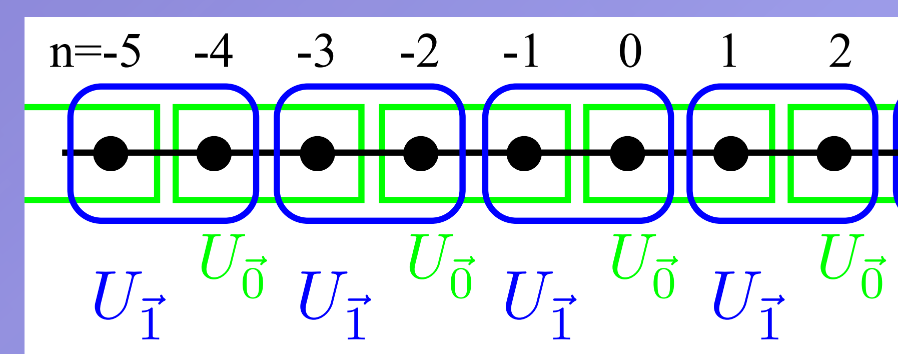


$$\mathcal{U} = U_{\vec{1}}U_{\vec{0}}$$

[Patel et al ('05), Falk ('13)]



$$U_{\vec{0},\vec{1}} = 2\sum_{n}\Pi_{n}^{\vec{0},\vec{1}} - \mathcal{I}$$

Coined Quantum Walk on a Line

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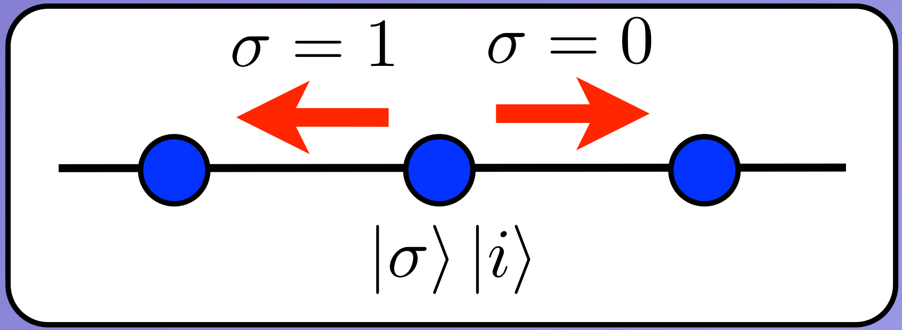

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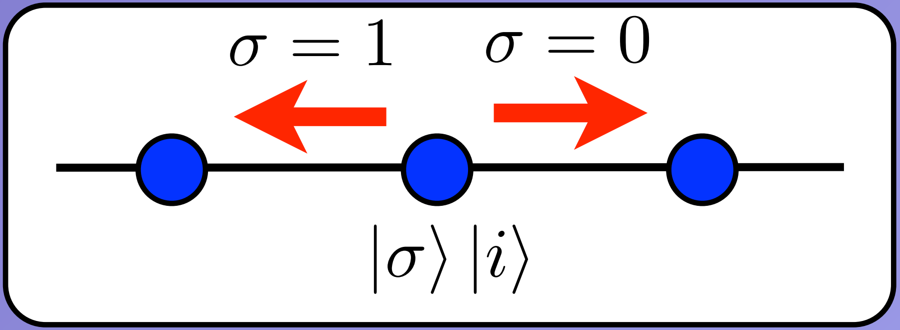

$$\mathcal{U} = \sum_n \sum_m \mathcal{P}_{n,m}\Delta_{n,m}\,|n\rangle\,\langle m|$$

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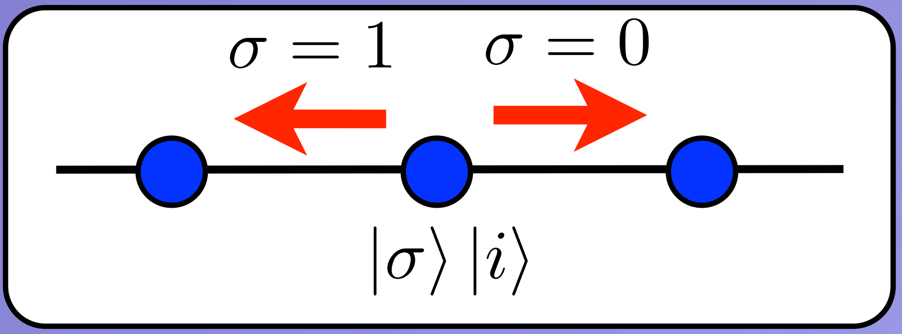

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Laplacian: $\Delta_{n,m}$

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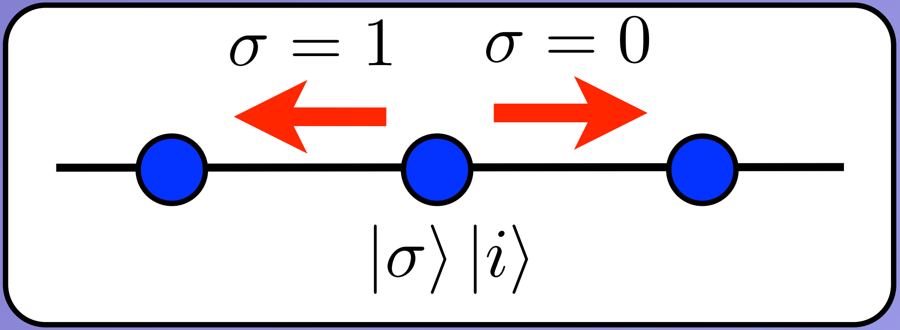

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Laplacian: $\Delta_{n,m}$

Hopping Parameters: $\mathcal{P}_{n,m}$

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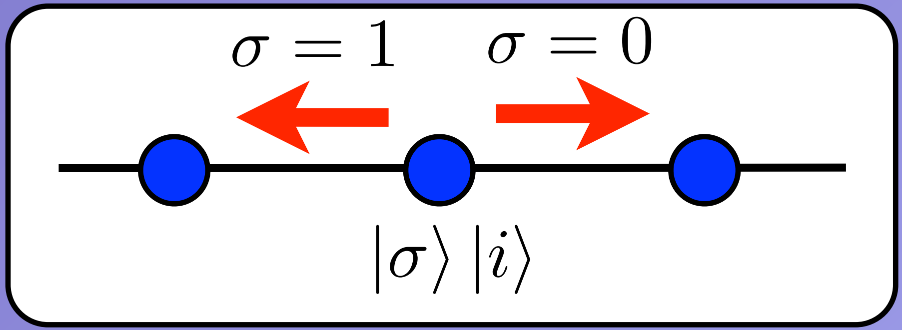

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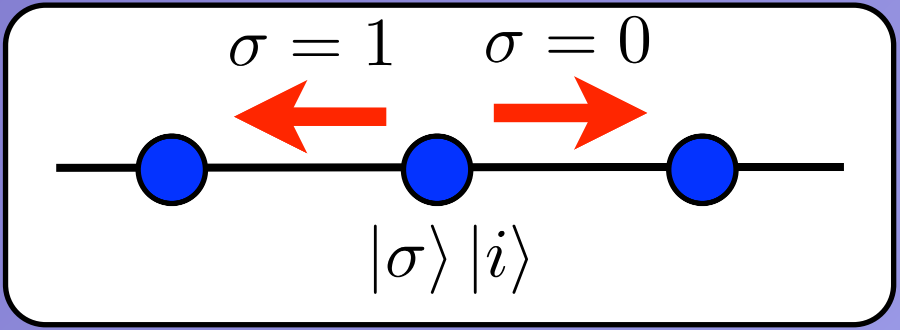

1d-line:

$$\mathcal{U} = \sum_n \left\{ A\,|n+1\rangle\langle n| + B\,|n-1\rangle\langle n| + M\,|n\rangle\langle n| \right\}$$

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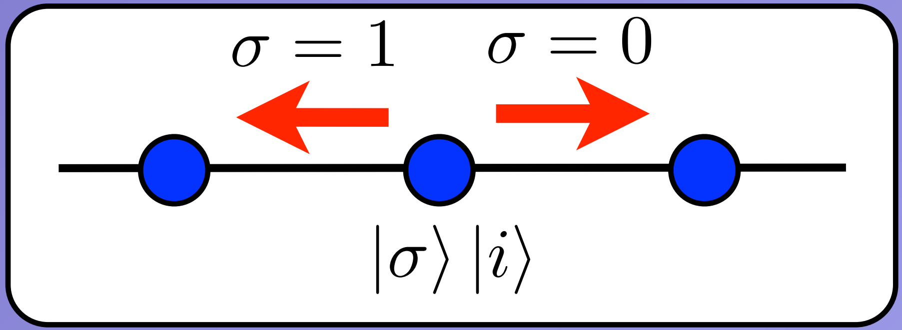

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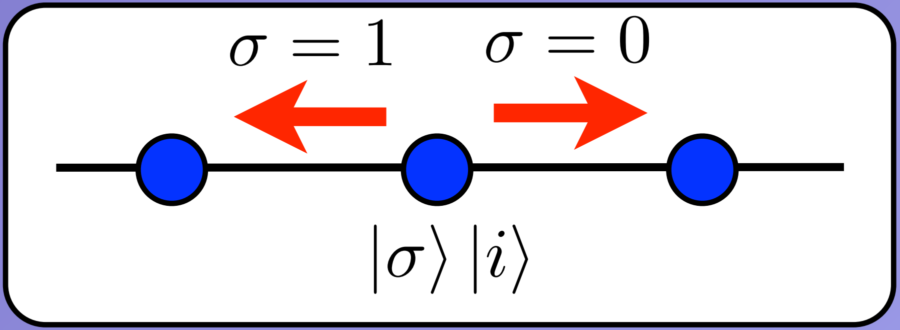

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Unitarity: $\mathcal{U}^\dagger\mathcal{U} = \mathcal{I} \quad\Rightarrow\quad \mathcal{I}_d = A^\dagger A + B^\dagger B + M^\dagger M$

$$0 = A^\dagger M + M^\dagger B,$$

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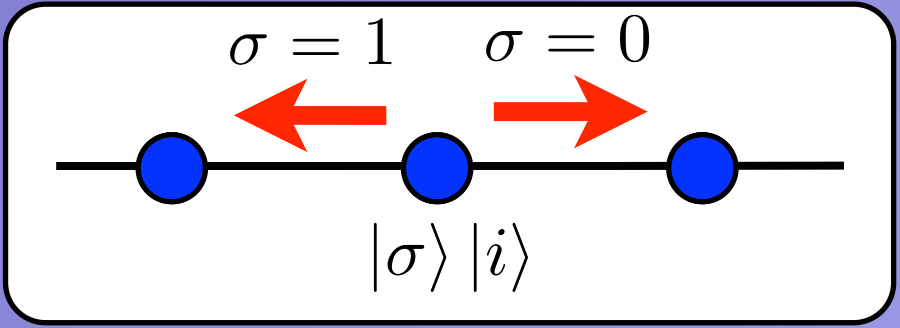

1d-line:

$$\mathcal{U} = \sum_n \left\{ A\left|n+1\right\rangle\left\langle n\right| + B\left|n-1\right\rangle\left\langle n\right| + M\left|n\right\rangle\left\langle n\right| \right\}$$

Unitarity: $\mathcal{U}^\dagger \mathcal{U} = \mathcal{I} \quad \Rightarrow \quad \mathcal{I}_d = A^\dagger A + B^\dagger B + M^\dagger M$

$$0 = A^\dagger M + M^\dagger B,$$

$$0 = A^\dagger B$$

Can **not** be scalar!

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Coined Quantum Walk on a Line

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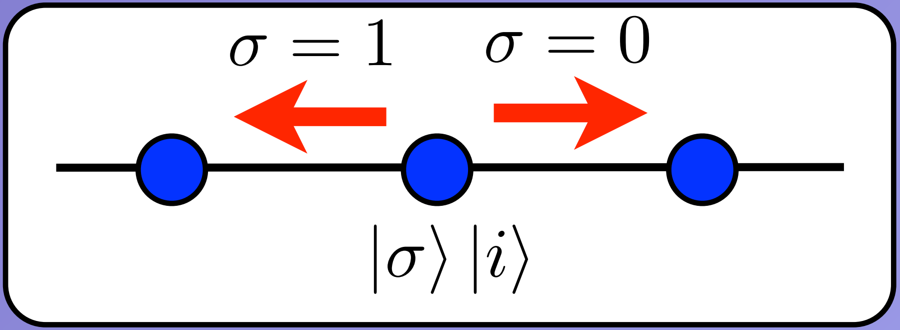

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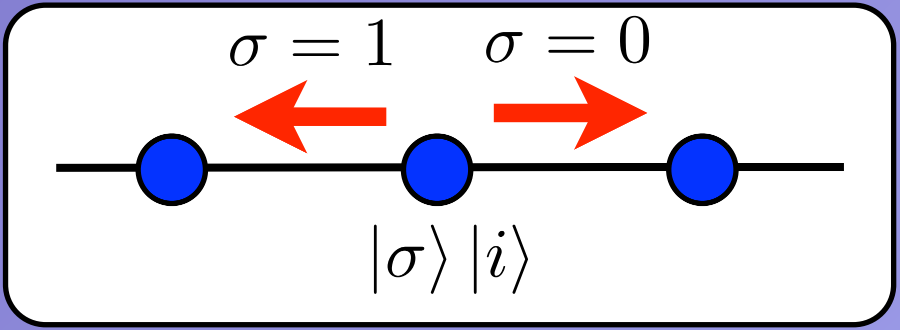

1d-line:

$$\mathcal{U} = \sum_n \left\{A\,|n+1\rangle\langle n| + B\,|n-1\rangle\langle n| + M\,|n\rangle\langle n|\right\}$$

Hopping: $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathcal{C}$ $\quad B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathcal{C}$ $\quad M = 0$

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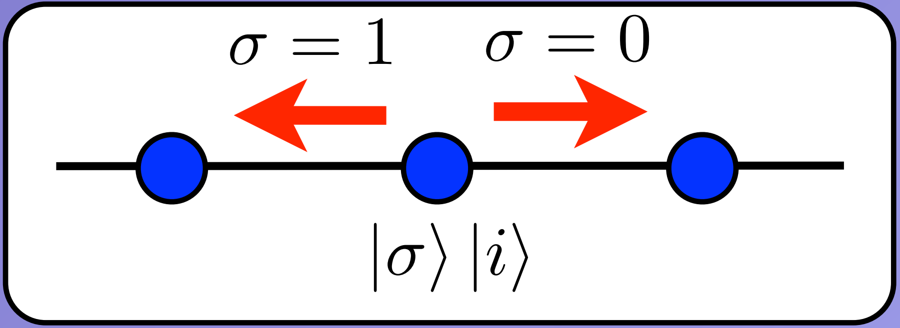

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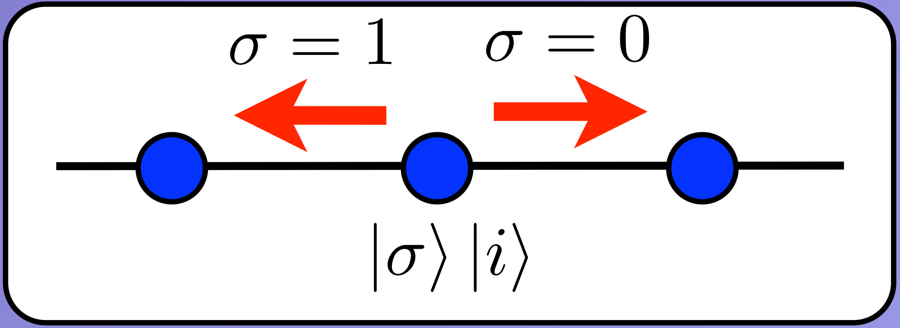
Q-Coin: $\mathcal{C} = \begin{pmatrix} \cos\rho & e^{i\phi}\,\sin\rho \\ e^{i\theta}\,\sin\rho & -e^{i(\phi+\theta)}\,\cos\rho \end{pmatrix}$

Coined Quantum Walk on a Line

Master Eq.: $$|\Psi_{t+1}\rangle = \mathcal{U}\,|\Psi_t\rangle$$



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Coined Quantum Walk on a Line

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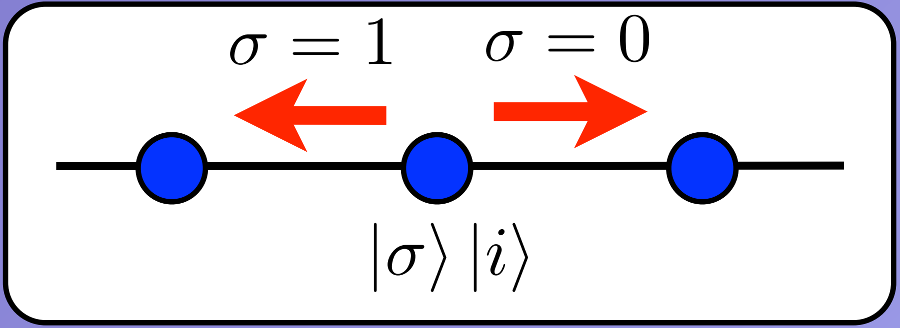
Real-Space:



$$\langle n|\Psi_{t+1}\rangle = \sum_m \langle n\,|\mathcal{U}|\,m\rangle\,\langle m|\Psi_t\rangle$$

$$\psi_{n,t+1} = \sum_m U_{n,m}\,\psi_{m,t}$$

Coined Quantum Walk on a Line

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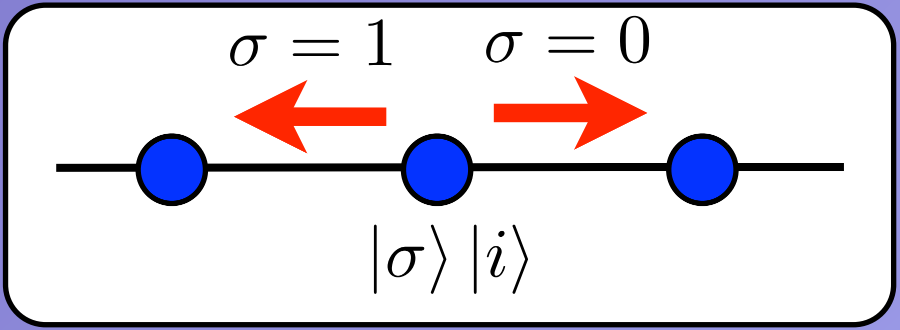

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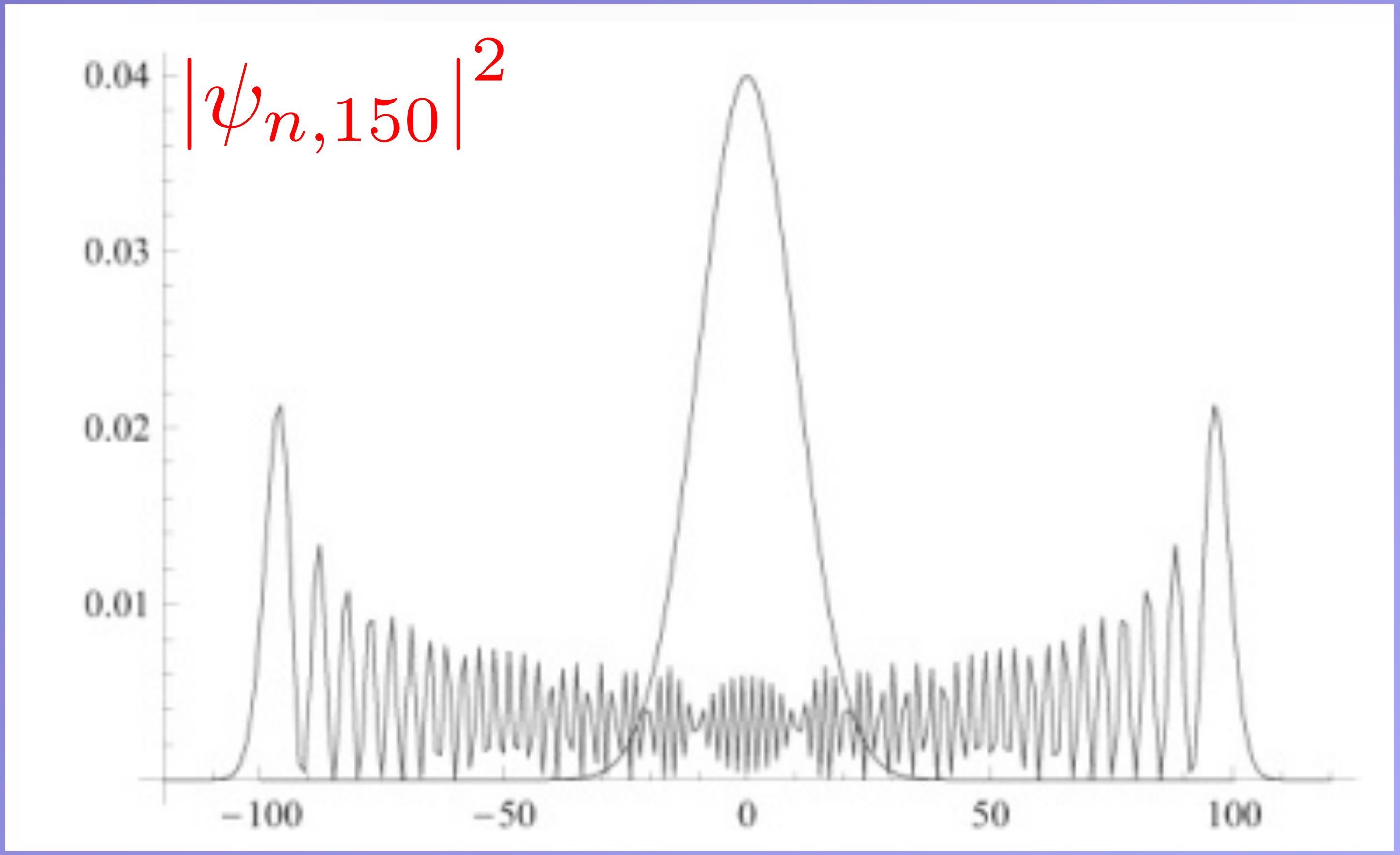

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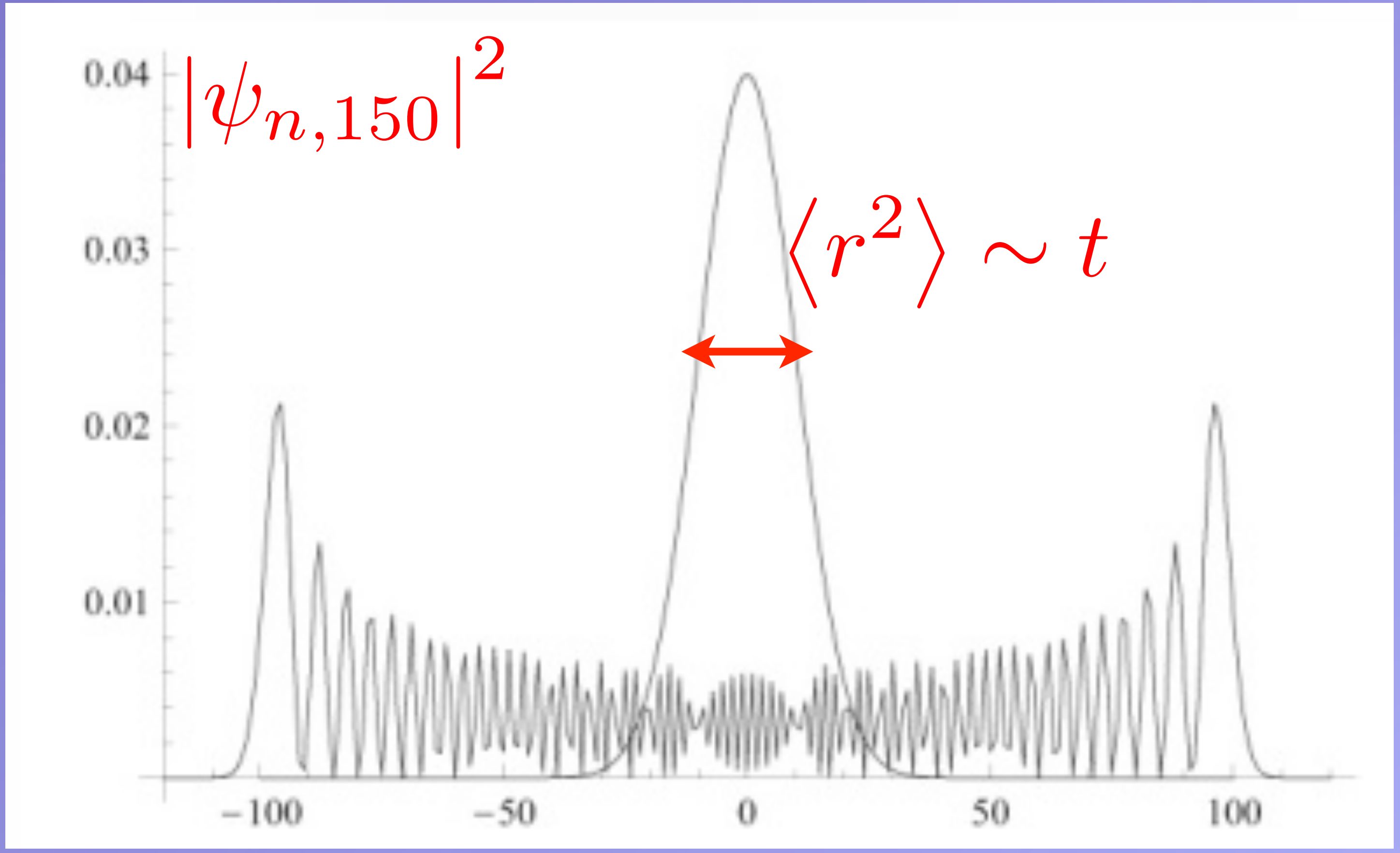

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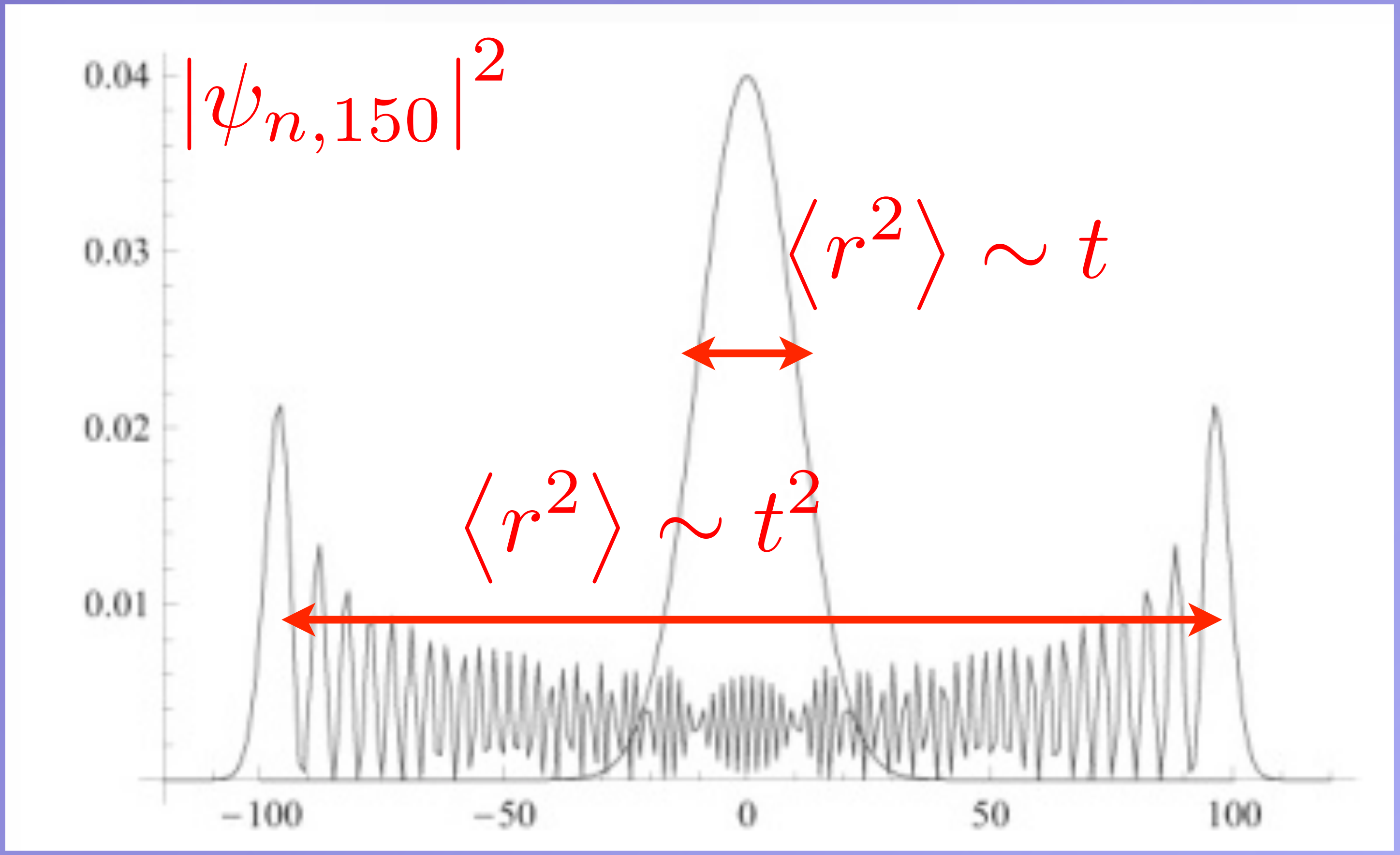

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Fourier:

$$\psi_{n,t} \sim \sum_{\mu} \int_0^{2\pi} \frac{dk}{2\pi}\, f_\mu(k)\, e^{-it\mathcal{H}_\mu(k)}$$

$$\mathcal{H}_\mu(k) = -vk + \omega_\mu(k)$$

$$v = \frac{n}{t}, \qquad \tilde{\mathcal{U}}(k)\,|\omega_\mu\rangle = e^{-i\omega_\mu(k)}\,|\omega_\mu\rangle$$

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Saddle Points:

$$k_\pm = \arccos\left(\frac{v\,\tan\rho}{\sqrt{1-v^2}}\right), \qquad v < v_{\max} = |\cos\rho|$$

Coined Quantum Walk on a Line

Master Eq.: $$\left|\Psi_{t+1}\right\rangle = \mathcal{U}\left|\Psi_t\right\rangle$$

Real-Space: $$\psi_{n,t+1} = A\,\psi_{n-1,t} + B\,\psi_{n+1,t} + M\,\psi_{n,t}$$

Fourier:

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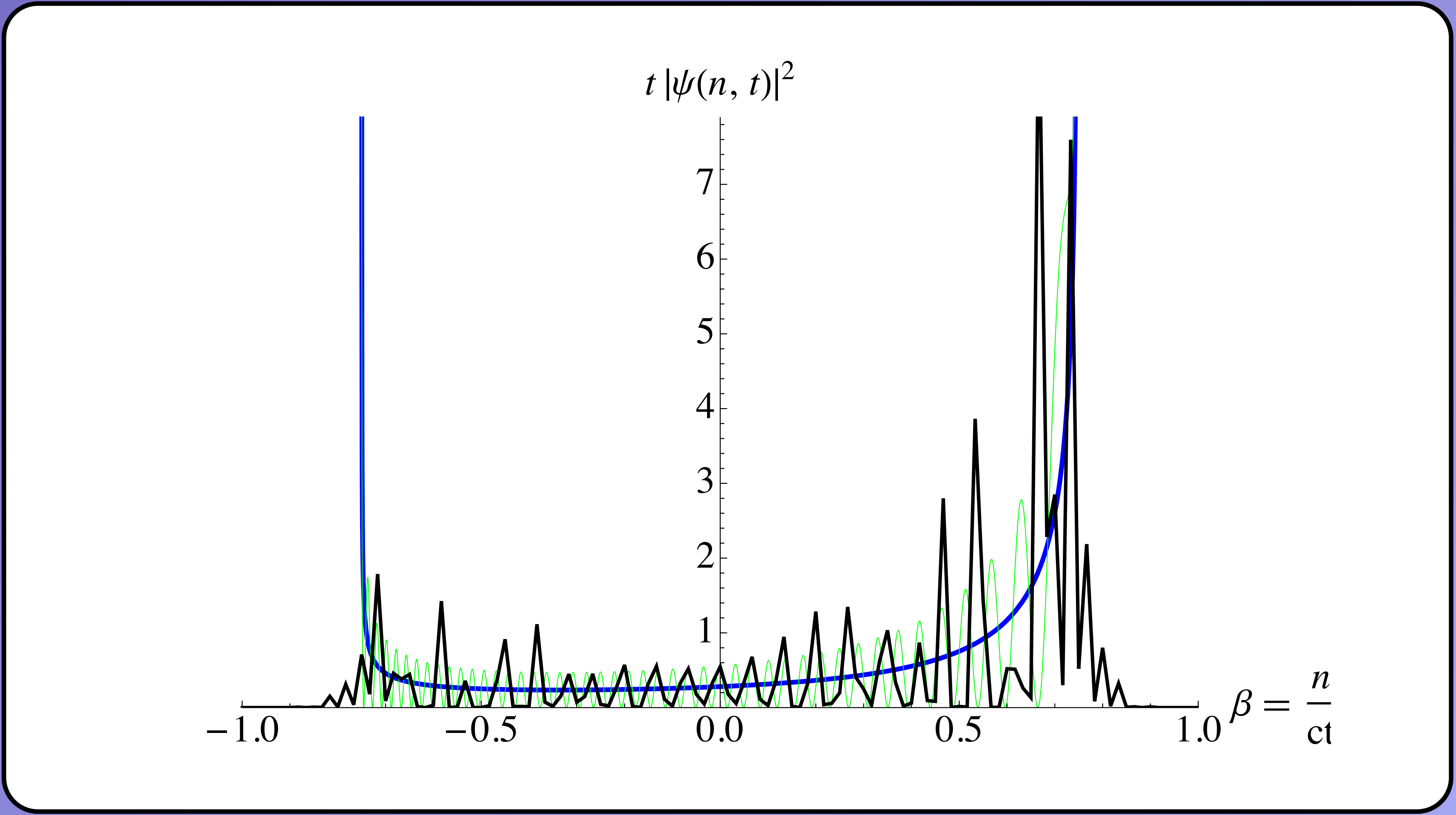

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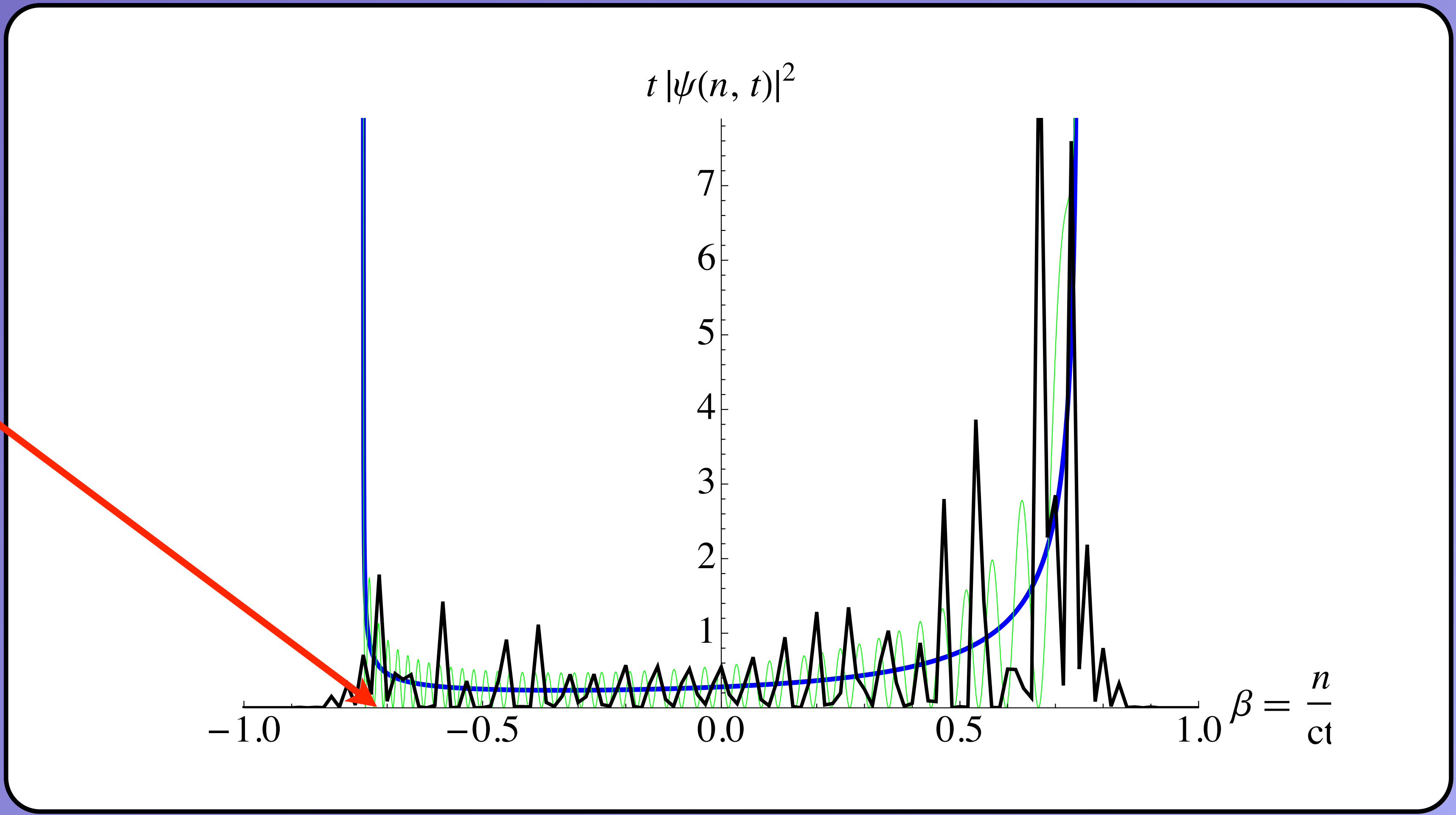
Fourier:

$v_{\max} = |\cos\rho| = \frac{3}{4}$



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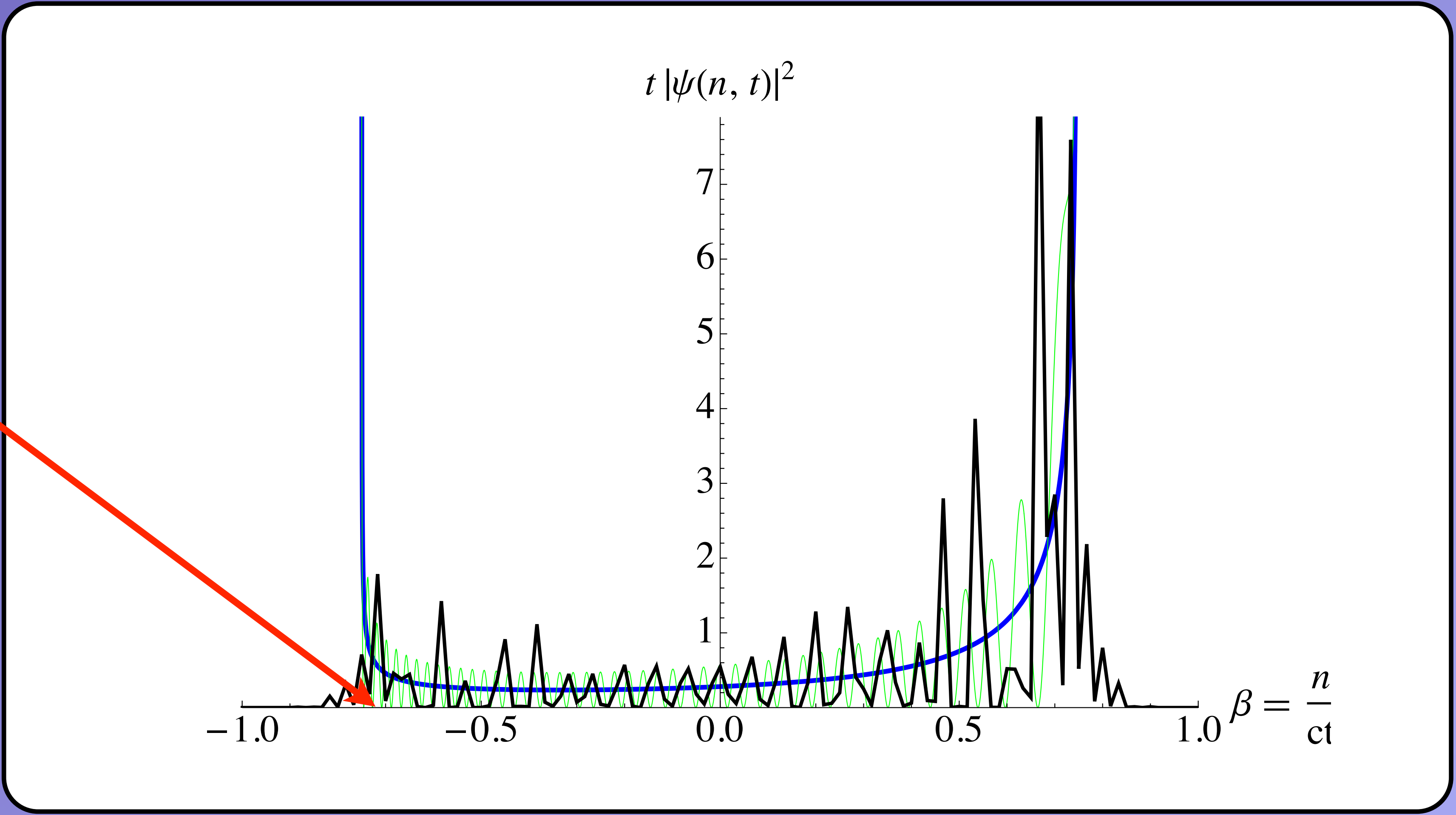

$$\sigma \sim \langle v\rangle_\rho\, t, \quad \langle v\rangle_\rho = \sqrt{(1-\sin\rho)\sin\rho}$$

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Coined Quantum Walk on a Line

Master Eq.: $|\Psi_{t+1}\rangle = \mathcal{U}\,|\Psi_t\rangle$

$\psi_{n+1,t} + M\,\psi_{n,t}$

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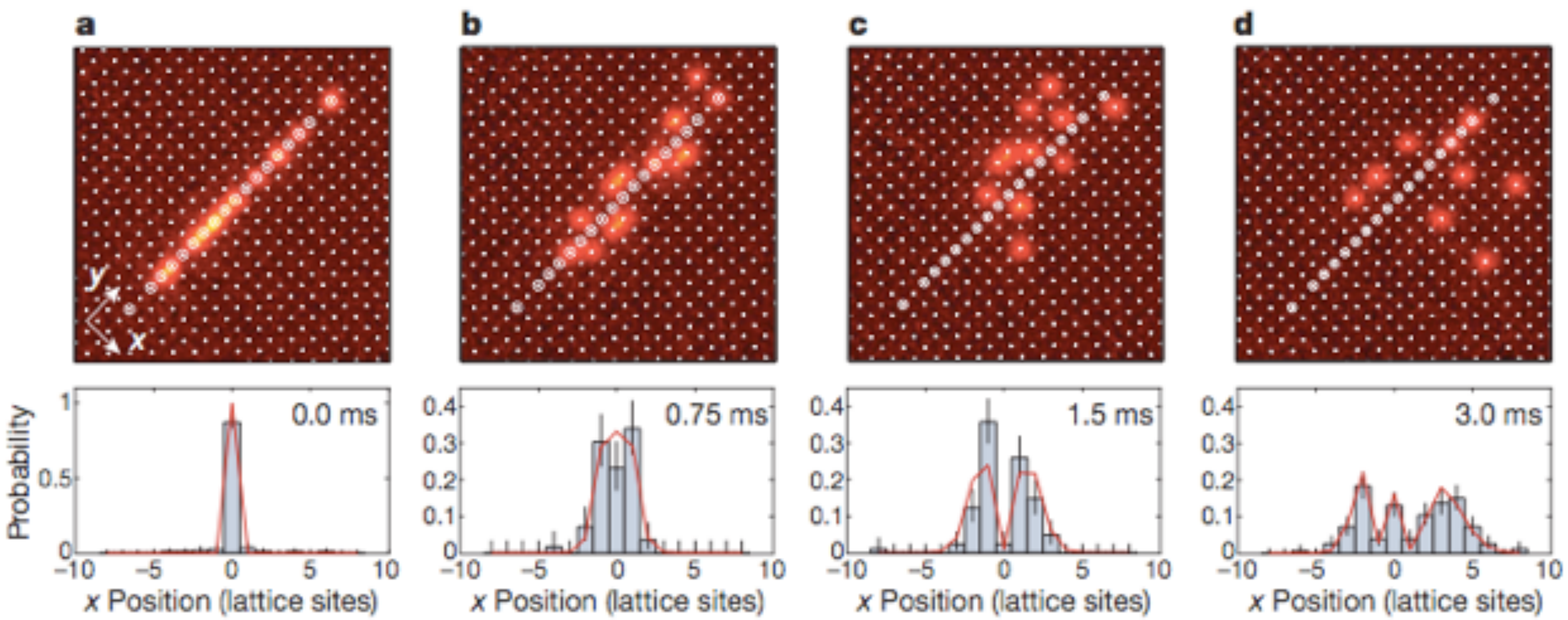

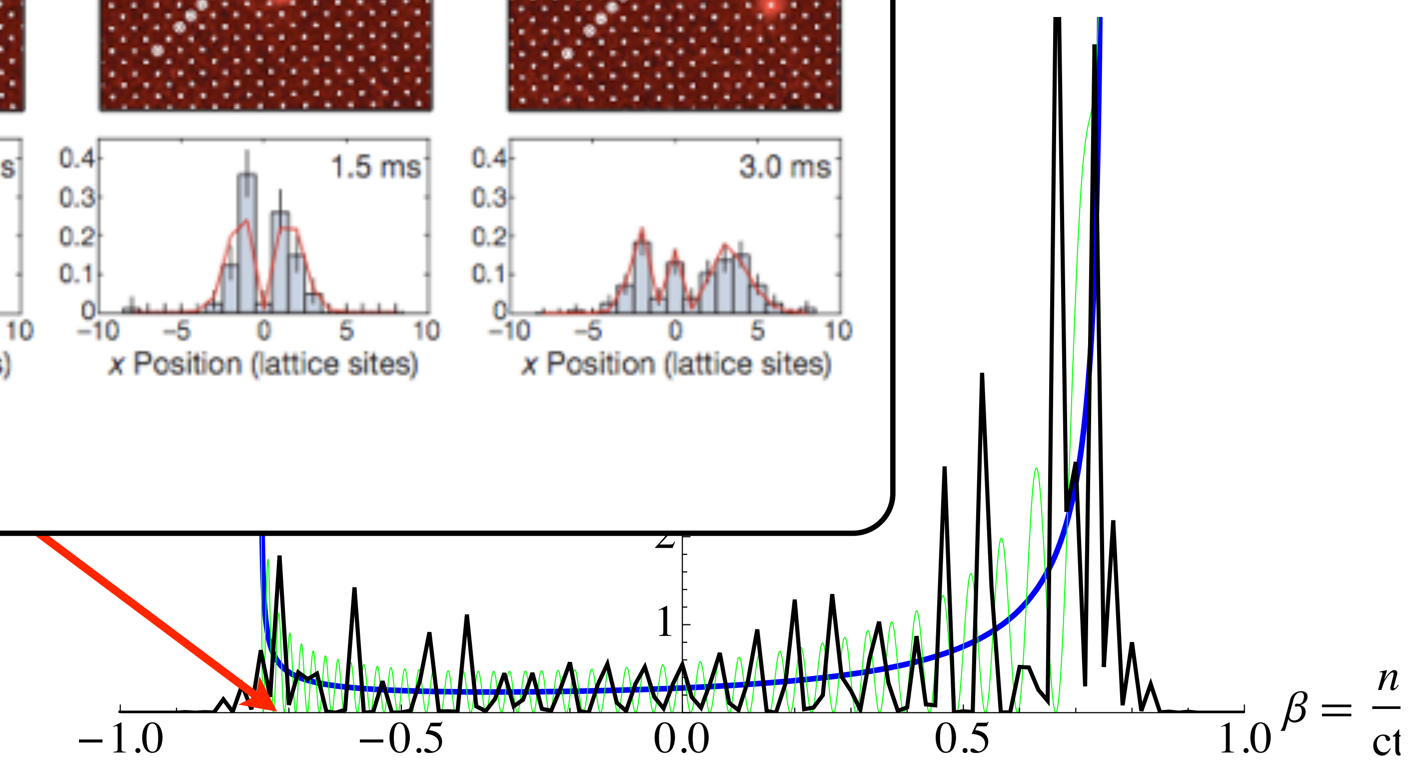

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RG for Quantum Algorithms

RG for Quantum Algorithms

Motivation

- Is $\langle r^2 \rangle \sim t^2$ on all translation-invariant Lattices?
- Is that true for all propagators/coins?
- Connection to Continuum QW?
- How sensitive is Universality to broken trans.-inv.?
- How sensitive is Universality to Disorder?
- Generally: What constitutes a Universality Class?

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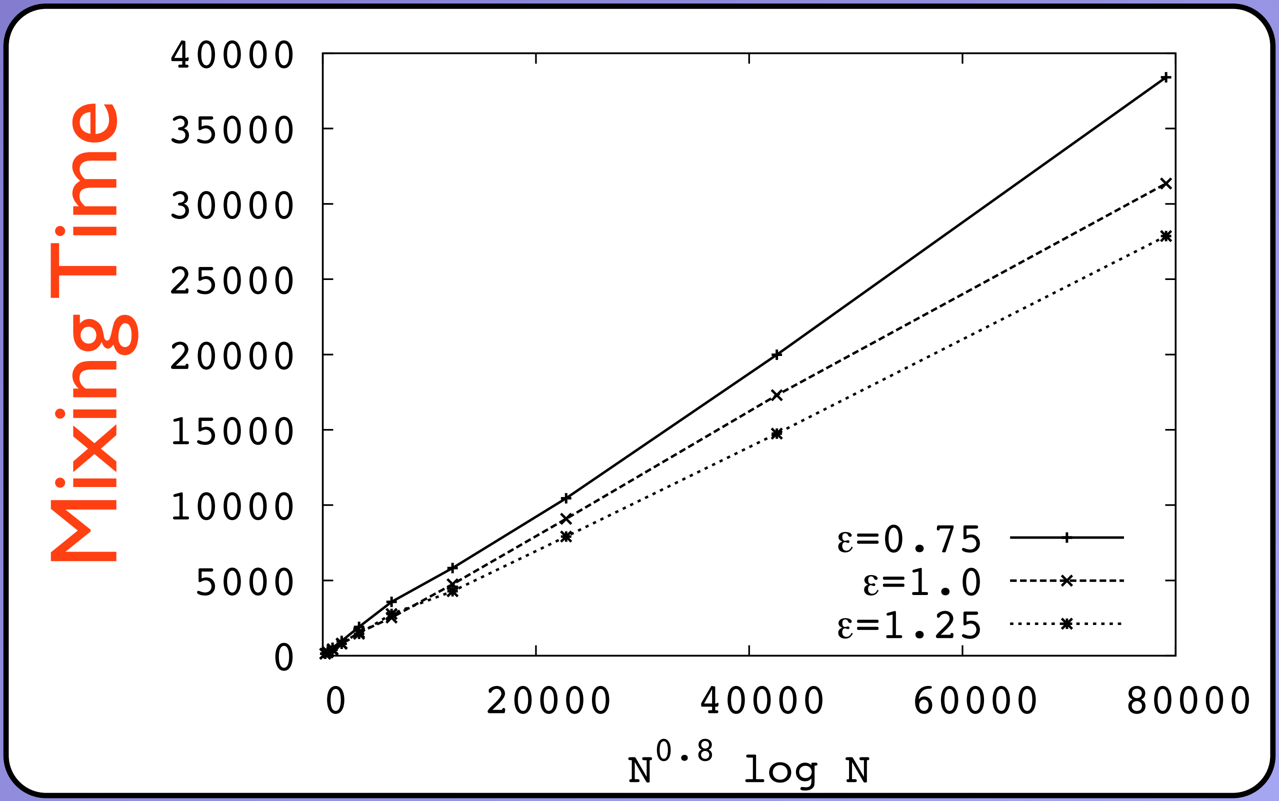

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Laplace: $$\tilde{\psi}_n = \sum_{t=0}^{\infty} z^t\,\psi_{n,t}$$

$$\tilde{\psi}_n = zA\,\tilde{\psi}_{n-1} + zB\,\tilde{\psi}_{n+1} + zM\,\tilde{\psi}_n + \psi_{IC}\delta_{n,0}$$

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$$\tilde{\psi}_n = zA\,\tilde{\psi}_{n-1} + zB\,\tilde{\psi}_{n+1} + zM\,\tilde{\psi}_n + \psi_{IC}\delta_{n,0}$$

Wanted:

$$\left\langle r^2\right\rangle \sim t^{\frac{2}{d_w}}$$

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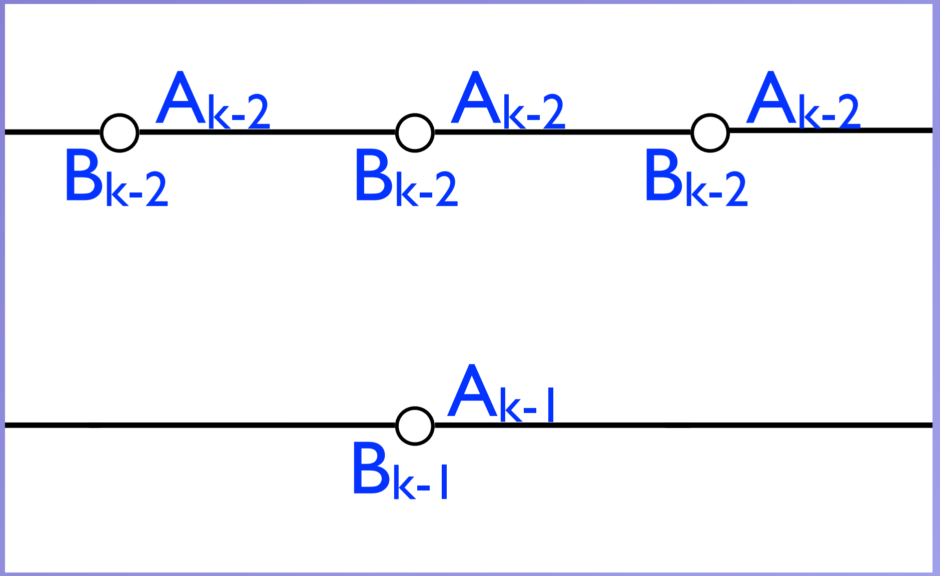
RG for Quantum Walks on a Line

Hierarchy of Algebraic Eq.:

$$\tilde\psi_{n-1} = A_k\,\tilde\psi_{n-2} + B_k\,\tilde\psi_n + M_k\,\tilde\psi_{n-1}$$
$$\tilde\psi_{n} = A_k\,\tilde\psi_{n-1} + B_k\,\tilde\psi_{n+1} + M_k\,\tilde\psi_{n}$$
$$\tilde\psi_{n+1} = A_k\,\tilde\psi_{n} + B_k\,\tilde\psi_{n+2} + M_k\,\tilde\psi_{n+1}$$

$$A_0 = zA, \qquad B_0 = zB, \qquad M_0 = zM$$



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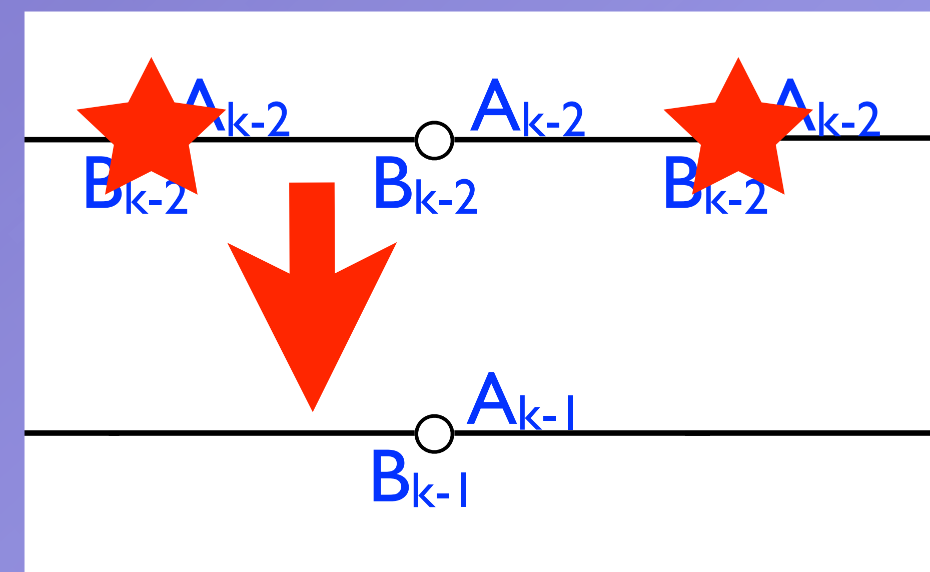

RG-Step

$$\tilde{\psi}_{n} = A_{k+1}\,\tilde{\psi}_{n-2} + B_{k+1}\,\tilde{\psi}_{n+2} + M_{k+1}\,\tilde{\psi}_{n}$$

$$A_{k+1} = A_k\,(I - M_k)^{-1}\,A_k, \quad B_{k+1} = B_k\,(I - M_k)^{-1}\,B_k,$$

$$M_{k+1} = M_k + A_k\,(I - M_k)^{-1}\,B_k + B_k\,(I - M_k)^{-1}\,A_k$$

RG for Quantum Walks on a Line

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$$\tilde{\psi}_{n-1} = A_k\,\tilde{\psi}_{n-2} + B_k\,\tilde{\psi}_n + M_k\,\tilde{\psi}_{n-1}$$
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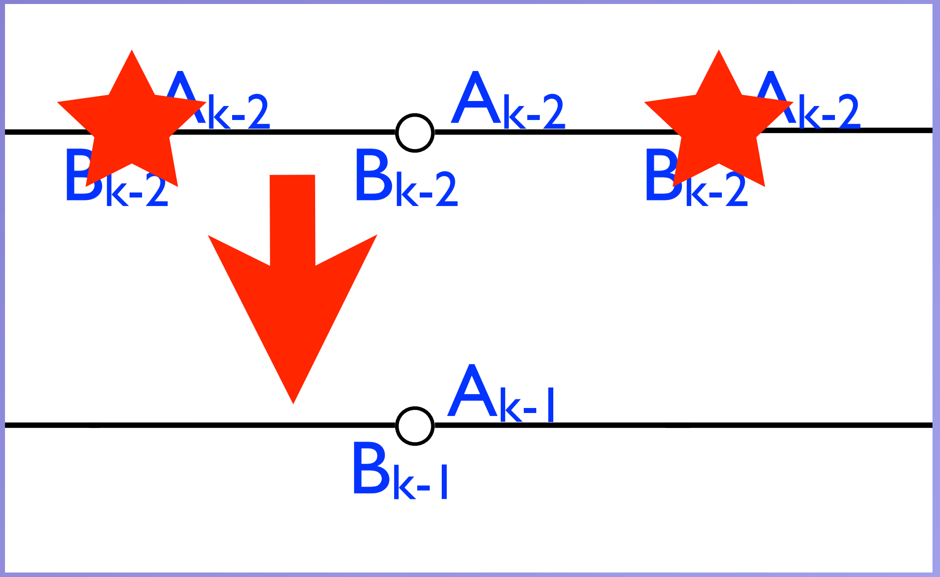

RG-Step

$$\tilde{\psi}_n = A_{k+1}\,\tilde{\psi}_{n-2} + B_{k+1}\,\tilde{\psi}_{n+2} + M_{k+1}\,\tilde{\psi}_n$$

RG-Flow:

$$A_{k+1} = A_k\,(I - M_k)^{-1}\,A_k, \quad B_{k+1} = B_k\,(I - M_k)^{-1}\,B_k,$$
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RG for Quantum Walks on a Line

RG for Quantum Walks on a Line

Example: RG for Classical Random Walk

$$a_{k+1} = \frac{a_k^2}{1-m_k}, \quad b_{k+1} = \frac{b_k^2}{1-m_k}, \quad m_{k+1} = m_k + \frac{2a_k b_k}{1-m_k}$$

$$a_0 = zp, \qquad b_0 = z(1-p), \qquad m_0 = 0$$

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Jacobian:

$$J_k=\det\left[\frac{\partial\left(a_{k+1},b_{k+1},\ldots\right)}{\partial\left(a_k,b_k,\ldots\right)}\right],\qquad \lambda=\max_{\mathrm{EV}}\left\{J_\infty\right\},\quad d_w=\log_2\lambda$$

$$a_\infty=b_\infty=m_\infty=0$$

$$a_\infty=0,\quad b_\infty=1-m_\infty,\quad \text{or}\quad a_\infty=1-m_\infty,\quad b_\infty=0$$

$$a_\infty\sim b_\infty\sim\alpha_k\,\epsilon^k\to 0,\quad m_\infty\sim 1-\mu_k\,\epsilon^k,\quad (\epsilon<1)$$

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Fixed Point 1:

$$a_\infty = b_\infty = m_\infty = 0$$

$\Longrightarrow$ $d_w = \infty$

$$a_\infty = 0, \quad b_\infty = 1 - m_\infty, \quad \text{or} \quad a_\infty = 1 - m_\infty, \quad b_\infty = 0$$

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$\Longrightarrow$ $d_w=\infty$

Fixed Point 2:

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$\Longrightarrow$ $d_w=1$

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Fixed Point 1:

$$a_\infty=b_\infty=m_\infty=0$$

$\Longrightarrow$ $d_w=\infty$

Fixed Point 2:

$$a_\infty=0,\quad b_\infty=1-m_\infty,\quad \text{or}\quad a_\infty=1-m_\infty,\quad b_\infty=0$$

$\Longrightarrow$ $d_w=1$

Fixed Point 3:

$$a_\infty\sim b_\infty\sim\alpha_k\,\epsilon^k\to 0,\quad m_\infty\sim 1-\mu_k\,\epsilon^k,\quad (\epsilon<1)$$

$\Longrightarrow$ $d_w=2$

RG for Quantum Walks on a Line

RG-Flow?

$$A_{k+1} = A_k \left(I - M_k\right)^{-1} A_k, \quad B_{k+1} = B_k \left(I - M_k\right)^{-1} B_k,$$

$$M_{k+1} = M_k + A_k \left(I - M_k\right)^{-1} B_k + B_k \left(I - M_k\right)^{-1} A_k$$

How to Parametrize? (Must satisfy Unitarity Condition!)

RG for Quantum Walks on a Line

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How to Parametrize? (Must satisfy Unitarity Condition!)

1) Guess $P_k=\begin{pmatrix} a_k & 0\\ 0 & 0\end{pmatrix},\qquad Q_k=\begin{pmatrix} 0 & 0\\ 0 & -a_k\end{pmatrix},\qquad R_k=\begin{pmatrix} 0 & b_k\\ b_k & 0\end{pmatrix}$

RG for Quantum Walks on a Line

RG-Flow?

$$A_{k+1} = A_k \left(I - M_k\right)^{-1} A_k, \qquad B_{k+1} = B_k \left(I - M_k\right)^{-1} B_k,$$

$$M_{k+1} = M_k + A_k \left(I - M_k\right)^{-1} B_k + B_k \left(I - M_k\right)^{-1} A_k$$

How to Parametrize? (Must satisfy Unitarity Condition!)

1) Guess $P_k = \begin{pmatrix} a_k & 0 \\ 0 & 0 \end{pmatrix}, \quad Q_k = \begin{pmatrix} 0 & 0 \\ 0 & -a_k \end{pmatrix}, \quad R_k = \begin{pmatrix} 0 & b_k \\ b_k & 0 \end{pmatrix}$

2) Insert Coin $A_k = P_k \mathcal{C}, \quad B_k = Q_k \mathcal{C}, \quad M_k = R_k \mathcal{C}.$

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3) Renormalize $\{A_{k+1}, B_{k+1}, \ldots\} = \mathcal{RG}\left(\{A_k, B_k, \ldots\}\right)$

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RG for Quantum Walks on a Line

RG-Flow:

$$a_{k+1} = \frac{a_k^2 \cos\rho}{1 - 2b_k \sin\rho + b_k^2}, \quad b_{k+1} = b_k + \frac{a_k^2\left(b_k - \sin\rho\right)}{1 - 2b_k \sin\rho + b_k^2}$$

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Jacobian:

$$J_k = \det\left[\frac{\partial\left(a_{k+1}, b_{k+1}, \ldots\right)}{\partial\left(a_k, b_k, \ldots\right)}\right], \qquad \lambda = \max_{\mathrm{EV}}\left\{J_\infty\right\}, \quad d_w = \log_2 \lambda$$

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Trivial FP: $a_\infty = b_\infty = 0$

$\Longrightarrow$ $d_w = \infty$

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Same as Exact!

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Problems for Quantum Walk RG

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$$P_i = \sum_{t=0}^{\infty} \left|\psi_{i,t}\right|^2 = \oint \frac{dz}{2\pi i z} \left|\tilde{\psi}_i(z)\right|^2$$

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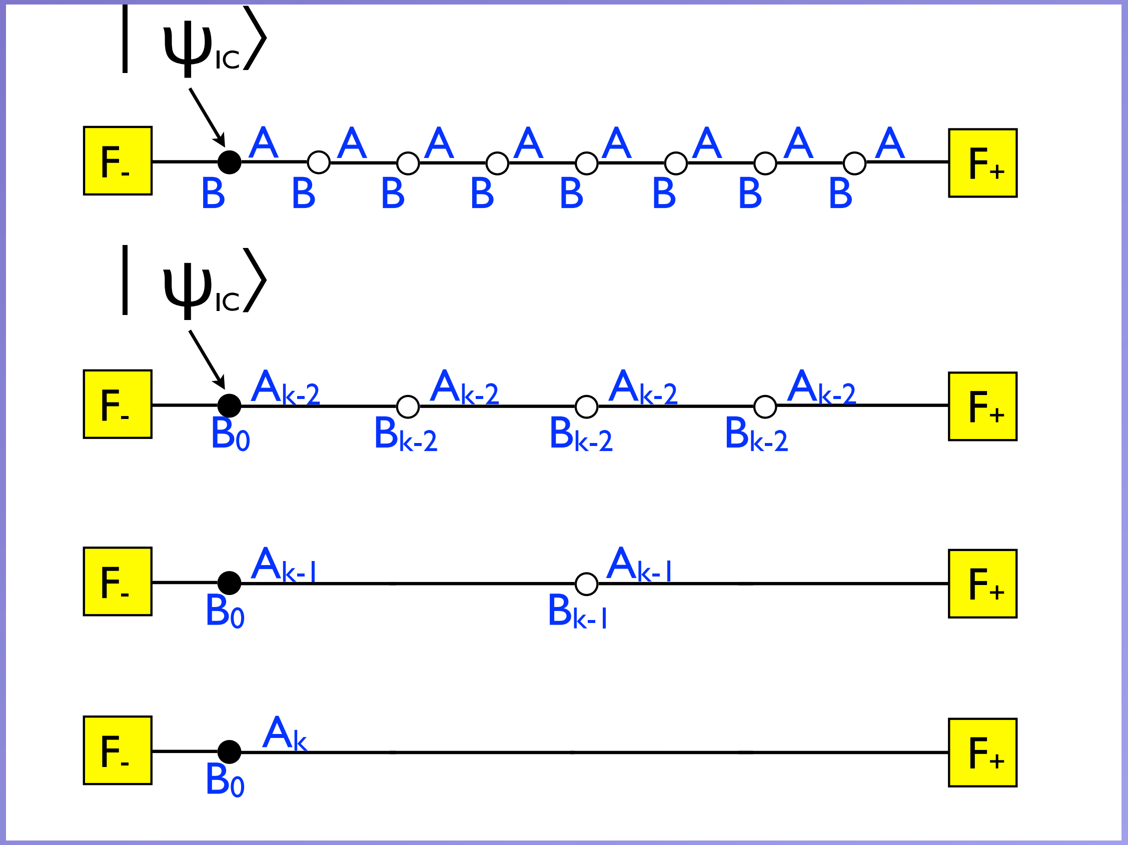

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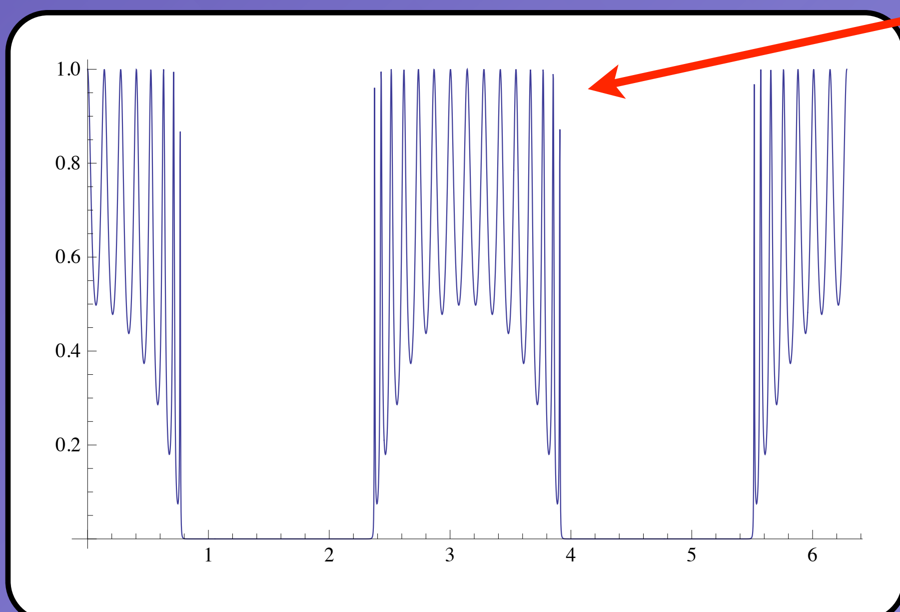

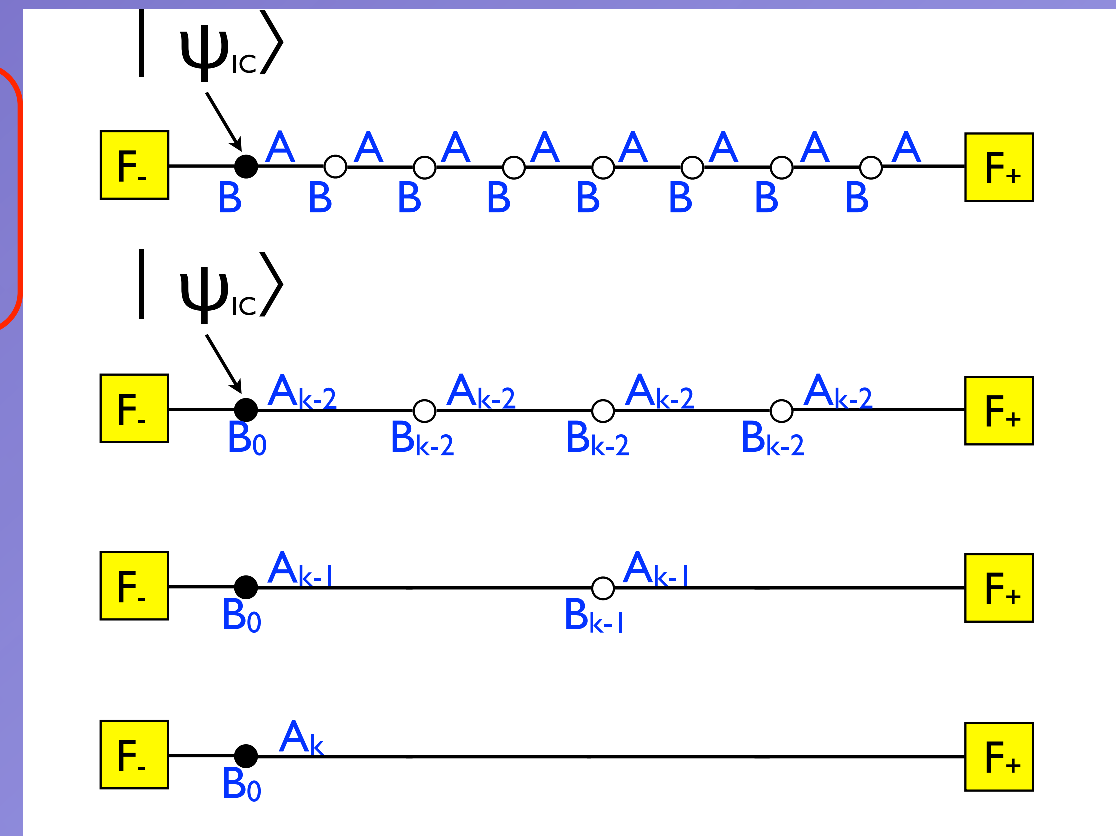

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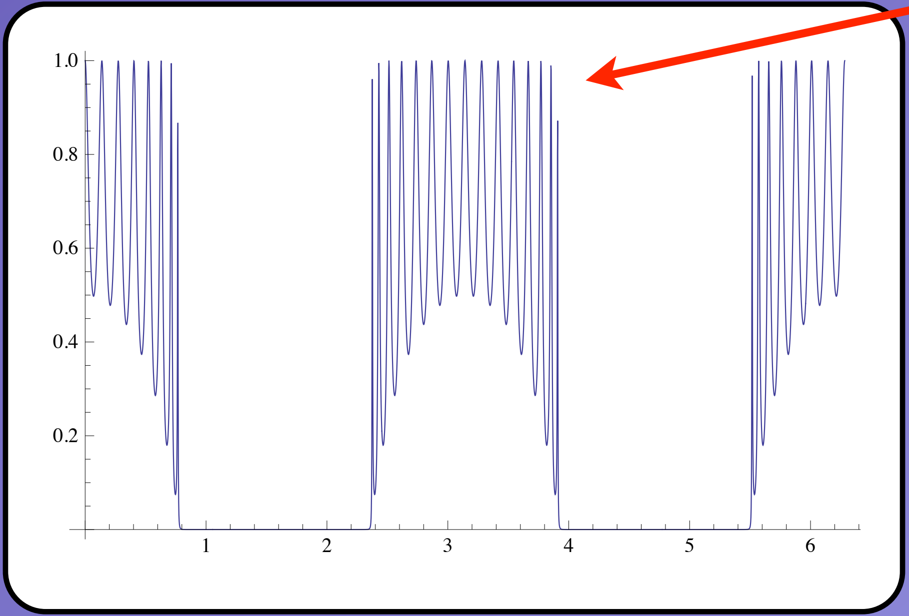

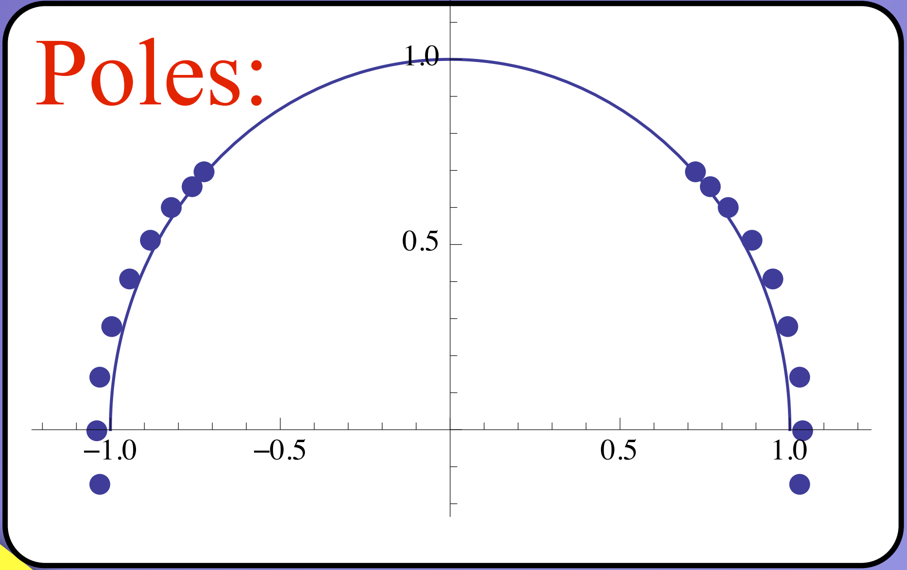

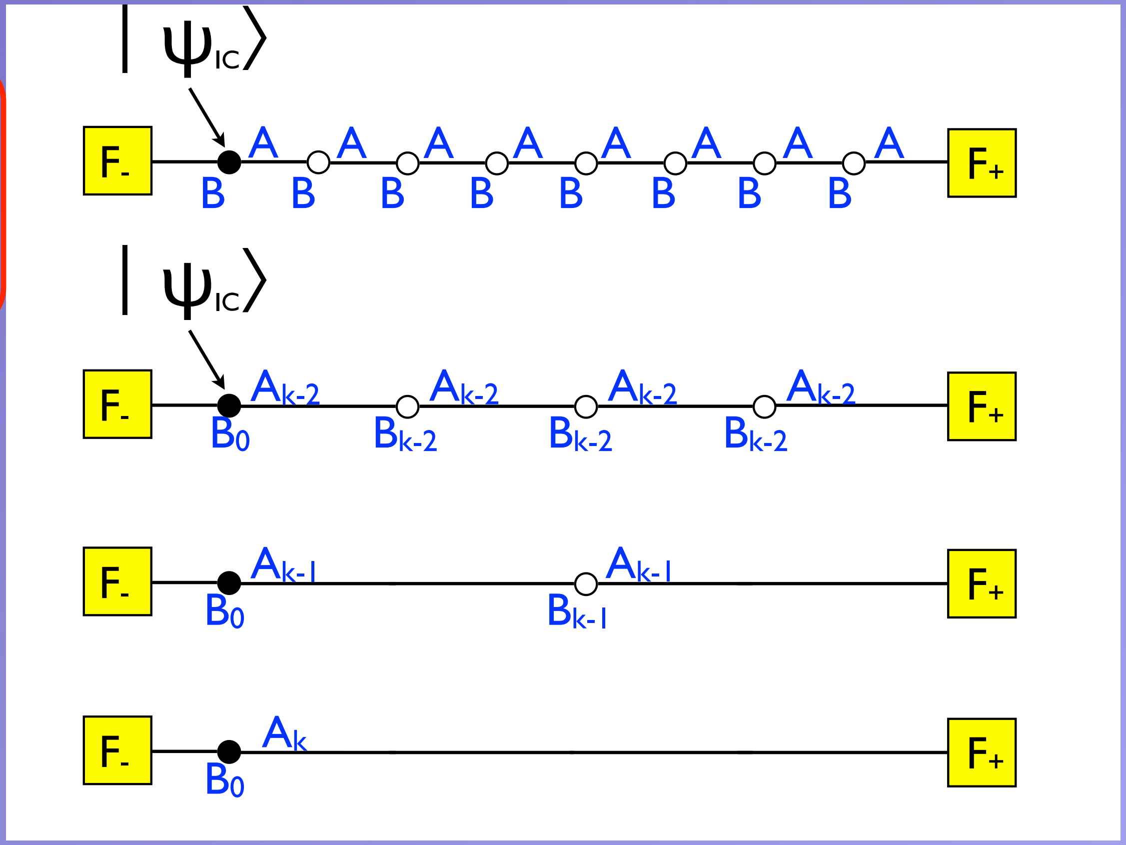

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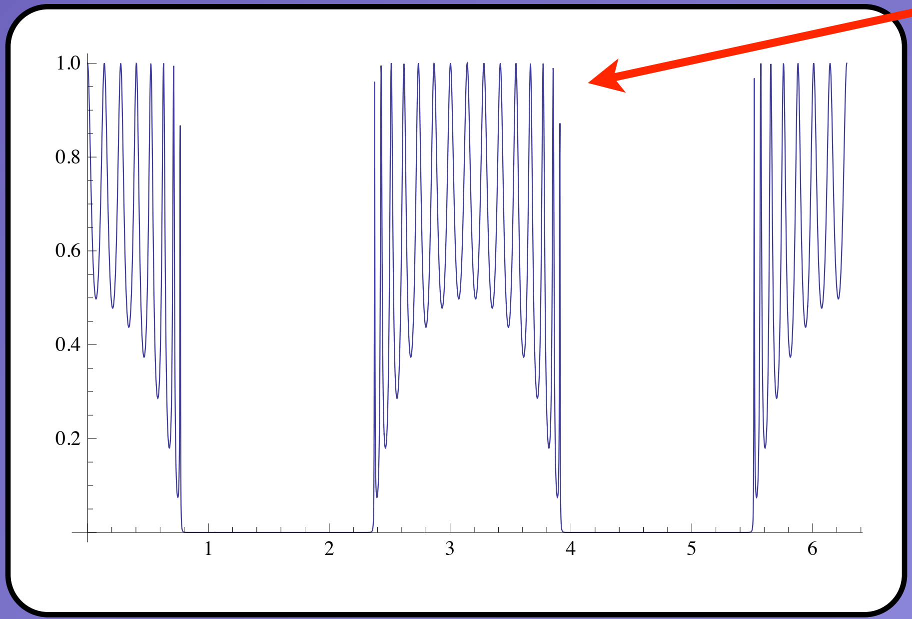
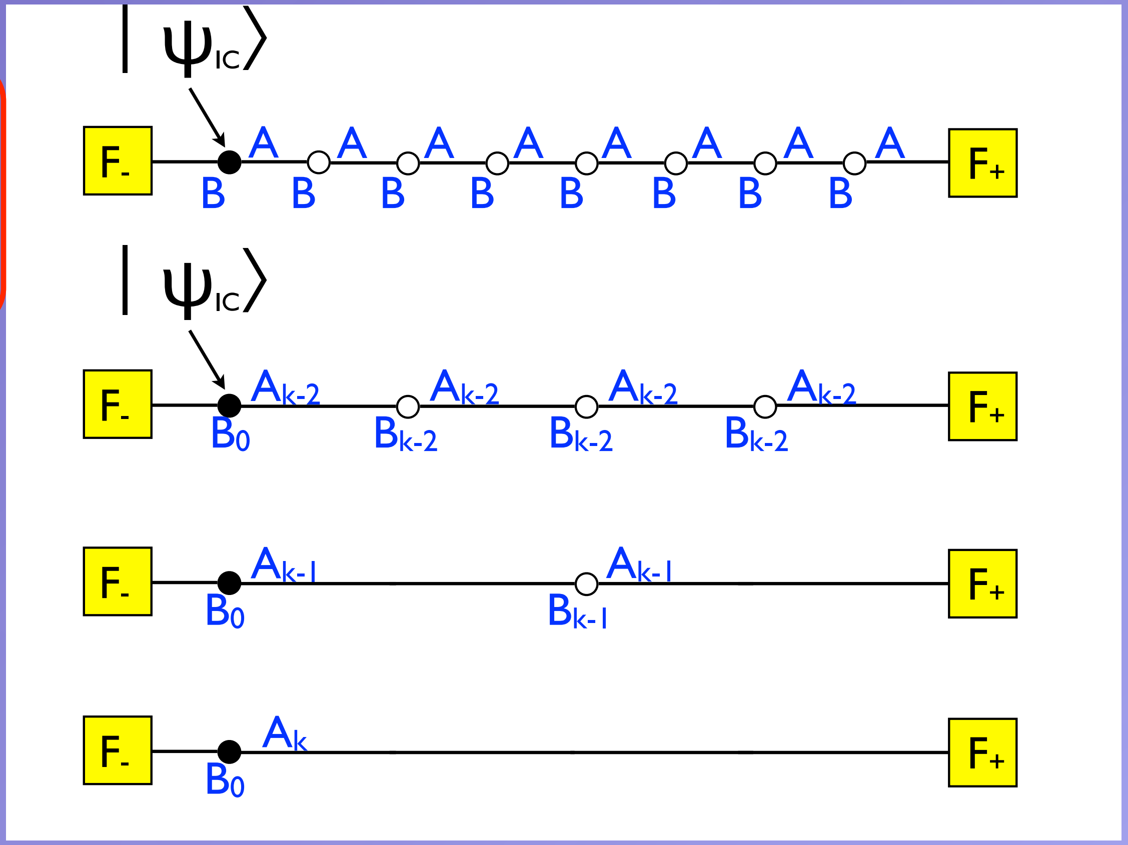
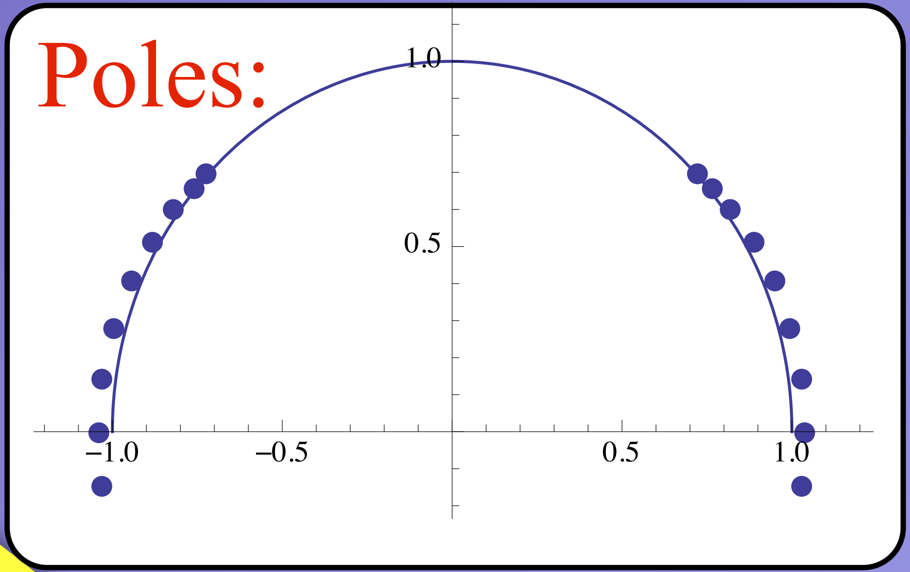
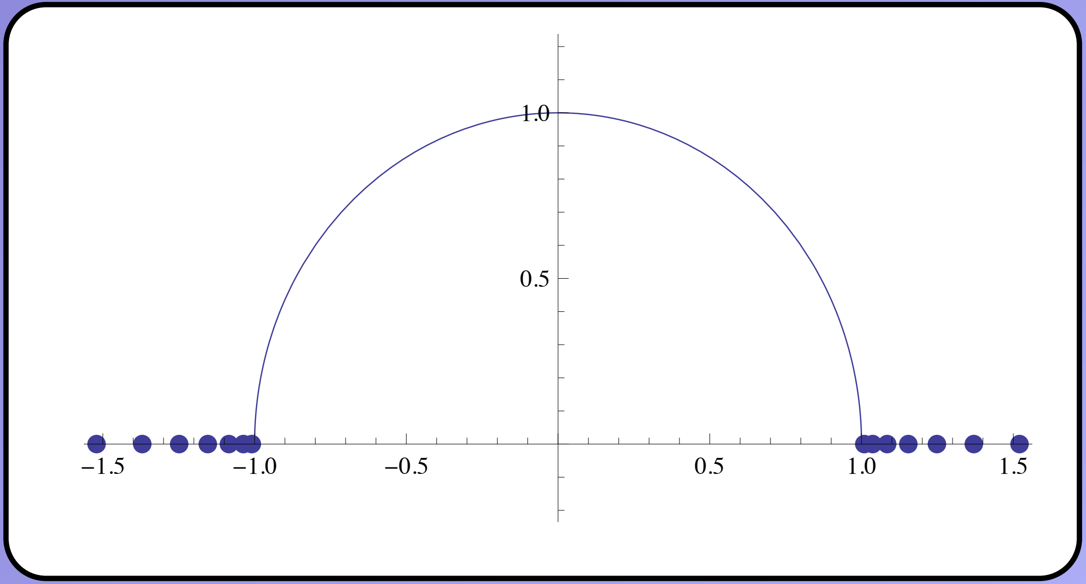






Classical RW:



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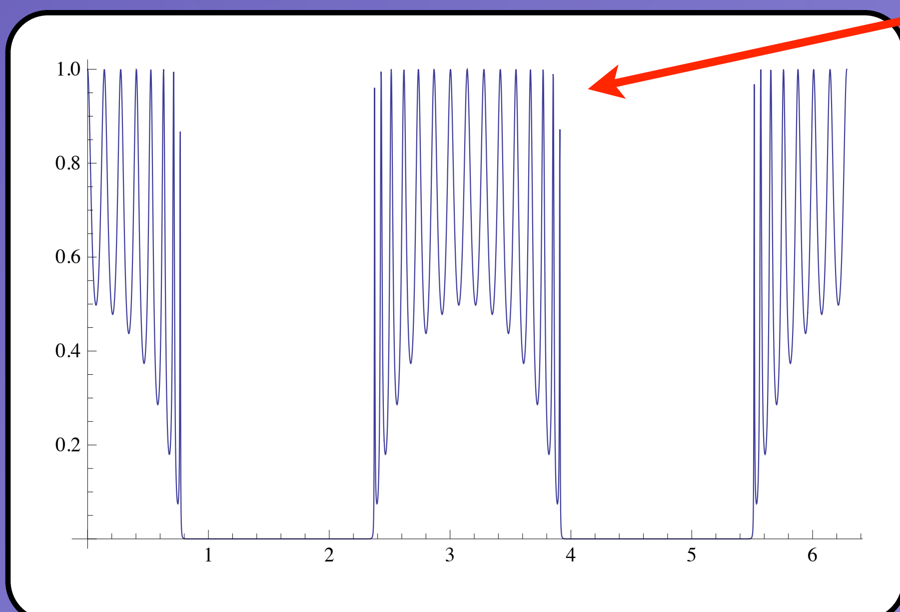

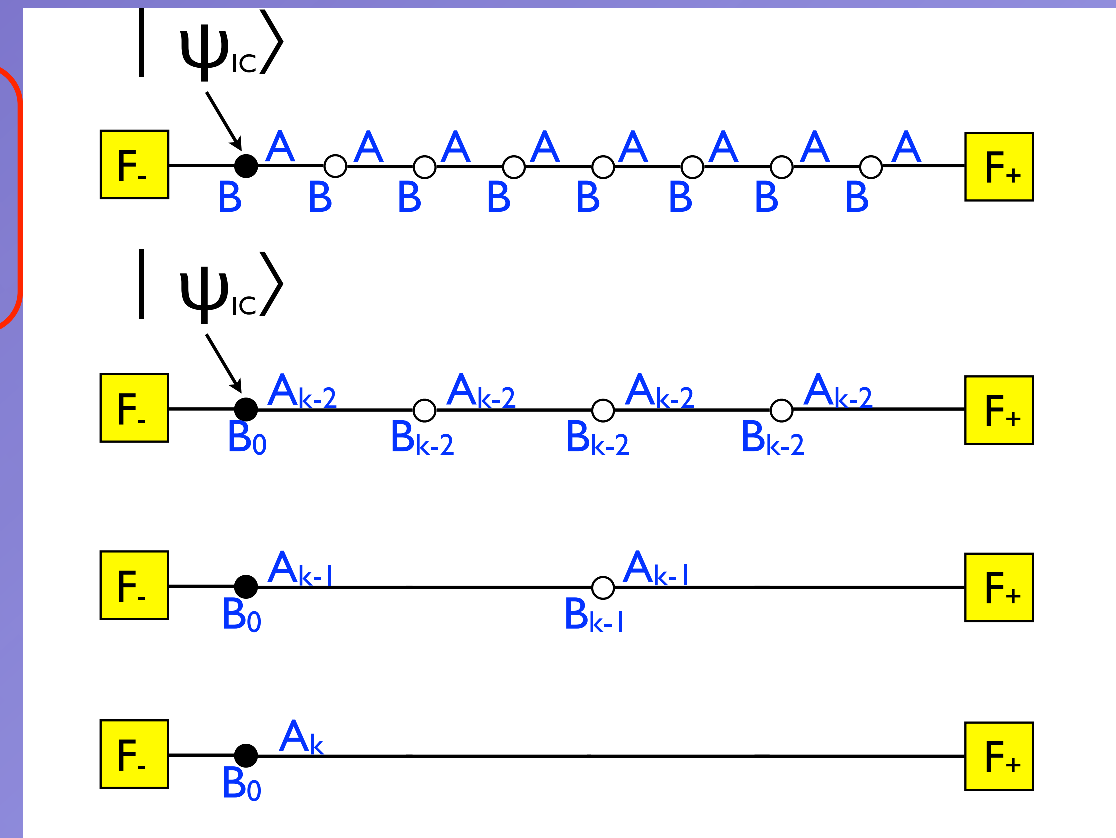

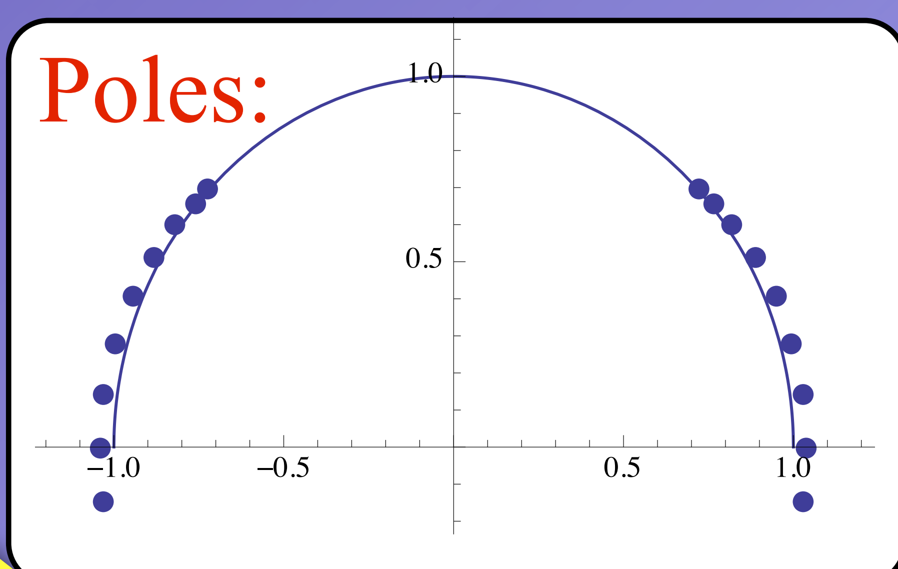

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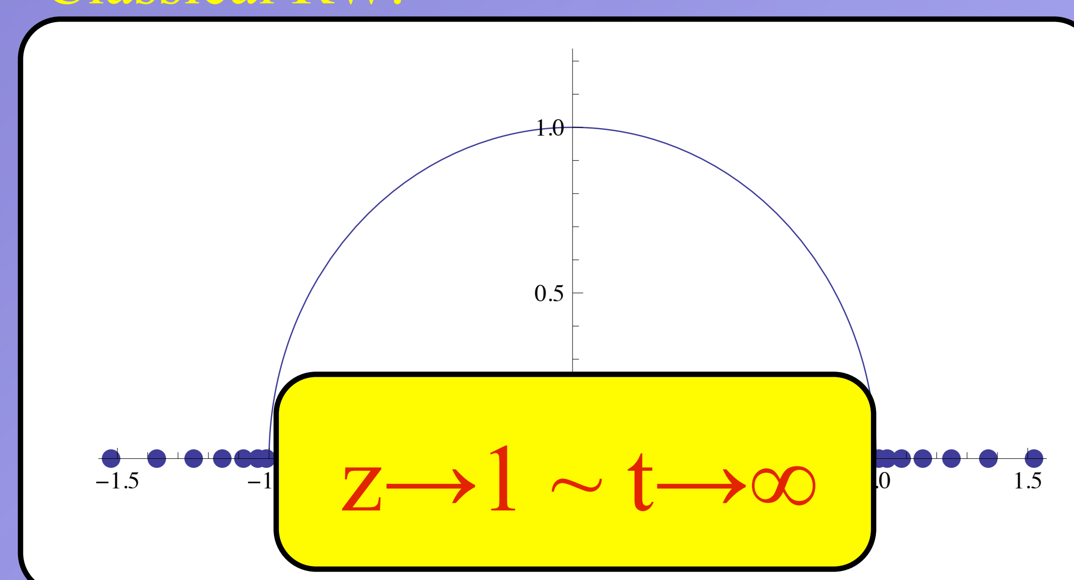

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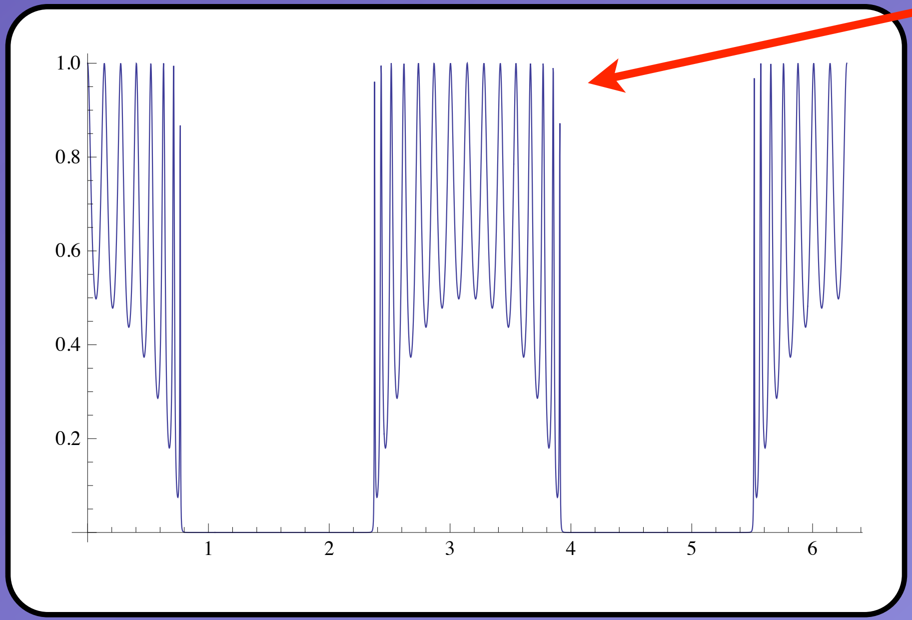

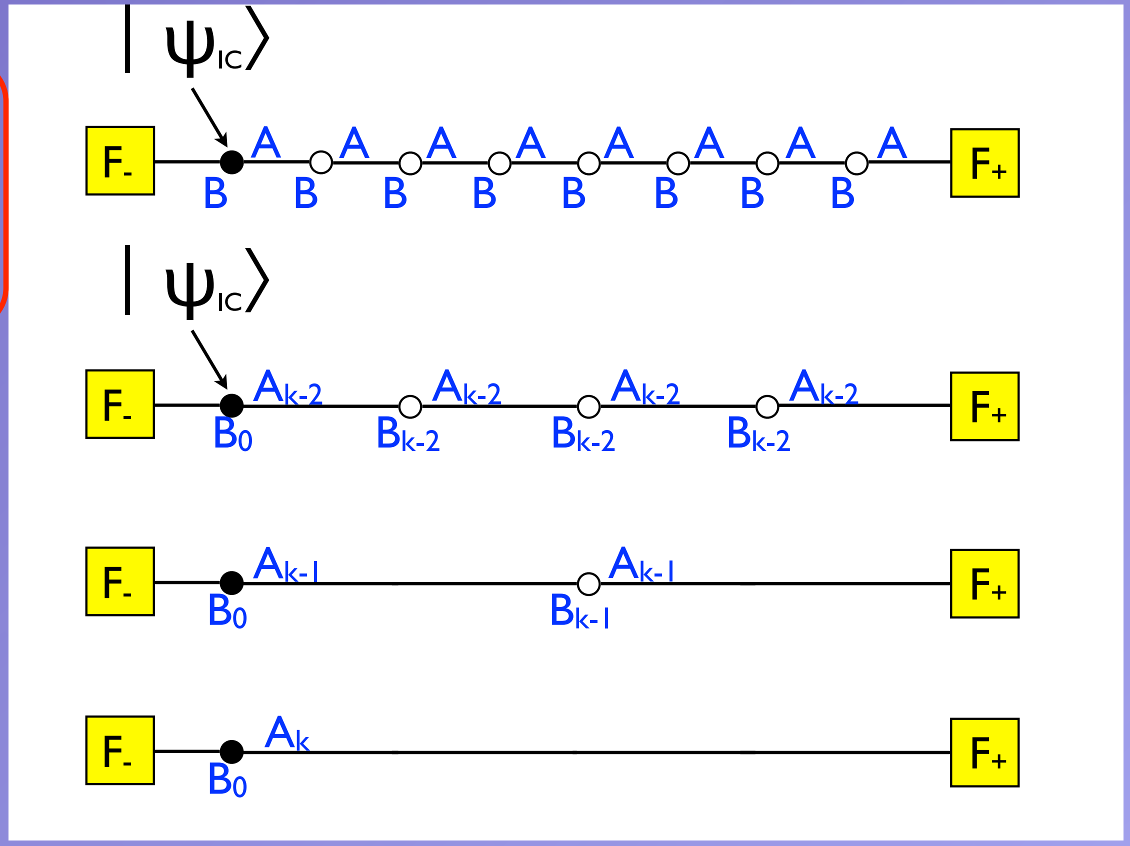

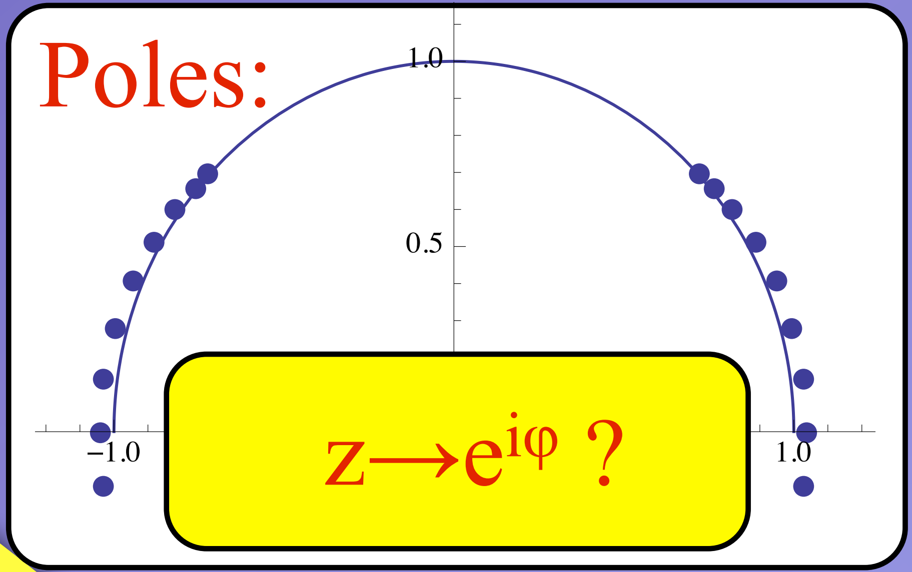

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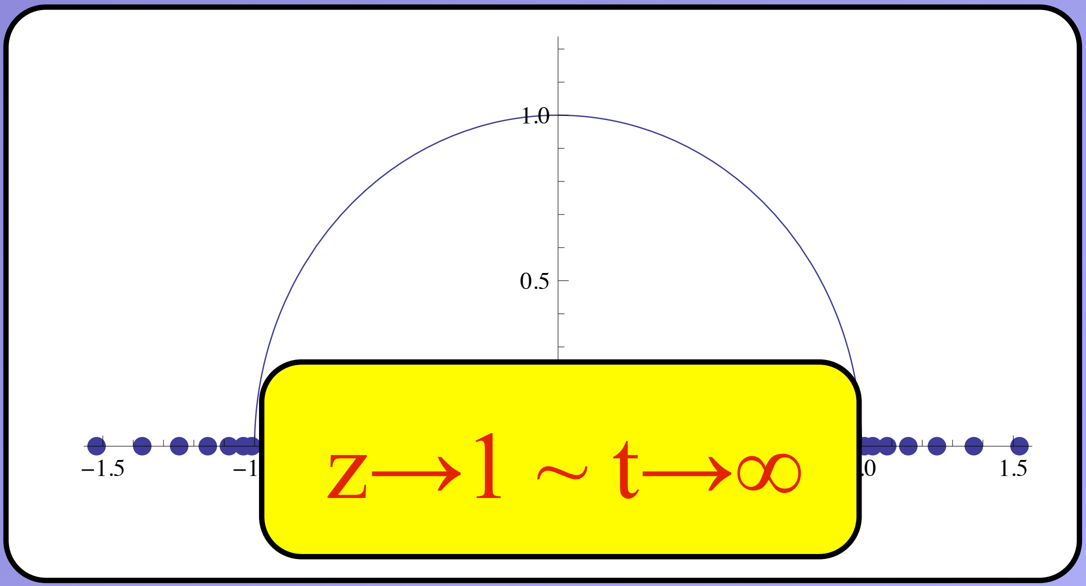

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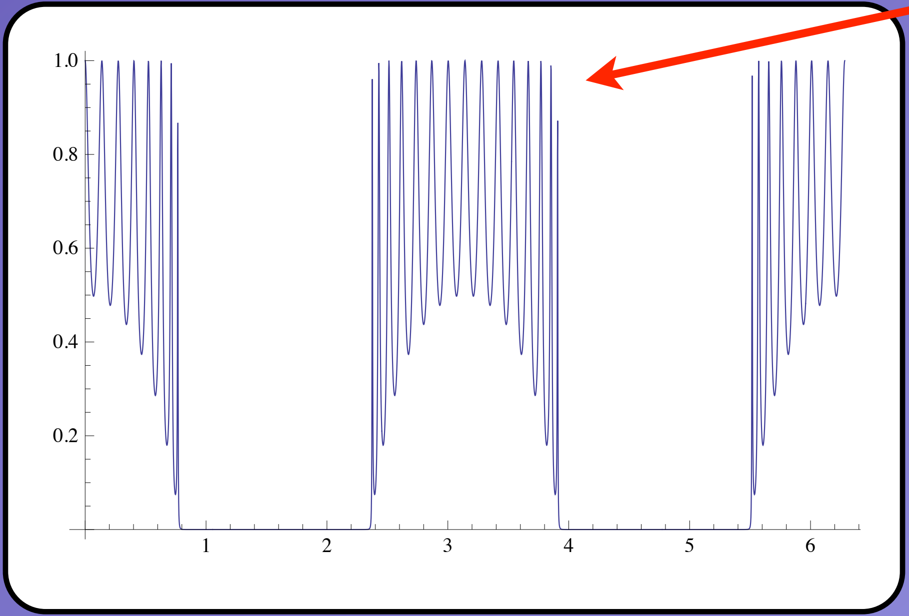

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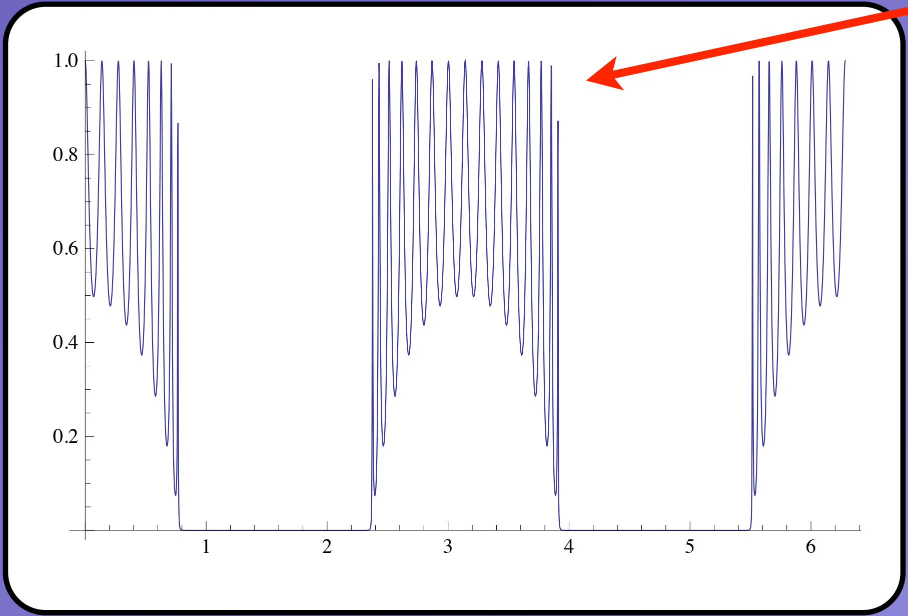

Lyapunov:

$$\Lambda_k = \prod_{i=0}^{k} J_i$$

$$\lambda = \lim_{k\to\infty} \left(\Lambda_k^\dagger \Lambda_k\right)^{\frac{1}{2k}}$$

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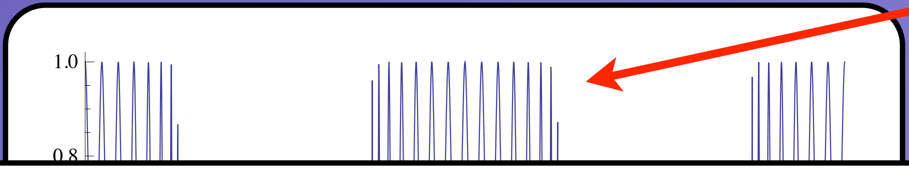

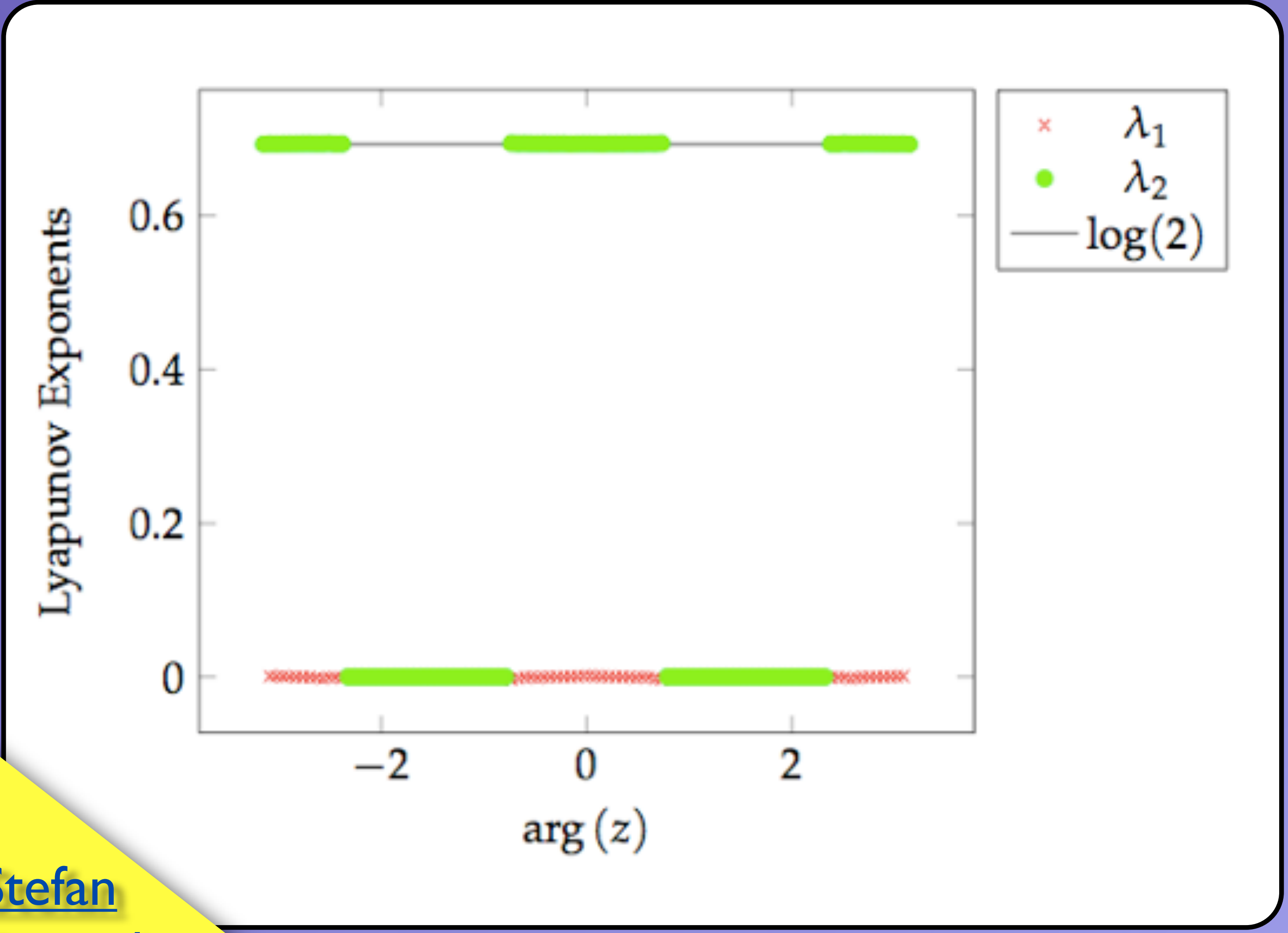

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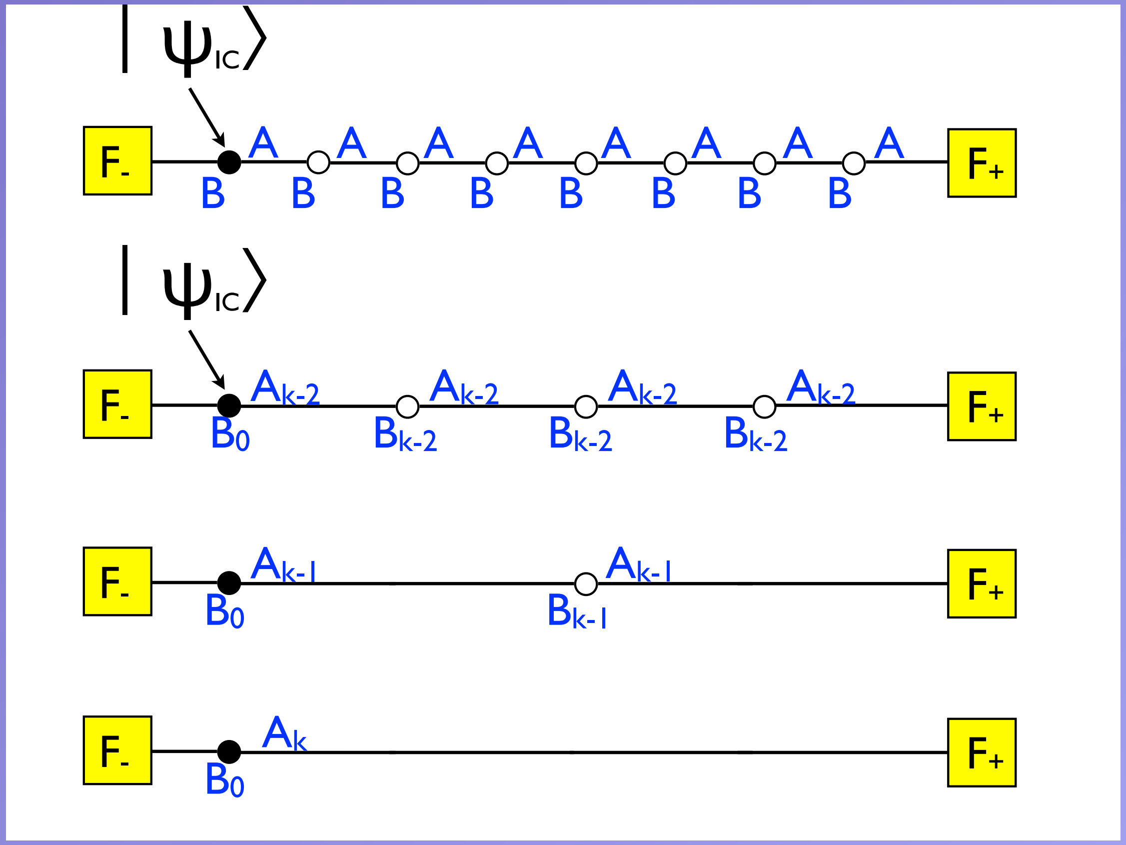

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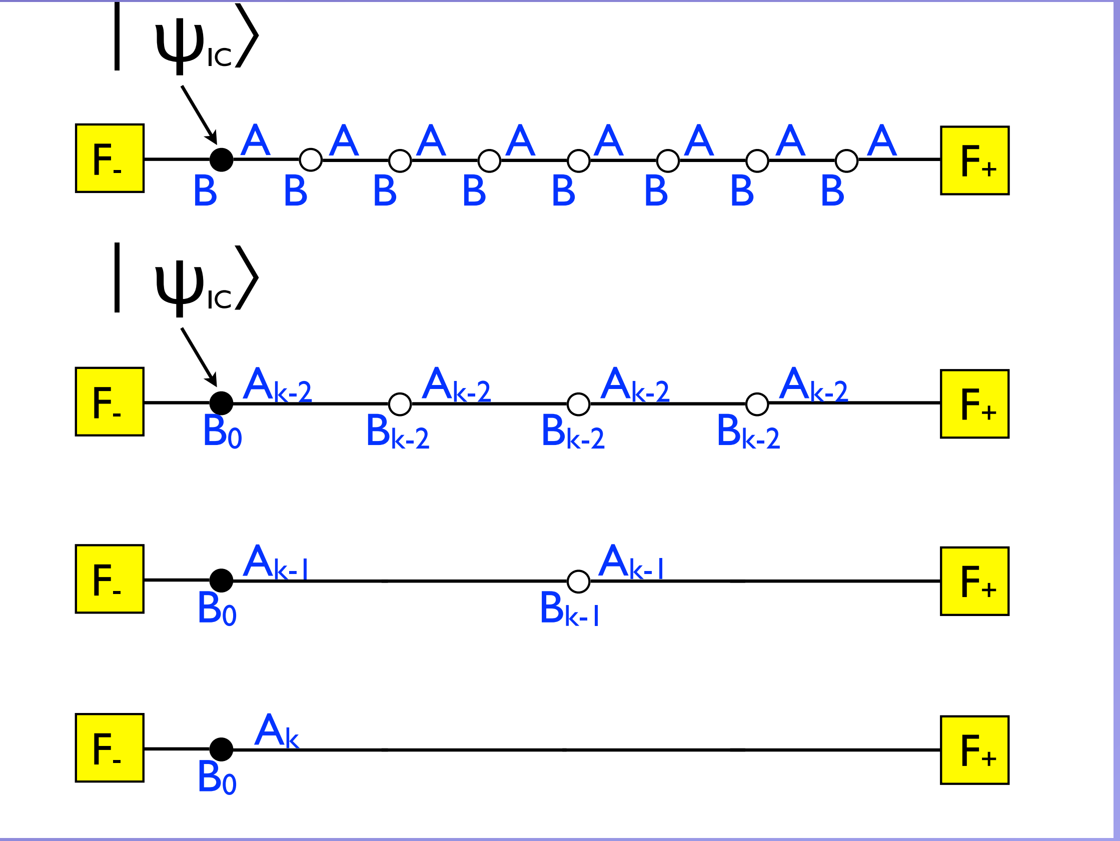

Polya???

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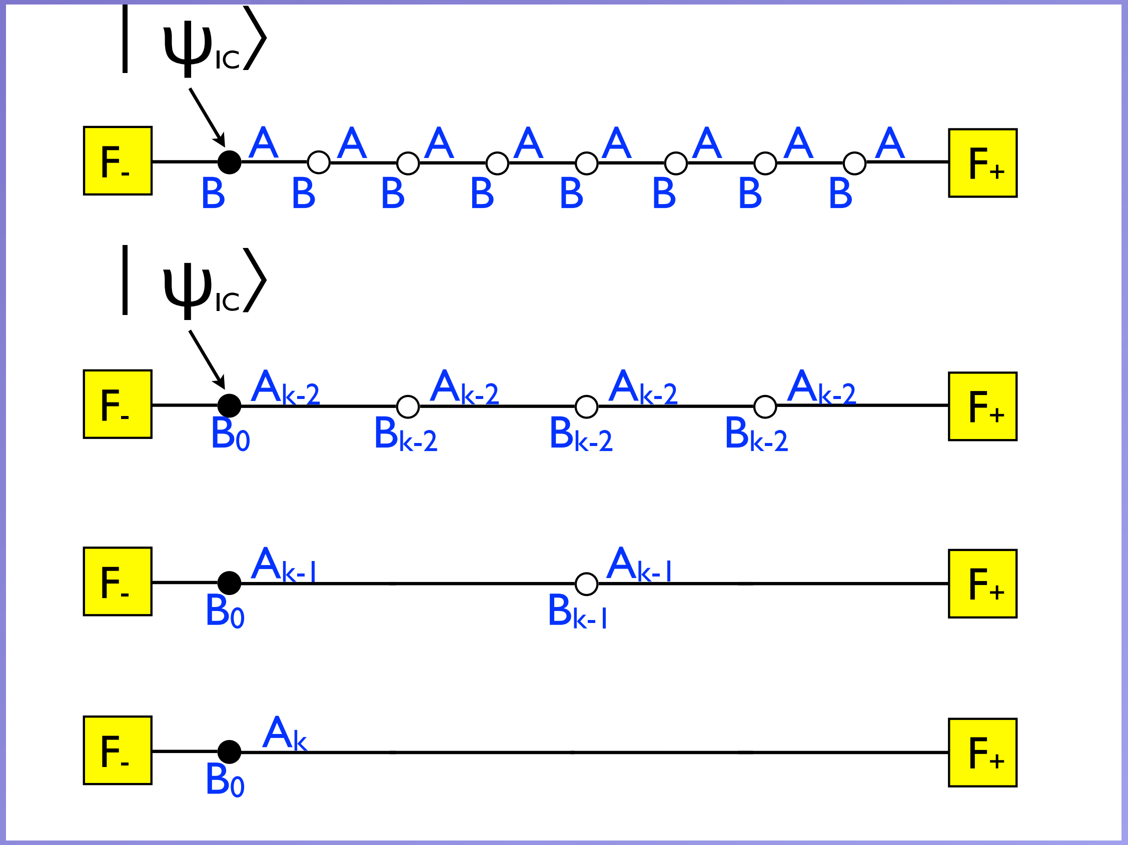

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Absorption in finite Domain:

$$F_- = \frac{1}{\sqrt{2}}$$

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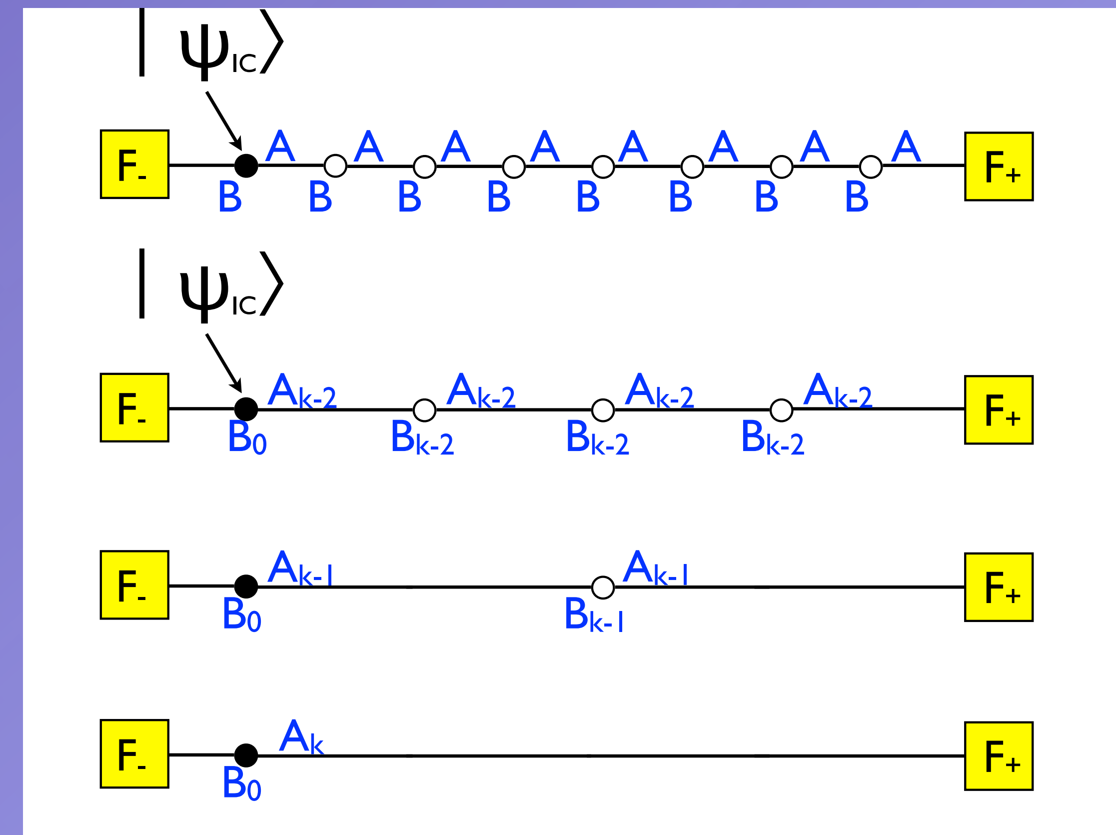

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Absorption in finite Domain:

$$F_- = \frac{1}{\sqrt{2}}$$

Absorption on semi-infinite Line:

$$F'_- = \frac{2}{\pi}, \quad (< 1 \text{ and } \neq F_- \text{ for } L \to \infty!)$$

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Problems for Quantum Walk RG

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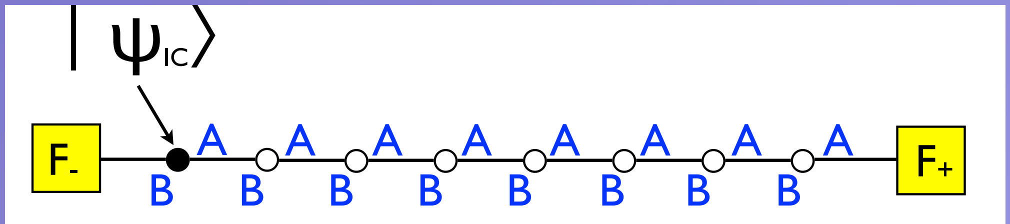
Problem 2: Localization

For 3x3-Coin, finite fraction of QW never reaches the wall!



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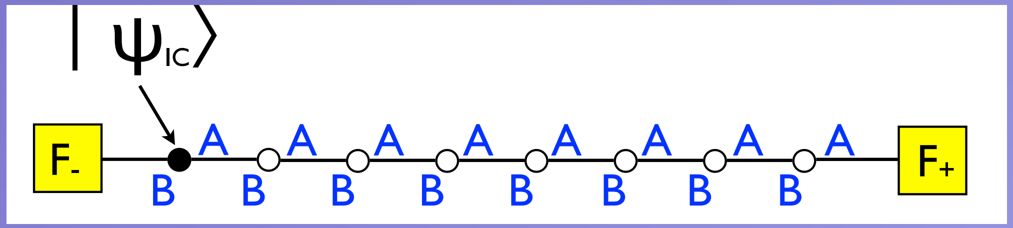
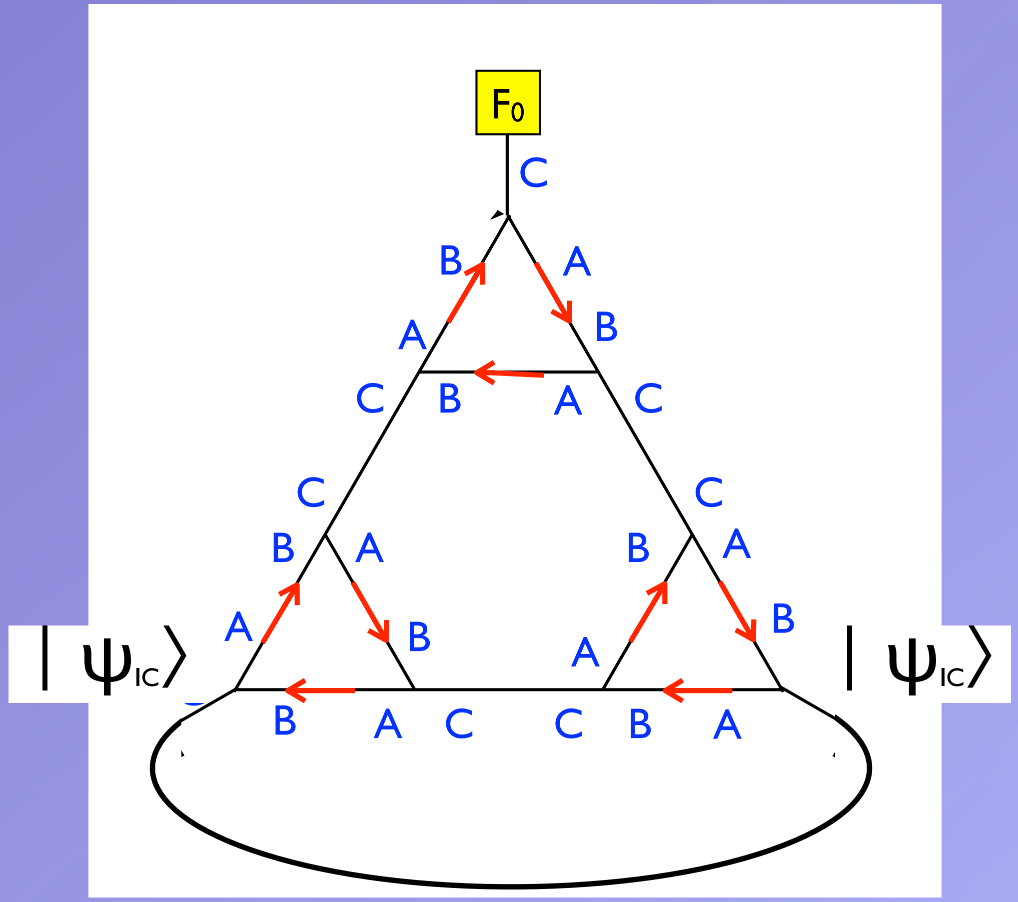


On Dual-Sierpinski:

For Grover-Coin, ***all*** of QW never reaches the wall for L→∞!



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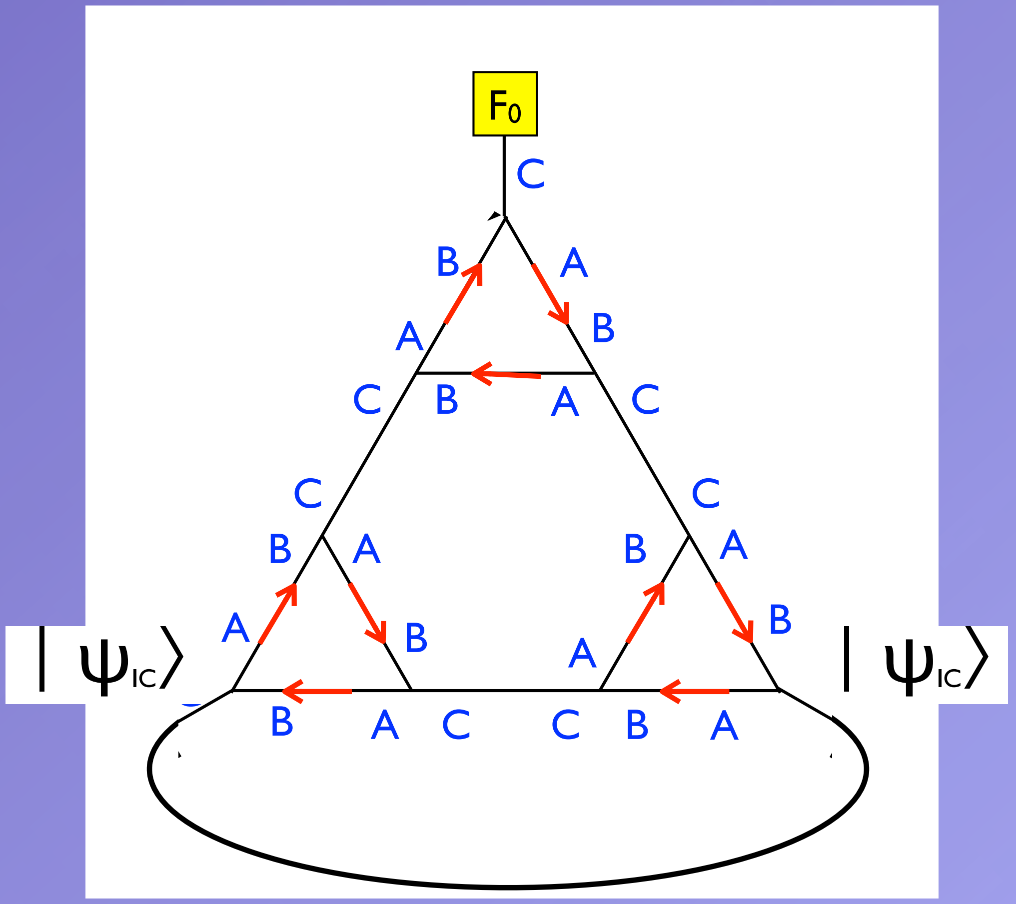

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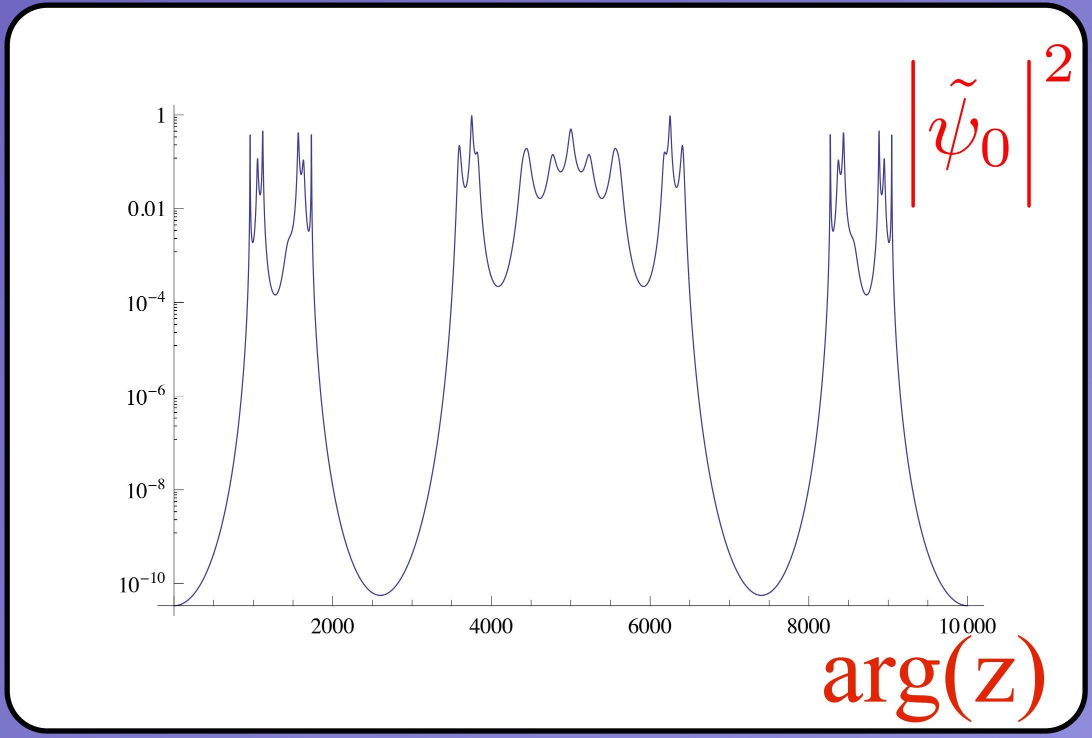

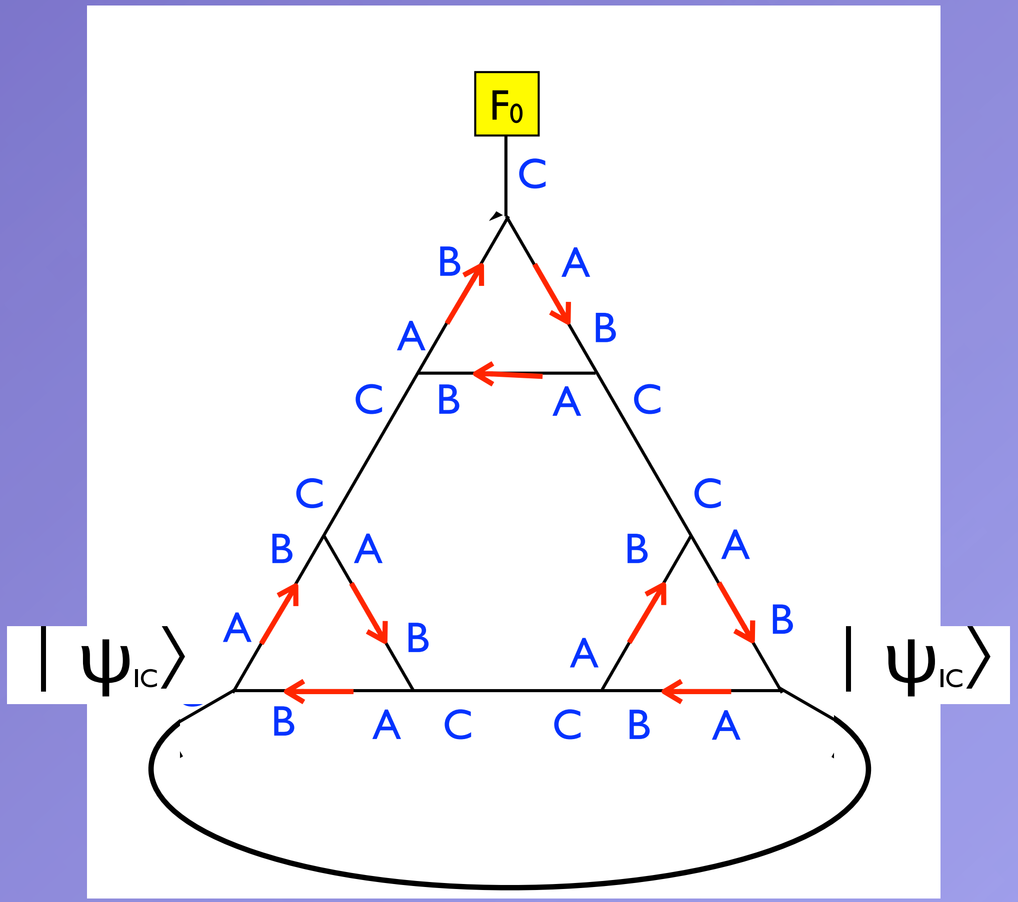

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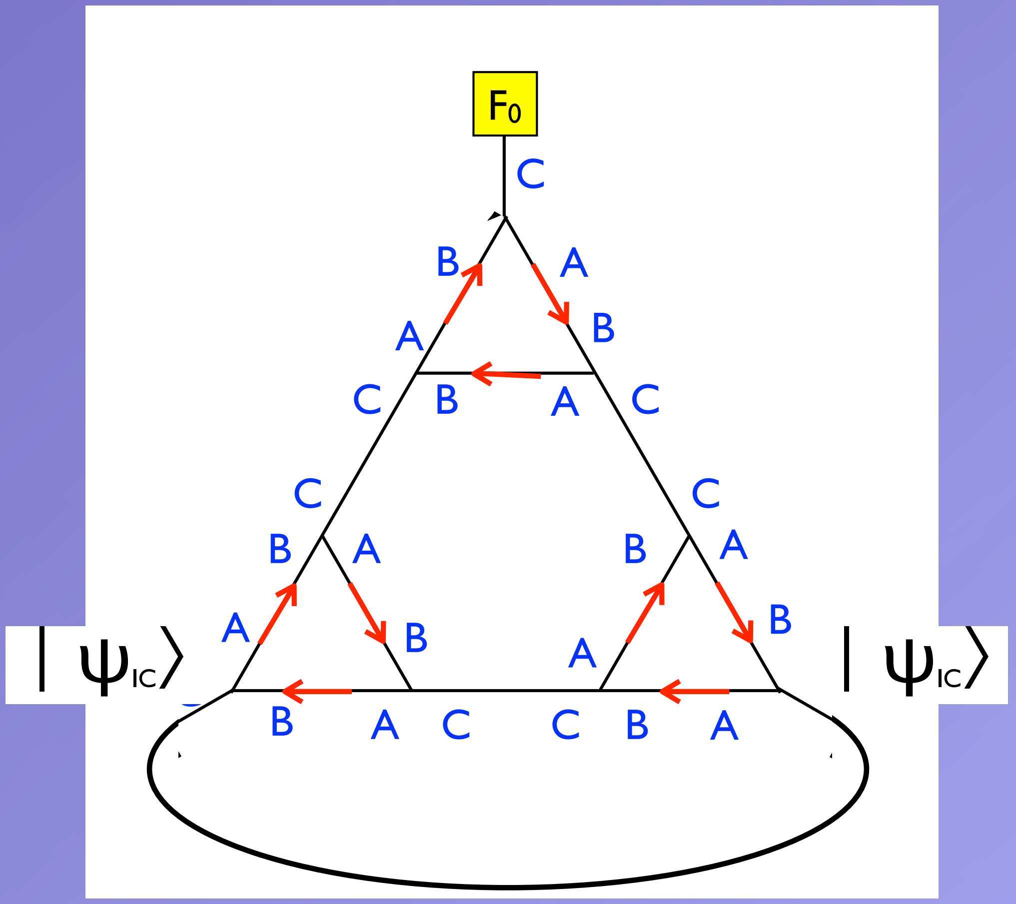
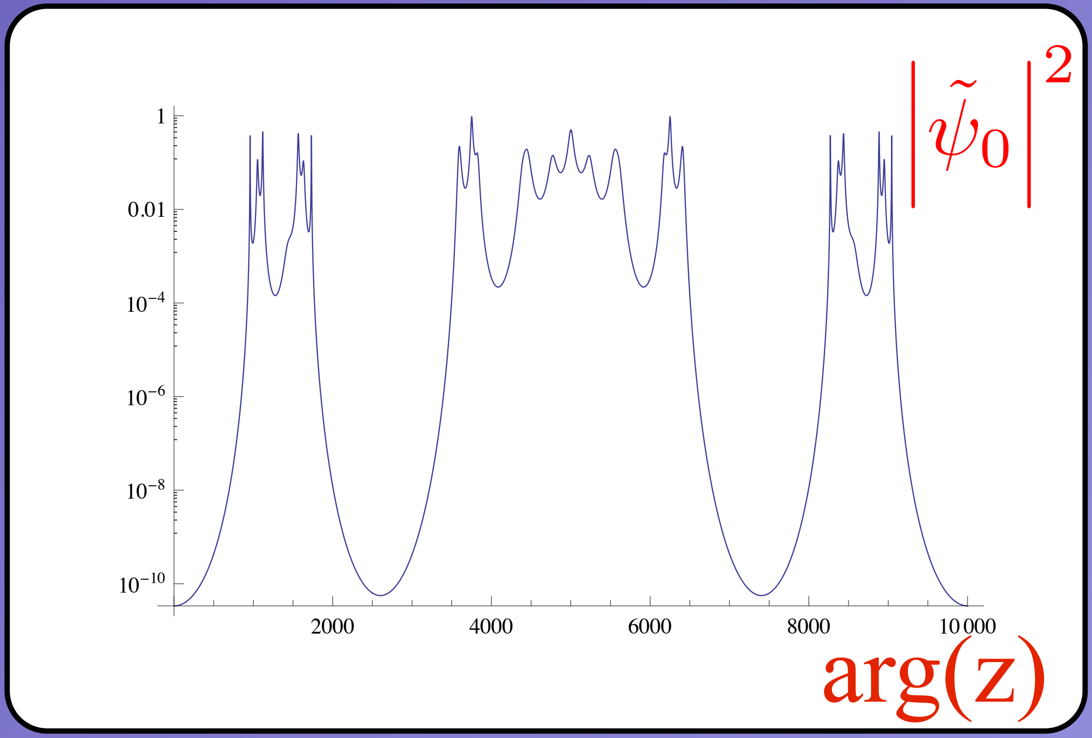
Problems for Quantum Walk RG

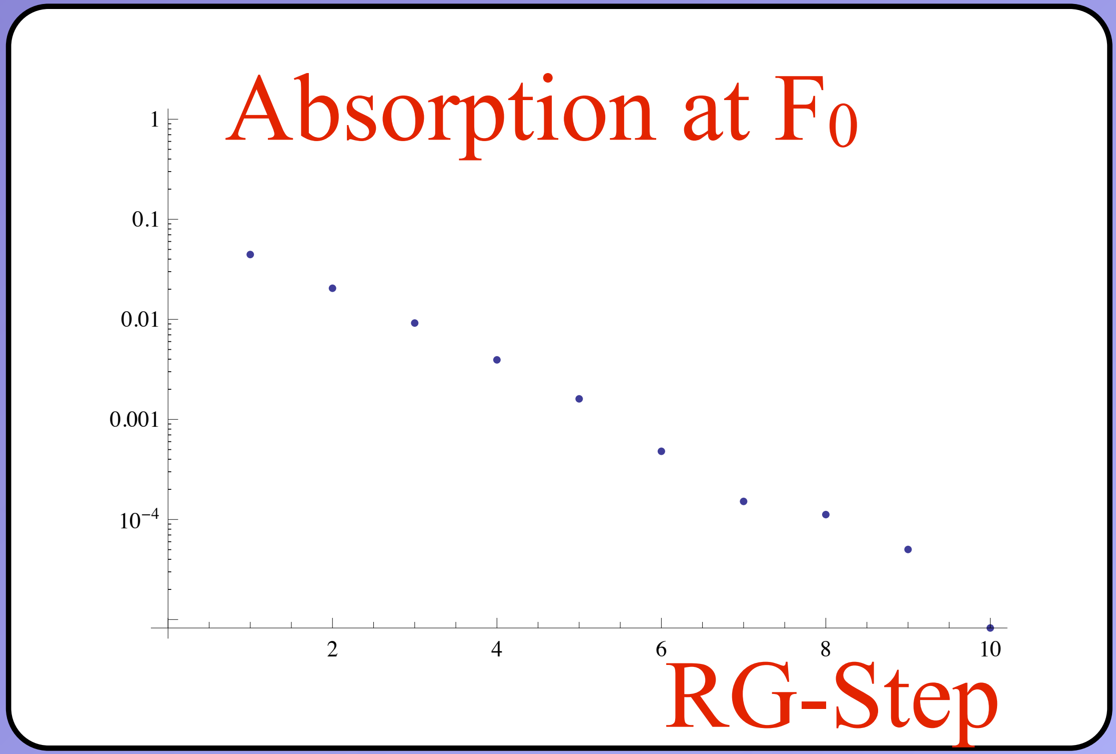
Problem 2: **Localization**

On Dual-Sierpinski:

For Grover-Coin, ***all*** of QW never reaches the wall for $L \rightarrow \infty$!







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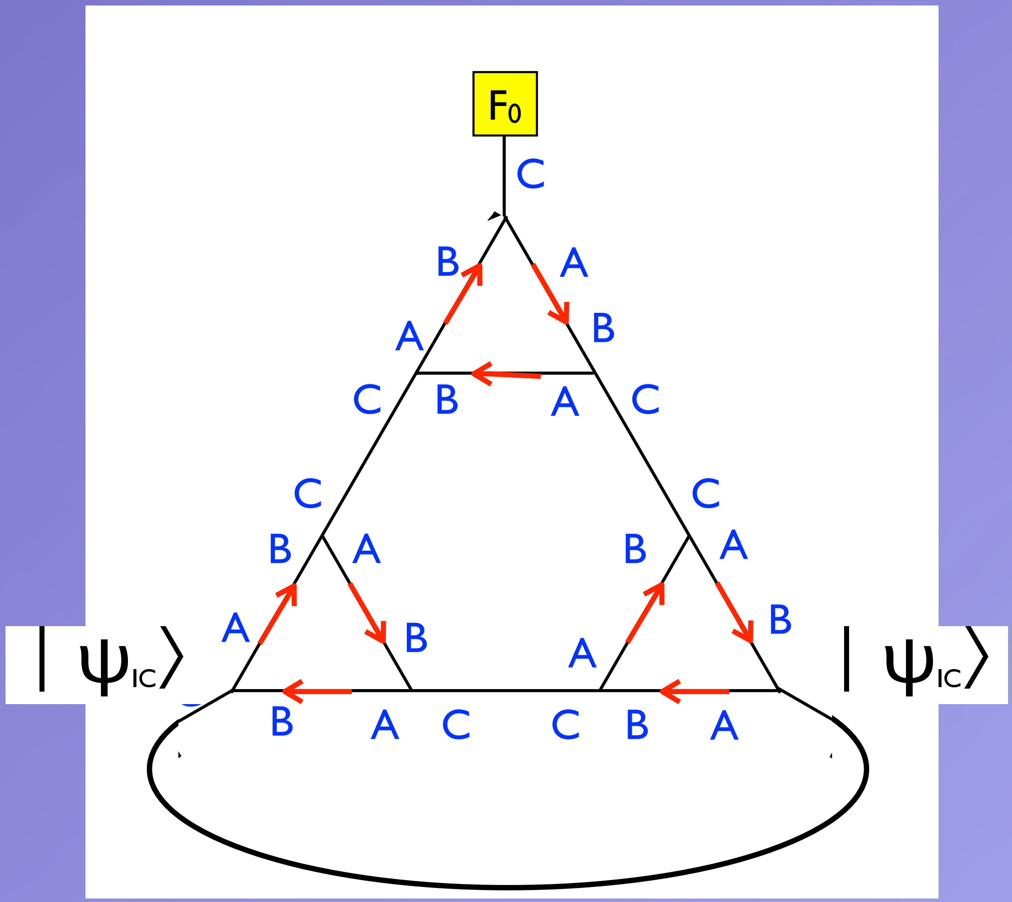

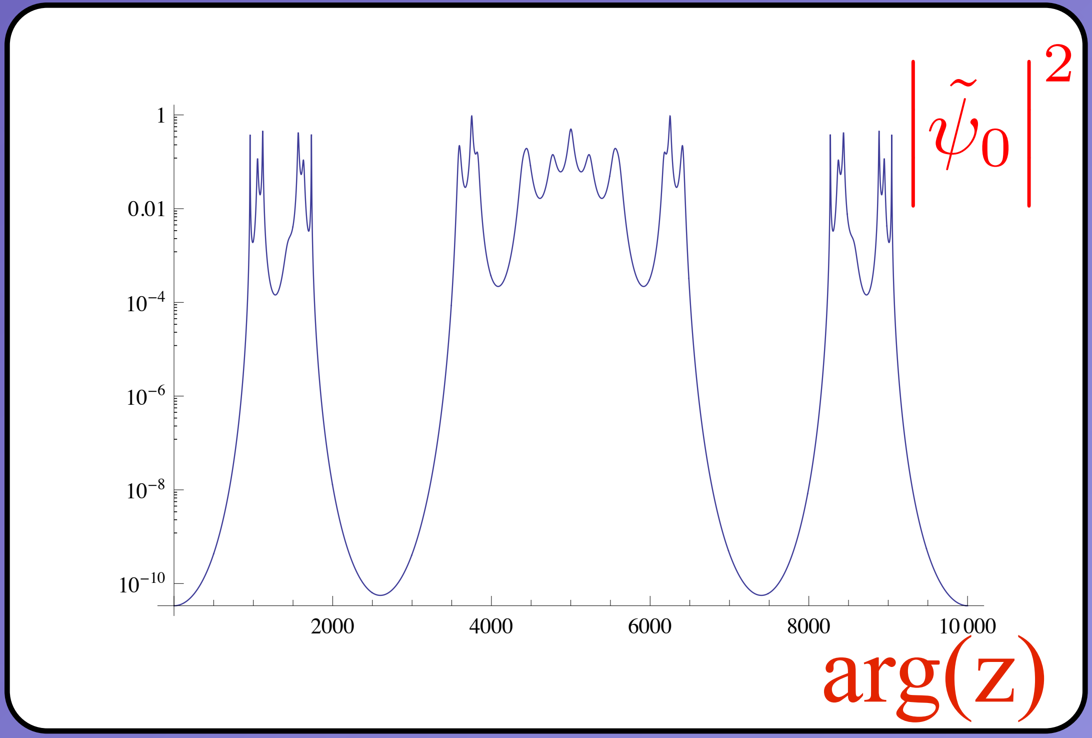

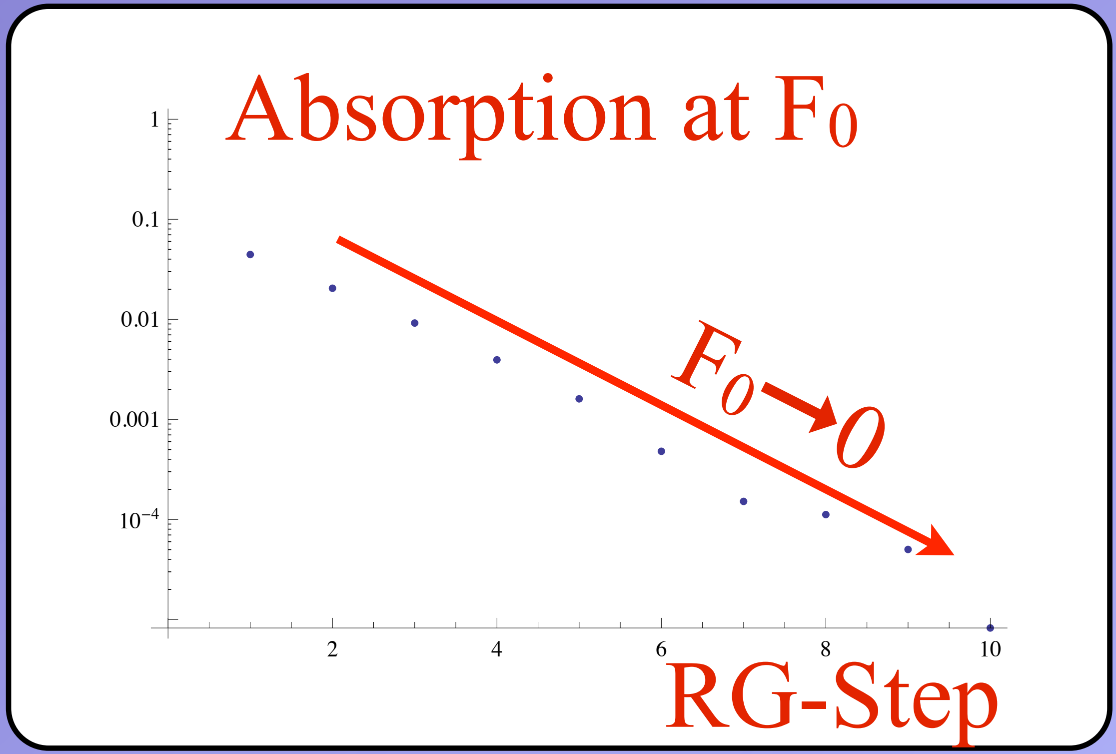

Conclusions:

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RG for Quantum Walks

- Encouraging Results for 1d-Line (universal $d_w=1$)
- Complex Fixed Point Analysis
- Can Lyapunov Procedure give Finite-Size Scaling?
- Can Localization be included?

General:

- What is Universality in Quantum Algorithms?