**NLTS A* of computer performance dynamics**

* nonlinear time-series analysis

bzip2 dynamics on an Intel Core2

Mytkowicz et al., Chaos 19:033124

bzip2 dynamics on an Intel Pentium 4

Mytkowicz et al., Chaos 19:033124

povray dynamics on an Intel Core2

Mytkowicz et al., Chaos 19:033124

Caveat: need enough data...

**If Δt is not uniform**

Theorem (Takens): for τ>0 and m>2d, reconstructed trajectory is diffeomorphic to the true trajectory.

Conditions: evenly sampled in time, smooth generic measurement function

**Interspike interval embedding**

idea: lots of systems generate spikes — hearts, nerves, etc.

if you assume that the spikes are the result of an integrate-and-fire system, then the Δt has a one-to-one correspondence to some state variable’s integrated value...

* in which case the embedding theorems still hold.

(with the Δts as state variables)

Sauer Chaos 5:127

**Prediction**
Predicting the path of a roulette ball...

The Santa Fe competition

- Weigend & Gershenfeld, 1992
- put a bunch of data sets up on an ftp server
- and invited all comers to predict their future
- chronicled in *Time Series Prediction: Forecasting the Future and Understanding the Past*, Santa Fe Institute, 1993 (from which the images on the following half-dozen slides were reproduced)

The Santa Fe competition: data

- Laboratory laser
- Medical data (sleep apnea)
- Currency rate exchange
- RK4 on some chaotic ODE
- Intensity of some star
- A Bach fugue

Embedding + patch models: (Sauer)

Neural net: (Wan)
Further out:

An even simpler prediction method: Lorenz’s method of analogues

A k-nearest neighbor modification of LMA
Using kLMA to predict computer dynamics

Noise...

Linear filtering: a bad idea if the system is chaotic
Nonlinear alternatives:
• use the stable and unstable manifold structure on a chaotic attractor...

Idea:
• If you have a model of the system, you can simulate what happens to each point in forward and backward time
• If your system has transverse stable and unstable manifolds, that does useful things to the noise balls
• Since all three versions of that data should be identical at the middle time, can average them
  • noise reduction!
  • Works best if manifolds are perpendicular, but requires only transversality

Results:

Noise:

Linear filtering: a bad idea if the system is chaotic
Nonlinear alternatives:
• use the stable and unstable manifold geometry on a chaotic attractor
• what about using the topology of the attractor?
Computational Topology

**Why:** this is the fundamental mathematics of shape, complements geometry.

**What:** compute topological properties from finite data

**How:**
- introduce resolution parameter
- count components and holes at different resolutions
- deduce topology from patterns therein


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Connectedness: definitions

- how many "lumps" in a data set?
- \( \varepsilon \)-connectedness (after Cantor)
- \( \varepsilon \)-connected components
- \( \varepsilon \)-isolated points:

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Connectedness: examples

If the data points are samples of a disconnected fractal like this:

The number of connected components looks like this:

(note obvious tie-in to fractal dimension...)

Robins et al., *Physica D* 139:276, Nonlinearity 11:913

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Connectedness: examples

If the data points are samples of a connected set like this:

The number of connected components looks like this:

Robins et al., *Physica D* 139:276, Nonlinearity 11:913

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Connectedness and filtering

The effect of noise is to add isolated points to the set and a shoulder to the C(x) curve:

So if you know that the object is connected — like the attractor of a flow — you can reasonably assume that any isolated points are noisy, and remove them by pruning with \( \varepsilon = \varepsilon^* \)

Robins et al., *Intelligent Data Analysis* 8:505, Chaos 14:305

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Continuity and filtering

**Mos:**
- deterministic, differentiable dynamics (maps & flows) are continuous

**Conjecture:**
- if the image of a connected set is not connected, more than one dynamics is at work

**Approach:**
- track connectedness over time

**Applications:**
- pulling apart interleaved dynamics, removing noise...

Alexander et al., CHAOS, 2012

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**Chaos and control**

Key concepts:
- Dense attractor coverage
- Exponential trajectory separation
- Un/stable manifold structure
- Local-linear control

Recall: local-linear control of a saddle point works successfully in a region whose geometry is defined by the $\lambda_i$ and the $W^s/W^u$, together with the sensor & actuator capabilities…

**Control:**
getting from A to B, minimizing some cost functional

**Lorenz System:**
denseness, reachability, and control

**Denseness & reachability in a real engineering application**
- Can control position/volume/density of attractor — within limits
- Possibly not reachable any other way
- Not for time-critical applications (that “eventually”)

Now what?
OGY control

- dense attractor coverage → reachability
- un/stable manifold structure + UPO denseness + local-linear control → controllability

Use local-linear control, designed using the eigenvalues and eigenvectors at that point to balance a chaotic system on a UPO passing through that point.

But you’re relying on denseness to get you into the controllable region, and that may take a while…

Lorenz System:
SDOIC-based targeting

OGY & co. have been used in tons of systems; see Shinbrot review paper.
Alfred Hubler has done a lot of cool stuff in this area as well.

Four R switches; 240X faster

Bradley, Cybernetics & Systems 26:299

Other cool ways to use invariant manifolds

Want to get a spacecraft onto a "halo orbit," which is a UPO of the dynamics.

Unstable Periodic Orbits (UPOs) have invariant manifolds:

- Stable Invariant Manifold (\(W^s\))
  - The set of all trajectories a particle could use to arrive onto the UPO.

- Unstable Invariant Manifold (\(W^u\))
  - The set of all trajectories a particle could take after a small perturbation from the UPO.
Low-energy (cheap) orbit transfers

- Depart along $W^{u}_{UL1}$ & arrive on $W^{s}_{UL2}$

Homoclinic orbits - The best case

- If a trajectory in Stable and Unstable intersect ("homoclinic connection")

Can we do any of that in spatially extended systems?

(i.e. harness the butterfly effect, exploit un/stable manifold geometry?)

A 2D jet

End view

room lighting

stop-action laser "slice"
aerosolized canola oil
Forcing the jet flow

Slit: 2.5 X 400 mm

The Butterfly effect in action…

no forcing 6Hz forcing

Forcing generates coherent structures that enhance entrainment and mixing

Does this have anything to do with reality?

Communication and chaos:

- Two coupled Lorenz systems will synchronize
- Robust to a small amount of noise
- Use this to transmit & receive information
- Chaotic carrier wave, so hard to intercept or jam
Another interesting application: chaos in the solar system

- orbits of Pluto, Mars
- Kirkwood gaps
- rotation of Hyperion & other satellites
- …

Solar system stability:

- recall: two-body problem not chaotic
- but three (or more) can be…

Exploring that issue, circa 1880:

An orrery, which is a mechanical computer whose gear ratios and circular platters simulate the orbits of the planets

Exploring that issue before the digital computer age…

An orrery, which is a mechanical computer whose gear ratios and circular platters simulate the orbits of the planets

Exploring that issue, circa 1980:

- write the n-body equations for the solar system
- solve them using symplectic ODE solvers on a special-purpose computer

The digital orrery
(Wisdom & Sussman)

Numerical Evidence That the Motion of Pluto Is Chaotic

[Graph showing data and analysis]
Should we worry?

- No.

Kirkwood gaps:

Evidence in favor of the conjecture:

Chaotic tumbling of satellites:

Voyager and Galileo saw this…

...so did Cassini:
Chaotic tumbling of satellites:

This happens for all satellites at some point in their history, unless they are perfectly spherical and in perfectly circular orbits (pf: KAM theorem; see Wisdom paper on syllabus)

Some of them are still tumbling chaotically because of their geometry, but most (like the earth and its moon) have settled down into tidal equilibria

More chaos in the solar system:

• obliquity of Mars (Bouma & Wisdom, Science 289:1294)

• etc.

Musical Variations from a Chaotic Mapping

Pitch sequence:
C, E, G, C, E, G, C, E, ...

Bradley & Stuart, Chaos 8:800

More chaotic variations on movement sequences

Dabby Chaos 6:95

Also see: https://www.youtube.com/watch?v=-B2XtE9YylCA
original cell itinerary

variation cell itinerary

medley — original

Rossler variation of medley

random variation of medley

original cell itinerary

variation cell itinerary

abrupt transition
Interpolation

Corpus-based approach

- graph captures motions of one joint
- note: specific to the genre of the corpus!

Initial state

Target state
Graph search

...for 44 joints in parallel!