Information Theory for Tralfamadorians: The Anatomy of a Bit: A Rope of Sand

Ryan G. James
Christopher J. Ellison
James P. Crutchfield

Complexity Sciences Center
Department of Physics
University of California, Davis
One Shields Avenue, Davis, CA 95616

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I’ve been ionized, but I’m okay now.

Say What?
I’ve been ionized, but I’m okay now.

Say What?

Kurt Vonnegut, *Slaughterhouse-Five*
I’ve been ionized, but I’m okay now.

Say What?

I am a Tralfamadorian, seeing all time as you might see a stretch of the Rocky Mountains.

- Kurt Vonnegut,
  Slaughterhouse-Five
Humans vs. Tralfamadorians

Humans:

I've been ionized, but I'm okay now.
I’ve been ionized, but I’m okay now.

Humans vs. Tralfamadorians

**Humans:**
- Time is sequential
Humans vs. Tralfamadorians

Humans:

- Time is sequential
- Use past measurements to inform us of the present
Humans vs. Tralfamadorians

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Tralfamadorians:
Humans vs. Tralfamadorians

Humans:
- Time is sequential
- Use past measurements to inform us of the present

Tralfamadorians:
- Time is “random access”
Humans vs. Tralfamadorians

**Humans:**
- Time is sequential
- Use past measurements to inform us of the present

**Tralfamadorians:**
- Time is “random access”
- The present can be placed in the context of its past as well as its future
Setting the Scene: Processes

\[ \mathcal{P} = (X, \mu) : \quad X \subseteq A^\mathbb{Z}, \sigma(\mathcal{P}) = \mathcal{P} \]

\[ \cdots X_{-3} \ X_{-2} \ X_{-1} \ X_0 \ X_1 \ X_2 \ X_3 \ \cdots \]

Additional properties:
- Ergodic
- Stationary
- Discrete
Setting the Scene: Processes

\[ \mathcal{P} = (X, \mu) : \quad X \subseteq A^\mathbb{Z}, \sigma(\mathcal{P}) = \mathcal{P} \]

\[ \begin{array}{cccccccc}
  & & & & & & & \\
  & & & & & & & \\
  & & & & & & & \\
  & & & \text{present} & & & & \\
  & & & & & & & \\
  \vdots & X_{-3} & X_{-2} & X_{-1} & X_0 & X_1 & X_2 & X_3 & \cdots \\
  & & & & & & & \\
  & & & & & & & \\
 \end{array} \]

Additional properties:

- Ergodic
- Stationary
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Setting the Scene: Processes

\[ \mathcal{P} = (X, \mu) : \quad X \subseteq A^\mathbb{Z}, \sigma(\mathcal{P}) = \mathcal{P} \]

\[ \begin{align*}
X_{-3} & \quad X_{-2} & \quad X_{-1} & \quad X_0 & \quad X_1 & \quad X_2 & \quad X_3 & \quad \cdots \\
\text{past} & \quad \text{present}
\end{align*} \]

Additional properties:
- Ergodic
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Setting the Scene: Processes

\[ \mathcal{P} = (X, \mu) : \quad X \subseteq \mathcal{A}^\mathbb{Z}, \sigma(\mathcal{P}) = \mathcal{P} \]

Additional properties:
- Ergodic
- Stationary
- Discrete
$H[X_0]$ is partitioned by the past and the future $\sigma \mu = I[X_0; X_1|X_0]$; evidence of internal states.
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H[X₀] is partitioned by the past and the future.
H[$X_0$] is partitioned by the past and the future.

$\sigma_{\mu} = I[X_{0}; X_{1:} | X_0]$: evidence of internal states.
Decompositions of the Present
The Human Decomposition
The Human Decomposition

- $\rho_\mu = I[X:0;X_0]$: predicted information
The Human Decomposition

- $\rho_\mu = I[X_0; X_0]$: predicted information
- $h_\mu = H[X_0|X_0]$ : unanticipated information
The Tralfamadorean Decomposition
The Tralfamadorian Decomposition

\( r_\mu = H[X_0|X_0, X_1]: \)  
ephemeral information
The Tralfamadorian Decomposition

- $r_{\mu} = H[X_0|X_0, X_1]:$ ephemeral information
- $b_{\mu} = I[X_0; X_1|X_0]:$ stochastic structure
The Tralfamadorian Decomposition

- \( r_\mu = H[X_0|X_0, X_1:] \): ephemeral information
- \( b_\mu = I[X_0; X_1|X_0] \): stochastic structure
- \( q_\mu = I[X_0; X_0, X_1:] \): who knows?
The Structural Decomposition
The Structural Decomposition

- \( r_\mu = H[X_0|X_0; X_1]: \)
  - irrelevant information
The Structural Decomposition

- $r_\mu = H[X_0|X_0; X_1:]$: irrelevant information
- $w_\mu = I[X_0; X_0, X_1:]$: structural information
The Recommended Decomposition

\[
\rho \mu = I[ X : 0 ; X 0 ]:
\]
an anticipated information

\[
\mu b = I[ X 0 ; X 1 : | X : 0 ]:
\]
unanticipated and relevant

\[
\rho r = H[ X 0 | X : 0 ; X 1 ]:
\]
unanticipated and irrelevant

\[
\rho \mu b \mu r
\]
The Recommended Decomposition

\[ \rho_{\mu} = I[X_0; X_0] : \text{anticipated information} \]
The Recommended Decomposition

- $\rho_\mu = I[X_0; X_0]$:
  anticipated information

- $b_\mu = I[X_0; X_1|X_0]$:
  unanticipated and relevant
The Recommended Decomposition

- $\rho_\mu = I[X_0; X_0]$: anticipated information
- $b_\mu = I[X_0; X_1; X_0]$: unanticipated and relevant
- $r_\mu = H[X_0 | X_0; X_1]$: unanticipated and irrelevant
And you know my Achilles tendon is my one Achilles’ heel

Like Humans Do

“How do I measure these?”
Asymptotic Rates: $H & T$

![Graph showing $H(\ell)$ and $T(\ell)$ as functions of block length $\ell$.

The graph displays two curves, one in green representing $H(\ell)$ and another in red representing $T(\ell)$. The y-axis represents information in bits, while the x-axis represents block length in symbols. The curves show how the information content changes with block length.](image-url)
Asymptotic Rates: R, B, & Q

The graph shows the curves of $R(\ell)$, $B(\ell)$, $Q(\ell)$, and $W(\ell)$ as functions of block length $\ell$ [symbols].
Entropy

$$H[X_{1:n}] = \sum_{x \in X} p_x \log(p_x)$$
Total Correlation

\[
T[X_{1:n}] = \sum_{i \in \{1...n\}} H[X_i] - H[X_{1:n}]
\]
Residual Entropy

\[ R[X_{1:n}] = \sum_{i \in \{1...n\}} H[X_i | X_{\{1...n\} \setminus i}] \]
Binding Information

\[ B[X_{1:n}] = H[X_{1:n}] - R[X_{1:n}] \]
$Q[X_{1:n}] = T[X_{1:n}] - B[X_{1:n}]$
Local Exogenous Information

\[ W[X_{1:n}] = T[X_{1:n}] + B[X_{1:n}] \]
**Pesin’s Theorem:**

\[ h_\mu = \max(0, \lambda) \]
Pesin’s Theorem: \( h_\mu = \max(0, \lambda) \)

Use a generating partition to gather statistics
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\[ h_\mu = \max(0, \lambda) \]

Use a generating partition to gather statistics

\[ h_\mu = r_\mu + b_\mu \]
Pesin’s Theorem:
\[ h_\mu = \max(0, \lambda) \]

Use a generating partition to gather statistics

\[ h_\mu = r_\mu + b_\mu \]
Logistic Map

- Pesin’s Theorem:
  \[ h_\mu = \max(0, \lambda) \]
- Use a generating partition to gather statistics
- \[ h_\mu = r_\mu + b_\mu \]
**Tent Map**

- Pesin’s Theorem: 
  \[ h_\mu = \max(0, \lambda) \]
- Use a generating partition to gather statistics 
- \[ h_\mu = r_\mu + b_\mu \]
It happens sometimes. People just explode. Natural causes.

So It Goes...

Thank You

Reference