Spiking Neural Networks on the Cusp of Chaos: Initial Report

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Abstract— Spiking neural networks, even of small order, show great variability in their behavioral patterns. This paper is an initial look at how structure of small spiking networks influence their measured behavior over time. Of particular interest are networks showing non-periodic behavior, one of the tell-tale indicators of chaos in the underlying dynamics.

***Index Terms*—Spiking neural networks, recurrent, periodicity, integrate and fire, chaos, structure-to-function mapping**

I. Introduction

CONSIDER that the brain is a network, constructed with hundreds of billions of neurons wired together with trillions of synapses. Implicit in the nominal operation of this network based biological machine is the capacity to accept, translate, store, and manipulate sensory information, ultimately allowing us to engage the environment around us in meaningful ways. The processes through which neural systems capture, code and use this information are of utmost importance to understand in the wider context of humanity's project to comprehend ourselves and the organisms from which we have evolved.

When modelling a system with billions of connected parts, each requiring possibly hundreds of variables in order to meet certain levels of accuracy, it can be daunting to know where to place one's focus. One such focus is on neural subsystems, small circuits of neurons modelled as visulizable networks with precisely defined connectivity. If it can be shown that such systems have functionality in their own right which can be utilized repeatedly by the larger system in which they exist, then it is not a far stretch to suggest that a comprehensive understanding of the computational and functional capacity of these subsystems is a key building block towards decoding the brain.

In this vein, software has previously been written [1] to study structure-behavior relationships in spiking neural network models. The interactive graphical interface of this software allows a researcher to construct small to medium sized networks, modify connectivity and observe temporal response of the network to a full range of initial conditions. Other recent work [] has used this software to characterize quantitatively, using information theory, the range of behaviors possible in spiking networks of various connectivity. Specifically, for small networks of six neurons, this work examined and compared entropies in the spiking behavior of networks with a range of connectivity patterns and noted trends in the types of spike patterns with which these entropies are correlated.

II. Background

A. Neurobiology and Modeling

Neural networks connect the worlds of neurobiology, mathematics and computer science. In short they are simplified mathematical models which try to capture essential properties of neurobiological systems in order to either understand such systems or use the inherent capabilities within them to do useful computation. An abbreviated introduction with links to more substantial sources can be found in [1].

Recent studies focusing on the computational capabilities of neurobiological networks have suggested that information may often be encoded in a more precise way than can be represented by a perceptron-like artificial neural networks. The latter sort of model employs a spike rate representation of a neural spike train, smearing the timing of individual spikes in the underlying biological system. Several studies have found that temporal resolutions of neural codes are on a millisecond time scale, which highlights the significance of the precision of spikes in the coding process.[]

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B. Previous Work

Software which models the activity of spiking neural networks has been built and used to investigate network behavior across a range of topologies. Specifically, previous work employed this software in studying the compelling information-rich activity which emerges in a transition regime between quiescence and constant saturated activity in the network as the mean degree is changed across a fairly narrow band of values.[]

This exploratory work produced several general observations about the behavior of these networks. Most obvious and immediately apparent was confirmation that each network, regardless of size, moves from a regime of absolute quiesence (no nodes spiking) to one of maximum constant homogeneous activity (all nodes spiking all the time) as one of several parameters is increased monotonically. Except when other parameters are set to extreme values (e.g. probability of edge = 0), each of the following parameters invoked this phase transition behavior as it is modulated accordingly: mean degree (or, equivalently, probability of an edge between any two nodes), size of stimulus delivered in response to an action potential, threshold to fire, and to a lesser extent (i.e. tighter constraints are necessary on other parameters) the decay rate of node voltage.

It was also observed in this work that each of the networks, regardless of configuration, is extremely sensitive to changes in these parameters. Not only will the system enter or fall out of quiescent or constant states quickly, but also one may notice significant qualitative changes in the type of patterns being displayed by the system for given parameter vectors. For instance, simply by changing the probability of edge presence by 1% in a 30 node network from 14% to 15% we see spiking patterns go from complex and possibly even chaotic to very simple and repetitive. This observation is significantly repeatable according to a limited number of observations made thus far.

III. METHODS

A. System Dynamics

The state space of the dynamical system being studied is simply the vector of node membrane potentials in the network at a given time. Network dynamics are based on the Leaky Integrate and Fire model. [] In this model, at every time step in the simulation, each node (or "neuron") in the network integrates electrical input from all of its neighbors and external inputs (if present). Electrical current in the node "leaks" out of or into the node over time exponentially according to a specified decay constant towards a given resting potential. If the potential of a node reaches a specified threshold due to input, the node "spikes" and delivers an stimulus of a specified voltage along its outgoing edges (or "synapses") to its neighbors.

In the software, dynamics in the system have been simplified as much as possible for both computational as well as conceptual efficiency. For instance, default values for several static parameters have been set to the simplest values imaginable (e.g., 0 and 1 for resting potential and threshold respectively). Since the chosen range parameters is isomorphic to the choice of units by neurobiologists, there is no danger of introducing inaccuracies. The reduction of complexity here permits increased focus on network topology while remaining neurobiologically relevant.

B. Experiment Set Up

The software mentioned above was modified to enable batch data collection across the complete range of network structures for a network of a given order (size) and across a limited set of initial conditions. In some experiments, the synaptic efficiency (i.e. the degree to which a spike from a pre-synaptic cell can effect a post-synaptic cell) was also tested across a range of values.

Prior to simulation, the Graph Theory package in SAGE [] was used to generate the set of all non-isomorphic networks of a given order.

Three varieties of batch experiments were run and analyzed.

1. Five-node networks: excitatory synapses only

All non-isomorphic graphs of order five (9607 in total) were generated and used to build an equal number of networks (with only excitatory edges). Each of these was simulated on a set of initial conditions (node voltages) that included all permutations of nodes either at rest (V=0) or just above threshold (spiking). These simulations were run for 2000 time steps, recording the resultant spike train (i.e. binary state over time where each node is either spiking or not at each time step). The periodicity of this data was calculated, after first tossing out the initial 200 time steps as transient.

2. Three-node networks: excitatory and inhibitory synapses

A complete set of non-isomorophic networks of order three (16 total) was generated as above. But while the values on the edges above were always +1, in the second batch of experiments the values on the edges were permuted so that all possible combinations of +1 and -1 were represented in the adjacency matrix given one of the original 16 matricies of only +1's. This change introduced *inhibitory* connections in the network, effectively creating the possibility of negative feedback.

Additionally, for each network in this new set, the synaptic efficiency variable was tested across a range of values (.4 to .875) in order to uncover a potentially larger range of behaviors. The total number of networks tested after these two changes was 4200.

Initial conditions were generated as in the first set of experiments, and simulation and analysis took place as before.

3. Four-node networks: excitatory and inhibitory synapses

The set of permuted four-node non-isomorphic networks was generated and tested as above. However, because of the large number of simulations required, these networks were run with synaptic efficiency at the nominal level of .7.

C. Network Structure Analysis

All networks were also analyzed structurally and these quantities were found for each network:

Average degree …

Clustering coefficient…

Strong connectedness ...

D. Software

In addition to a Python base install, the Python packages NetworkX and MatPlotLib were used for network analysis and generating figures, respectively.

IV. RESULTS

A. Five-node networks: excitatory only

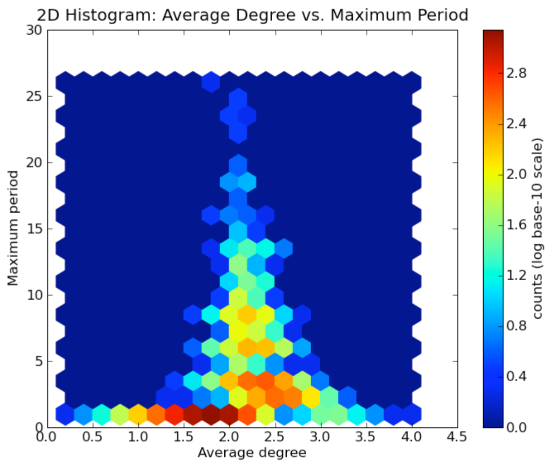

Fig. 1. A Hexbin (2D Histogram) plot of Average Degree vs. Maximum Period (maximized across initial conditions for a given network) for the set of all 9607 5-node excitatory networks. Note: color bar is on log scale.

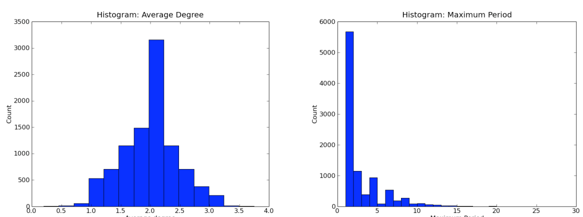

Fig. 2. Two histograms: Average Degree and Maximum Period for the set of all 9607 5-node non-isomorphic excitatory networks.

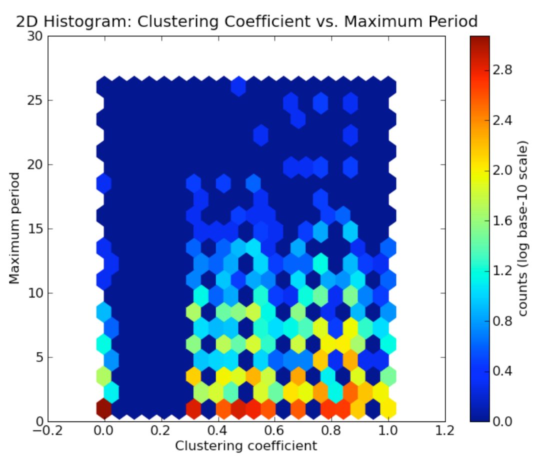

Fig. 2. A Hexbin (2D Histogram) plot of Clustering Coefficient vs. Maximum Period (maximized across initial conditions for a given network) for the set of all 9607 5-node excitatory networks. Note: color bar is on log scale.

B. Three-node networks: excitatory and inhibitory synapses

C. Four-node networks: excitatory and inhibitory synapses

V. DISCUSSION

When considering the space of possible networks of a given order, much of the variability in behavior can be attributed to the structure of the network. Here we have explored the spaces of networks of several different orders and examined specifically the periodicity in their behavior starting from a wide range of initial conditions. As we have seen, there is a narrow band of average degrees in which networks are able to achieve (relatively) large periodic spiking output. While these periods themselves are not indicative of chaos, one might wonder if maximum periodic behavior grows exponentially with network order to the point where a period, if it exists, is not measurable. This would be reminiscent of the period doubling route to chaos as seen in the logistic map.

Additionally, the narrow range of average degree values which seem to admit larger periods is reminiscent of an “edge” near chaos. …

ACKNOWLEDGMENT

The author would like to thank the Santa Fe Institute for support, guidance and resources; particularly the devoted individuals who organized, managed and created content for the Complex Systems Summer School. Special thanks also to Greg Leibon, Aric Hagberg, Rosemary, Angela, Corinne, Max, and Andrew for the enlightening conversations and ideas.

REFERENCES

[1] R. Watson. (2009). A Spiking Model Exploration Tool [Online]. Available http://www.math.ucdavis.edu/~watson/spiking .