Detection of low-dimensional chaos in quasi-periodic time series: The 0-1 test

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Abstract

The 0-1 test is a novel test that has been recently suggested to detect low-dimensional chaos in time series. The test has already been applied successfully on a variety of time series. The goal of the present paper is to study the capability of the test to detect the presence of long-term chaos in time series presenting a strong periodic or quasi-periodic component. To achieve that goal, we apply the test using chaotic (quasi-periodic) and nonchaotic (periodic) Rössler time series. The method succeeds in detecting the presence or absence of chaos, even when significant white noise is added to the time series.

The 0-1 test along with the classic method of calculating the maximum Lyapunov exponent (MLE) are then applied for the first time to a time series obtained by surface electromyography (sEMG). Results from both tests are consistent with the presence of low-dimension chaos. The two tests, however, give the same result when applied to a surrogate nondeterministic time series resembling closely to the sEMG dataset. Thus, although the presence of chaos in the sEMG dataset is consistent with the results from two disparate tests, we have by no means proved it.
1 Introduction

Recently, a new test has been proposed to detect the presence of low-dimensional chaos in time series (Gottwald and Melbourne 2003, Gottwald and Melbourne 2005, Gottwald and Melbourne 2008b; Section 2). The method has the advantage to be easy to implement and does not need the underlying equations, the reconstruction of the phase space, or the dimension of the actual system. The test has been applied successfully on theoretical time series, both with and without noise, from various dynamical systems (van der Pol, Kortweg de Vries, Lorenz, the logistic map), as well as on experimental data (Falconer et al. 2007).

The goal of the present article is to apply this new test specifically for the case of time series containing a strong periodic or quasi-periodic component. To achieve this, we use Rössler time series in periodic and chaotic regimes as well as with or without white noise (Section 3). The test succeeds in every case to differentiate between chaotic and nonchaotic dynamics.

We then apply the 0-1 test, as well as the classic method of calculating the maximum Lyapunov exponent (MLE), to an experimental time series obtained by surface electromyography (sEMG) as well as to a surrogate nondeterministic time series resembling closely to the sEMG dataset (Section 4). Results from both tests suggest that chaos is present in both time series. We conclude that although the classic and novel tests suggest the presence of chaos in the experimental dataset, the failure of the tests when applied to the surrogate time series undermine that result (Section 5).

2 The 0-1 test

Unlike the classic method of calculating MLE, the 0-1 test does not need any information about the dynamics or the dimension of the system, which is clearly an advantage. It is, furthermore, relatively easy to implement. As the name implies, the
test only gives two possible results: 0 for a nonchaotic system, and 1 otherwise.

Given an observable $\phi$ and a parameter $c$ arbitrarily chosen between 0 and $2\pi$, one define

$$p_c(n) = \sum_{j=1}^{N} \phi(j) \cos(jc),$$  \hspace{1cm} (1)

$$q_c(n) = \sum_{j=1}^{N} \phi(j) \sin(jc).$$  \hspace{1cm} (2)

Results from ergodic theory (Gottwald and Melbourne 2003 and references therein) state that $p_c(n)$ and $q_c(n)$ are bounded if the underlying dynamics generating $\phi$ is nonchaotic. On the other hand, $p_c(n)$ and $q_c(n)$ behave asymptotically like a Brownian motion and are unbounded if the underlying dynamics is chaotic. The definition $p_c$ and $q_c$ shows the universality of the test, and hence, the origin and type of data used in the test are a priori irrelevant.

To determine the growth of $p_c$ and $q_c$ it is convenient to look at the mean square displacement (MSD), defined as

$$M_c(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} [(p_c(j + n) - p_c(j))^2 + (q_c(j + n) - q_c(j))^2]$$  \hspace{1cm} (3)

with $n \ll N$. Consequently, not all data points can be used in that calculation. In practice, $M_c$ is computed over roughly the first 10% of the time series. This highlights the fact that even though the 0-1 test does not need any information about the actual dynamics of the system, it still needs enough data points to work properly. However, if this condition is fulfilled, the MSD of a nonchaotic system will grow slower than the number of data points, unlike the case of a chaotic system.

The $p_c$ and $q_c$ terms have, by construction, an oscillating component that can be removed using a linear transformation of $M_c$, as it is described in the most recent version of the 0-1 test (Gottwald and Melbourne 2008b). With $p_c$ and $q_c$ bounded in a nonchaotic system, the correlation $K_c$ between $n$ and $M_c$ is zero for $n \to \infty$. 

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Otherwise, the correlation is equal to one for a chaotic system, since \( M_c \) grows as fast as the number of data points itself. Thus when \( K_c \) is close to zero, the test suggests that the time series is nonchaotic; close to one and the test suggests that the time series is chaotic.

A major pitfall, however, that is common also to the classic method of calculating MLE, is that the test would detect the presence of chaos in a nondeterministic time series. This problem is illustrated in Section 4.4 in the case of the experimental dataset and is the principal reason why we cannot convincingly conclude in the presence of chaos in that dataset besides the results from the 0-1 and MLE tests. It has also been noticed that the 0-1 test may fail as well in some specific regimes, such as when the time series corresponds to a process near the edge of chaos (Hu et al. 2005; Gottwald and Melbourne 2008a). In addition, as with all approaches, the 0-1 method is impracticable if there are extremely long transients or if the dimension of the attractor is too large (Gottwald and Melbourne 2004).

### 3 Rössler time series

The Rössler attractor has both a periodic and chaotic regime. Hence, it is ideal to study the capability of the 0-1 test to detect the presence or not of chaos in time series that appear periodic or quasi-periodic. The Rössler system is described by the governing equations:

\[
\frac{dx}{dt} = -(y + z), \\
\frac{dy}{dt} = x + ay, \\
\frac{dz}{dt} = b + z(x - d).
\]

A doubly-periodic time series is constructed using \( a = b = 0.1 \) and \( d = 6 \) (Fig. 1a). A chaotic time series is constructed by using \( d = 18 \) instead (Fig. 1b). In both cases,
Figure 1: Rössler time series: (a) doubly-periodic and (b) chaotic. Only the 10% of the original time series over which $M_c$ and $K_c$ are computed are shown. The blue dots show the re-sampling executed to avoid over-sampling: every 110 points in (a) and 155 points in (b).
Figure 2: $K_c$ in the case of (a) doubly-periodic and (b) chaotic Rössler time series: (blue) noiseless and (red) noisy time series.
Eqs. (4-6) are integrated with time starting from the initial condition \((x_0, y_0, z_0) = (-7.12, 5.54, 0)\), with a time increment \(dt = 0.01\) and for \(N=1e5\) points.

The time series thus constructed are oversampled which might cause the 0-1 test to fail (Gottwald and Melbourne 2008b). Using the method of averaged mutual information (AMI, Abarbanel 1993), the time series are re-sampled so that a minimum number of points are used without losing much variability (Fig. 1).

Two noisy time series are also constructed by adding 10% of white noise to each of the noiseless original time series (not shown). The previous re-sampling is applied to those noisy time series and is again optimum to capture most of their variability.

The 0-1 test succeeds to detect the presence or lack of chaos in every case (Fig. 2). The median \(K\) of \(K_c\) values obtained are close to zero in the nonchaotic case and close to one in the chaotic case: \(K\) is 0.02 and 0.17 in the doubly-periodic case without and with noise respectively, and 0.71 and 0.74 in the chaotic case without and with noise respectively. Notice, however, that \(K\) in the chaotic case are much less closer to 1 than expected from previous studies on other theoretical dataset.

From this result, the 0-1 test appears to be reliable to detect the presence of chaos in time series that appear periodic or quasi-periodic even in the presence of significant white noise. In the next section, the 0-1 test is used in parallel with the calculation of MLE to detect the presence or not of chaos in an experimental dataset.

### 4 sEMG time series

Surface Electromyographic (sEMG) signals record the electrical activity of the muscles under the skin. Such sensors are used to understand the dynamics of muscle activity and how muscular forces are generated by the neuromuscular system. The sEMG signal used in this analysis was obtained during a procedure that induces a spinal wave. The sEMG electrode was placed on the cervical region of a quadriplegic.
subject. A spinal wave, a physical standing wave along the spinal cord of the subject, was induced and the muscle activity of the neck region recorded through the surface electrode. The sEMG records continuously and an analog-to-digital (A/D) converter is used to sample the data at 4 KHz. The resulting time series is unitless. A sample of 30-second long is analysed here (Fig. 3a).

\[ \begin{array}{c}
\text{(a)} \\
20 \\
25 \\
30 \\
35 \\
40 \\
45 \\
\text{x 1e3}
\end{array} \]

\[ \begin{array}{c}
\text{(b)} \\
0 \\
3 \\
6 \\
9 \\
12 \\
15 \\
18 \\
21 \\
24 \\
27 \\
30 \\
\text{t (seconds)} \\
\text{x 1e3}
\end{array} \]

Figure 3: (a) sEMG time series and (b) a surrogate.

4.1 Fourier analysis

The dataset is the sum of a periodic component and a 'noisy' background. The periodic component has a peak centered at the 0.0200-second period throughout the time series (Fig. 4) with an amplitude varying by as much as 35% over time (Fig. 5). Interestingly, there is also consistently a lack of energy at the 0.0166-second period (Fig. 4).

The amplitude and temporal structure of the background, however, change with...
Figure 4: Power density spectrum computed from successive 3-second long segments: (a) averaged over all 10 segments and (b) corresponding to segment between $t=9$ to 12 seconds (blue) and that between $t=12$ and 15 seconds (red).
Figure 5: Amplitude of the periodic component (0.0200-second period) over the series of 3-second long segments.

time in a more dramatic way. An illustration is given in Fig. 4b by comparing the power density spectrum for the segment between $t=9$ and 12 seconds and that between $t=12$ and 15 seconds. The ‘energy’ of the background has increased between the two segments. The increase is furthermore not uniform throughout the frequency space: the ‘energy’ increases by less than one order of magnitude for frequencies smaller than 0.01 KHz and by about two orders of magnitude for frequencies between 0.01 and 0.1 KHz and larger than 1 KHz, while it stays nearly constant around the 0.5-KHz frequency band. The variation in the amplitude of the background is responsible for the nonstationary aspect of the dataset (Fig. 3a). Are the background together with the periodic component a manifestation of chaotic behavior?

4.2 Calculation of MLE

The MLE is computed using the TISEAN software package (Hegger et al. 1999; http://www.mpipks-dresden.mpg.de/tisean/Tisean_3.0.1/index.html). The sensitivity of the results is studied by performing the computation on a series of segments that increase in length: first segment is between $t=0$ and 3 seconds, second segment
between $t=0$ and 6 seconds, etc. The last segment of the series corresponds to the entire 30-second long sEMG dataset.

First, the AMI is determined: the time series is re-sampled every 19 time steps, that is every $4.7e^{-3}$ second, without much loss of variability. The embedding dimension is then computed using the technique of false nearest neighbors (FNN). The percentage of FNN versus the embedding dimension converges with increasing segment length (Fig. 6), suggesting that the 30-second long time series is long enough to capture the entire attractor. For every segment, the result suggests that the variability can be well represented by a 4-dimension phase space.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{Percentage of false nearest neighbors (FNN) versus the embedding dimension for the series of segments increasing with length. The profile corresponding to the shortest segment is in blue and that to the longest one is in red.}
\end{figure}

The phase space is reconstructed and projected onto the dimension-1/dimension-2 plan in Fig. 7a. The trajectory appears noisy. Indeed, its overall behavior have the same characteristics than the trajectory obtained from a surrogate nondeterministic time series (Fig. 7b) that resembles the sEMG dataset (Section 4.4).

The Lyapunov exponents are also calculated for the series of segments (Fig. 8) using the algorithm of Rosenstein et al. (1993). As for the embedding dimension,
Figure 7: Reconstructed phase space, with dimension-1 plotted against dimension-2, for (a) the sEMG time series and (b) its surrogate.

Figure 8: Logarithm of the stretching factor versus the number of iterations for the series of segments increasing with length. Color code is the same as in Fig. 6. The thick magenta dashed line have been computed for the entire 30-second segment of the surrogate time series.
the profiles converge confirming that the 30-second long time series is long enough to capture the entire dynamics. In every case, the MLE is positive suggesting that the dynamics are 4-dimensional and chaotic.

4.3 0-1 test

The 0-1 test is applied over another series of segments. As previously, those segments have different lengths and positions in time. The series is here constructed by dividing the dataset into \( n \) segments of equal length, with \( n \) being successively 10, 5, 2 and 1. \( M_c \) and \( K_c \) are then computed for values of \( c \) varying from \( \pi/5 \) to \( 4\pi/5 \) with an increment of 0.01 and the median value \( K \) of \( K_c \) is deduced (Fig. 9).

![Diagram](image)

**Figure 9:** Median value \( K \) of \( K_c \) calculated for different segments of the dataset: (blue) 3-second long, (red) 6-second long, (magenta) 15-second long and (black) 30-second long segments. Each square is plotted at the central location in time of the segment. The black square at \( t=15 \) seconds corresponds to the entire record.

Except for one 3-second long segment, all median value \( K \) are above 0.75. The median \( K \) obtained for the three longest segments (the entire record plus two 15-second long segments), that is for the three most accurate calculations, are all above 0.98. Furthermore \( K_c \) obtained in the case of the entire record is larger than 0.75 for
more than 90% of all values of \(c\) chosen. The robustness of the result with respect to the segment’s length and position suggests that the variability in the dataset has a chaotic component, confirming the result from the calculation of MLE.

### 4.4 A surrogate time series

In this section, the calculation of MLE and the 0-1 test are performed over a surrogate nondeterministic time series that resembles closely the sEMG dataset. Because the sEMG time series is composed of a periodic component with a period \(T=0.02\) second and a ‘noisy’ background, one constructs the following surrogate:

\[
S = A \cos\left(\frac{2\pi}{T}t\right) + B_S
\]

where \(A\) is the amplitude of the Fourier component at \(T=0.02\) second obtained from the sEMG dataset.

![Figure 10: sEMG dataset (blue) and its surrogate (red) between \(t=3\) and 3.6 seconds.](image)

In Eq. (7), \(B_S\) is a surrogate for the background \(B\) of the sEMG dataset constructed following the algorithm 1 of the surrogate time series technique (Theiler et al. 1992): the two share the same power spectrum but the phases from the Fourier transform...
of the original background have been shuffled in the surrogate. The background \( B \) has been constructed by replacing the amplitude and phase of the 16 Fourier components between \( T=0.0199 \) and 0.0201 second by those of Fourier components chosen randomly from the surrounding vicinity. The power spectrum of \( B \) (and \( B_S \)) are thus the same than that of the sEMG dataset without the peak at \( T=0.02 \) second 'cut' at the level of the surrounding background (not shown).

The surrogate is plotted over the entire 30 seconds in Fig. 3b and a close-up between \( t=3 \) and 3.6 seconds is shown in Fig. 10. Visually, the surrogate time series reproduces well the characteristic variability of the sEMG dataset. Furthermore, the surrogate is found to have the same embedding dimension and its reconstructed phase space (Fig. 7b) has the same topological characteristics than that of the sEMG dataset.

The MLE is calculated and shown in Fig. 8 (thick dashed magenta line). Positive MLE is found as in the case of the sEMG dataset. The 0-1 test is applied over the entire surrogate and one finds that the median \( K \) is 0.99. Thus, both methods suggest that the surrogate is also chaotic which is not the case.

5 Conclusion

A novel test, called the 0-1 test, to detect chaos has been applied to theoretical and experimental time series that contain periodic or quasi-periodic components. Rössler time series in the periodic and chaotic regimes are used with or without the presence of significant white noise. In every case, the 0-1 test succeeds to detect the presence or not of chaos.

Based on those results, the test is used along with the classic calculation of MLE for the first time to an experimental time series obtained from sEMG. Results from both tests suggest that the presence of chaos in the time series.
To build confidence in our results, the two tests are applied to a surrogate non-deterministic time series that maintain the main characteristics of the variability of the sEMG dataset. Unfortunately, results from both tests suggest wrongly the presence of chaos as well.

Although it is known that the calculation of MLE and the 0-1 test cannot differentiate between real chaos from a deterministic time series from false chaos from a nondeterministic time series, the result from the surrogate time series illustrates the vulnerability of those tests to the presence of nondeterministic noise or high-dimensional chaos. It also suggests that although the presence of chaos in the sEMG dataset is consistent with the results found from two disparate tests, we have by no means proved it.
6 References


Gottwald G. A., and I. Melbourne, 2008b: On the implementation of the 0-1 test for chaos, draft.

