The dependence of the dissipation rate of open and dissipative systems on noise and the phenomena of spontaneous wire formation

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Abstract

Several systems or networks display self-organizing behavior that is characterized by a state with the maximum production of entropy. Such systems are complex in that they are open, i.e. they are not thermally isolated and therefore, the system has access to a continuous supply of energy, and dissipative, i.e. they output energy to external energy reservoirs. The nature of entropy production in these systems is explored by analyzing an example first proposed by Prigogine. An expression for the rate of entropy production in an R-C circuit will be used to demonstrate the behavior of entropy production observed in experiment. The insight provided will then be applied to the phenomena of self-organizing wires made of carbon nanotubes particles dissolved in a non-polar dielectric fluid.

Contents

1 Introduction

1.1 Origins of entropy

The idea of entropy can be traced back to Lazare Carnot's paper of 1803 - *Fundamental Principles of equilibrium and Movement* - where he discusses the propensity of usable energy to dissipate. Shortly thereafter, – in 1824 – his son, Nicholas Leonard Sadi Carnot,in his paper *Reflections on the motive power of fire* presents his observation that heat flow – then thought of as a fluid called 'caloric' – can do work, in the technical sense, or generate power. Rudolf Clausius, the next principal investigator of this concept, investigated this intrinsic behavior of a thermodynamic system to lose energy after it has performed work. Rudolf then coined the term 'entropy' defined as the energy used by the thermodynamic system during a change of phase. Moreover, the notion that entropy always increases, i.e. the directionality of heat flow, was also proposed: "There can be no process the sole result of which is a flow of energy from a colder to a hotter body." [1][2][5]

This definition of entropy did not rely on the assumption that the objects of these systems were made of small, indivisible components called atoms.[5] Hence, this classical formulation only considered the *macroscopic* quantities of systems. In contrast, Ludwig Boltzman, among others (Josiah Willard Gibbs, and Jonas Clerk Maxwell), gave a probabilistic formulation of entropy – one that depended on the idea of how an collection of ideal gas particles could be 'arranged', i.e., an "atomistic" [5] [6] view of the system which considered the *microscopic* states of such a system. Boltzman's result presented in 1877 demonstrated that the measure of entropy was proportional to (the natural log of)the number of microstates, i.e., the number of possible combinations of states of each atom for a given *macroscopic* state of the system. [2]

After the turn of the century, Claude Shannon quantified the "lost information" in communication (across phone lines) while working at Bell laboratories. His formulation was named *information entropy* due to its similarity with Boltzman's entropy. This name, recommended by John Van Neumann, has lead to some confusion in the scientific community, since Shannon's description is a more general notion of a measure of uncertainty in a system.[2]

1.2 The variation and maximum entropy principles

To know the probability of obtaining some value of an attribute, i.e. the distribution function for some observable property, for a complex statistical system constrained by a set of conditions, one must use the maximum entropy principle. This set of conditions often include path-dependence, memory, and strong interaction between elements of the system [7]

The maximum entropy principle is a child of the variational principle, i.e., the method of determining the dynamics or state of a system by considering a critical point (minimum, maximum, or 'saddle point') of a function. Pierre Louis Maupertuis, a French philosopher and mathematician,first encapsulated this idea by his assertion that, nature is thrifty in all its actions. [4] In optics, it is seen in the form of Fermats principle. Euler asserted in his paper*Reflexions sur quelques loix generales de la nature* (1748) that effort, i.e. potential energy, is minimized when a system of objects is allowed to rest. William Rowan Hamilton, motivated by the work of Joseph Louis Lagrange, used this principle of variation to derive the Lagrangian equations of motion. [3]

1.3 Summary of current work

Earlier this year, Rudolf Hanel, among others, demonstrated that for statistical systems the entropy associated with the distribution function is a maximum, if information about the system is properly accounted for – in keeping with the principle of maximum entropy. They showed that the "Boltzman-Gibbs-Shannon entropy" is related to the number of "independent random processes" and that "relaxation of the condition of independence leads to the most general entropy", i.e. the (c,d)-entropies.In essence, they successfully gave the first "exact definition of a generalized entropy" by taking into account the microscopic attributes of random processes which are path-dependent.[7]

Similar work has been done by Wissner-Gross and Freer where they establish that intelligence, using the principle of entropy maximization, is a "causal generalization of entropic forces through configuration space". Through several quintessential demonstrations of cognitive or accommodative ability, they show that highly intelligent or "adaptive" behavior emerges even in simple systems. They call such behavior in this context "adaptive as a nonequilibrium process in open systems". [8] They also show that noise has a stabilizing effect for an inverted pendulum system. This idea extends to our current work.

In a well written article by Hiroshi Serizawa and others, they prove that the maximization of entropy production leads to tree-like structures, i.e. "low entropy dissipative structures". They conclude that in accordance with both the second law of thermodynamics and the principle of maximum entropy, entropy is radiated from these "dissipative structures" to the surrounding environment.[9]

The recent work of Lucia suggests that the principles of minimum and maximum entropy production are different approaches to analyze a system because the given set of parameters and constraints for each approach differs. Yet, the principle of variation underlies these approaches - allowing one to describe a system by observable quantities regardless of the "frame of reference" or method used. Moreover, she asserts that the "Prigogine" or minimum entropy production principle is relevant for only "non-equilibrium dissipative systems" and the "entropy generation principle" is a more general and versatile concept that applies to "...all real open systems". Hence, she successfully connects these two approaches so that both remain correct within their appropriate framework. [10]

2 Power consumption for electrical circuits

2.1 A simple circuit

Consider a circuit comprised of two resistors, R_c and R_s, connected in series with a constant voltage supply V_0. Using Kirchoff's voltage and current laws, the following equation is obtained:

$$I = \frac{V_0}{R_s + R_c}. \tag{2.1}$$

With $P_c = I^2 R_c$, the power consumption across resistor R_c is given by

$$P_c = \frac{V_0^2 R_c}{(R_c + R_s)^2}. \tag{2.2}$$

Let us now find the critical points of P_c with respect to R_c by taking its derivative (with V_0 and R_s both constant and not equal to zero). Therefore,

$$\frac{\partial P_c}{\partial R_c} = -V_0^2 \frac{(R_c - R_s)}{(R_c + R_s)^3} := 0, \tag{2.3}$$

which implies that a global maximum of P_c exists at the point $R_c = R_s$. Thus $P_c(R_s)$ is the maximum value of P_c, denoted as P_0.

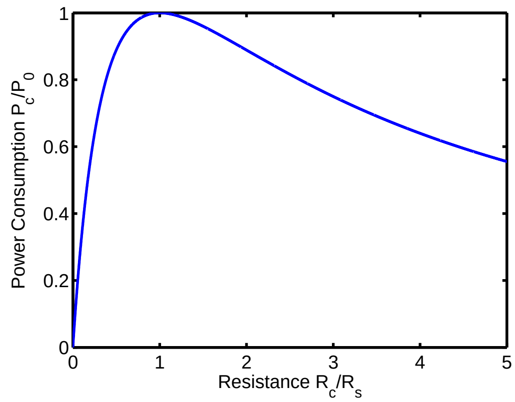

Figure 1: A summary of power consumption for a simple circuit

Equation 2.2 may be normalized, as expressed in 2.4 below.

$$\frac{P_c}{P_0} = \frac{4R_cR_s}{(R_c + R_s)^2}. \tag{2.4}$$

Moreover, P_c/P_0 may be written as a function of R_c/R_s.

$$\frac{P_c}{P_0} = \frac{4(R_c/R_s)}{(1 + R_c/R_s)^2}. \tag{2.5}$$

Figure 1 is a plot of Eq. 2.5 and clearly shows that P_c reaches a maximum at $R_c = R_s$.

Now, let the resistance of the capacitor is proportional to the gap size d between the plates and inversely proportional to the area of the plate A. Therefore, with ρ as a proportionality constant, let

$$R_c = \frac{\rho d}{A}. \tag{2.6}$$

where d and A are given in meters and squared-meters respectively.

In the limit as time goes to infinity, the behavior of a capacitor may be reduced to that of a resistor. Hence, with Eq. 2.6 and Eq. 2.2, the power consumption of a circuit with a capacitor P_c as a function of gap size d is

$$P_c = Ax\rho \left(\frac{V_0}{d\rho + AR_s}\right)^2 \tag{2.7}$$

Setting $\frac{\partial P_c}{\partial d} = 0$ to find a maximum,

$$\frac{\partial P_c}{\partial d} = \frac{AV_0^2\rho(-d\rho + AR_s)}{(d\rho + AR_s)^3} = 0, \tag{2.8a}$$

when $x = \frac{R_sA}{\rho}\, m$.

2.2 Power consumption of a parallel plate capacitor: A simple calculation

2.2.1 Description

Consider a circuit composed of three components in series: a battery, resistor, and parallel-plate capacitor. Suppose that the gap size d between the parallel plates be adjusted by moving one of the plates. In this example, d ranges between 0 and 1 centimeters. Moreover, for ease of calculation, let the voltage of the battery V_0 be 1 volt, the resistance of the resistor R_s be 1 ohm, and the resistance of the capacitor R_c be 1 kilo-ohm per centimeter in gap size. The problem may be modeled as a circuit constructed such that R_c and the capacitor are wired in parallel with respect to the voltage source and resistor R_s.

2.2.2 Solution

The power consumed by the capacitor P_c as a function of the gap size d for $0\ cm \leq d \leq 1\ cm$ may be expressed as

$$P_c(t,d) = I(t,d)V_c(t,d), \tag{2.9}$$

where $I(t,d)$ is the current and $V_c(t,d)$ is the voltage drop across the capacitor for a certain time and gap size.

The problem may be modeled as a circuit constructed such that R_c and the capacitor are wired in parallel with respect to the voltage source and resistor R_r. Once more, consider the behavior of the system after the capacitor is fully charged so that the capacitor may be reduced to a resistor R_c as given in Eq. 2.6. In this way, V_c is only a function of the gap size, i.e.

$$V_c = \frac{\rho d V_0}{R_s A + \rho d} \tag{2.10}$$

with the power consumption of the capacitor given by Eq. 2.7.

First, via Kirchoff's voltage law,

$$V_c = V_0 - V_{R_s} = V_0 \left(1 - \frac{R_s}{R_c + R_s}\right), \tag{2.11a}$$

which becomes,

$$V_c = \left(1 - \frac{1}{1000d + 1}\right), \tag{2.11b}$$

with the values given in this example.

Now, combining Eq. 2.11b and Eq. 2.2,

$$P_c = -\alpha^2 + \alpha, \tag{2.12}$$

where $\alpha = (1 + 1000d)^{-1}$.

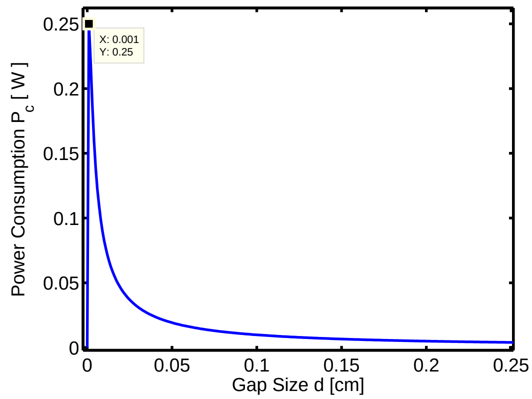

Figure 2: Power consumption of a capacitor

Considering only those values for which the gap size ranges from zero to one centimeter, Fig. 2 is a plot of Eq. 2.12 as a function of d and shows a maximum value is reached at $10^{-3}\ m$, or equivalently, when ρ/A is $10^3\ \frac{\Omega}{m}$.

This value for the gap size corresponds to a resistance of one ohm, i.e. when $d = 10^{-3}\ m,\ R_c = R_r = 1\ \Omega$. Therefore, in this case, the maximum power consumption for a capacitor is attained when the gap size is adjusted so that the resistance of the capacitor is equal to that of the resistor.

2.3 The parallel plate capacitor as a dynamical system

2.3.1 Description

The R-C circuit mentioned in the previous section is a dynamical system. The forces acting upon it, the parameters that determine its state, and the relationships between them determines the behavior of this system.

The previous model involved a capacitor connected in series with a resistor. Such an arrangement implies that for either of the two plates of the capacitor, each plate is directly connected with the portion of the circuit associated with the 'accumulation of positive charge' or the portion of the circuit associated with the accumulation of negative charge. Now consider a capacitor such that one large conducting plate is not in direct contact with any portion of the circuit and is mobile; the other side of the capacitor constitutes two equally sized – roughly half of the large plate – stationary plates that are directly connected to one of the two ends of a resistor circuit, so that one accumulates positive charge and the other negative charge. This arrangement creates a 'charge differential' across the larger plate so that, in effect, two equivalent capacitors are connected in series. Such an arrangement may be reduced to an effective capacitor with two plates of equal area A. This is illustrated in Fig. 3.

2.3.2 Formulation

In this system, there is a (1) inward-directed or attractive force, F_A, which acts to decrease the separation between the capacitor plates, (2) contact force (with a capital 'C'), F_C, that in accordance with Newton's third law, is an outward-directed or repulsive force that is zero when $x \neq 0$, and (3) random force F_N that will serve as a source of noise. Firstly, let us consider the force F_A in terms of the position of the mobile plate x and capacitance of the effective capacitor, C. The force F_A is given by

$$F_A = QE. \tag{2.13}$$

where F_A is the electrostatic force on the mobile plate with total area A.

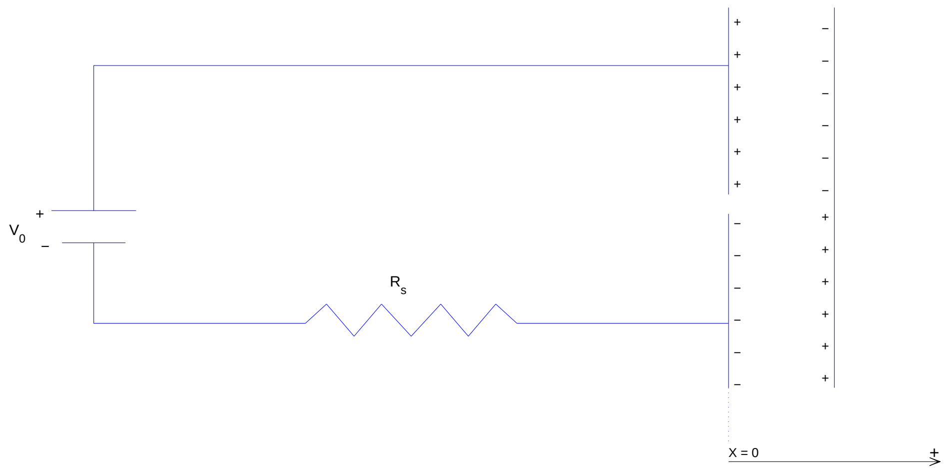

Figure 3: A RC circuit

The charge held on a plate Q is the potential V_c across the plates times the capacitance C. Moreover, there is a dielectric of dielectric constant k inserted between the plates so that $C = kC_{vacuum}$. Therefore, capacitance may be expressed as

$$C = \frac{k\epsilon_0 A}{x}. \tag{2.14}$$

Thus,

$$Q = \frac{k\epsilon_0 A}{x} V_c, \tag{2.15}$$

In general, the electric field E contributed by one plate with a dielectric of dielectric constant k may be written as

$$E = \frac{\sigma}{2k\epsilon_0}, \tag{2.16a}$$

where σ is the charge density in coulombs per area and x is small compared to A.

With $\sigma = Q/A$,

$$E = \frac{Q}{2k\epsilon_0 A}. \tag{2.16b}$$

Thus, Eq. 2.13 is

$$F_A = QE = \frac{Q^2}{2k\epsilon_0 A} \tag{2.17a}$$

with Eq. 2.15, and appending a minus sign to denote the direction of the force,

$$F_A = -\frac{k\epsilon_0 A}{2} \left(\frac{V_c}{x}\right)^2 \tag{2.17b}$$

With Eq. 2.10 and setting $d = x$, the final expression for the full capacitor is given by

$$F_A = -\frac{Ak\epsilon_0}{2}\left(\frac{\rho V_0}{R_s A + \rho x}\right)^2 \tag{2.17c}$$

Note that for $x \geq 0$, a maximum value of F_A is reached at $x = 0$, and the $\lim_{x\to\infty} F_A = 0$.

In general, Eq. 2.17c has the form plotted in Fig. 4 with $k = 2$, $A = 4\ m^2$, $R_s = \frac{1}{4}\ \Omega$,$\rho = 2\ \Omega\, m$,

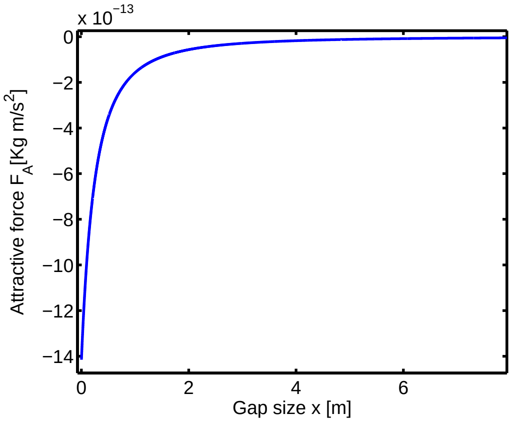

Figure 4: Attractive force as a function of gap size.

and $V_0 = 1\ V$. Note that at $x = 0$, F_A is a finite value.

The electrostatic potential energy stored in the capacitor $U_c = QV_c/2$ with Eq. 2.10 and Eq. 2.15 becomes

$$U_c = \frac{Ak\epsilon_0 x}{2}\left(\frac{\rho V_0}{R_s A + \rho x}\right)^2 = -F_A x \tag{2.18}$$

which is plotted in Fig. 5 with $k = 2$, $A = 4\ m^2$, $R_s = \frac{1}{4}\ \Omega$, $\rho = 2\ \Omega\, m$, and $V_0 = 1\ V-$ which shall be henceforth called the default set of values for a given plot. Note that with Eq. 2.7, we find that

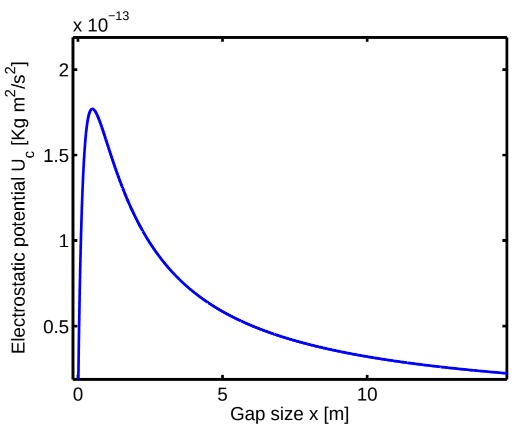

Figure 5: Potential energy stored as a function of gap size.

the dissipation, i.e. the power consumption of the capacitor P_c, is proportional to the stored electrostatic potential. Hence,

$$U_c = \frac{k\epsilon_0\rho}{2}\,P_c \tag{2.19}$$

The effective potential may also be calculated by integrating the expression for the work done up to a gap size of x. With F_A given by Eq. 2.17c,

$$U_{\mathrm{E}} = -\int_0^x F_A(\tau)\,\mathrm{d}\tau = \int_0^x \frac{Ak\epsilon_0}{2}\left(\frac{\rho V_0}{R_s A + \rho\tau}\right)^2 \mathrm{d}\tau \tag{2.20}$$

where τ is the variable of integration. Thus, the final expression is given by

$$U_{\mathrm{E}} = -\frac{Ak\epsilon_0}{2}\left(\frac{\rho V_0^2}{R_s A + \rho\tau}\right)\Bigg|_0^x = \frac{k\epsilon_0 x}{2R_s}\left(\frac{\rho V_0}{\sqrt{R_s A + \rho x}}\right)^2 \tag{2.21}$$

Taking the limiting value of Eq. 2.21, we find that

$$\lim_{x\to\infty}\left[\frac{k\epsilon_0 x}{2R_s}\left(\frac{\rho V_0}{\sqrt{R_s A + \rho x}}\right)^2\right] = \frac{k\epsilon_0 \rho}{2R_s}\,V_0^2$$

Equation 2.21 is plotted in Fig.6 below. Note that U_{E} approaches a value of 7.08×10^{-13} J in accordance with the value of the limit calculated above with the default set of values.

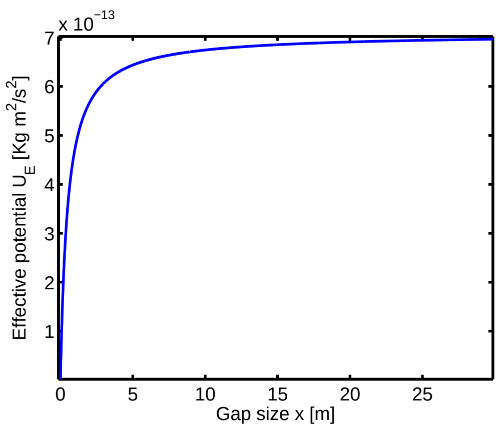

Figure 6: Effective potential as a function of gap size.

Consider the dissipation as a function of gap size. The dissipation will be proportional to the 'frictional force' present in the system. Suppose that the dielectric material between the plates is a kind of oil so that drag forces are present when one of the capacitor plates move. According to newton's second law,

$$m\ddot{x} = F(x) - \eta\dot{x} \tag{2.22}$$

where m is the mass of the single moving plate, $-\eta\dot{x}$ is the frictional force, and $F(x)$ is the force resulting from physical and electrostatic interactions. If the rhs of Eq. 2.22 is much larger than the lhs, then the approximation,

$$0 = F(x) - \eta\dot{x} \tag{2.23}$$

is valid. Therefore, the following first order differential equation is obtained

$$\dot{x} = \frac{F(x)}{\eta}. \tag{2.24}$$

$F(x)$ is made up of the attractive force between the plates, and a random force F_N which will be a source of noise. This is summarized in expression 2.25 below with the condition that $x \geq 0$.

$$\dot{x} = \frac{F_N + F_A}{\eta} \tag{2.25}$$

where η is the effective viscous friction constant. N.B. that the attractive force F_A acts in the 'negative' or inward direction, and F_N in either with a randomly generated value in some interval determined by the experimentalist.

2.3.3 Simulation

2.3.4 Method 1: Euler's method - A simple approach

Equation 2.25 may be integrated using Euler's method and implemented using a simple iterative algorithm. N.B. that x is restricted to positive values.

Figure 7 shows the value of x for fifteen time steps given that the noise is nonexistent ($F_N := 0$) with an initial gap size, chosen arbitrarily, of 8 m,

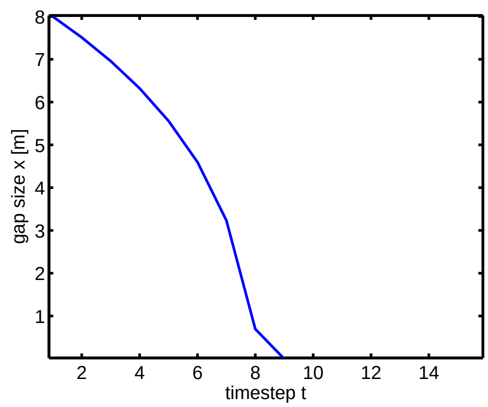

Figure 7: Gap size over time ($F_N = 0$)

where $k = 2$, $A = 4\ m^2$, $R_s = \frac{1}{4}\ \Omega$, $\rho = 2\ \Omega\, m$, $\eta = 1$, and $V_0 = 10^9\ V$.

Note that the system settles at a gap size of $x = 0$. This is not the case if noise is added to the system. Let $F_N(x)$ be a random number that can take any value in the interval $(-|F_A(x)|, |F_A(x)|)$ for some value of x. In this way, the noise in the system is comparable to the attractive force. Figure 8 shows how the system evolves over discrete time for ten thousand steps with an initial gap size of 8 m.

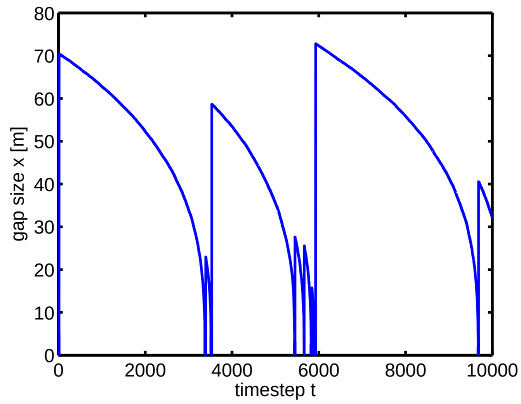

Figure 8: Gap size over time with F_N comparable to F_A

Now suppose that F_N is not a function of x and takes a range of values in some fixed interval $I := [-|F_A(0)|, |F_A(0)|]$ with $|F_A(0)|$ being the maximum value of the magnitude $|F_A|$. Equation 2.25 is solved numerically and the solution to the ODE in some interval is determined by an initial point with noise added. Figure 12, Fig. 13 , and Fig. 14 display the time evolution of the system with intervals I , $2.5 \times 10^2 I$, and $2.5 \times 10^{-2} I$ respectively.

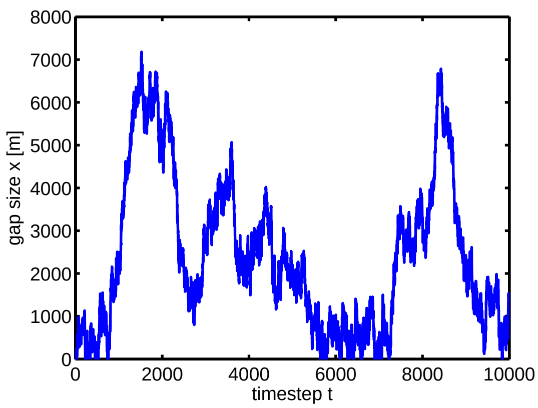

Figure 9: $F_N \in I$

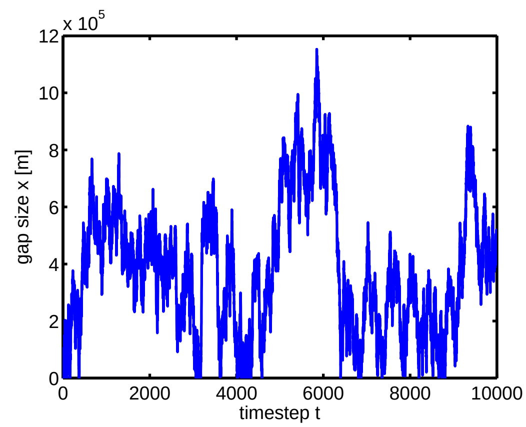

Figure 10: $F_N \in 2.5 \times 10^2 I$

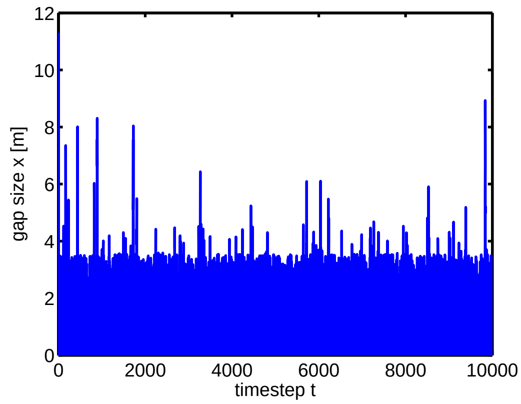

Figure 11: $F_N \in 2.5 \times 10^{-2} I$

2.3.5 Method 2: Explicit Runge-Kutta - Dormand-Prince Pair

Equation 2.25 is now solved numerically using the Dormand-Prince pair technique. After each 10 second interval, noise is added and this value becomes the new initial value of x with which we solve for the next interval of time. Figures 12, 13, and 14 display the time evolution of the system for the given intervals.

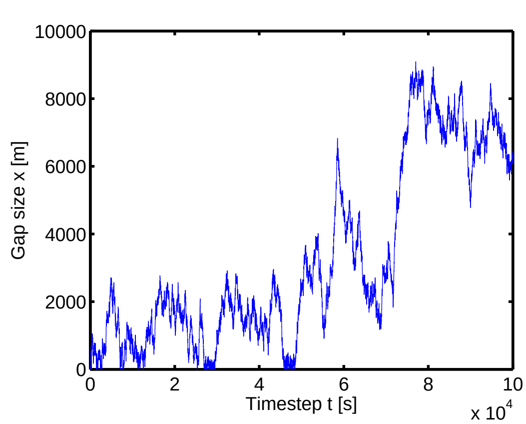

Figure 12: $F_N \in I$

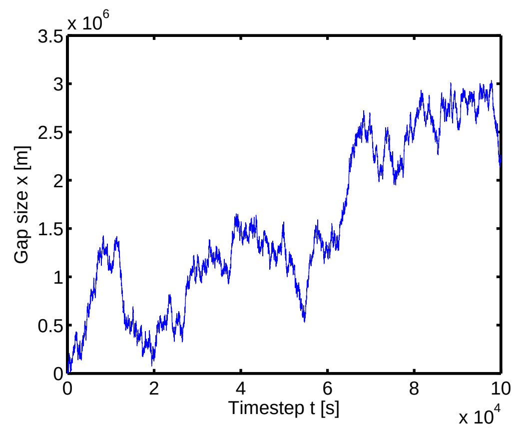

Figure 13: $F_N \in 2.5 \times 10^{2} I$

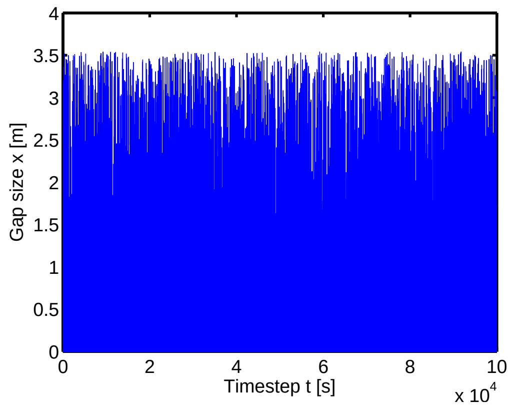

Figure 14: $F_N \in 2.5 \times 10^{-2} I$

2.4 Analysis

2.4.1 Expected value for the gap size x - Empirical approach

To get a clearer sense for the behavior of the system (via method 2) in response to the noise, Fig. 15 is a plot of the expectation value of x versus $a = 0, 1, 2, ...2.5 \times 10^2$ given $a|F_N(0)| = aF_0$ is the magnitude of the force i.e.

$$\big(-aF_0, aF_0 \big) = I(a).$$

and 10,000 seconds of runtime.

The roughly linear trend seen in the logarithmic plot, Fig. 15, suggests a power law.

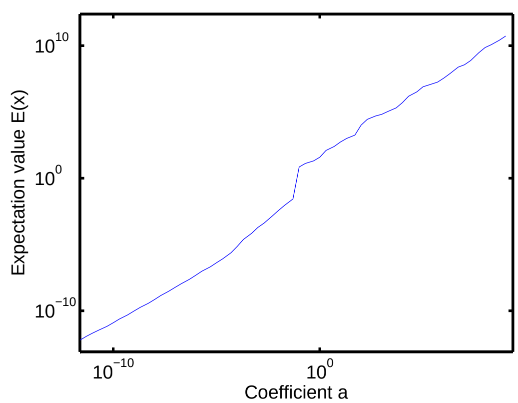

Figure 15: Expectation value of x given $F_N \in I(a)$ - Logarithmic plot

Figure 16 is a plot of expectation values for 10^3, 10^4, and 5×10^4 seconds of runtime.

After close examination, one will observe that for a larger window for which we calculate the expected value of the position of the plate, the expectation value is nearly the same for values of a less than some value. For values of a greater than that value, the system diverges i.e. an increase in noise implies an increase in the expectation value. Moreover it is at this particular value of a that the behavior of the system divides into two distinct classes or 'phases'.Note that when $x = \frac{R_s A}{\rho} = .5$, - the point at which the system has the maximum level of dissipation –, the expectation value of x is invariant with respect to time i.e. it exists in

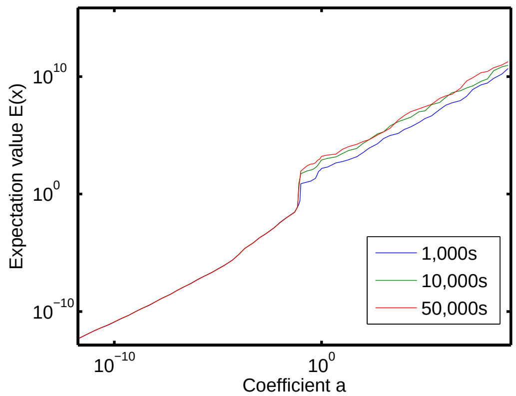

Figure 16: Expectation value of x for several run-times given $F_N \in I(a)$ - Logarithmic plot

the region immediately before the expectation values diverge. This suggests that before this region there is a 'stable' state for noise levels below, and a divergent state for levels above.

2.4.2 Expected value for the gap size x - Analytical approach

The expected value x denoted by $\langle x \rangle$ is given by

$$\langle x \rangle = \int_0^\infty x\, f(x) \mathrm{d}x \tag{2.26}$$

where $f(x)$ is the probability distribution for x, which is also found in the corresponding Fokker-Planck equation for the system. First, however, the Langevin formulation of Eq. 2.25 is given in the development below. [1]

$$\dot{x} = \frac{F_A}{\eta} + \frac{F_N}{\eta} = g(x) + \Psi \tag{2.27}$$

Where $g(x) = -F(A)$, and

$$\Psi = \frac{F_N}{\eta} = \frac{1}{\eta} \sum_j \phi_j \delta(t - t_j)(\pm 1)_j$$

where $\phi_j \in I(a)$ is the magnitude of the force at a moment t_j. The terms under summation represent all of the contributions due to the random force.

The corresponding Fokker-Planck equation is given by

$$\dot{f} + \frac{\mathrm{d}}{\mathrm{d}x}\left(fg - \frac{Q}{2}\frac{\mathrm{d}f}{\mathrm{d}x}\right) = 0 \tag{2.28}$$

with f and g as given above and Q the diffusion coefficient given by

$$Q = \frac{F_0^2}{\eta^2 t_0}\, a^2 \tag{2.29}$$

where $a \in \mathbb{R}^+$.

Suppose that the probability distribution $f(x)$ is time-independent, therefore

$$fg - \frac{Q}{2}\frac{\mathrm{d}f}{\mathrm{d}x} = \text{constant} \tag{2.30}$$

It is easy to show that the solution to Eq. 2.30 is given by

$$f(x) = \mathcal{N} \exp\left(-2U_\mathrm{E}/\eta\, Q\right) \tag{2.31}$$

with U_E as given in Eq. 2.21 and $\mathcal{N}$ as the normalization constant.

Finally, setting $\beta := k\, \epsilon_0\, t_0\, \rho\, V_0^2\, \eta / F_0^2\, R_s$ Eq. 2.31 becomes

$$f(x, a) = \mathcal{N} \exp\left(\frac{-\beta\, x}{a^2\,(R_s\, A/\rho + \, x)}\right) \tag{2.32}$$

In the limit as x goes to infinity, Eq. 2.31 reduces to

$$\mathcal{N} \exp\left(-2 \lim_{x\to\infty} U_\mathrm{E}/\eta\, Q\right) = \mathcal{N} \exp\left(-\frac{\beta}{a^2}\right) \tag{2.33}$$

Note that as $a \to \infty$, $f \to 1$ implying that the distribution function diverges for large noise. Moreover, as $a \to 0$, $f \to 0$ implying that the distribution converges for small noise. Figures 17 and 18 demonstrate this observation.

[1] See [11] Pg. 147-167

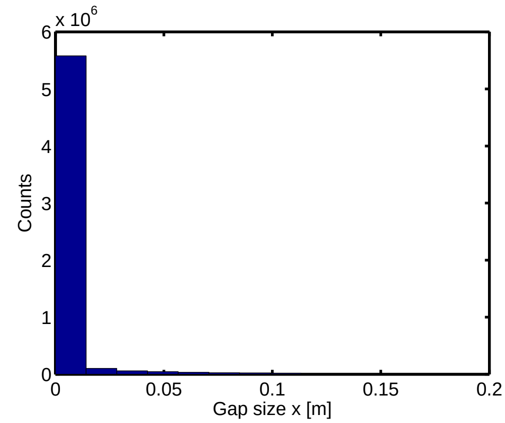

Figure 17: Histogram of gap size x for $a = 10^{-3}$ ($\approx 6 \times 10^6$ counts)

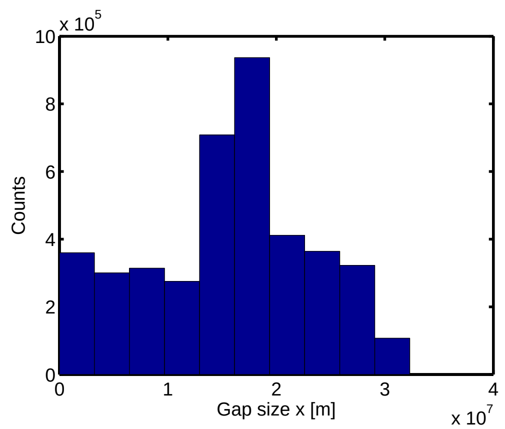

Figure 18: Histogram of gap size x for $a = 10^3$ ($\approx 4 \times 10^6$ counts)

3 Conclusion

The insight provided demonstrates that if a process exists which slowly reduces the noise level, the first stable state of the system would exist near the state of maximum entropy production. In the context of the carbon nanotube wire experiment, this statement suggests that for this dissipative system the most stable state - resistant to fluctuations - exists exactly at this state of maximum entropy production. This conclusion is consistent with experimental findings, where a quasi-stable configuration of wires is first formed at this point.

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