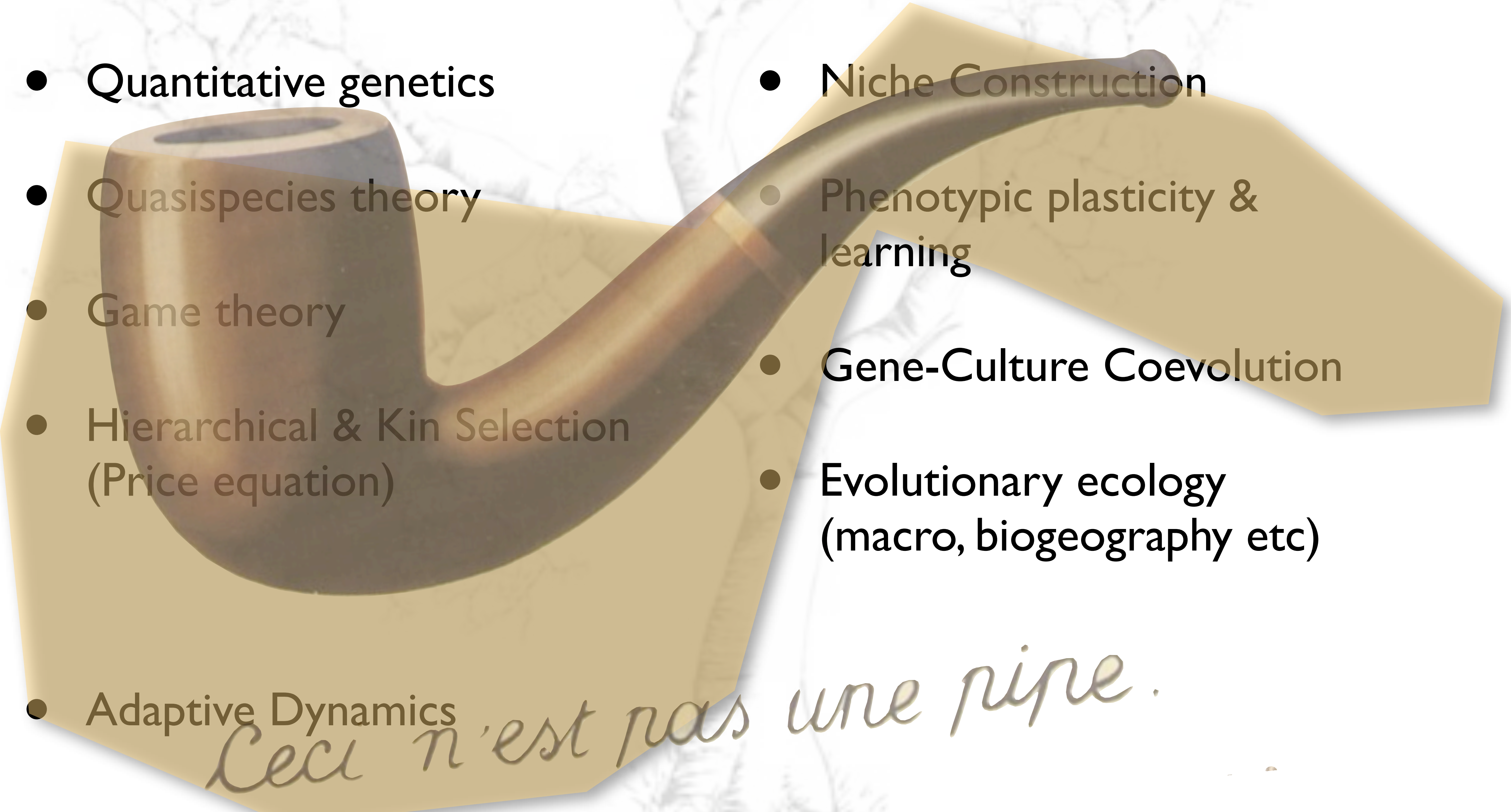
Evolutionary Dynamics

Evolutionary Theory - specialities

- Population genetics/neutral theory
- Quantitative genetics
- Quasispecies theory
- Game theory
- Hierarchical & Kin Selection (Price equation)
- Adaptive Dynamics
- Phylogenetic reconstruction/ inference
- Niche Construction
- Phenotypic plasticity & learning
- Gene-Culture Coevolution
- Evolutionary ecology (macro, biogeography etc)



The Intellectual Foundations are simple,
The Orbit of influence is wide...

Evolutionary Stoichiometry

replication

$$g_i \xrightarrow[\text{Energy + Resources}]{r_i} 2g_i$$

Evolutionary Stoichiometry

competition

$$g_i + g_j \xrightarrow{c_{ij}} g_j$$

Evolutionary Stoichiometry

mutation

$$g_i \xrightarrow[\text{Radiation}]{m_{ij}} g_j$$

$$m_{ij} = \mu^{H(i,j)}(1-\mu)^{L-H(i,j)}$$

Evolutionary Stoichiometry

recombination

$$g_j + g_l \xrightarrow{b_{ijl}} g_i$$

$$b_{ijl} = 1, \quad \text{if} \;\; i = j = l$$

$$b_{ijl} = \left(\frac{1}{2}\right)(1-c) + c\left(\frac{1}{2}\right)^{H(j,l)} \quad \text{if} \quad i = j \quad \text{or} \quad i = l$$

$$b_{ijl} = c\left(\frac{1}{2}\right)^{H(j,l)} \quad \text{if} \quad H(i,j) + H(i,l) = H(j,l)$$

Evolutionary Stoichiometry

Development

ontogenetic : $$g_j + g_l \xrightarrow{d_{ijl}} p_i$$

Typically Treated Thus

$$g_i \xrightarrow{d_i} p_i$$

Replicator Equation

$$g_i \xrightarrow{r_i} 2g_i$$

$$g_i + g_j \xrightarrow[r_j]{c_{ij}} g_j$$

n genomes

$$\dot{g}_i = g_i(r_i - \bar{f})$$

where $\bar{f} = \sum_i^n r_i g_i$ and $c_{ij} = 1$

$$\sum g_i(t=0) = 1$$

Evolutionary Game Theory: Frequency dependent Replicator Equation

$$g_i \xrightarrow{r_i(\mathbf{g})} 2g_i$$

$$g_i + g_j \xrightarrow[r_j]{c_{ij}} g_j$$

n genomes

$$\dot{g}_i = g_i(r_i(\mathbf{g}) - \bar{f})$$

where $\bar{f} = \sum_{i}^{n} r_i(\mathbf{g}) g_i$ and $c_{ij} = 1$

Evolutionary Game Theory: Frequency dependent Replicator Equation

$$\dot{g}_i = g_i(r_i(\mathbf{g}) - \bar{f})$$

$$\text{Payoff Matrix } \; P = [p_{ij}]$$

with linear payoffs:

$$r_i(\mathbf{g}) = \sum_{j}^{n} g_j p_{ij}$$

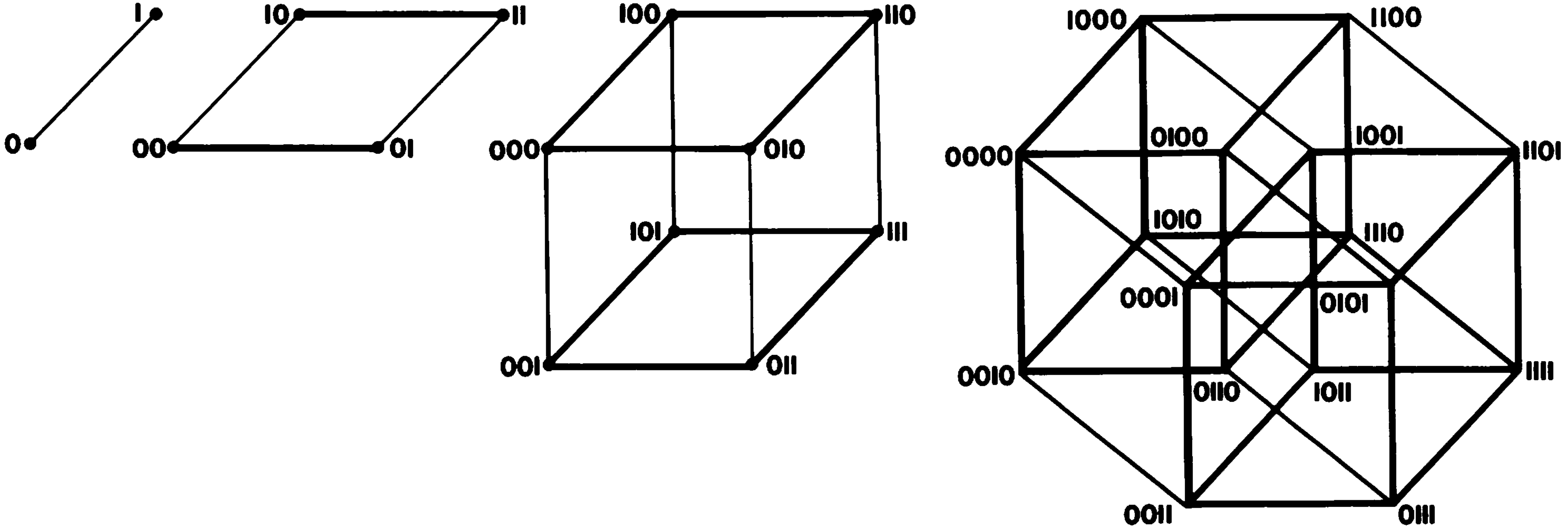
Evolutionary Game Theory: Frequency dependent Replicator Equation

$$\dot{g}_i = g_i(\sum_{j}^{n} g_j p_{ij} - \sum_{j}^{n} g_j \sum_{k}^{n} g_k p_{jk})$$

Stable Equilibria of Replicator Eq. and Nash Equilibria

Mathematica Demo

Mutation & Sequence Space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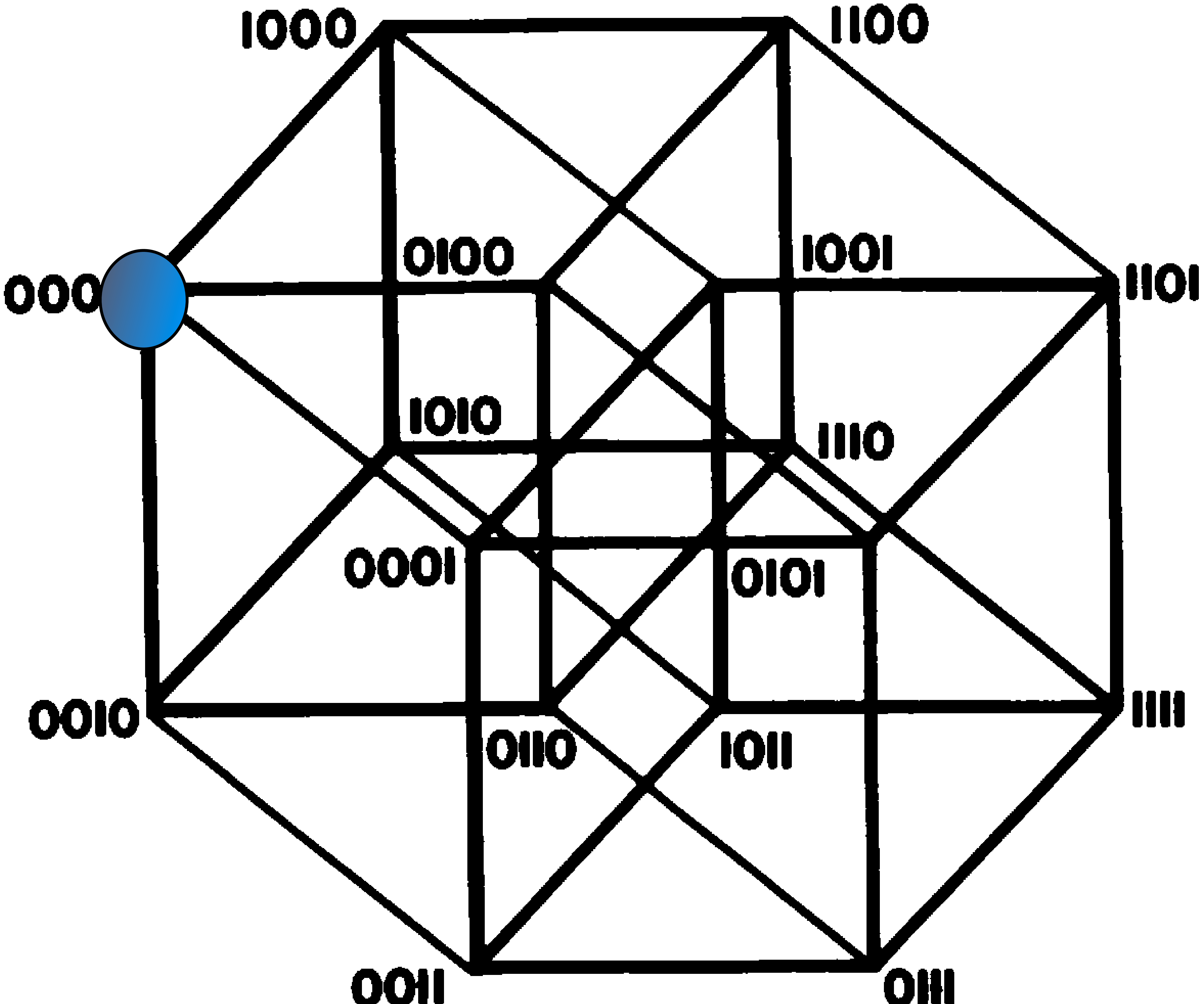
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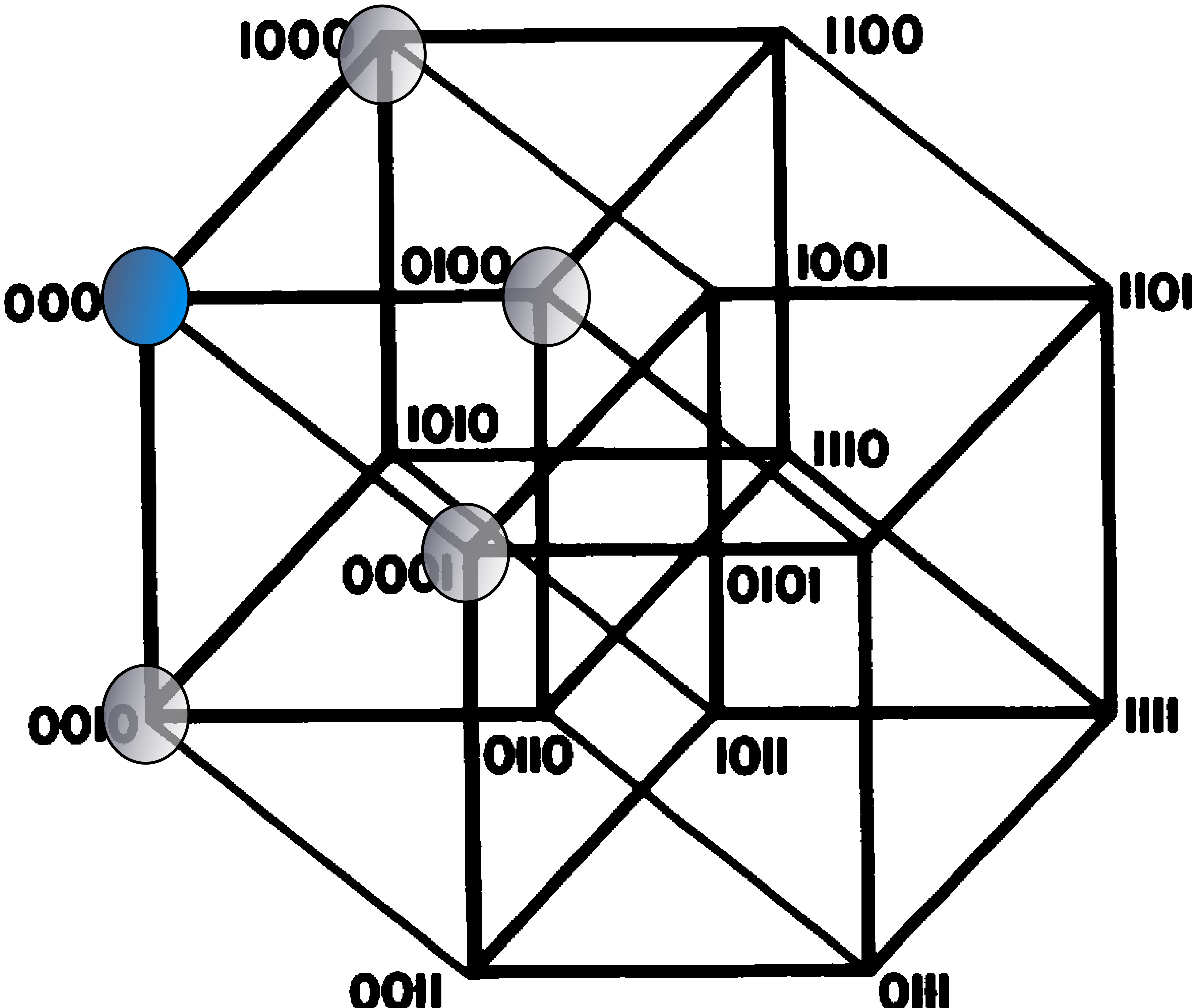
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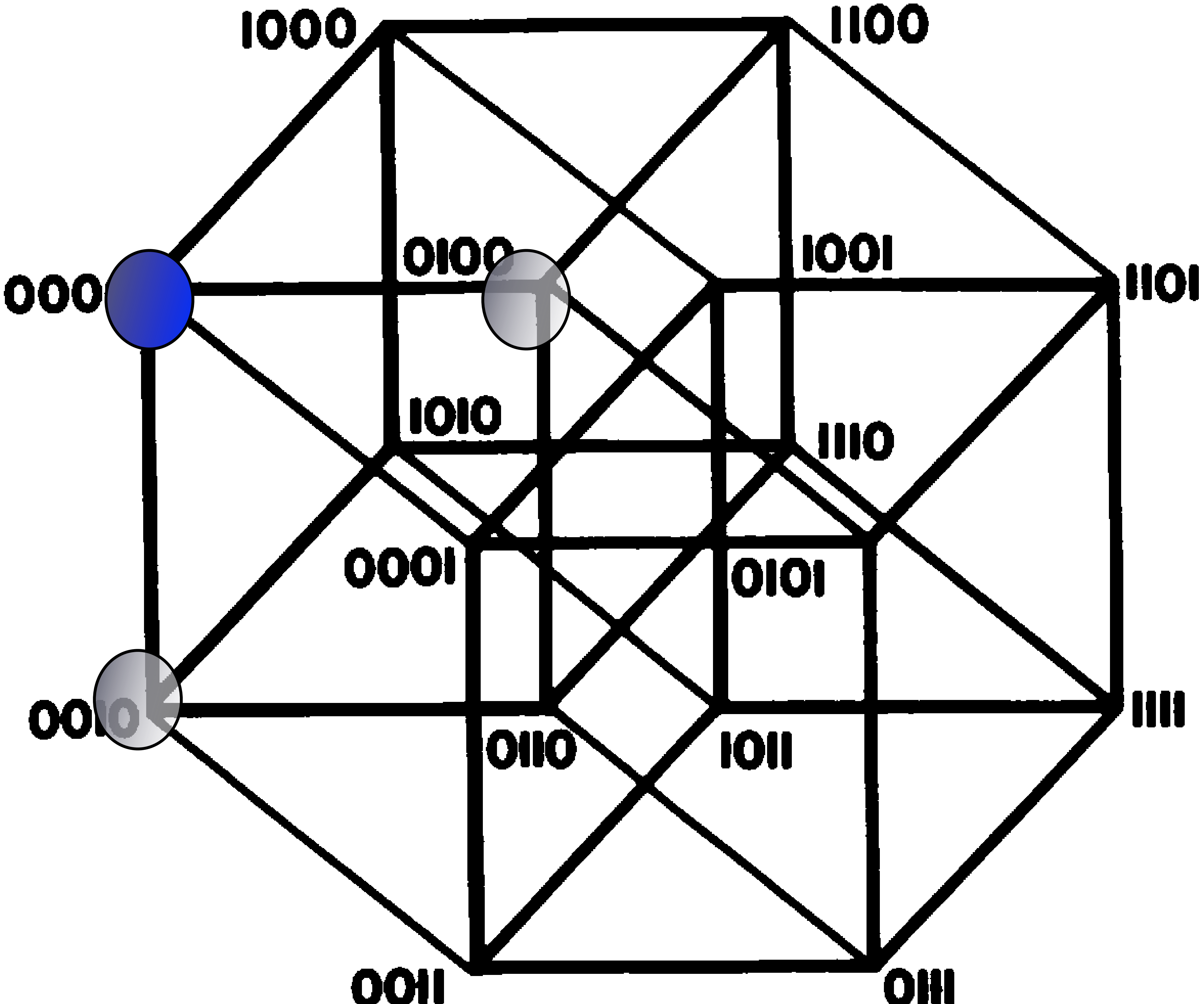
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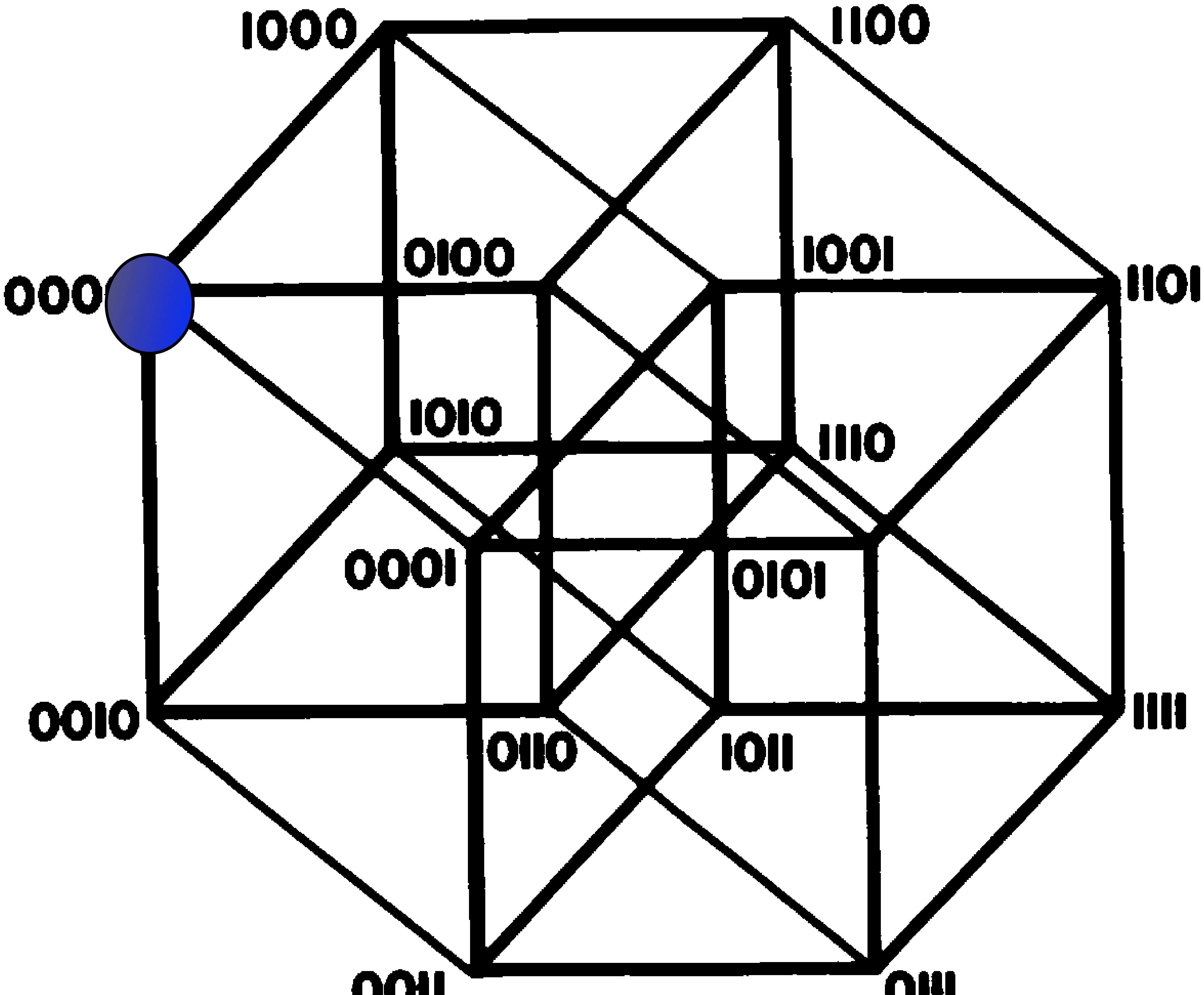
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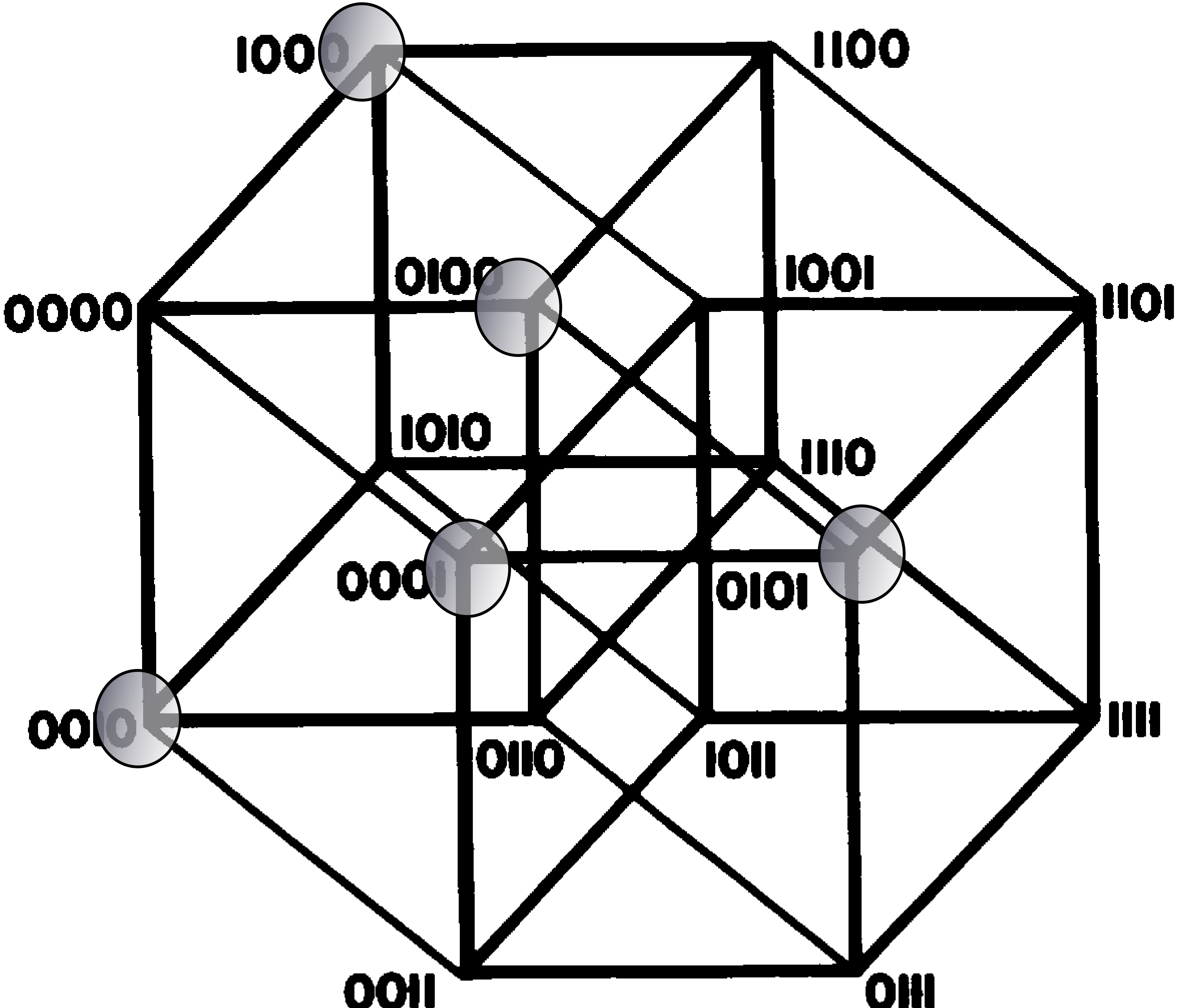

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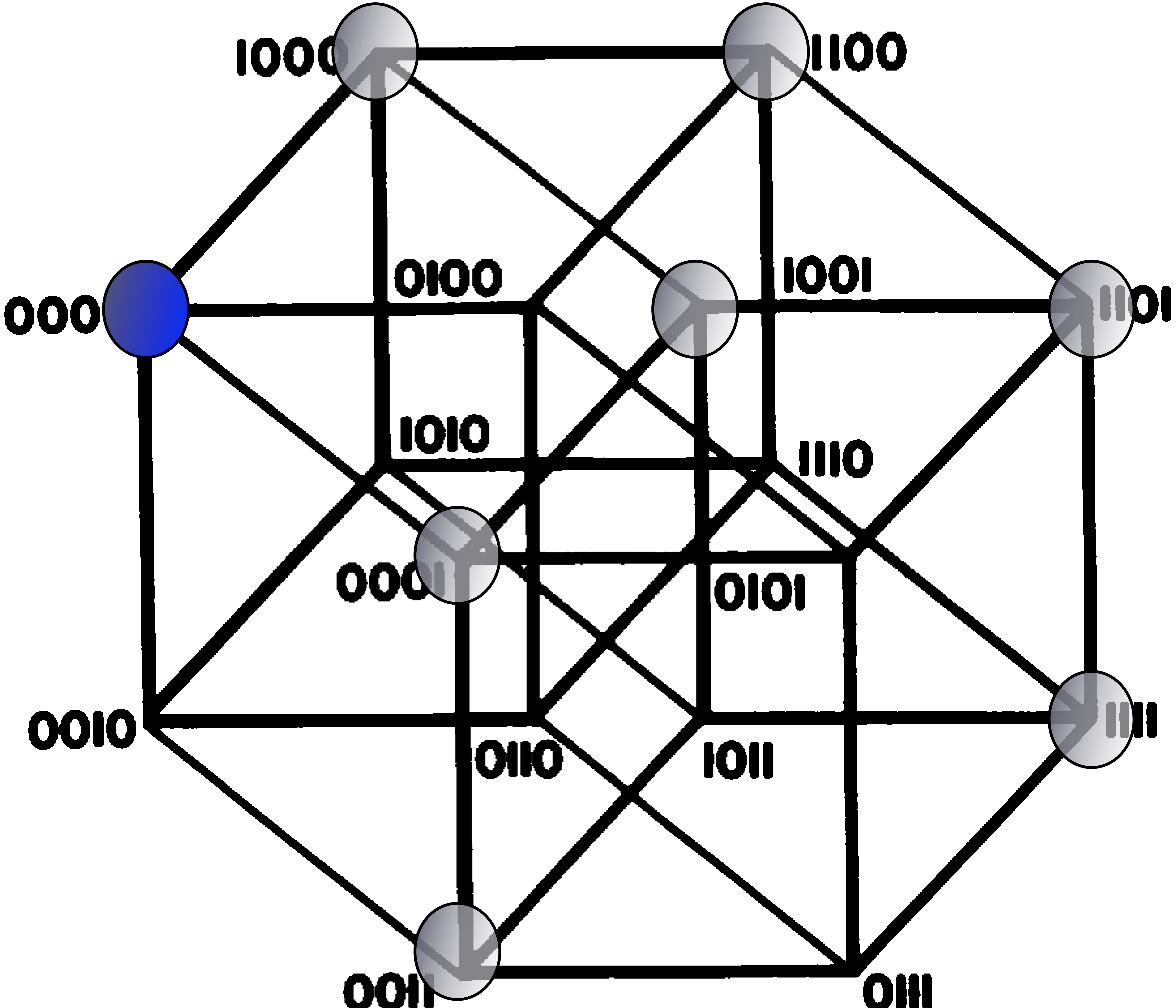
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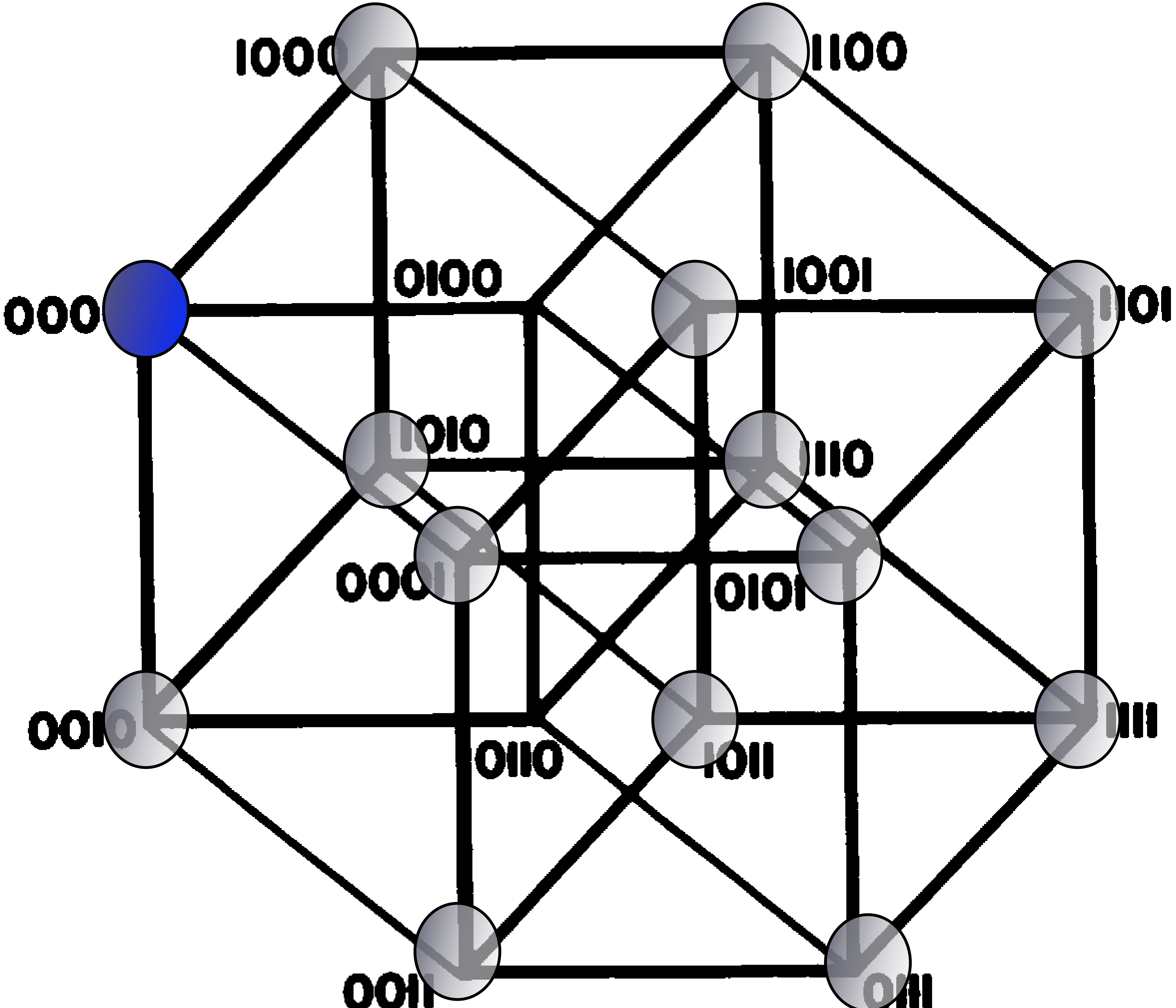
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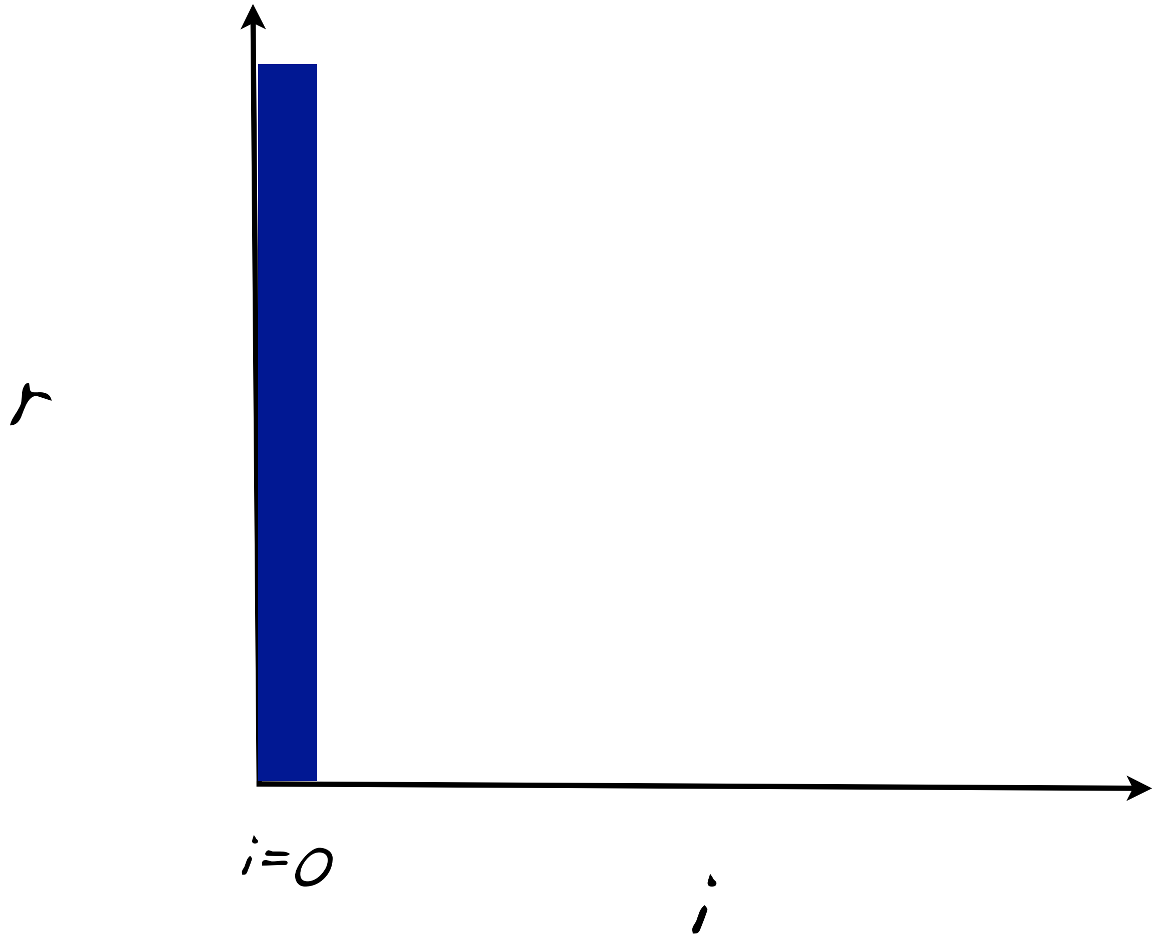
Replicator-Mutator Equation

$$\dot{g}_i = \sum_{j}^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

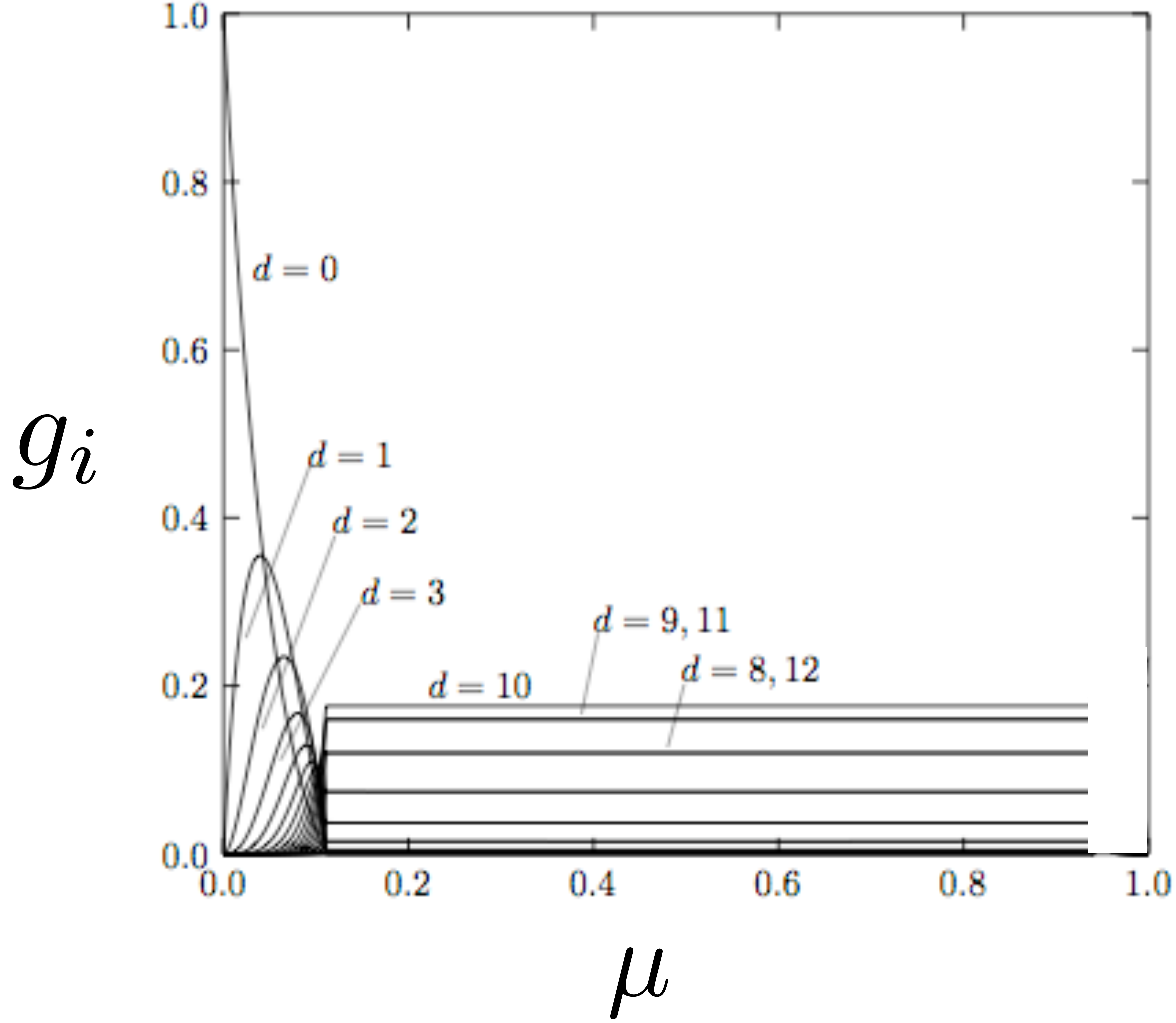
$$m_{ij} = \mu^{H(i,j)} (1 - \mu)^{L - H(i,j)}$$

Curious Implications.....

Delta-function landscape



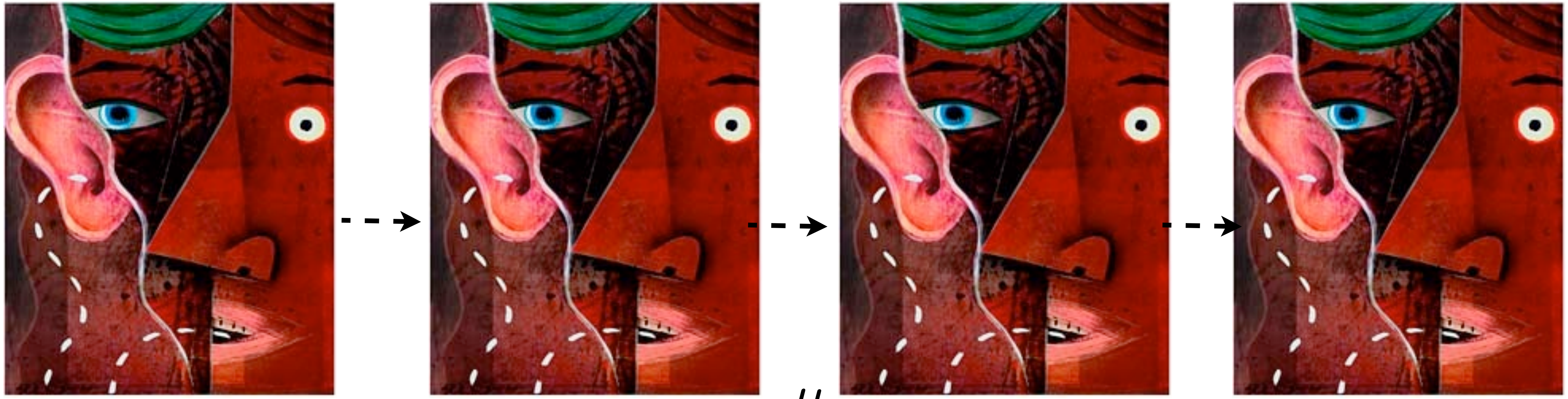
Error Threshold



Error Threshold

Delta function:

$$\mu < \frac{s}{L} = \frac{1}{L}$$



μ μ μ

$L = 1$ $L = 2$ $L = 3$ $L = 4$

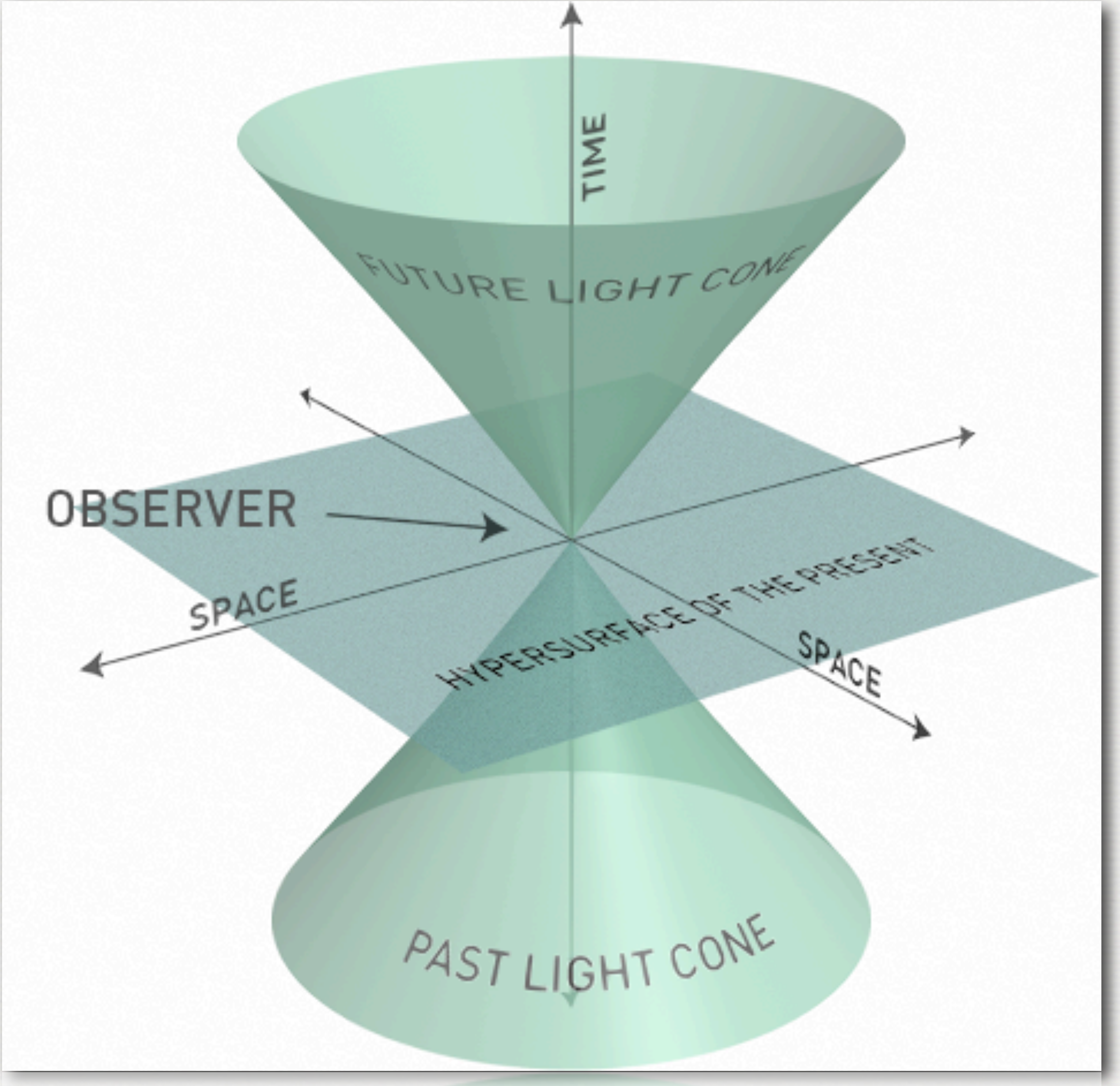
Fitness Landscape

Delta function:

$$\mu < \frac{s}{L} \approx \frac{1}{L}$$

Multiplicative function:

$$\mu < s$$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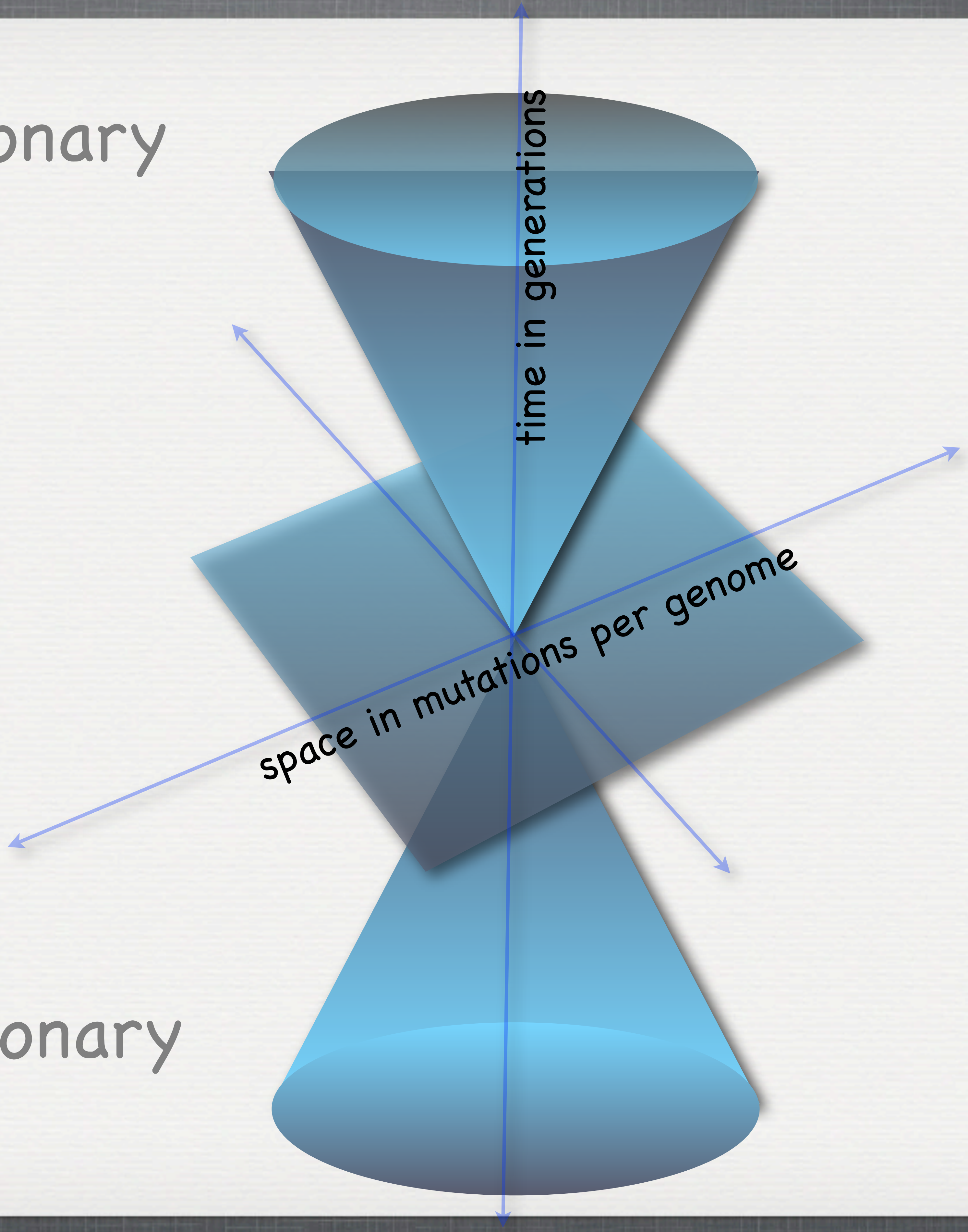
PAST LIGHT CONE

The evolutionary future

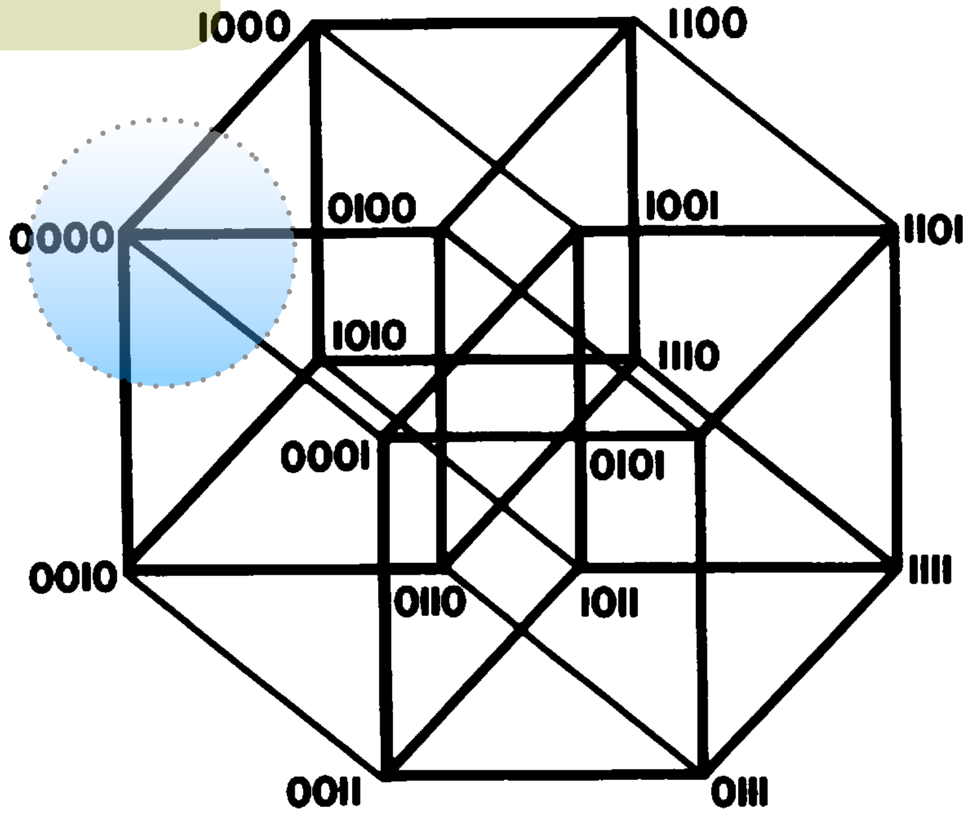


The evolutionary past

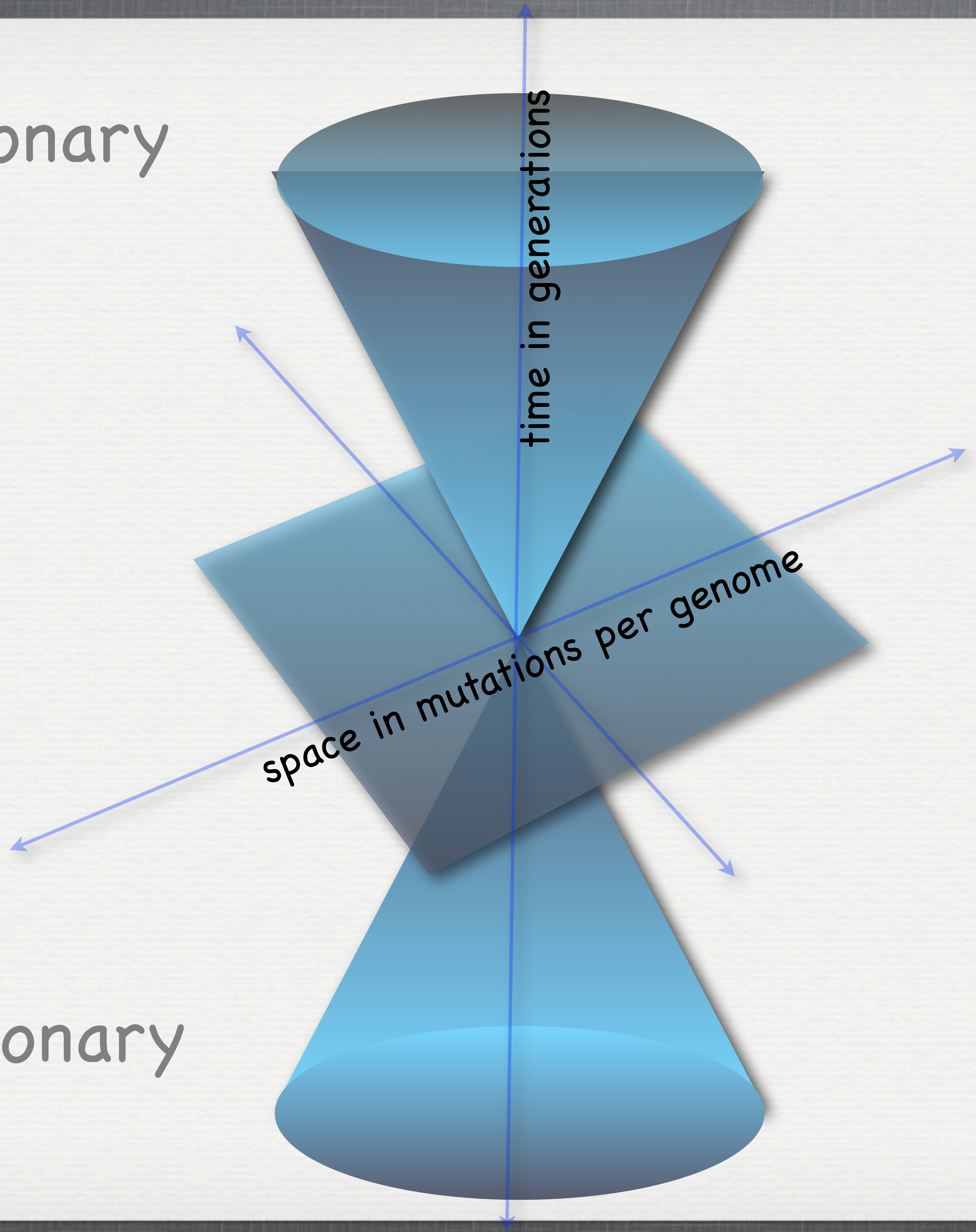
Max n subject to preserving g_I in the locally stable equilibrium distribution

$$\dot{g}_i = \sum_j^{2^n} g_j r_j(\mathbf{g}) m_{ij} - g_i \bar{f}$$

$$m_{ij} = \mu^{H(i,j)} (1-\mu)^{L-H(i,j)}$$

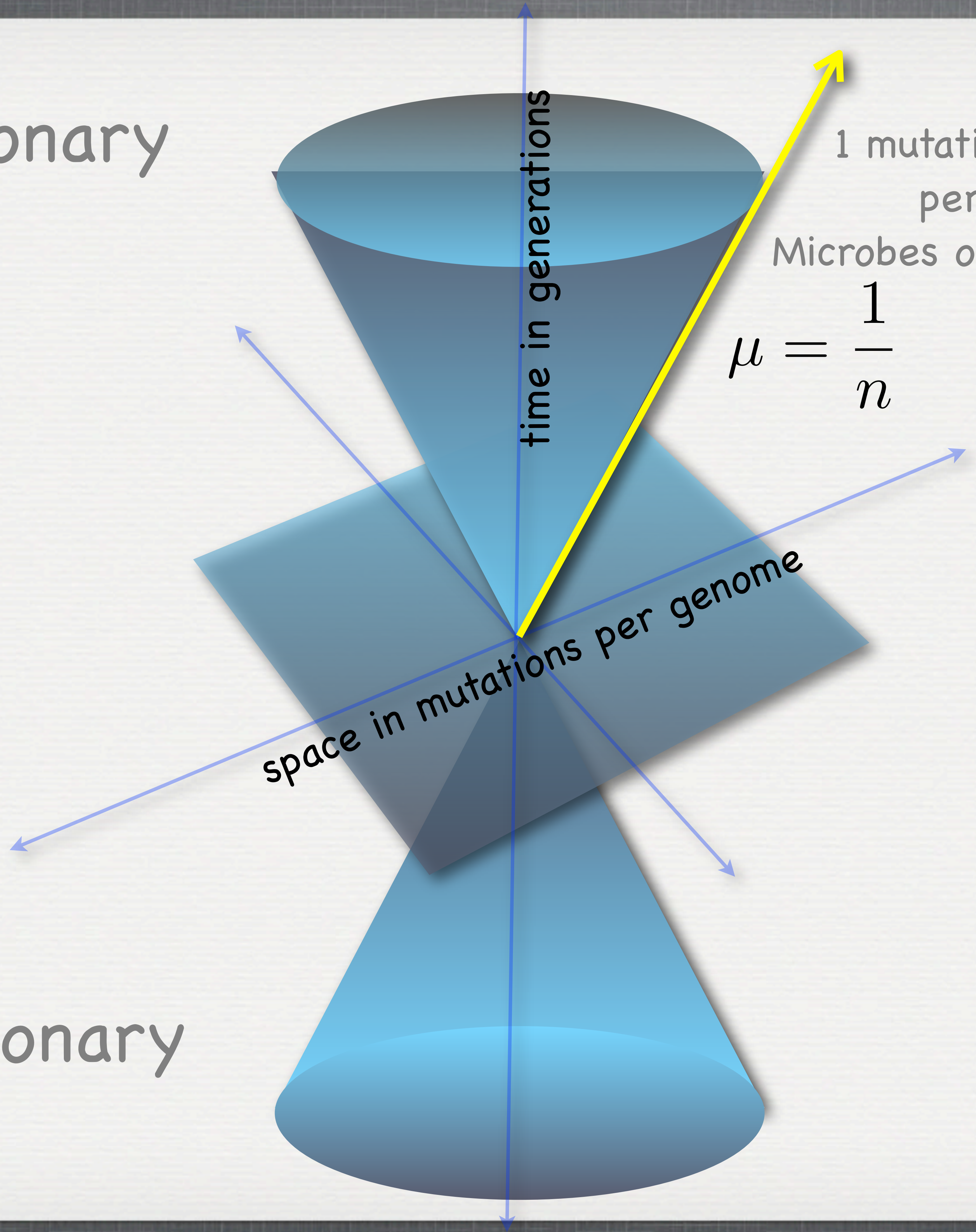


The evolutionary future



The evolutionary past

The evolutionary future



1 mutation per genome per generation

Microbes obey the Eigen Law

$$\mu = \frac{1}{n}$$

The evolutionary past

Testing the Prediction:

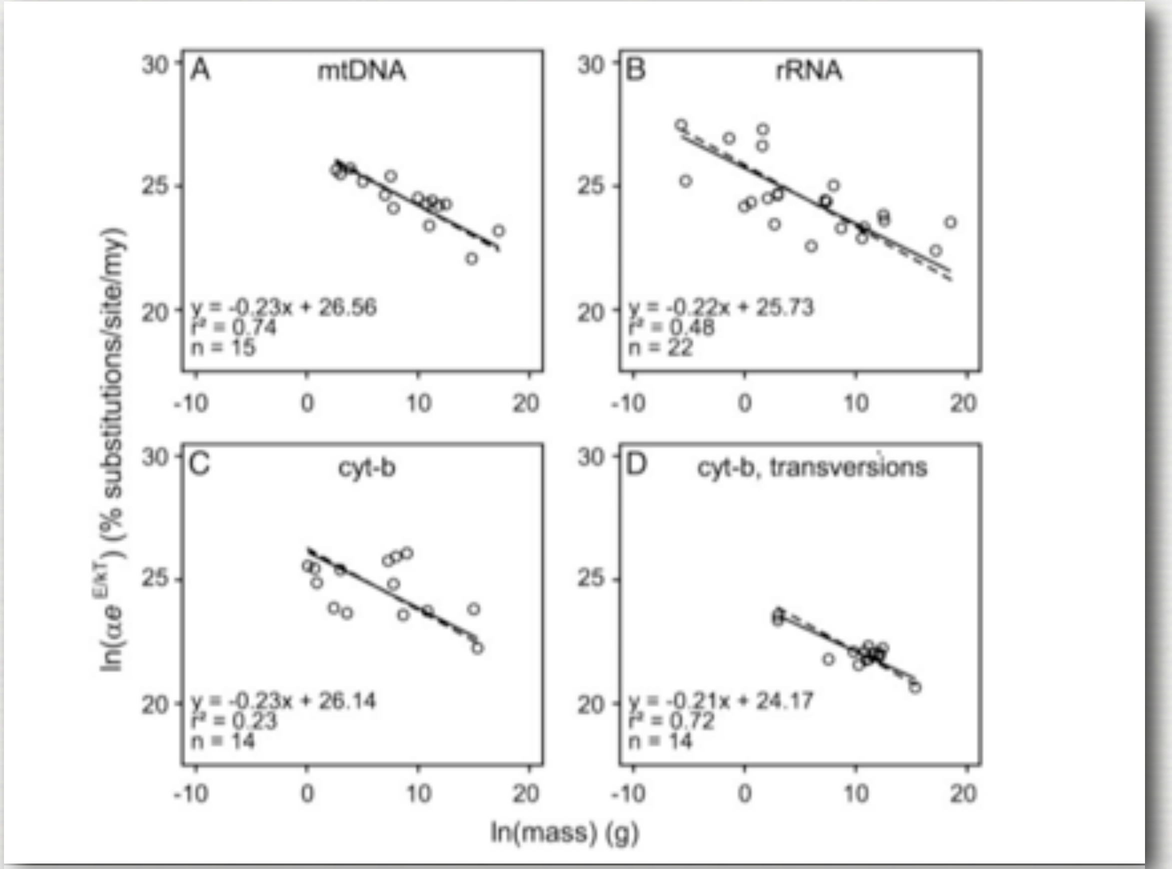
$$n = \frac{1}{\mu}$$

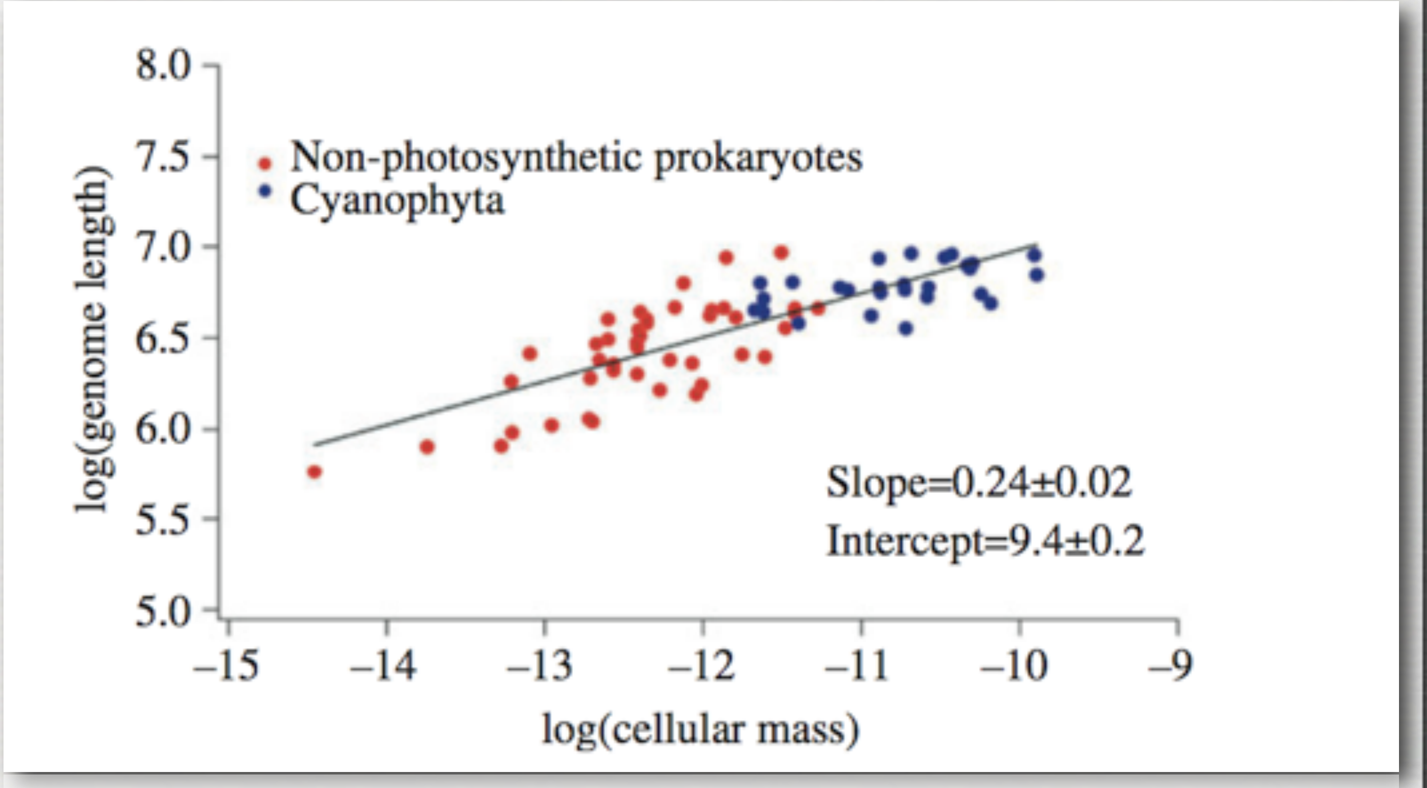
Evol. Maximization

metabolic constraint

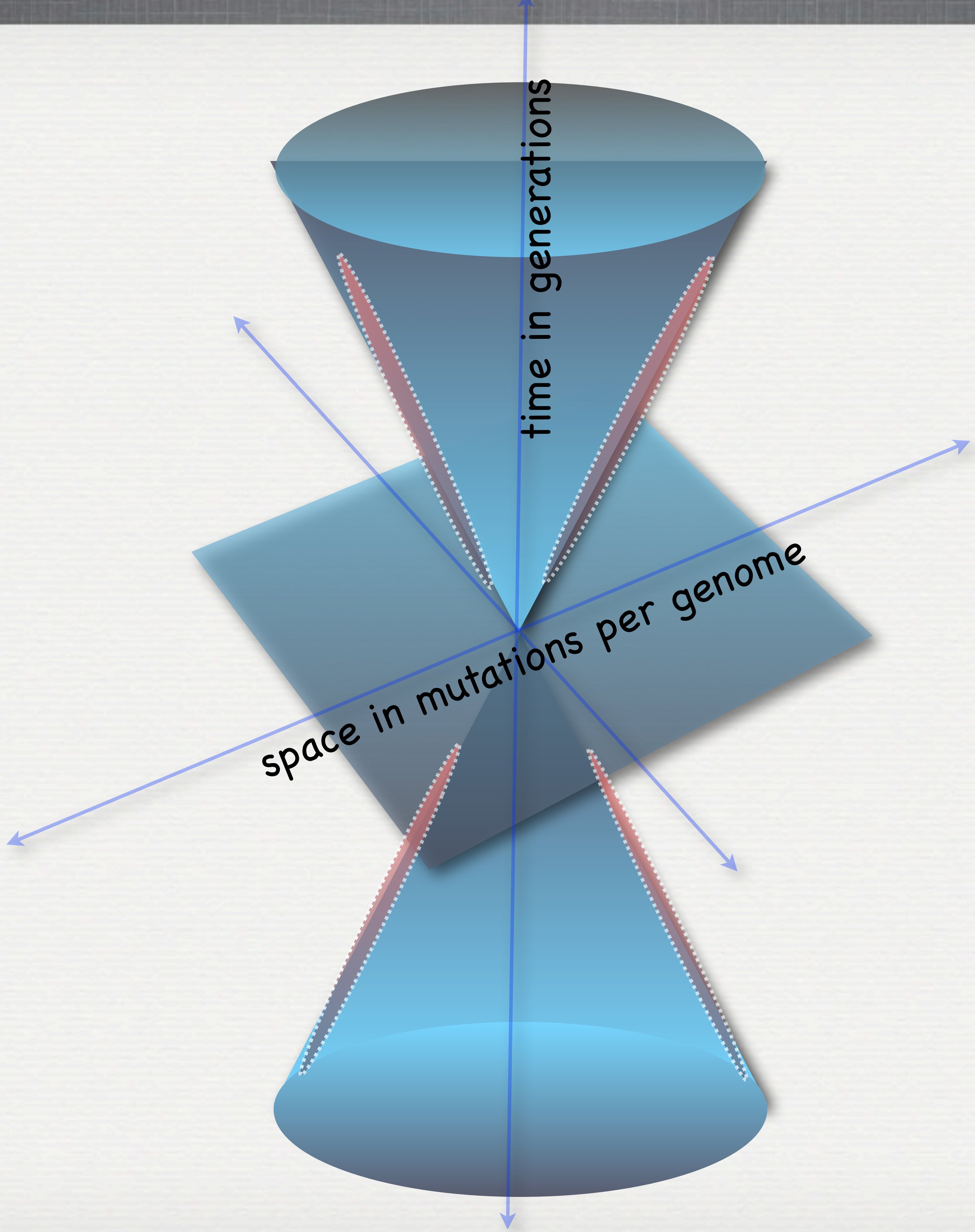
$$\mu = k m^{-\frac{1}{4}}$$

$$n = k m^{\frac{1}{4}}$$





Prokaryotic life evolves on the membrane of life cone



Evolutionary dynamics in microbes is extracting adaptive information at the maximum possible velocity

Selection as Statistical Inference

$$\frac{\Delta g_i(t)}{\Delta t} = g_i(t-1)(r_i(\mathbf{g}) - \bar{f})$$

$$P(X|Y) = P(X)\frac{P(Y|X)}{P(Y)}$$

$$P(X|Y) = P(X)\frac{L_X}{\bar{L}}$$

$$\bar{L} = P(Y) = \sum_{x\in\omega} P(Y|X)P(X)$$

$$P_X(t) = P_X(t-1)\frac{L_X}{\bar{L}}$$

$$\Delta P_X(t) = P_X(t-1)(\frac{L_X}{\bar{L}} - 1) = P_X(t-1)\frac{1}{\bar{L}}(L_X - \bar{L})$$

$$\Delta P_X(t) = P_X(t-1)(f_t - \bar{f}), \quad \text{where} \quad f_t = L_X/\bar{L}$$

Selection as Learning:
(Imitation & Operant)

Biological Complexity is the mechanical/inferential support
for making more accurate hypotheses
about an expanding number of environmental regularities

But Evolutionary Light Speed Constrains the Maximum Rate of Inference

Introducing Phenotypes and hierarchical selection: deriving the *Price Equation*

$$\dot{E}[p] = Cov(r, p) + E[\dot{p}]$$

Insights from Page, Sigmund & others

$$\dot{g}_i = \sum_j^{2^n} g_j r_j(\mathbf{g}) m_{ji} - g_i \bar{r}$$

$$g_i \rightarrow p_i \qquad \sum_i g_i(t=0) = 1 \qquad \bar{r} = \sum_i r_i g_i$$

$$\bar{p} = E[p] = \sum_i p_i g_i$$

$$\dot{E}[p] = \sum_i p_i \dot{g}_i + \sum_i g_i \dot{p}_i$$

$$\dot{E}[p] = \sum_i p_i [\sum_j^{2^n} g_j r_j(\mathbf{g}) m_{ji} - g_i \bar{r})] + \sum_i g_i \dot{p}_i$$

$$\dot{E}[p] = \sum_{ij} p_i g_j r_j m_{ji} - \bar{r}\bar{p} + E[\dot{p}]$$

$$\dot{E}[p] = \sum_j p_j g_j r_j - \bar{r}\bar{p} + \sum_j g_j r_j \sum_i m_{ji}(p_i - p_j) + E[\dot{p}]$$

$$Cov(x, y) = E[(x - \mu)(y - \nu)] = E[xy] - \mu\nu$$

$$Cov(r, p) = \sum_j p_j g_j r_j - \bar{r}\bar{p}$$

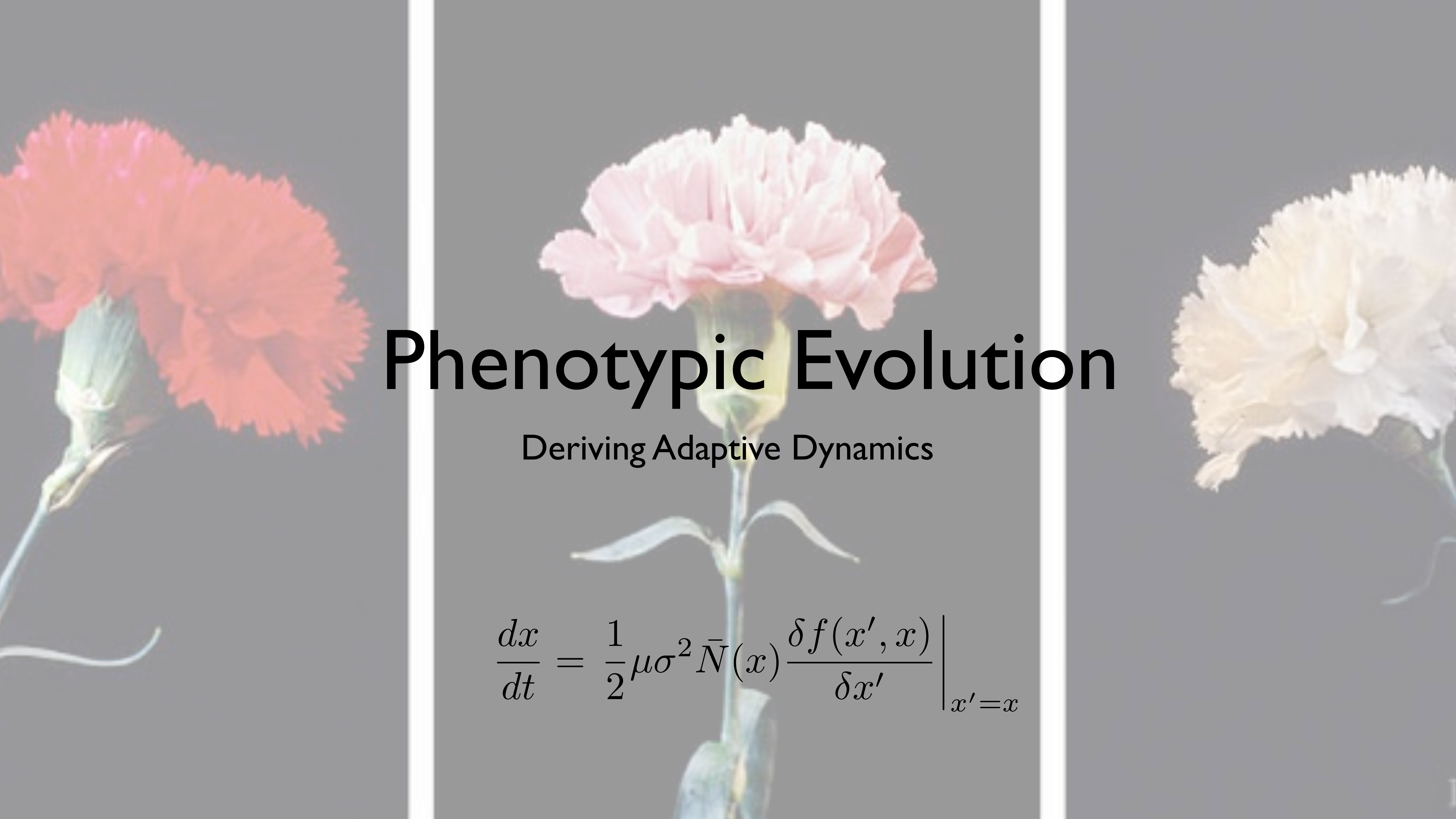
$$E[r\Delta_m p] = \sum_j g_j r_j \sum_i m_{ji}(p_i - p_j)$$

$$\dot{E}[p] = Cov(r, p) + E[\dot{p}] + E[r\Delta_m p]$$

The Price Equation

Uses of Price Equation

- Fisher’s fundamental theorem
- Kin selection (Hamilton’s rule)
- Group selection
- Evolution of cooperation

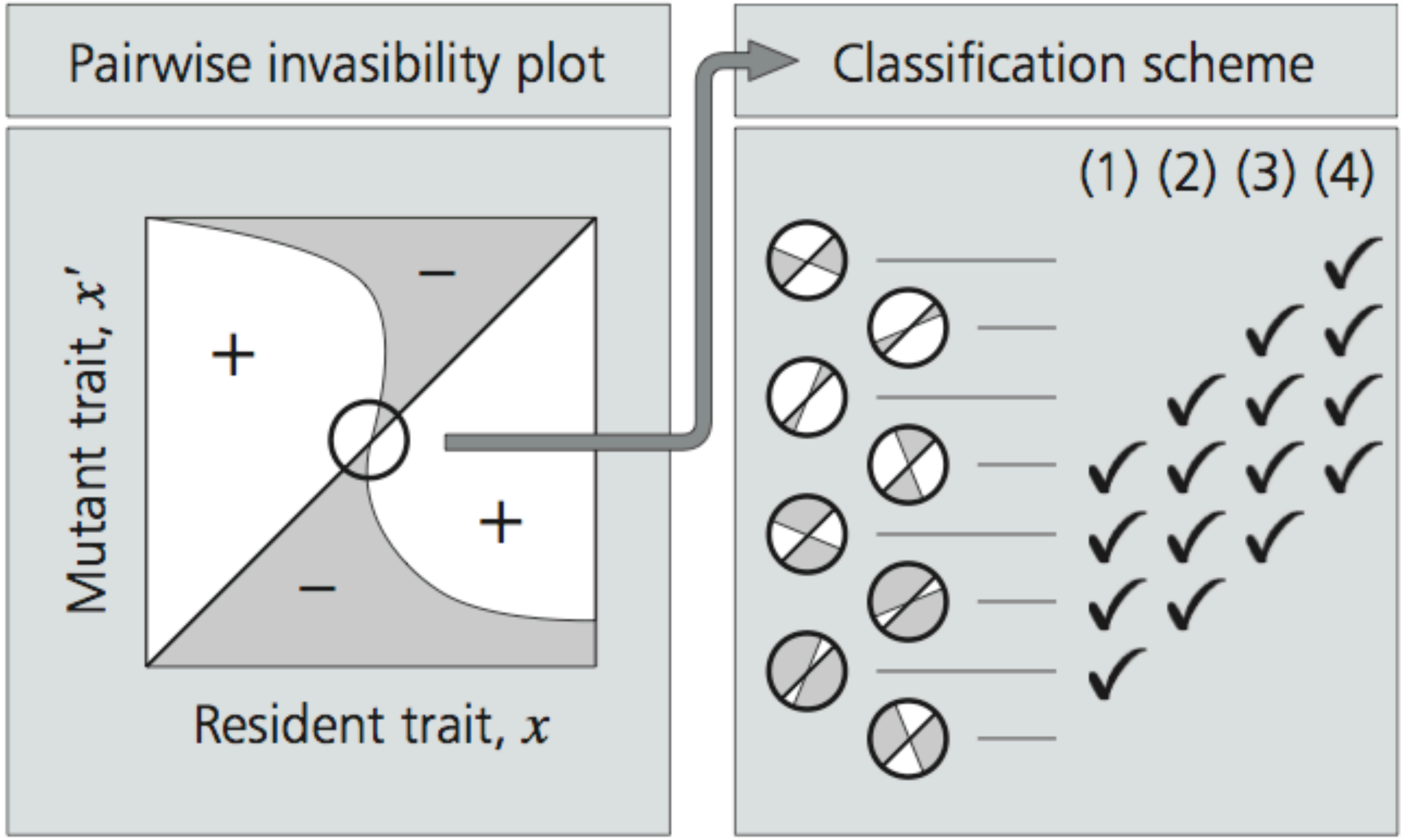


Phenotypic Evolution

Deriving Adaptive Dynamics

$$\frac{dx}{dt} = \frac{1}{2}\mu\sigma^2\bar{N}(x)\frac{\delta f(x',x)}{\delta x'}\bigg|_{x'=x}$$

Evolutionary Game Adaptive Dynamics for Continuous Traits



$$\frac{dx}{dt} = \frac{1}{2}\mu\sigma^2\bar{N}(x)\frac{\delta f(x',x)}{\delta x'}\bigg|_{x'=x}$$

$$\dot{E}[p] = Cov(r,p) + E[\dot{p}] + E[r\Delta_m p]$$

$$E[r\Delta_m p] = 0$$

$$E[\dot{p}] = 0$$

$$\dot{E}[p] = Cov(r,p) = Cov(r(p;g),p)$$

Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$r(p;g) = r(\bar{p};g) + \frac{\partial r(a,g)}{\partial a}|_{a=\bar{p}}(p-\bar{p})$$

$$\dot{E}[p] = Cov[p, r(\bar{p};g) + \frac{\partial r(a,g)}{\partial a}|_{a=\bar{p}}(p-\bar{p})]$$

$$\dot{E}[p] = Var(p)\frac{\partial r(a,g)}{\partial a}|_{a=\bar{p}}$$

Conclusions 1.

- Simple stoichiometry allows us to derive many of the fundamental, equations of evolutionary dynamics
- How genomes change in frequency as a result of frequency-dependence, density dependence and mutation.
- These equations provide insights into how total genomic information is constrained by mutation rates - Eigen law
- Allow us to study game dynamics in an evolutionary and ecological framework
- Through AD & Price Eq. provide the basis for many current studies on cooperation, kin & group selection, microbial dynamics and cultural/language evolution.

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