Long-Memory in an Order-Driven Market

Blake LeBaron and Ryuichi Yamamoto

October 13, 2006

Abstract

This paper introduces an order-driven market with heterogeneous investors, who submit limit or market orders according to their own trading rules. The trading rules are repeatedly updated via simple learning and adaptation of the investors. We analyze markets with and without learning and adaptation. The simulation results show that our model with learning and adaptation successfully replicates long-memories in trading volume, stock return volatility, and signs of market orders. We also discuss why evolutionary dynamics are important in generating these long memory features.

1. Introduction

This paper provides a simulation analysis of an order-driven market to explain the following empirical properties. First, trading volume is persistent over time (Lobato and Velasco (2000)). Second, stock return volatility has long-memory, meaning that the stock price fluctuations have positive autocorrelations (Ding et al. (1993), Mantegna and Stanley (1995, 1997), Engle (1982), Pagan (1996)).[1] Third, there is a positive contemporaneous correlation between volatility and volume (Tauchen and Pitts (1983)). Finally, the signs of market orders follow a long-memory process (Lillo, Mike, and Farmer (2005)); yet the market is efficient (Lillo and Farmer (2004)).

We modify an order-driven market with heterogeneous investors, which was originally constructed by Chiarella and Iori (2002). In the order-driven market of Chiarella and Iori, investors set bids and asks and submit limit or market orders according to exogenously fixed rules. Our key modification is that the trading rules are repeatedly updated via learning and adaptation of the investors. At a given point in time, investors look back at the past performance of their predictions. They realize their past mistakes on these predictions and update their prediction methods to new ones by *imitating* strategies

[1] Long memory is detected in absolute returns of S&P 500 (Bollerslev and Mikkelsen (1996)) and volatility of nominal exchange rates (Baillie, Bollerslev and Mikkelsen (1996)).

of more successful investors. The success of others is measured by their past predictive accuracy. Thus, investors learn from the past and adapt their behavior to improve their future performances in the market. We compare the dynamics that emerge from this economy with and without imitation. Further, we examine how the model can explain the previously mentioned empirical features of the stock market.

Our market, with imitation on trading strategies, successfully replicates the previously mentioned four empirical regularities on the stock market. We first find that volume and volatility have long-memory only with imitation. This implies that imitative behavior is correlated with long-memory processes. To interpret this result, first note that imitation of trading strategies implies that agents submit similar types of orders over some periods. Large (small) numbers of market orders are followed by similar numbers of market orders for the subsequent periods. Since our economy assumes that agents can trade one stock at a time, the numbers of market orders are just trading volume. So, trading volume becomes persistent. Therefore, similarity of trading behavior is important to understand the long-memory in volume. It will emerge in an economy with certain types of evolutionary behavior, such as imitation.

We then demonstrate that persistence in the number of transactions is crucial to understanding long-memory of volatility. In order to investigate this relation, we first show a significant and positive contemporaneous correlation between them. Then, we discuss their long term relation. We show that a positive contemporaneous correlation exists in our order-driven market *only with imitation*. In an economy with imitation, agents are likely to imitate the strategies of others so that their trading behavior becomes similar among agents. For example, as some agents decide to submit limit orders to buy, others also enter the same types of orders. When many agents place unexecuted limit orders, the price does not move much because orders are mostly limit orders going to the order book. Even if some orders are executed, price changes would be small since the limit order book is getting thicker around the best prices. When most agents submit more executing limit orders, the number of transactions increases, the order book thins, and price changes increase. The similarity of trading behavior implies the positive correlation between transaction frequency and volatility.

We then investigate what drives the long-memory on stock return volatility in a market with evolution. We partition the sample into sub-periods, which contain equal numbers of transactions. This set-up eliminates the impact of long-memory in trading volume on persistence in volatility. The autocorrelations of volatility over such sub-periods are examined. We find that the autocorrelations become substantially weaker (but still exist with short lags) with such divisions of the sample, indicating that persistence in the transaction frequency is important for long-memory features in volatility.

Signs of order flows are also persistent in our economies with evolution. We show that the imitative behavior of investors generates persistence in the sequences of buyer or seller initiated trades. This appears as long memory in our signed order flow series. In our market we show that this long range dependence can coexist with a relatively efficient market in that return series are relatively uncorrelated.

Several recent papers have attempted to replicate these features in an environment with zero intelligence agents.[2] In these models random order flow is sent to the market clearing mechanism, and the resulting time series are analyzed. These random behavior models are an important benchmark for understanding which time series features could be driven by institutional features alone. By putting simple distributional assumptions on order placements these models generate sharp and testable predictions about functional forms on spread, price diffusion rate, and order flow dynamics (Farmer, Patelli, and Zovko (2004)). Rationality-based theories rarely make predictions about such functional forms, and as a result, their predictions are harder to

[2] Those studies include Daniels, Farmer, Gillemot, Iori, and Smith (2003), Farmer, Patelli, and Zovko (2005), Farmer, Gillemot, Lillo, Mike, and Sen (2004), Gillemot, Farmer, and Lillo (2005), and Iori, Daniels, Farmer, Gillemot, Krishnamurty and Smith (2003).

test.

We move from the zero intelligence world, to one of ``low intelligence'', or bounded rationality for several reasons. First, we are interested in having some economic motivations for our agents generating buy and sell orders. This could protect us from results being driven by wildly irrational trading behavior. Second, we seek an endogenous explanation for the extreme persistence in order signs, volume, and liquidity. Finally, our agents are motivated by economic objectives to at least work hard to ensure an efficient market. We are interested in seeing how efficient these markets can be in terms of prices, while generating the other persistent time series results. We will do this in a single market with simple, but adaptively rational agents.

The rest of the paper proceeds as follows. Section 2 summarizes price determination mechanism of an actual order-driven market. Section 3 presents the market structure. Section 4 describes the statistical methodology to detect long-memory on a time series. Section 5 gives simulation analyses and the results. The last section concludes.

2 A Brief Sketch on the Pricing Mechanism in an Order-Driven Market

Before introducing our artificial stock market, this section briefly summarizes the mechanism on price determination in an order-driven market without market makers. So, the following explanation would be similar to that in many markets with automated clearing mechanisms.

We analyze an order-driven market where price is determined according to the double continuous auction. In the double continuous auction, which is a standard price formation mechanism in financial markets, agents place two kinds of orders: limit orders, and market orders. Limit orders are requests to sell/buy at a specific price or better. They represent the lowest/highest price at which the investor is willing to sell/buy. Market orders are requests to buy or sell a given number of shares immediately at the best available price on the market. Market orders are executed more easily than limit orders. Limit orders, which are not executed immediately, are stored in the limit order book. We call buy/sell limit orders as bids/asks (or offers). The difference between the lowest ask, and the highest bid is called the spread. Investors place orders at any price on a pre-specified grid called the tick size, which is the increment by which prices can move. Market orders are matched with limit orders of the opposite side on the order book, and executed using time priority at a given price and price priority across prices. A transaction removes an ask or bid order when they are matched with incoming limit orders or market orders. Limit orders are also removed after they are canceled, or an expiration date has passed.

Here is an example of how the stock price is determined.[3] Let's consider the following tables which illustrate the order book. In Table 1, the best bid is currently 300 shares at $178, and the best ask is 1000 shares at $179. A new order is matched with the orders already placed on the book. Suppose that a market buy order for 100 shares is entered on the book. This is matched with the sell order with the highest priority, which is for 1000 shares at $179. So the 100 shares are bought at $179, which leave 900 shares at $179 in the ask side of the book.

[3] Tokyo Stock Exchange (2004) explains the details of an order-driven market.

Table1: An Example of Limit Order Book (1).

Bid	Price	Ask
100	Market Order	
	180	400
	179	1000
300	##178	
400	177	
2000	176	

Next, suppose that a buy order of 1000 shares at a limit price of $180 is placed as in Table 2. This is matched with the sell limit order of 900 shares at a limit price of $179. The remaining 100 shares are then matched with 400 ask at $180. The book then looks like Table 3, and the price is now $180. These transactions are carried out continuously.

Table 2: An Example of Limit Order Book (2).

Bid	Price	Ask
	Market Order	
1000	180	400
	##179	900
300	178	
400	177	
2000	176	

Table 3: An Example of Limit Order Book (3).

Bid	Price	Ask
	Market Order	
	##180	300
	179	
300	178	
400	177	
2000	176	

3. Market Structure

This section describes an artificial stock market based on the market outlined in Chiarella and Iori (2002). Chiarella and Iori (2002) introduce a continuous double auction market where agents submit orders on the market sequentially and at random times. Orders are matched and executed according to the time and price priorities. A single asset is traded in the market, and its fundamental value is assumed to be constant. Agents are assumed to know the fundamental value p^f and the past history of the prices. The price p_t is determined at the level where any transaction occurs. When there is no transaction, a proxy of the price is given by the average of the lowest ask (the quoted ask, a_t^q) and the highest bid (the quoted bid, b_t^q). We use the number of events as a measure of time.

Investors decide to enter the market with a probability, λ. They observe all the past price series and determine their strategies based on their expectations of future prices. They make their predictions based on the following equation,

$$\hat{r}^i_{t,t+\tau} = g^i_1\left(\frac{p^f - p_t}{p_t}\right) + g^i_2\bar{r}_{L_i} + n_i\varepsilon_t \tag{1}$$

where g^i_1, g^i_2, and n_i are fundamentalist, chartist, and noise-induced components, respectively, and initially these are randomly assigned according to the following distributions,

$$g^i_1 \sim |N(0,\sigma_1)|, \qquad g^i_2 \sim N(0,\sigma_2), \quad n_i \sim |N(0,n_0)| \qquad \varepsilon \sim N(0,1).$$

The sign of g^i_2 represents a trend chasing if g^i_2>0, and a contrarian strategy if g^i_2<0.

The value, $\bar{r}_{L_i}$, is given as:

$$\bar{r}_{L_i} = \frac{1}{L_i}\sum_{j=1}^{L_i}\frac{p_{t-j} - p_{t-j-1}}{p_{t-j-1}}. \tag{2}$$

It is the average return over the interval, L_i, which is randomly drawn from a uniform and independent distribution across agents over the interval $(1, L_{\max})$.

Agents expect future prices at time $t+\tau$ according to those of the future returns as:

$$\hat{p}^i_{t+\tau} = p_t e^{\hat{r}^i_{t,t+\tau}}. \tag{3}$$

This paper assumes that all agents simultaneously make future price predictions at a particular point in time, but trade sequentially.

An agent buys (sells) one unit of stock at a price b_t^i (a_t^i) when she expects an increase (decrease) in the price. b_t^i and a_t^i are given as follows,

(4)
$$\begin{aligned} b_t^i &= \widehat{p}_{t,t+\tau}^i (1-k^i) \\ a_t^i &= \widehat{p}_{t,t+\tau}^i (1+k^i) \end{aligned},$$

where k^i is randomly and uniformly assigned across agents in the interval $(0, k_{\max})$, with $k_{\max} \leq 1$. k^i is fixed over time, but varies over agents.

Agents are assumed to submit a limit buy (sell) order at b_t^i (a_t^i) for one unit, if b_t^i (a_t^i) is smaller (larger) than the current quoted ask, a_t^q (bid, b_t^q). If the bid(ask) price is higher(lower) than they best ask(bid) then the order executes immediately at current best quoted price on the book. The trade is executed for one share, and the order is cleared from the book. This immediately executed limit order has some connections to market orders in real markets in that the agent sees the current price, and then decides to execute. Since all transactions are for one share only, there is no possibility that the buyer or seller will walk up the book. Every limit order has a lifetime τ, so that after that period, the limit orders are removed from the book.

Events in this artificial stock market proceed as follows. First, all agents observe the past history of prices and make predictions on future prices at time t. Based on their own predictions, they decide to enter or not to enter the market. These decisions are made sequentially. When they enter the market, they submit orders and the price is recorded. After all agents make some decisions on entering the market, they revise their expectations on future price based on the history of the prices, and another trading round starts again.

Agents' trading strategies evolve over time. After five trading rounds have been completed, all agents look back to their records on their past forecasts, and update their forecast methods (in particular, their parameters in equation (1)) to improve their forecasting ability. They are more likely to select forecast methods, which performed well in the past, to make more accurate predictions in the future. Better strategies are likely to be imitated by agents. Such learning and adaptation are described with the following real-valued genetic algorithm. Once agents update their parameters, the next trading round starts. This process, which has five trading rounds for each GA run, is repeated 50 times. A simulation consists of 250 trading rounds in total.

The real-valued GA:

Step 1) Selection

After trading five rounds, all agents calculate their own fitness, which is given by

(5) $$fitness_i = \sum_{5\ rounds} \left(p_t - E^i(p_t)\right)^2$$

Equation (5) is the sum of squared prediction errors for agent *i* for five rounds.

The probabilities of being copied into the next round is given by,

(6) $$P_i = \frac{fitness_i}{\sum_{j=1}^{1000} fitness_j}$$

We select 1000 parameters using these probabilities. The parameters that produced better forecasts in the past are more likely to be used by other agents. The bad strategies are less likely to be copied for the next generations. The selected parameters are then subject to the following mutation operator.

Step 2) Mutation[4]

Step a) Select randomly a parameter set among g_1^i, g_2^i, L_i ,and n_i. Mutation

[4] This mutation, which replaces the old parameter with the new one with a small probability, is based on that in Arifovic and Mason (1999).

(Experimentation) takes place with p_m (=0.08).

Step b) A random number ~ U[0,1]. (A random number is drawn from a uniform distribution in the interval [0,1].)

Step c) If it is less than p_m, a new parameter value is drawn from an associated distribution, i.e., $g_1^i \sim |N(0,\sigma_1)|$, $g_2^i \sim N(0,\sigma_2)$, $n_i \sim |N(0,n_0)|$, or $L_i \sim U(1,L_{\max})$ and that number replaces the old one.

Step d) Otherwise, the investor *i* keeps the old value.

This paper analyzes the long-memory in two types of the economies: economies with and without evolution (note that the market structure is the same in both economies). The long-memory is examined using simple plots and a statistical test, which is described in the next section.

4. A Statistical Method to Detect Long-Memory

A random process has long-memory when its autocorrelation function has an asymptotically decaying form with a power law form,

(7) $\rho(k) \sim ck^{2d-1} \qquad as\ k \to \infty$

where d<0.5 and c is a non-zero constant term. A long-memory process implies that the future value is significantly predictable from the past value. Several papers show that many financial time series are characterized by long memory processes. For example, long memory is detected in absolute returns of the S&P 500 (Bollerslev and Mikkelsen (1996)) and volatility of nominal exchange rates (Baillie, Bollerslev and Mikkelsen (1996)).

The "rescaled range" or "range over standard deviation" (R/S) statistic, which is originally developed in Hurst (1951) can be used to detect long-memory.[5,6] In this paper, we estimate Lo's modified rescaled range (R/S) statistic (Lo (1991)) and test its significance. The Lo's modified R/S statistic is given as follows.

Consider a time series X_1, X_2,……, X_n, and let $\overline{X}_n$ be the sample mean, $(1/n)\sum_{j=1}^{n} X_j$. Let $\hat{\sigma}_x^2$ and $\hat{\gamma}_x$ be the sample variance and autocovariance estimators of X.

---

5 Hurst (1951) found persistence in autocorrelations in hydrological data.

6 The Hurst exponent is often calculated to describe the memory. It is confined within an interval (0,1), and interpreted as follows. A stochastic process is a random walk, which is a statistically independent and uncorrelated process, if the Hurst exponent (H) is 0.5. 0.5 of H implies that we cannot predict future values using past values of the series. If H is greater than 0.5 but less than or equal to 1, the series implies the persistence. If H is less than 0.5 but greater than or equal to 0, the series is described with anti-persistence so that it has a trend reverting tendency. The interpretation of the Hurst exponent is well described in Hampton (1996).

Then the Lo's modified R/S statistic, denoted by Q_n is defined as:

(8)
$$Q_n \equiv \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\le k\le n}\sum_{j=1}^{k}\left(X_j - \overline{X}_n\right) - \min_{1\le k\le n}\sum_{j=1}^{k}\left(X_j - \overline{X}_n\right)\right]$$

where

(9)
$$\begin{aligned}\hat{\sigma}_n^2(q) &\equiv \frac{1}{n}\sum_{j=1}^{n}\left(X_j - \overline{X}_n\right)^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left\{\sum_{i=j+1}^{n}\left(X_i - \overline{X}_n\right)\left(X_{i-j} - \overline{X}_n\right)\right\} \\ &= \hat{\sigma}_x^2 + 2\sum_{j=1}^{q}\omega_j(q)\hat{\gamma}_j, \qquad \omega_j(q) \equiv 1 - \frac{j}{q+1}, \qquad q < n\end{aligned}.$$

The first term in brackets in equation (8) is the maximum over *k* of the partial sum of the first *k* deviations of the X_j from the sample mean. The *k*th partial sum is defined as $\sum_{j=1}^{k}\left(X_j\right)$, *k = 1, 2,….., n*. The maximum must be positive since the sum of all *n* deviations of the X_j's from their mean is zero. The second term in equation (8) is the minimum over *k* of the partial sums, so it is always non-positive. So, the R/S statistic is described with the range of partial sums of deviations of a time series from its mean, rescaled by its standard deviation. The statistic is designed to compare the maximum and minimum values of running sums of deviations from the sample mean, re-normalized by the sample standard deviation. The deviations are greater in a presence of a long-memory process than with no long-memory case.

The Lo R/S statistic, Q_n, differs from the classical R/S statistic of Hurst in terms of the denominator, $\hat{\sigma}_n(q)$. $\hat{\sigma}_n(q)$ in the classical statistic is the sample standard deviation, $\left[\frac{1}{n}\sum_{j=1}^{n}\left(X_j-\overline{X}_n\right)^2\right]^{1/2}$. The Lo R/S statistic includes the weighted autocovariance up to lag q to capture the effects of short-range dependence. A time series is characterized to have short-range dependence if the maximal dependence between events at any two dates becomes trivially small as the time span between those two dates increases.[7]

Lo suggests a data-dependent formula for calculating the value q:

(10) $$q=[k_n] \qquad k_n \equiv \left(\frac{3n}{2}\right)^{1/3}\left(\frac{2\hat{\rho}}{1-\hat{\rho}}\right)^{2/3}$$

where $[k_n]$ denotes the greatest integer less than or equal to k_n, and $\hat{\rho}$ is the estimated first-order autocorrelation coefficient of the data. If the process has finite fourth moments and a short-range dependence, a random variable, $V_n \equiv \frac{Q_n}{\sqrt{n}}$ asymptotically follows:

(11) $$F_V(\upsilon)=1+2\sum_{k=1}^{\infty}\left(1-4k^2\upsilon^2\right)e^{-2(k\upsilon)^2} .$$

The critical value under the null hypothesis that the time series is short-range dependent is found with this formula. We perform one tailed tests of the null hypothesis against the alternative that the time series has long memory. From Table II in Lo (1991), if the statistic V_n is above 1.747, we can reject the null hypothesis with 95% confidence.[8]

---

[7] It implies that the long-range dependence produces autocorrelations, which decay very slowly. Both anti-persistent and persistent series are characterized as long-range dependence since both have slowly-decaying autocorrelations although those of anti-persistent series collapse to zero.

[8] We can conduct two-tailed test if our alternative hypothesis is long-range dependence. The

The Lo modified R/S statistic is known to have some advantages over the classical R/S statistic. First, Lo provides the sampling theory so that the modified R/S statistic can be tested statistically with its limiting distribution. Second, The Lo modified R/S statistic distinguishes between short-range and long-range dependence in a series. Although recent stock return time series have short-range dependence, the classical R/S statistic does not consider such characteristics, and it often produces some biased results in the presence of short-range dependence. Lo showed that the power of the classical R/S test is weak so that it often does not reject the null hypothesis of short-memory. As Lo suggests, any empirical investigation of long-memory in financial time series must account for the presence of higher frequency autocorrelations.

5. Simulation Analyses on Long-Memory

This section emphasizes that an economy with evolution can produce the following properties: (1) trading volume is persistent, (2) volatility of stock returns has long-memory, (3) there is a contemporaneous positive correlation between volatility and volume, and (4) signed order flow has long-memory, but the market is efficient. We will

lower tail rejection indicates that the autocorrelations collapses to zero although it decays very slowly.

present evidence that suggests the imitation of strategies is key to understanding these empirical properties in an order-driven market.

We run simulations on two types of economies, i.e., economies with and without imitation, with parameter values in Table 4.

Table 4: Parameters

Std of fundamentalists component: σ_1	1
Std of chartist component: σ_2	1.5
Std of noise-trader component: σ_n	0.5
Order life: τ	200
Probability to enter the market: λ	0.5
$k_{\max}$	0.5
Fundamental value: p_f	1000
Maximum time horizon in chartist component: $L_{\max}$	100
Tick size: Δ	0.1

The number of agents is 1000. The trading rounds are set to 250.[9] In an economy with imitation, agents invoke the GA every 5 trading rounds.

We simply plot the autocorrelation functions, and conduct the Lo modified R/S

[9] We count 1 trading round when all 1000 agents make some decisions, i.e., entering or not entering the market.

test to examine long-memory processes. We consider different values for numbers of lags in autocovariance in equation (9), q = 4, 6, 8, and 10.

Volume and Volatility:

In our market, agents can trade only one unit of stock per transaction. In order to investigate autocorrelations on trading volume, we need to assign a pseudo-clock time. We count one when agents make at least some decisions. Those decisions include not entering the market, entering and submitting market or limit orders. We consider 50 decisions for one clock time.[10] Volume is calculated for each pseudo-clock time. We measure return volatility simply by taking standard deviations per pseudo-clock time.

The bold numbers in Table 5 and 6 give the averages of the R/S statistic over 10 simulations. The symbol, **, indicates that we reject the null hypothesis of short-range dependence at the 95% confidence level in at least 70% of the simulations over 10 simulations. The symbol, *, indicates the rejection of the null with 50% or greater but less than 70% of the simulations over 10 simulations.

[10] We also investigated 25 and 100 trading decisions for a unit of pseudo-clock time. However the results are quite similar.

Table 5: Lo's modified R/S tests for an Economy with Evolution

Bold numbers are the averages of R/S statistic over 10 simulations. ** indicates that we reject the null hypothesis of short-range dependence at the 95% confidence level in at least 70% of the simulations over 10 simulations. * indicates the rejection of the null with 50% or greater but less than 70% of the simulations over 10 simulations. When there is no signs of ** or * next to the bold number, it indicates that we do not reject the null at all.

	Volume	Volatility	Signs of Market Orders	Returns
q=4	**7.919****	**2.521****	**4.660****	**0.974**
q=6	**6.723****	**2.137****	**4.142****	**1.047**
q=8	**5.946****	**1.889***	**3.773****	**1.091**
q=10	**5.388****	**1.710***	**3.492****	**1.121**

Table 6: Lo's modified R/S tests for an Economy without Evolution

Bold numbers are the averages of R/S statistic over 10 simulations. ** indicates that we reject the null hypothesis of short-range dependence at the 95% confidence level in at least 70% of the simulations over 10 simulations. * indicates the rejection of the null with 50% or greater but less than 70% of the simulations over 10 simulations. When there is no signs of ** or * next to the bold number, it indicates that we do not reject the null at all.

	Volume	Volatility	Signs of Market Orders	Returns
q=4	**0.702**	**0.458**	**2.785****	**0.826**
q=6	**0.595**	**0.389**	**2.456****	**0.885**
q=8	**0.525**	**0.344**	**2.226****	**0.927**
q=10	**0.476**	**0.311**	**2.053****	**0.957**

The second columns in Table 5 and 6 are the results for volume series. The short-range dependence is often rejected for any values of q in an economy with evolution, while it is not rejected in an economy without evolution. Figure 1 is the plot of autocorrelation functions in an economy with evolution, while Figure 2 is that without evolution. The autocorrelations are very high even with long lags in the evolutionary economy, but they are near zero after 50 lags in an economy without evolution. The figures demonstrate that volume is persistent over pseudo-clock time only in an economy with evolution.

Since agents can only trade one stock at a time, volume just reflects the number of transactions, or executed limit orders. Persistence in volume means that the numbers of transactions per pseudo-clock time are auto-correlated over time. Since order execution depends on agents' expectations of future prices,[11] the number of transactions becomes persistent if the expectations are similar among a particular number of agents over some periods. Agents trade sequentially in our market, so similarity of the expectations would generate persistence in order types. Such similarity of expectations is more likely as agents are allowed to imitate each other. An economy with evolution shows higher and longer autocorrelations on volume since it allows imitative agent

---

[11] As discussed in Section 3, agents submit a market order of one unit of stock if b_t^i (a_t^i) is larger (smaller) than the current quoted ask, a_t^q (bid, b_t^q). b_t^i and a_t^i depend on the expectations as in equation (4) as follows.

$$(4)\quad \begin{aligned} b_t^i &= \hat{p}_{t,t+\tau}^i(1-k^i) \\ a_t^i &= \hat{p}_{t,t+\tau}^i(1+k^i) \end{aligned}.$$

behaviors.

Figure 1:
Autocorrelation Functions of Trading Volume with Evolution

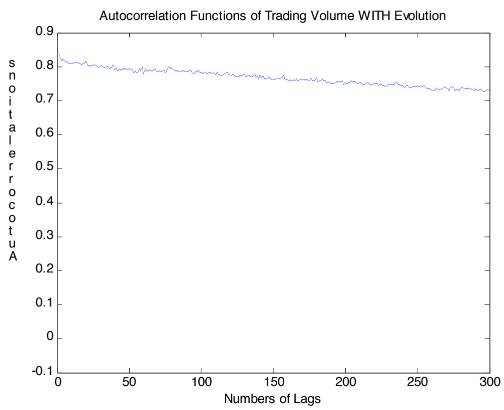


Figure 2:
Autocorrelation Functions of Trading Volume without Evolution

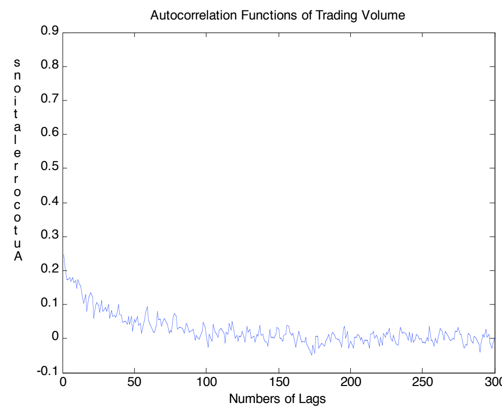


Figure 3 and 4 present the autocorrelation functions for volatility in economies with and without evolution, respectively. The third columns in Table 5 and 6 are the results of the Lo's R/S test for different values of q. The statistical results and the figures show that the long-memory is found only in an economy with evolution.

Figure 3:
Autocorrelation Functions of Volatility with Evolution

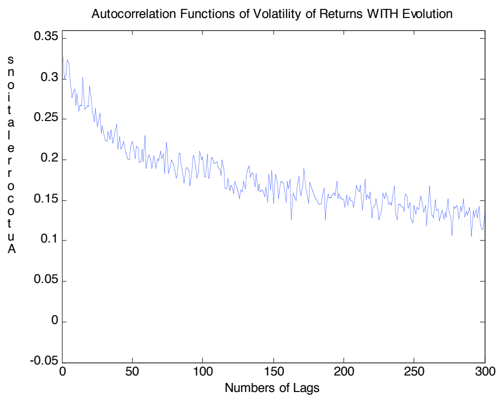
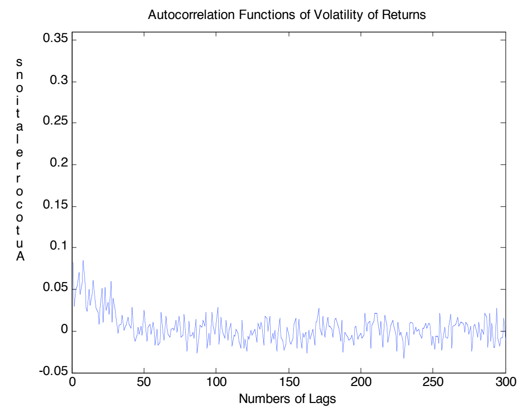


Figure 4:
Autocorrelation Functions of Volatility without Evolution



Now we show that persistence in the number of transactions is important to understand long-memory in volatility. In order to investigate this relation, let us first investigate whether volume is contemporaneously correlated with volatility. Then we will discuss their long term relation.

Previous empirical studies demonstrated that there is a positive contemporaneous correlation between volatility, measured as absolute or squared price changes, and volume.[12] Now we show that such a correlation exists in our order-driven market *only with evolution*. First, we simply calculate the correlation coefficients between the standard deviations of returns and trading volume for pseudo-clock times, which are

---

[12] For example, Tauchen and Pitts (1983) present empirical evidence on a positive correlation between volatility and volume.

given in Table 7. Bold numbers are means of the estimates over 10 runs. Numbers in parenthesis are the standard deviations of the estimates over 10 runs. We found a significant contemporaneous correlation between volatility and volume, about 42%, in an evolutionary economy. However, there is no statistical evidence on the contemporaneous correlation in an economy without evolution since the estimate is 0.012 but its standard error is 0.03. Since agents can only trade one stock at a time, volume is given by the number of transactions. Our result in an economy with evolution indicates that the number of transactions, or trading volume, has a positive correlation with volatility. This is consistent with the findings such as Easley and O'Hara (1992).

Table 7: Contemporaneous correlations (Volatility vs. Volume)

Bold numbers are means of the estimates over 10 runs. Numbers in parenthesis are the standard deviations of the estimates over 10 runs.

With Evolution	Without Evolution
0.418	**0.012**
(0.06)	(0.03)

Our observation of high positive correlation between volume and volatility in an economy with evolution could occur because agents imitate trading strategies. They decide their trading strategies based on their expectations of future prices. Their expectations depend on the past prices and parameters as in equation (1). In an economy with evolution, agents are allowed to change their parameter values in their expectation structure, and imitate the weights of others. As a result, they are likely to submit similar orders in terms of bid or ask prices. During periods in which many orders are executed, the book becomes sparser around the best prices. As the book becomes sparser, the change in the executing price will be larger. During periods with many unexecuted limit orders coming in, the book becomes thicker as orders stay on the book until they are removed. Price changes in these periods are small because of the market depth around the best prices. Therefore, similarity of the trading behavior of agents explains the positive correlation between trading volume and volatility. An economy without evolution cannot produce this result, since such an economy does not generate the homogeneous sequence of orders necessary for this result.

We now demonstrate that long-memory on the number of transactions is also connected to long memory in volatility. The long-term relation between trading volume and volatility can be analyzed by removing the effect of volume autocorrelation on that of volatility. This can be done by splitting the sample into sub-periods which include equal numbers of executed orders per sub-period. Autocorrelation functions for the standard deviations of the returns are calculated over the sub-periods. We analyze the relation between volume and volatility only in the economy with evolution since in Table 7 we showed that evolution was necessary for the connection between volume and volatility. Figure 5 shows much weaker autocorrelations than in Figure 1. Lo's modified R/S test results are presented in Table 8 which show no evidence for long memory in volatility after adjusting to a trading volume/transaction time scale. In this model speed of transaction activity appears to drive long memory in both volume and volatility.

Figure 5:
Autocorrelation Functions of Volatility for a Fixed Number of Transactions with Evolution

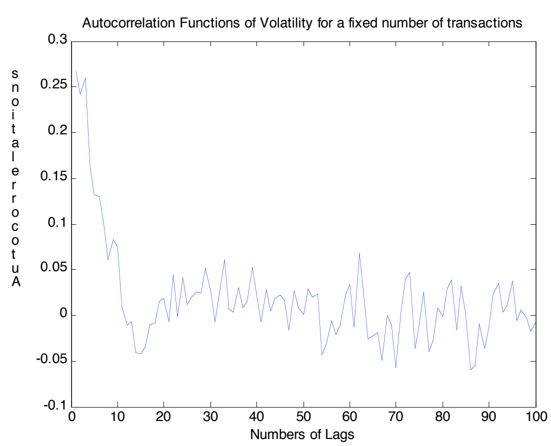


Table 8: Lo's modified R/S tests for Volatility eliminating Volume effect

Bold numbers are the averages of R/S statistic over 10 simulations. ** indicates that we reject the null hypothesis of short-range dependence at the 95% confidence level in at least 70% of the simulations over 10 simulations. * indicates the rejection of the null with 50% or greater but less than 70% of the simulations over 10 simulations. When there is no signs of ** or * next to the bold number, it indicates that we do not reject the null at all.

	Volatility without Volume effect
q=4	**0.589**
q=6	**0.500**
q=8	**0.442**
q=10	**0.400**

Figure 5 does show evidence for a weaker short range persistence in the volatility process which the volume time scale has not removed. It is therefore possible that there is a second piece to volatility dynamics that is driven by something other than volume, but does not have long memory persistence.

Market Orders and Efficient Market:

We also find long-memory on signed order flow as demonstrated in Lillo, Mike, and Farmer (2005). We again divide the sample per pseudo-clock time, and denote the period as +1 (-1) when the majority of orders are buyer (seller) initiated in a period. We first plot the autocorrelation functions for signed orders in Figure 6 and 7. Figure 6 and 7 are the autocorrelations with and without evolution, respectively, which show some memory of market orders in both economies. However, only the economy with evolution displays positive autocorrelations that do not decay to zero at long lags.

Lo's R/S statistics are given with different values of q in the fourth columns in Table 5 and 6. Long-memory is detected both in economies, but its effect is stronger in an economy with evolution than without evolution. The test statistics are around 3-4 for different values of q in an evolution economy, but only around 2 in an economy without evolution.

Figure 6:
Autocorrelation Functions of Signs of Market Orders with Evolution

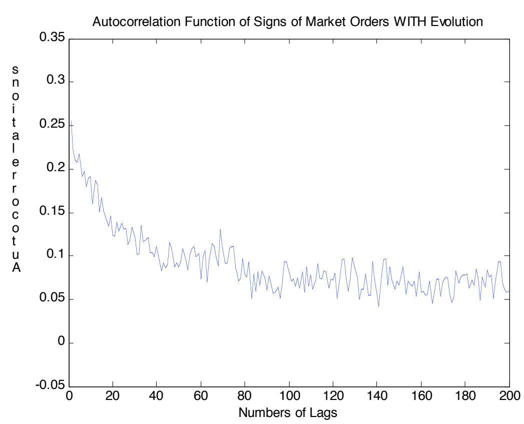
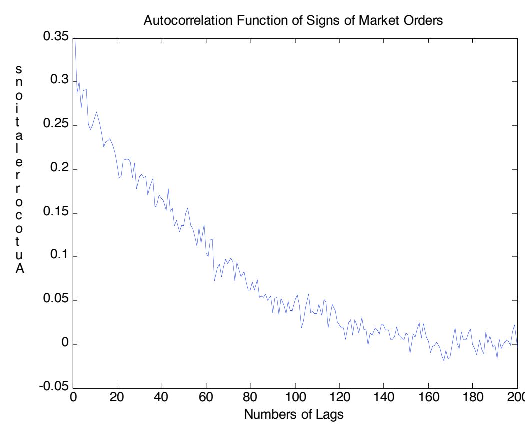


Figure 7:
Autocorrelation Functions of Signs of Market Orders without Evolution



What causes this difference in results across the economies? We propose that imitative behavior of agents is important in generating long memory on order flows.

When imitative behavior exists in the market, groups of agents will expect similar price behavior in the future. Such imitative behavior produces a sequence of buy or sell orders. Our economy with evolution can generate such imitative behavior. Fundamentalists and chartists exist in both economies, and their behavior is different with the different values of the parameters, g_1^i, g_2^i, and L_i in equation (1) and (2).[13] The values of these parameters are initially assigned different random values that are fixed over time in an economy without evolution. However, the values will possibly be similar when agents imitate each other, causing persistent order flow.

The signs of market orders are persistent, but the market is actually efficient. In order to demonstrate this result, we plot the autocorrelation functions of returns with a partition of 50 pseudo-clock time in Figure 8 and 9 and show the R/S test results in the fifth columns in Table 5 and 6.

---

[13] For convenience, we repeat equations (1) and (2) here:

(1) $\hat{r}_{t,t+\tau}^i = g_1^i\left(\frac{p^f - p_t}{p_t}\right) + g_2^i \bar{r}_{L_i} + n_i \varepsilon_t$ and (2) $\bar{r}_{L_i} = \frac{1}{L_i}\sum_{j=1}^{L_i}\frac{p_{t-j} - p_{t-j-1}}{p_{t-j-1}}$.

Figure 8:
Autocorrelation Functions
of Returns with Evolution

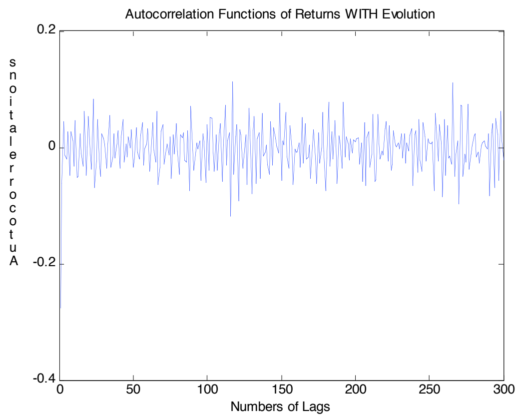


Figure 9:
Autocorrelation Functions
of Returns without Evolution

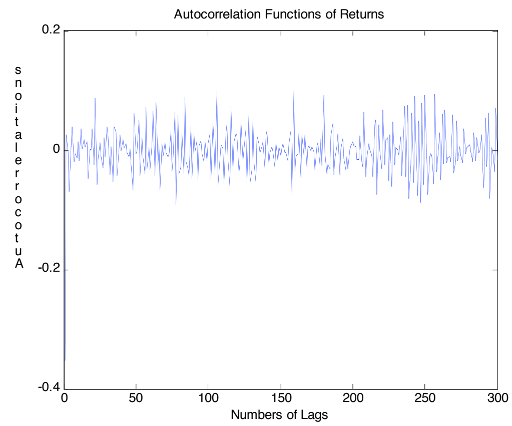


Figure 8 and 9 indicate that the autocorrelations are always around zero, and R/S test results show that we do not always reject the null hypothesis of short-range dependence in both economies. We conclude that the returns have no correlations over time so that the market is close to efficient.

In any financial setting with adaptive agents there will be a tension between imitative behavior and contrarian behavior, which breaks free, and takes advantage of the rest of the herd. Our market must contain some of both of these, maintained in a balance through evolution. Strategies that rely too much on herd chasing will eventually fail, but we know there must be a lot of this going on since it is central to driving many of our results. However, evolutionary pressure appears to be strong enough to keep the group behavior in the market from generating easily detectable linear predictability in our market return series. This is an interesting contrast to our benchmark experiments without evolution. There, return behavior is also random, but randomness in those markets must be driven more through randomness in initial strategies, and not through an adaptive process working to predict future prices. The fact that evolution can maintain a relatively efficient market while generating other time series features is important.

6. Conclusion

This paper analyzes order-driven market with and without evolution, and shows that a market with evolution can replicate well long-memory features on stock return volatility, market orders, and volume. We also find that a contemporaneous positive and significant correlation between volatility and volume in an economy with evolution, and persistence in the number of transactions is related to long-memory on volatility. Although the signs of market orders are persistent, we conclude that the market is efficient. We emphasize that imitative behavior of the agents on trading strategies are important to understand the differences in results from the economies with and without evolution.

References

Arifovic, Jasmina and Masson, Paul. “Heterogeneity and Evolution of Expectations in a Model of Currency Crisis.” Technical Report, Simon Fraser University, Vancouver, BC, Canada, (1999).

Baillie, R.T., T. Bollerslev, and H.-O. Mikkelsen. “Fractionally integrated generalized autoregressive conditional heteroskedasticity.” *Journal of Econometrics* 74 (1996): 3-30.

Bollerslev, T. and H.-O. Mikkelsen. “Modeling and pricing long-memory in stock market volatility.” *Journal of Econometrics* 73 (1996): 151-184.

Chiarella, C. and Iori, G. “A simple microstructure model of double auction markets.” *Quantitative Finance* Vol. 2, no. 5, (2002): 346-353.

Daniels, M. G., J. D. Farmer, L. Gillemot, G. Iori, and E. Smith. "Quantitative Model of Price Diffusion and Market Friction Based on Trading as a Mechanistic Random Process." *Phys. Rev. Lett.* 90(10) (2003): 108102.

Ding, Zhuanxin, C.W.J. Granger, and R.F. Engle. “A long memory property of stock market returns and a new model.” *Journal of Empirical Finance* 1 (1993): 83-106.

Easley, D. and O'Hara M. “Time and the process of security price adjustment.” *The Journal of Finance*, 47(2) (1992):577-605.

Engle, Robert F. “Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.” *Econometrica* 50 (1982): 987-1008.

Farmer, J. D., P. Patelli, and I. I. Zovko. "The Predicitive Power of Zero Intelligence in Financial Markets" *PNAS USA* 102(11) (2005): 2254-2259.

Farmer, J. D., L. Gillemot, F. Lillo, S. Mike, and A. Sen. "What Really Causes Large Price Changes?" *Quant. Fin.* 4(4) (2004): 383-397.

Gillemot, L., J. D. Farmer, and F. Lillo. "There's More to Volatility than Volume." Santa Fe Institute Working Paper 05-12-041.

Hampton, J. "Rescaled Range Analysis: Approaches for the Financial Practitioner, Part 3," *NeuroVe$t Journal*, Vol. 4, No. 4 (1996): 27-30.

Hurst, H. "Long Term Storage Capacity of Reservoirs." *Transactions of the American Society of Civil Engineers* 116 (1951): 770-799.

Iori, G., M. G. Daniels, J. D. Farmer, L. Gillemot, S. Krishnamurty and E. Smith. "An Analysis of Price Impact Function in Order-Driven Markets." *Physica A* 324(1-2) (2003): 145-151.

Lillo, F., and J. D. Farmer. "The Long Memory of the Efficient Market." *Studies in Nonlinear Dynamics & Econometrics* 8 (3) article 1 (2004).

Lillo, F., S. Mike, and J. D. Farmer. "Theory for Long Memory in Supply and Demand." *Phys. Rev. E* 7106 (2005): 287-297.

Lo, A.W. "Long-term memory in stock market prices." *Econometrica* 59 (1991): 1279-1313.

Lobato, I. and Velasco, C. "Long memory in stock-market trading volume", *Journal of Business and Economic Statistics* 18 (2000): 410–426.

Mantegna, Rosario N., and Stanley, Eugene H. "Scaling Behavior in the Dynamics of an Economic Index." *Nature* 376 (1995): 46-49.

Mantegna, Rosario N., and Stanley, Eugene H. "Stock Market Dynamics and Turbulence: Parallel Analysis of Fluctuation Phenomena." *Physica A* 239 (1997): 255-266.

Pagan, Adrian. "The Econometrics of Financial Markets." *Journal of Empirical Finance*

3 (1996): 15-102.

Tauchen, G. E. and Pitts, M. "The price variability-volume relationship on speculative markets." *Econometrica*, 51(2) (1983): 485-505.

Tokyo Stock Exchange *"Guide to TSE trading methodology"*, 3rd edition, (2004).