Gateway to Emergence:
The Paradigms of Nonlinear Science

Presentation at the ICAM/SFI Workshop on
Gateways to Emergence in Science and Society

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Goal

To provide an (almost) equation-free introduction to the critical role of emergent behavior in nonlinear systems. The key will be the use of images and metaphors, but behind all of these are solid physical principles and mathematical equations.

Distinction between Linear and Non-Linear

Mathematically

Linear *versus* Non-Linear

- Can add two separate solutions to form new solution: superposition principle => systematic methods for solving linear problems, independent of apparent complexity: break into small "simple" pieces and add solutions together

- Can *not* add together solutions to form new solution: failure of superposition => must solve nonlinear system *in toto*

- $(a+b)^2 = a^2+2ab+b^2$
  $\neq a^2+b^2$

Distinction between Linear and Non-Linear

Physically

Linear *versus* **Non-Linear**

Linear	Non-Linear
• Smooth, regular motion in space/time and as function of parameters	• Transitions from smooth motion to erratic "chaotic" motion, "random" behavior
• Response in proportion to stimulation	• Self-sustaining oscillations, response can *differ* from stimulation
• Initial pulses/lumps (typically) spread and decay (dispersion)	• Highly coherent, stable localized spatial structures

The Fundamental *Paradigms* of Non-Linear Science

1) Deterministic Chaos and Fractals:

   Apparently random behavior in deterministic systems: non-trivial structure on all scales

2) Solutions and Coherent Structures:

   Persistent, localized spatial structures

3) Patterns: Formation, Selection, Competition

   Formation of complex spatial patterns and dynamical competition to select among them

Role of analytics, computations, and experiments in providing common language, precise and "interdisciplinary"

Emergence in the Paradigms of Non-Linear Science

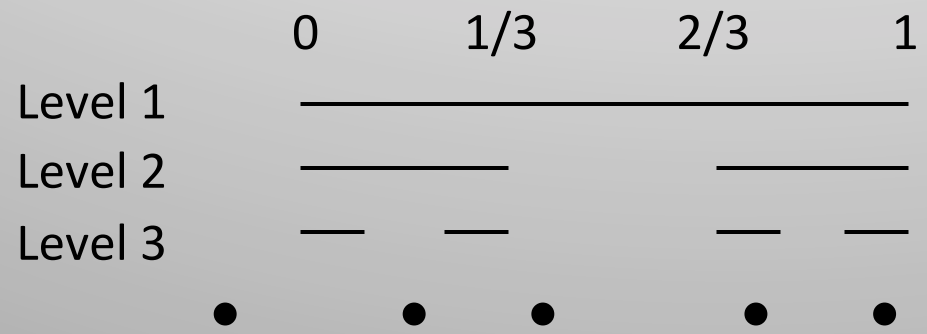
Chaos and Fractals

Chaos: Seemingly random behavior in deterministic nonlinear dynamical system, such as a damped and driven simple pendulum. Very counterintuitive at first but follows from “sensitive dependence on initial conditions” (cf Poincaré – 1890(!). Motions that start near each other separate exponentially in time (stretch) and if the motion is confined (cannot escape to infinity) then it must turn back on itself (fold). This “stretching and folding” creates fractal structures that are like *mille feuille* pastry with many separate layers. These are fractals that emerge as “fossils” of the chaotic motion.

Fractals

“Fossils of Chaotic Motion”

- Fractals: geometrical objects with “non-trivial” structure *on all scales*: typically “self-similar” (i.e., same non-trivial structure on all scales) and with “fractional” dimension
- Example: Cantor Set

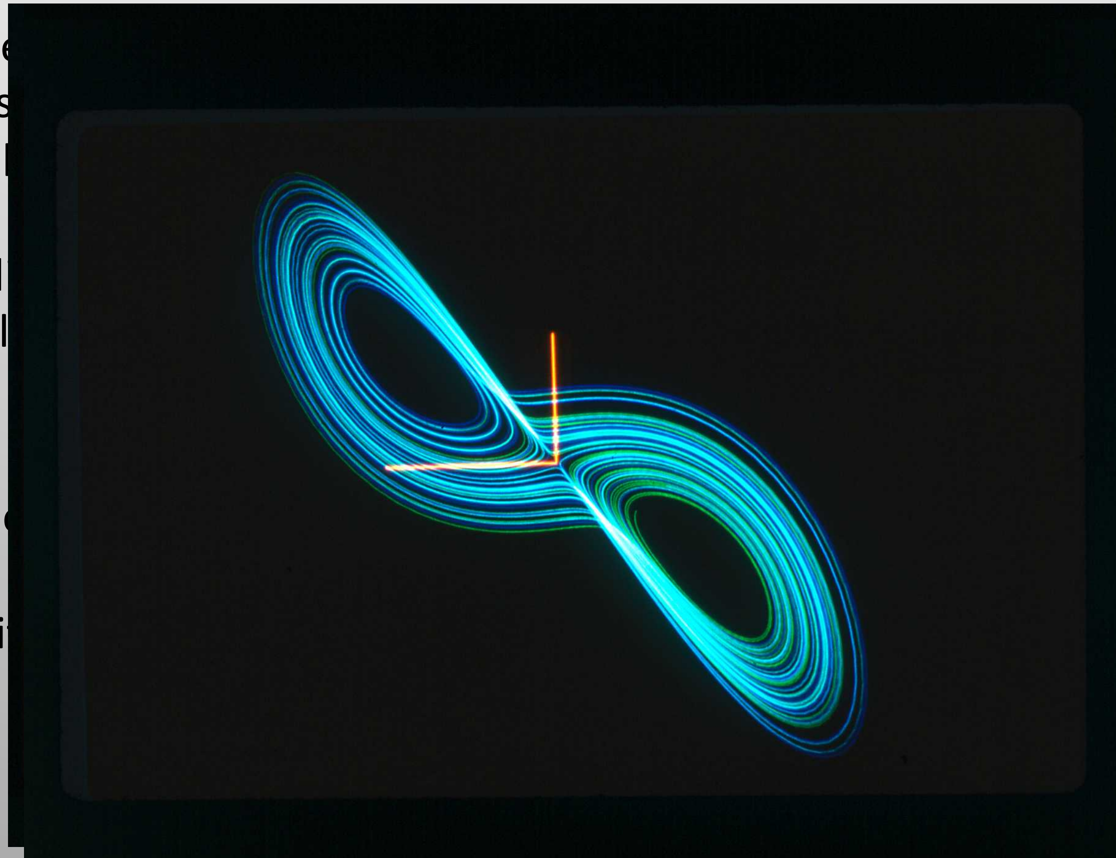


Resulting Cantor set formed as n-> ∞ has “fractal” dimension “between” a point and a line):

$$D_F = \ln 2/\ln 3 \sim 0.63\ldots$$

Fractals in Mathematical Models

- Edward Lore
“determinis … ion in
the atmosp … ucture
that can be … ension
~ 2.06. As d
exponential … *feuille*
pastry: thin … a
bagel.
- Lorenz coin … f the
weather to
“Predictibili … rnado
in Texas?



Three views of the Lorenz attractor: suggestive of a butterfly.

Fractals in Mathematical Models

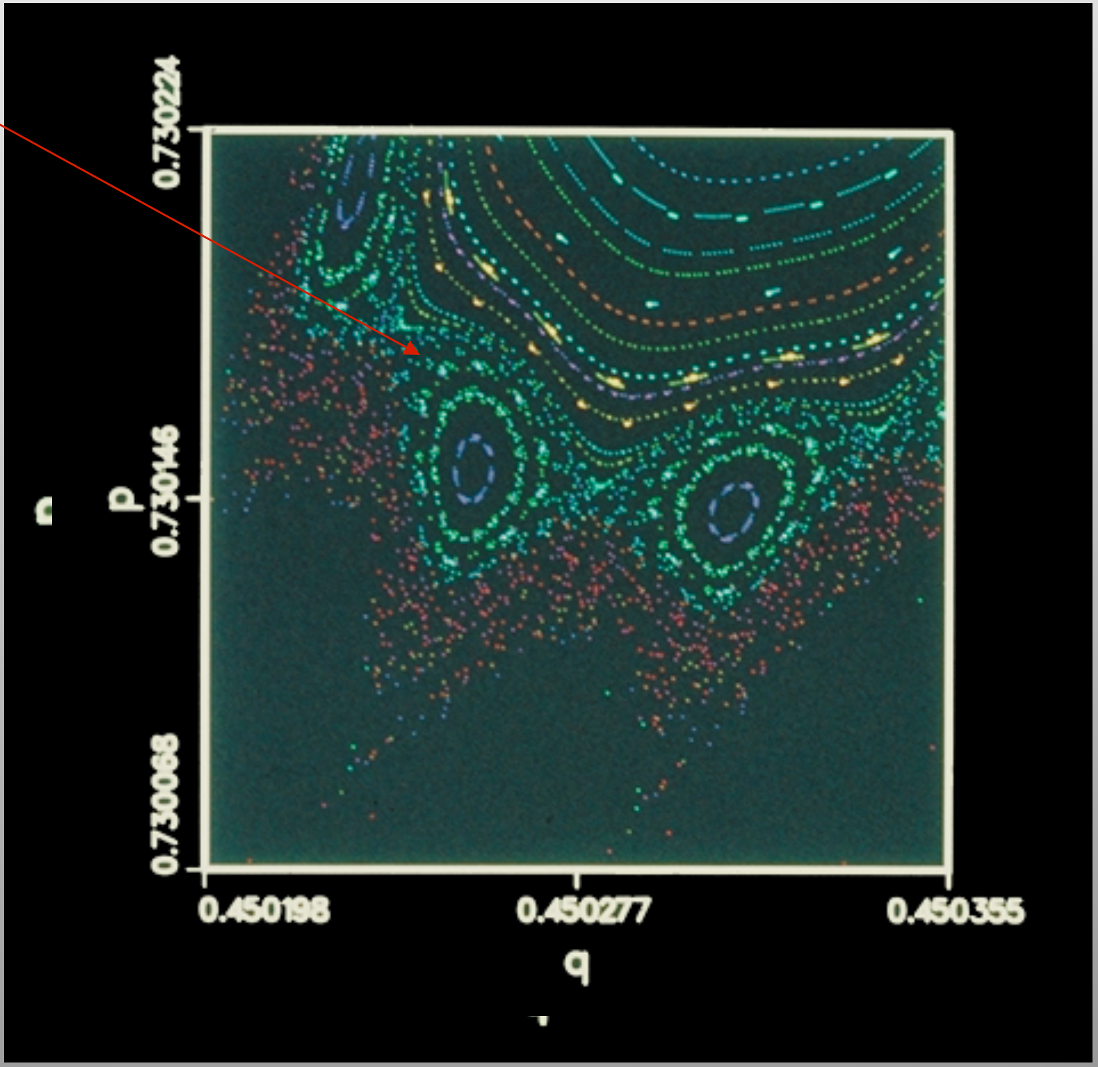
- The "standard map" is a mathematical model for particle trajectories in a large, accelerator ring, like the Large Hadron Collider at CERN. It is a two-dimensional map that, in essence looks at a cross section of the ring and plots the sequence of points that arise from the intersection of the accelerator beam with that cross section as the beam circulates in the ring.

- First studied by Boris Chirikov in USSR in 1960's. Contains a "nonlinearity" parameter typically called "k." For k=0, map is linear. As k is increased, nonlinearity causes chaos to occur in ever increasing portions of the map and gives rise to a (fat) fractal that has structures on all scales.

Chaos and Fractals in the Standard Map

- For ***k*** = 1.1

Expand small region

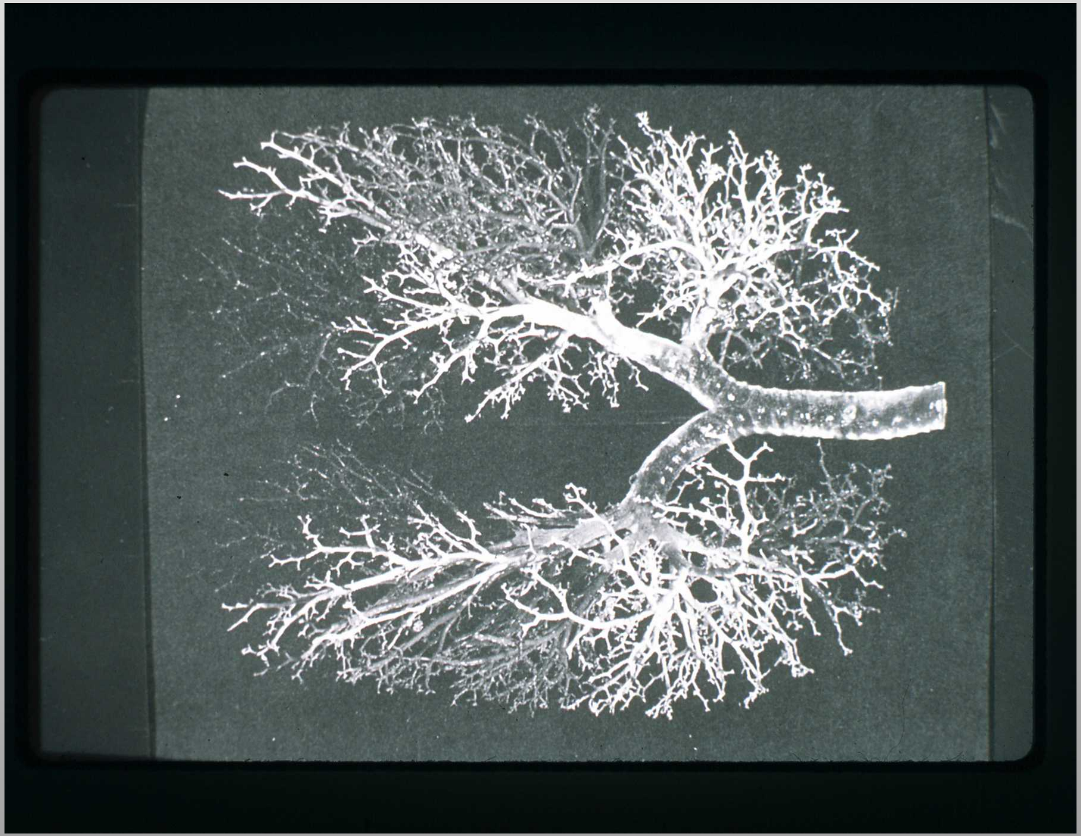


Mixture of regular and irregular motion on all scales, "chaos all the way down"

Implications for solar system (Laskar), orbits of asteroids

Fractals in Nature

- Human lung

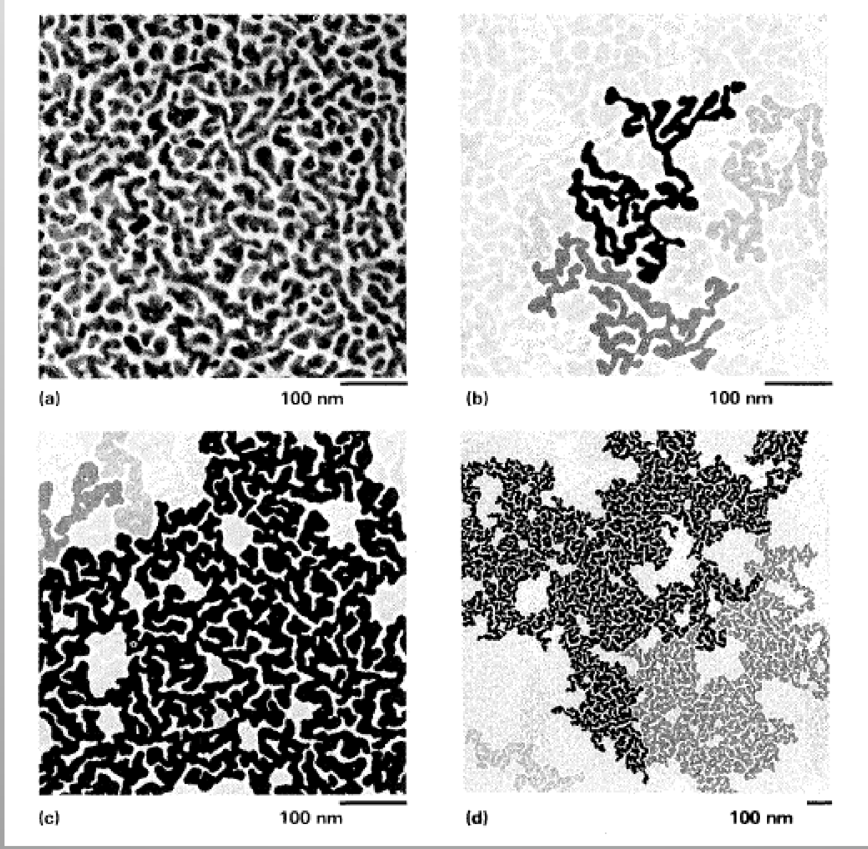


Fractals in Nature

Dielectric breakdown ("artificial lightning") in irradiated plexiglas

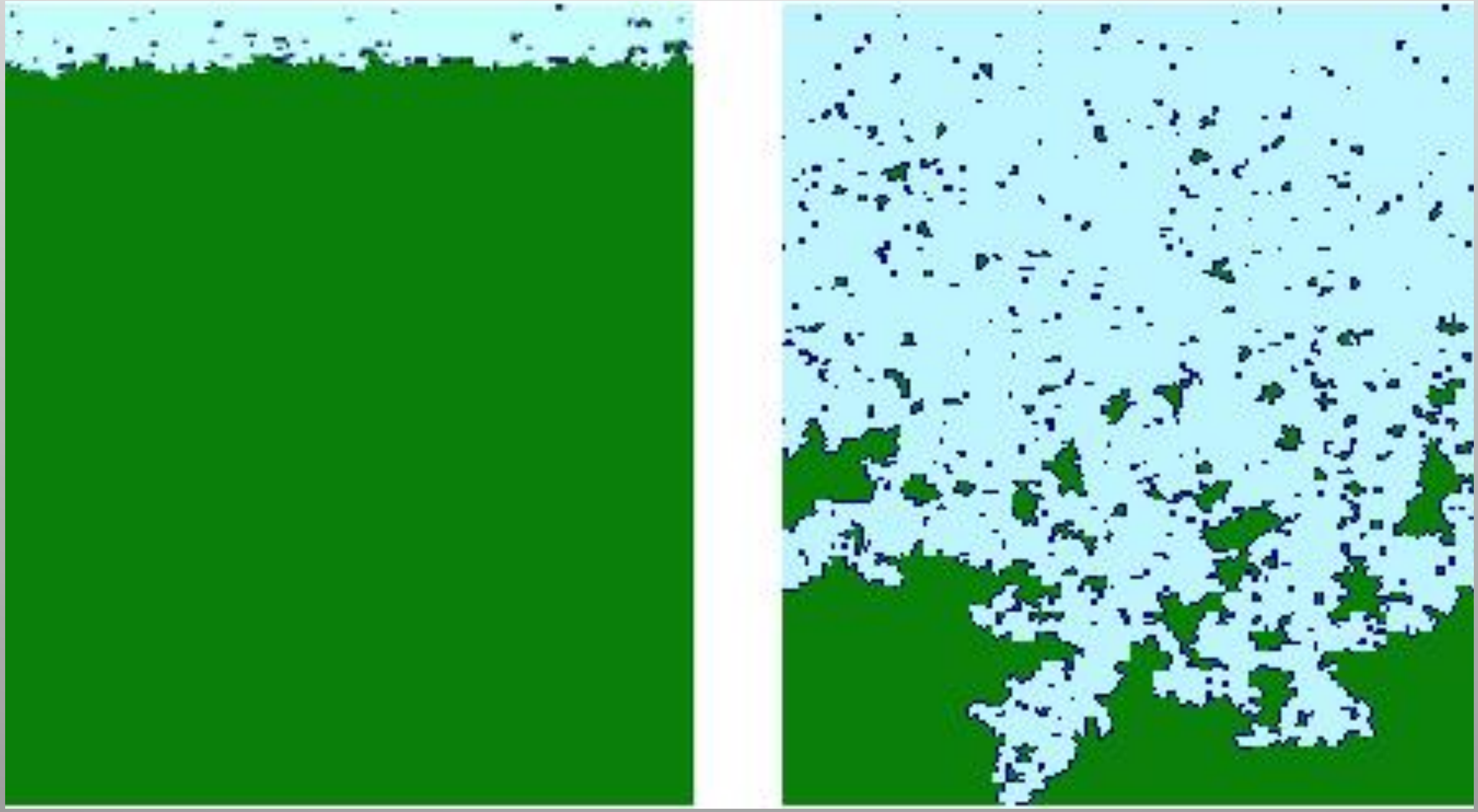
Fractals in Nature

- Thin gold films near percolation threshold



Fractals in Nature

Creation of stable fractal shoreline by ocean wave erosion



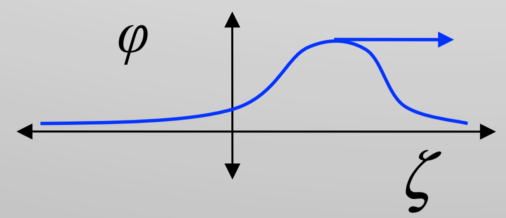
Practical Chaos and Fractals

- Practical Chaos
  - Systems in nonlinear regime can show much more interesting behavior than in linear regime
  - Importance of chemical oscillations for reactive control, pattern selection, etc.
  - Nonlinear systems in chaotic regime are even more interesting
- Chaotic Control/Controlling Chaos
  - Intuition: chaotic motion "explores" much of the "space" of the system. Further, chaotic regime is filled with unstable fixed points and periodic orbits. Central idea of (one form of) chaotic control is to
  - 1) identify a desired unstable motion in that is in/near some region visited by chaotic system and then
  - 2) alter the "parameters" of the system to try to stabilize this already existing orbit
- Chaos => Flexibility
  - N.B.: Chaos may be desirable for: enhanced mixing in chemical reactions and/or spreading out/distributing wear
- Fractals for data compression, flow in porous media

Emergence of Solitons and Coherent Structures

Consider continuum, wave-like motion, in 1 space dimension and time

A "***solitary wave***" is a localized, traveling wave



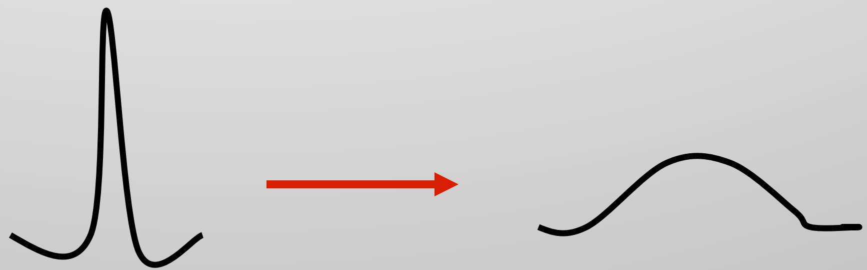
$$\phi\left(\zeta = x - \mathrm{v}t\right)$$

A "***soliton***" is a solitary wave that preserves *exactly* its amplitude, shape and velocity after collisions with all other waves. For nonlinear systems, this is very surprising, because when to solitons interact you would expect their superposition *not* to be a solution of the wave equation.

The term ***"coherent structure"*** is more generally used for localized excitations (like vortices) in spatial dimensions greater than one.

Intuition behind Emergence of Solitons

- In nonlinear systems, the "natural" tendency of waves to "spread out" (disperse) can be balanced by tendency to grow where they are already large and "break:" this balance can be viewed as the organizing principle for both solitons and coherent structures.
  - Dispersion

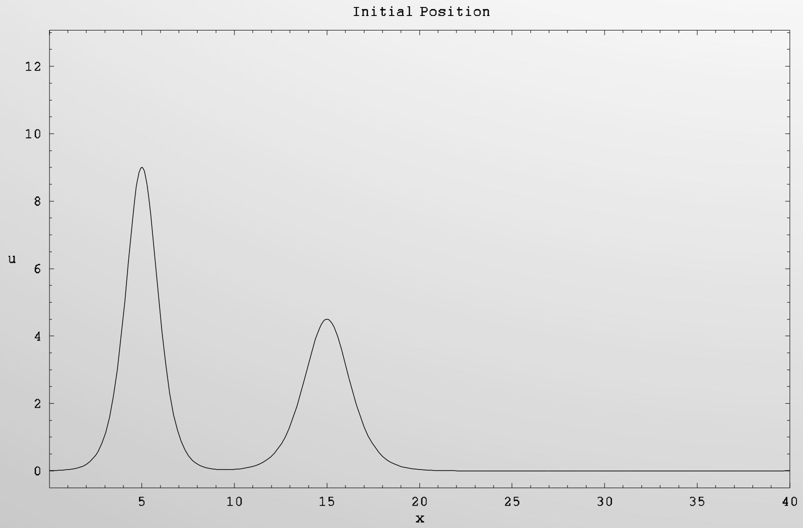


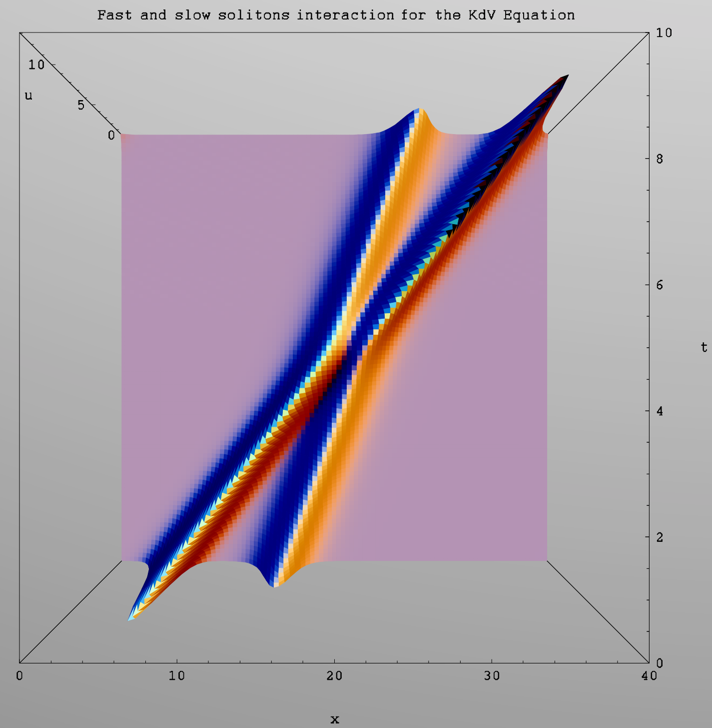
  - Nonlinearity

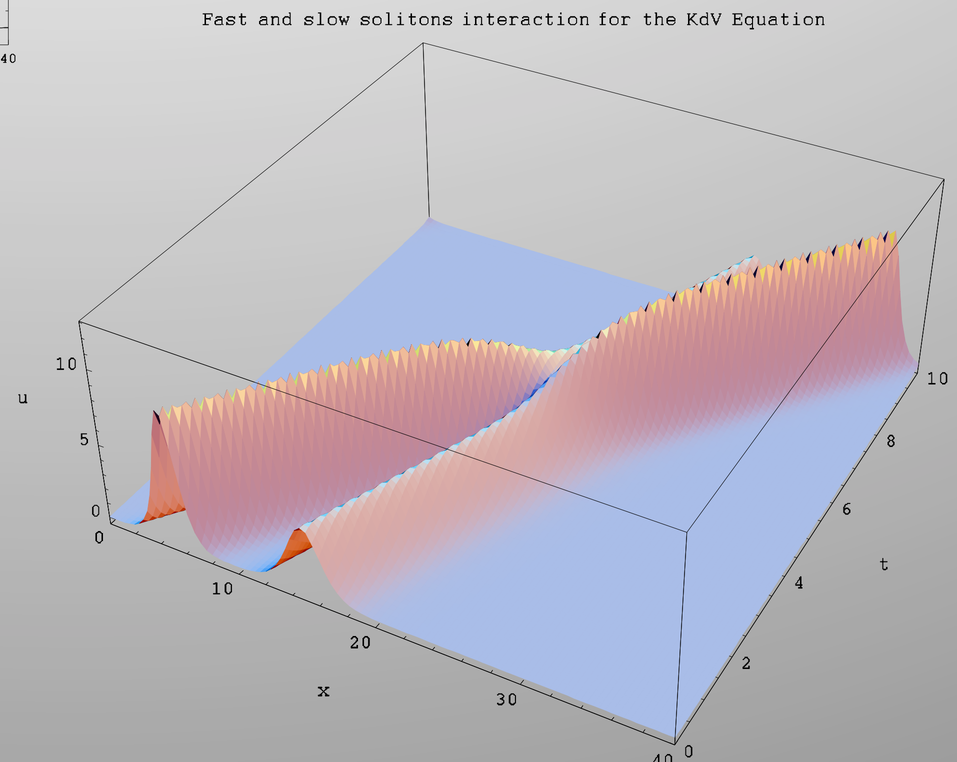
  - Combined effects lead to Korteweg-de Vries (KdV) equation with soliton solution as shown in next two slides.

KdV Soliton collision:

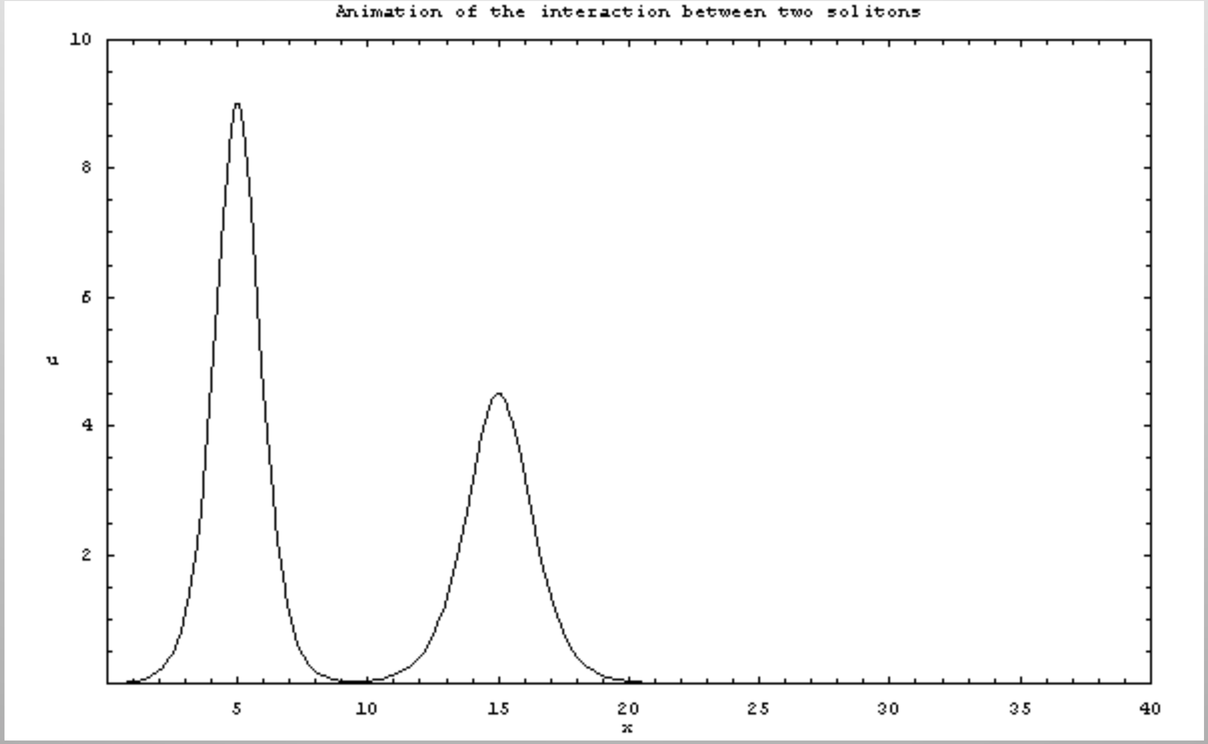
$V_{large} = 3, V_{small} = 1.5$





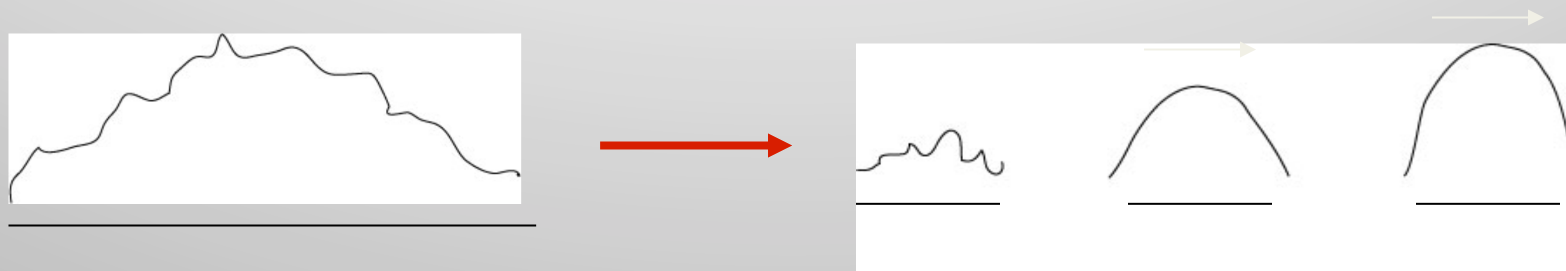


Animation of KdV soliton collision



Why is emergence of solitons so special?

1. That they exist at all in nonlinear equations is amazing; expect nonlinearity would destroy, particularly in view of our experience with low-dimensional dynamical systems.
2. Solitons—more generally, coherent structures—can dominate asymptotic form of solution.



Random Junk t = 0

Separated Solitons t = T >> 0

3. Many physical systems are well-approximated by soliton equations and conversely soliton-like excitations are observed and studied widely in nature: in physics, for instance, “Skyrmions” in nuclear and condensed matter physics, “monopoles” in particle physics, cavitons in atmosphere, vortices in fluids, *etc*.

4. Organizing principle for true solitons is hidden symmetry, as their equations contain an infinite number of conservation laws.

Coherent Structures in Nature

In natural world, don't expect exact soliton behavior: more general concept of ***coherent structures*** – persistent, localized spatial structures in extended nonlinear systems--is relevant.

Coherent Structures are observed on all scales in nature

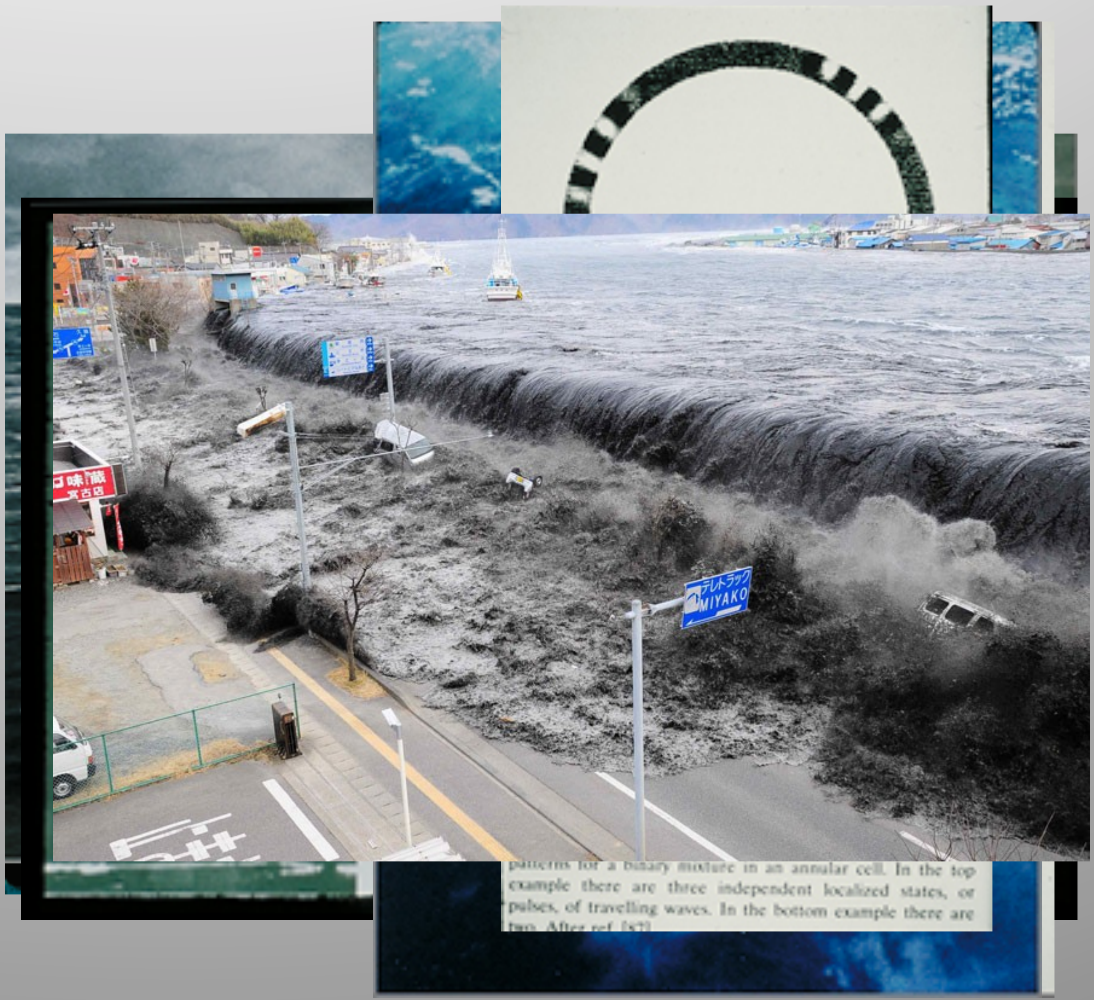
- Red Spot of Jupiter

- Earth Ocean Waves
  - Tsunamis
  - Apollo – Soyuz image
  - Waves on a Beach

- Laboratory Fluid Expts
  - Smoke rings
  - Binary Convection

- Pulses in optical fibers

Important in Technology



Practical Solitons/Coherent Structures

- Solition optical communications
  - Paradigm: Balance of dispersion with nonlinear index of refraction ("Kerr effect") in a glass (SiO_2) fiber
- Practicality:
  - Low loss optical fiber (<1dB/km)
  - Appropriate laser frequency to allow (small) nonlinear effects to balance dispersion
  - Reliable, cheap, low-power amplification mechanism (erbium doped sections of fiber)
- Bottom line:
  - All optical system, glass fiber, erbium doped sections every 20-50km. Solitions of 10-20ps duration and ~20mw can be used to send 10-20Gbit/s rate. Rates of 300 Gbit/s possible with "sliding guiding filtering" and "wavelength division multiplexing (WDM)!
  - Utilized in fiber cables in Europe and US

Emergence of Patterns

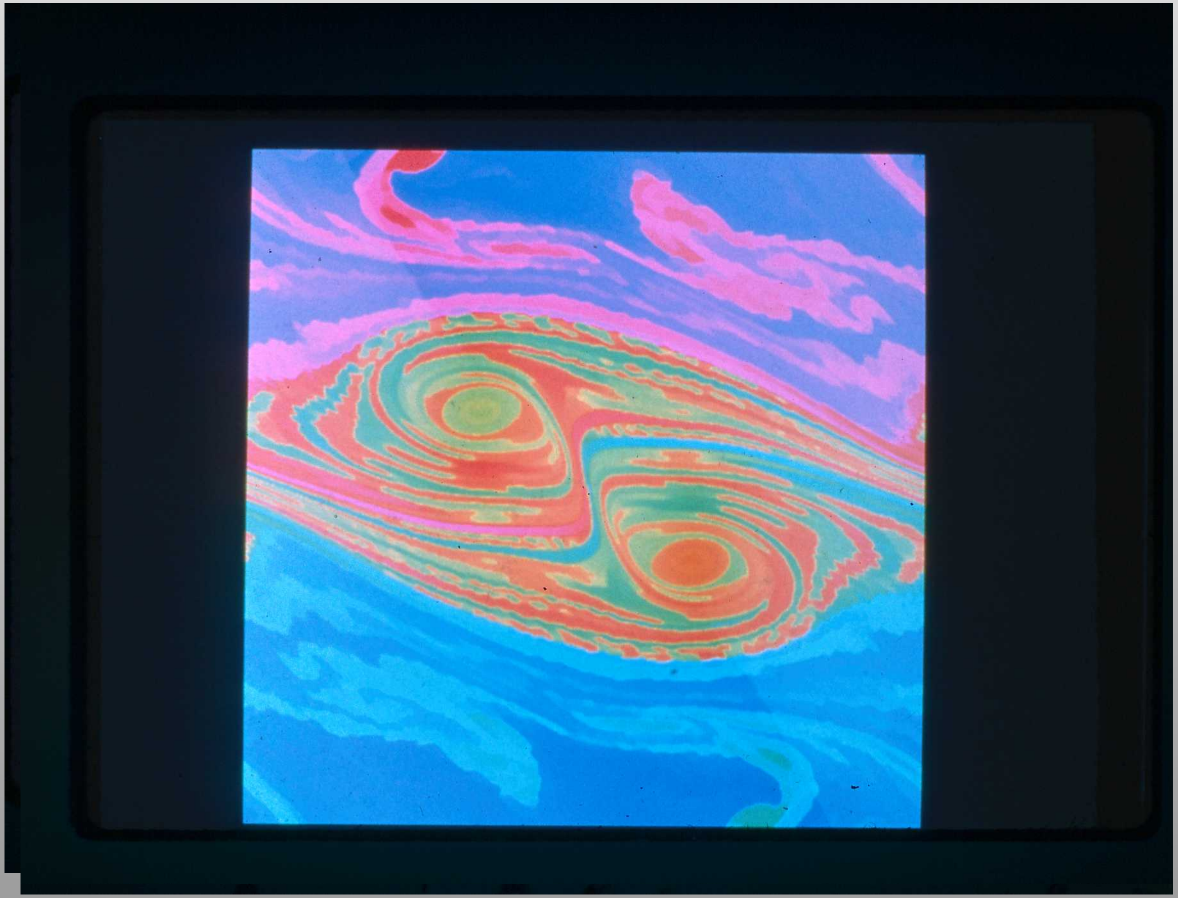
- Patterns and complex configurations arise in spatially extended nonlinear systems from competition between coherent structures and chaotic dynamics. Organizing principle is instability of the homogeneous state and emergence of a new inhomogeneous state: not unlike phase transitions, but out of equilibrium.

- Many examples in next slides

  - Kelvin-Helmholtz instability in counter-flowing fluids—entrapment of boundary (P. Woodward)
  - Model for coupled array of damped, driven Josephson junctions (P. Lomdahl)
  - Rayleigh-Benard Convection Experiments (R. Ecke)
  - Targets and spirals in chemical reactions (Belusov-Zhabotinskii) and in slime mold colonies (A. Winfree, P. Newell)
  - Turing patterns (H. Swinney, Q. Ouyang)

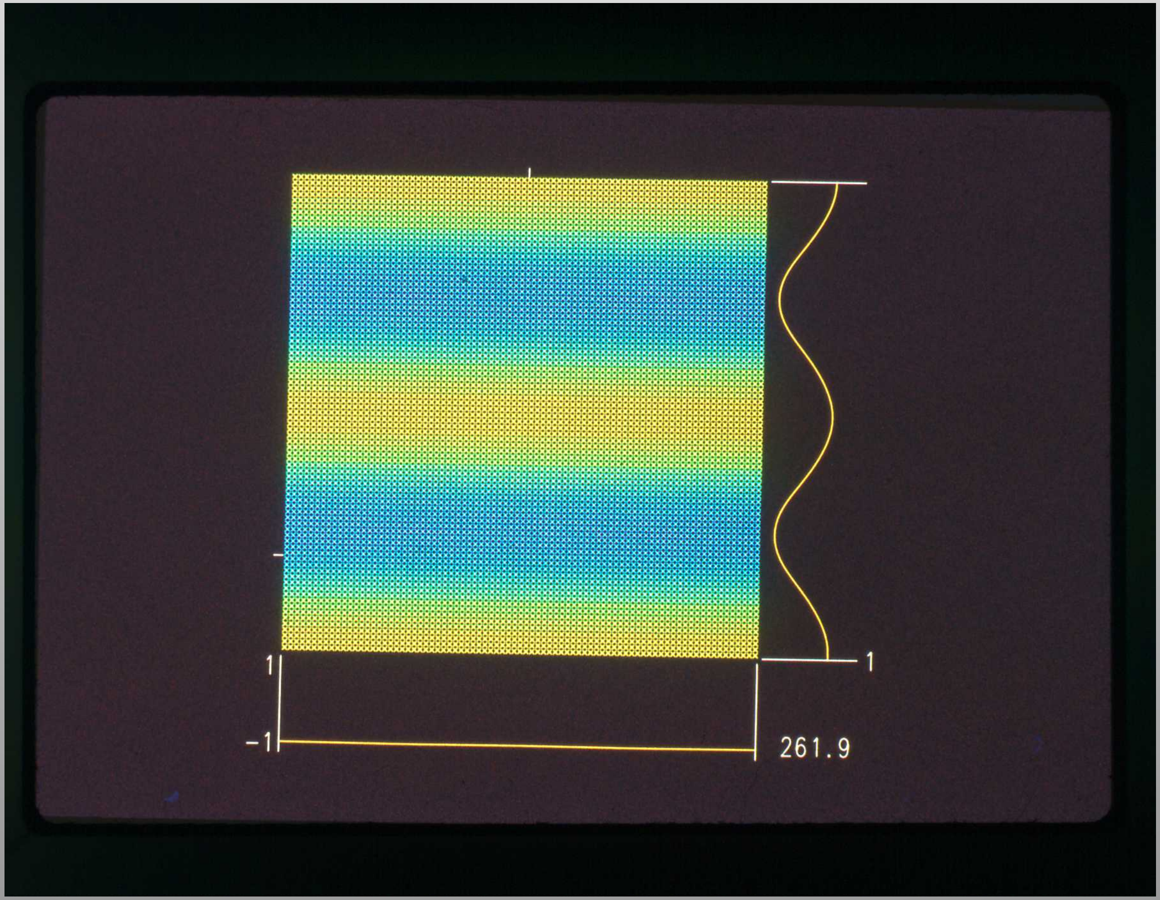
Patterns: Kelvin Helmholtz Instability

Two counter flowing fluids (upper =pink, lower = greenish blue) boundary colored yellow: note "chaotic" stretching and folding of boundary and emergence of coherent structures (vortices) (Simulation: P. Woodward)



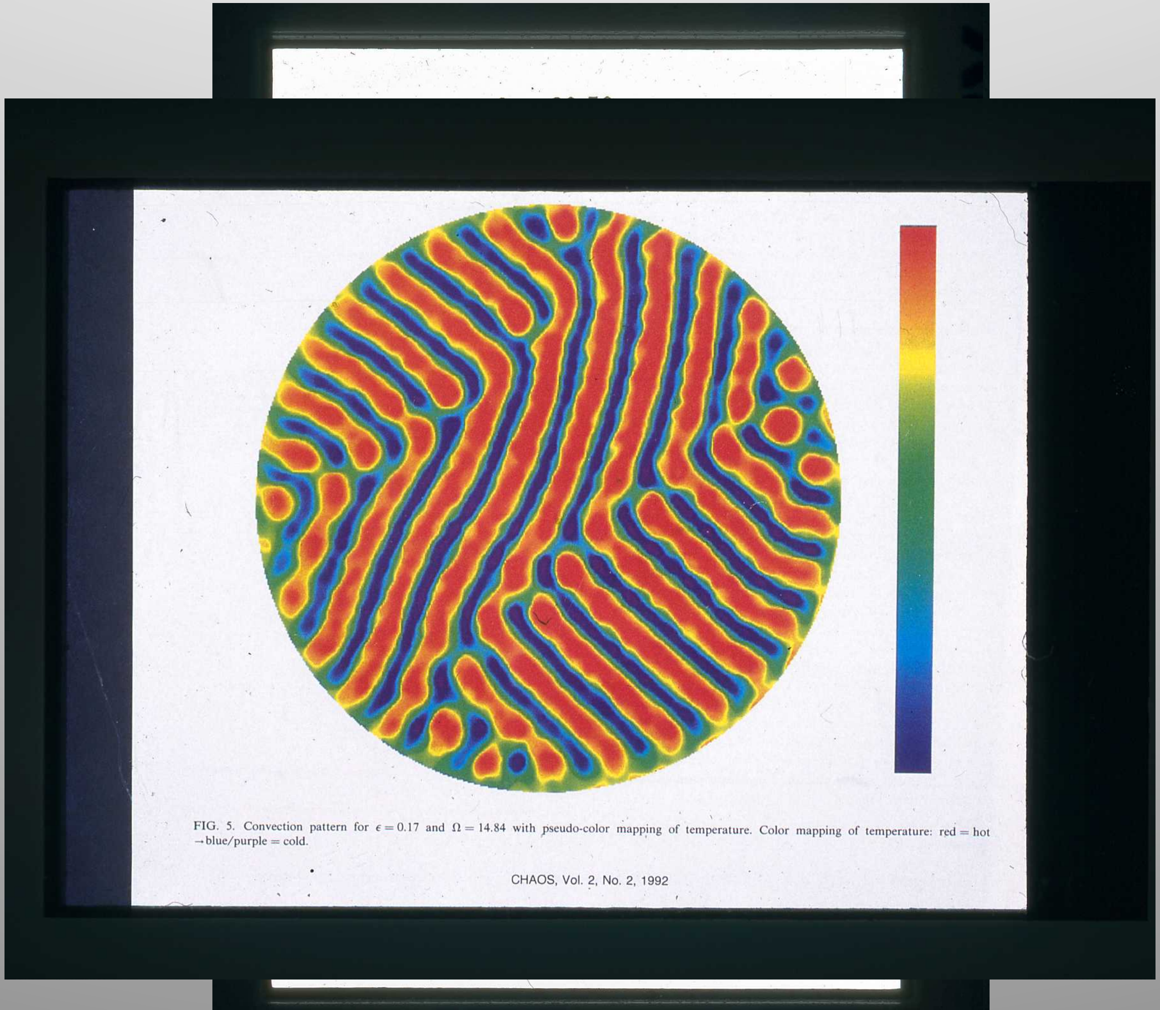
Patterns: Damped, Driven Josephson Junction

- Simulation of Model for Damped, Driven Josephson Junction: (Partial Differential Equation in (x,y,t): Color indicates value of dependent variable: Passes through transient patterns before reaching final state: (Simulation: P. Lomdahl)



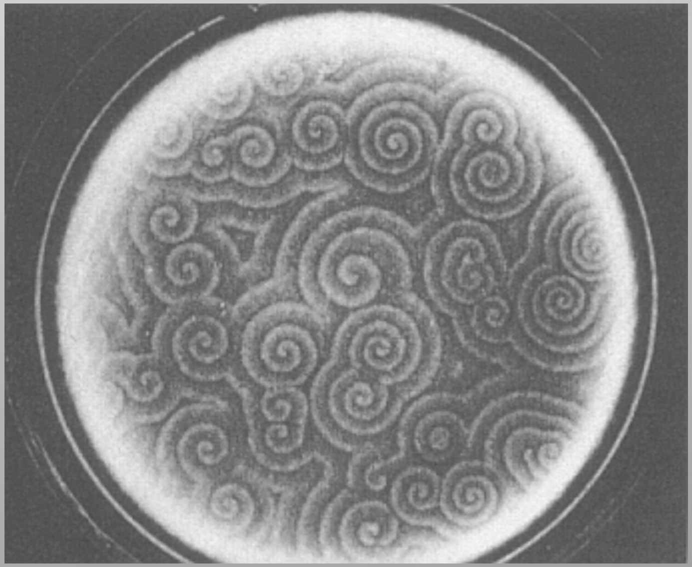
Patterns: Rayleigh-Benard Convection

- Water heated from below in a rotating disk; convection "rolls" made visible by shadowgraph techniques; note pattern evolution with time (R. Ecke)



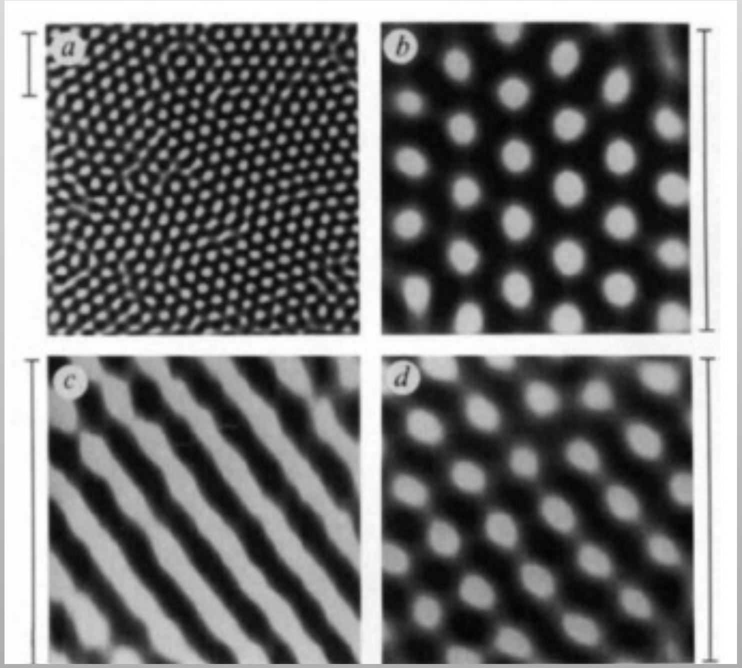
Patterns: Chemical Reactions and Slime Mold

Striking similarity between target and spiral patterns formed in Belusov-Zhabotinsky chemical and in slime mold (dictyostelium discoideum)



Patterns: Turing Patterns

– Turing patterns in chemical traveling waves (Q. Ouyang and H. Swinney)



Practical Patterns

"Practical patterns" exist almost everywhere, from nanostructures in materials science through morphogenesis in biology to geological formations, networks, and economics: they are an important form of "emergence" and underlie much of the behavior of complex systems.

Summary

Nonlinear science provides an excellent gateway to emergence and emergent phenomena through its paradigms. We have learned

- That nonlinear phenomena can produce incredible irregularity and seemingly random motion, through chaos and sensitive dependence on initial conditions leading to emergent fractal structures.
- That nonlinear phenomena can also produce remarkable order and regularity, solitions and coherent structures***,*** *emergi*ng unexpected in nonlinear wave equations and observed in nature on all scales
- That **patter**ns and complex configurations *emerge* in typical extended nonlinear systems and are basis for structure observed in natural world

Concluding Image

Close with one image that sums up the gateway to emergence via nonlinear science