Santa Fe Institute
2007 Complex Systems Summer School

Week I: Introduction to Nonlinear Dynamics

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Syllabus:

1. Introduction; Dynamics of Maps — chs 1 & 10 of [50]
   - a brief tour of nonlinear dynamics — [32] *(in [17])*
   - an extended example: the logistic map
     - how to plot its behavior
     - initial conditions, transients, and fixed points
     - bifurcations and attractors
     - chaos: sensitive dependence on initial conditions, λ, and all that
     - pitchforks, Feigenbaum, and universality — [22] *(in [17])*
     - the connection between chaos and fractals — [23], ch 11 of [50]
     - period-3, chaos, and the u-sequence — [31, 34] *(latter is in [17])*
     - *maybe:* unstable periodic orbits — [2, 25, 49]
2. Dynamics of Flows

   [50], sections 2.0-2.3, 2.8, 5, and 6 (except 6.6 and 6.8)

   - maps vs. flows
     - time: discrete vs. continuous
     - axes: state/phase space — [9]
   - an example: the simple harmonic oscillator
     - some math & physics review — [8]
     - portraying & visualizing the dynamics — [9]
   - trajectories, attractors, basins, and boundaries — [9]
   - dissipation and attractors — [42]
   - bifurcations

- how sensitive dependence and the Lyapunov exponent manifest in flows
- anatomy of a chaotic attractor: [23]
  - stretching/folding and the un/stable manifolds
  - fractal structure and the fractal dimension ch 11 of [50]
  - unstable periodic orbits [2, 25, 49]
  - shadowing
  - *maybe:* symbol dynamics [26] *(in [13])*; [28]

3. Tools [1, 9, 37, 40]

- ODE solvers and their dynamics [8, 33, 35, 44]
- *maybe:* PDE solvers [8, 44]
- Poincaré sections [27]
- stability, eigenstuff, un/stable manifolds and a bit of control theory
- embedology [29, 30, 39, 46, 47, 45, 52] *([39] is in [37] and [45] is in [53];)*
- *maybe:* calculating Lyapunov exponents and fractal dimensions [1, 9, 37, 40]

4. Applications [13, 37, 38]

- prediction [3, 4, 5, 14, 15, 53]
- filtering [20, 21, 24]
- control [7, 6, 11, 36, 48] *([36] is in [37])*
- communication [16, 41]
- classical mechanics [10, 43, 51, 54, 55]
- music, dance, and image [12, 18, 19]

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References [1, 3, 4, 13, 15, 17, 28, 37, 50, 53] are in the CSSS library.

More Resources:

`www.cs.colorado.edu/~lizb`

`amath.colorado.edu/faculty/jdm/faq.html`

`www.mpipks-dresden.mpg.de/~tisean/Tisean_3.0.1/index.html`