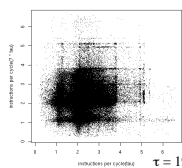
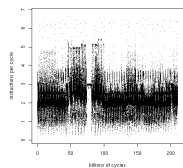


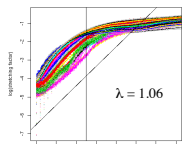
NLTSA* of computer performance dynamics

* nonlinear time-series analysis



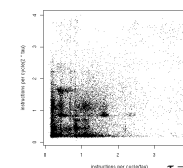
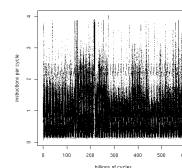
$\tau = 194, m = 10$

bzip2 dynamics on an Intel Core2



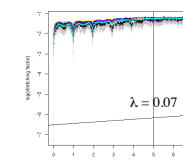
$\lambda = 1.06$

Mytkowicz et al., *Chaos* 19:033124



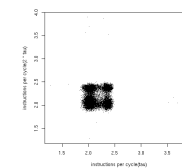
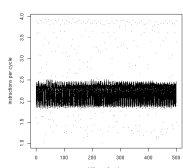
$\tau = 973, m = 12$

bzip2 dynamics on an Intel Pentium 4



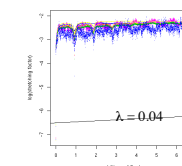
$\lambda = 0.07$

Mytkowicz et al., *Chaos* 19:033124



$\tau = 930, m = 8$

povray dynamics on an Intel Core2



$\lambda = 0.04$

Mytkowicz et al., *Chaos* 19:033124

Caveat: need enough data...

If Δt is not uniform

~~Theorem (Takens): for $\tau > 0$ and $m > 2d$, reconstructed trajectory is diffeomorphic to the true trajectory~~

~~Conditions: evenly sampled in time~~

Interspike interval embedding

idea: lots of systems generate spikes — hearts, nerves, etc.

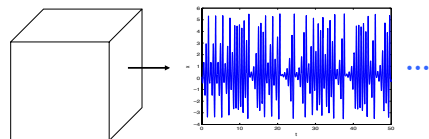
if you assume that the spikes are the result of an integrate-and-fire system, then the Δt has a one-to-one correspondence to some state variable's *integrated* value...

in which case the embedding theorems still hold.

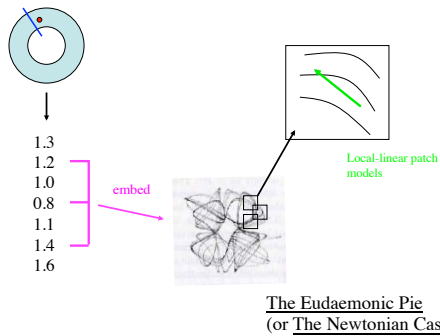
(with the Δt s as state variables)

Sauer *Chaos* 5:127

Prediction

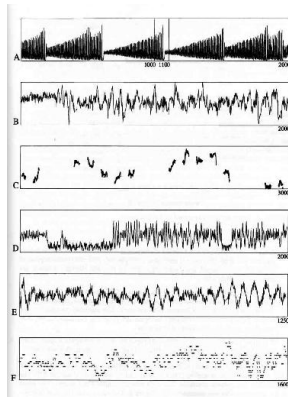


Predicting the path of a roulette ball...



The Santa Fe competition

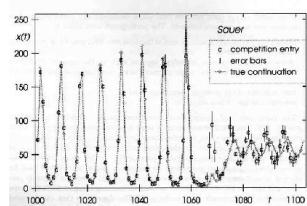
- Weigend & Gershenfeld, 1992
- put a bunch of data sets up on an ftp server
- and invited all comers to predict their future
- chronicled in *Time Series Prediction: Forecasting the Future and Understanding the Past*, Santa Fe Institute, 1993 (from which the images on the following half-dozen slides were reproduced)



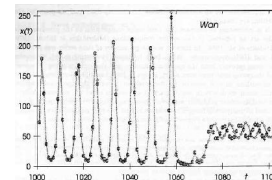
The Santa Fe competition: data

- Laboratory laser
- Medical data (sleep apnea)
- Currency rate exchange
- RK4 on some chaotic ODE
- Intensity of some star
- A Bach fugue

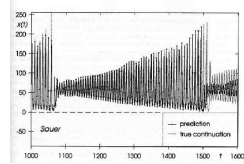
Embedding + patch models: (Sauer)



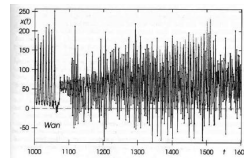
Neural net: (Wan)



Further out:

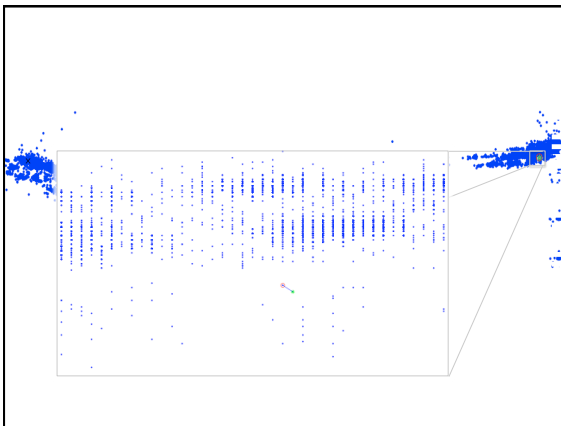
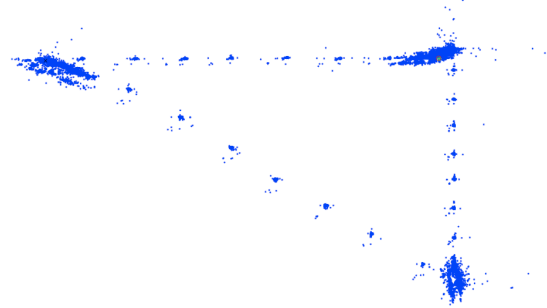


Sauer

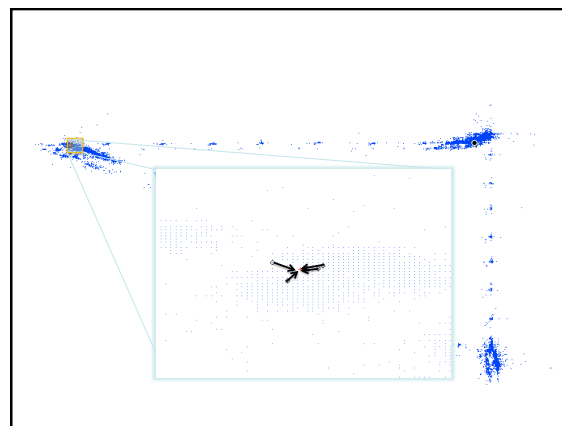
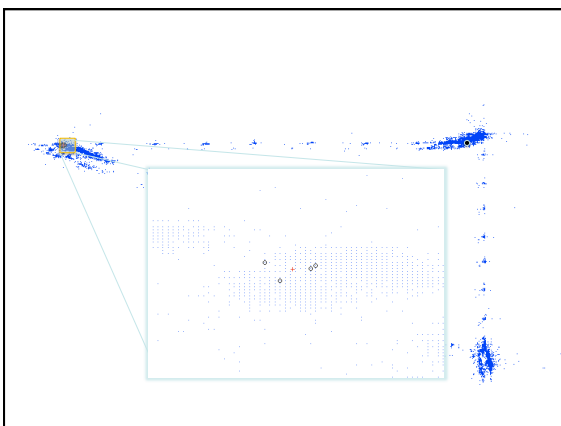
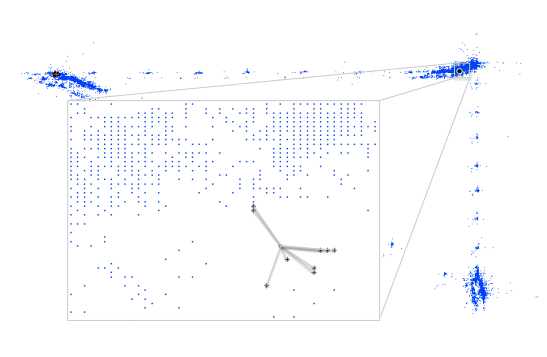


Wan

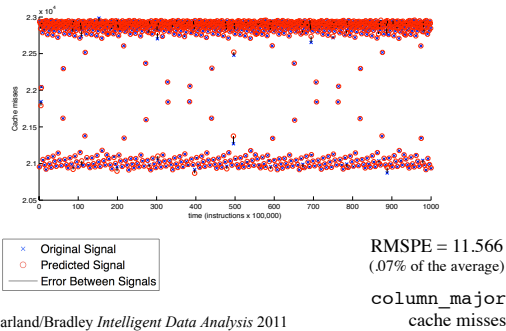
An even simpler prediction method:
Lorenz's method of analogues



A k -nearest neighbor modification of LMA



Using kLMA to predict computer dynamics

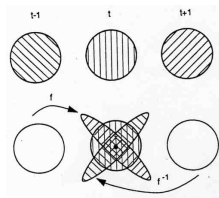


Noise...

Linear filtering: a bad idea if the system is chaotic

Nonlinear alternatives:

- use the stable and unstable manifold structure on a chaotic attractor...

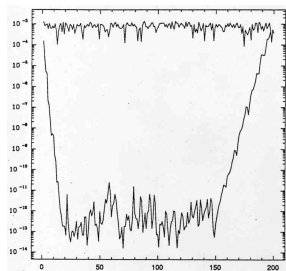


Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

Idea:

- If you have a model of the system, you can simulate what happens to each point in forward *and backward* time
- If your system has transverse stable and unstable manifolds, that does useful things to the noise balls
- Since all three versions of that data should be identical at the middle time, can average them
- ➡ noise reduction!
- Works best if manifolds are perpendicular, but requires only transversality

Results:



Farmer & Sidorowich, in *Evolution, Learning and Cognition*, World Scientific, 1983

Noise...

Linear filtering: a bad idea if the system is chaotic

Nonlinear alternatives:

- use the stable and unstable manifold geometry on a chaotic attractor
- what about using the *topology* of the attractor?

Computational Topology

Why: this is the fundamental mathematics of shape. complements geometry.

What: compute topological properties from finite data

How:

- introduce resolution parameter
- count components and holes at different resolutions
- deduce topology from patterns therein

V. Robins Ph.D. thesis, UColorado, 1999



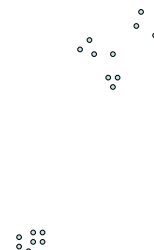
Connectedness: definitions

• how many “lumps” in a data set:

• ϵ -connectedness (after Cantor)

• ϵ -connected components

• ϵ -isolated points:

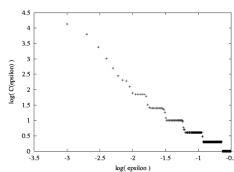


Connectedness: examples

If the data points are samples of a disconnected fractal like this:



The number of connected components looks like this:



(note obvious tie-in to fractal dimension...)

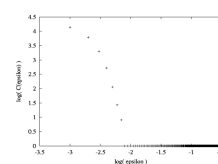
Robins et al., *Physica D* 139:276, *Nonlinearity* 11:913

Connectedness: examples

If the data points are samples of a connected set like this:



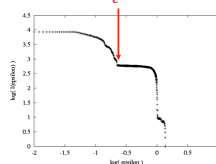
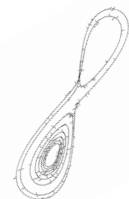
The number of connected components looks like this:



Robins et al., *Physica D* 139:276, *Nonlinearity* 11:913

Connectedness and filtering

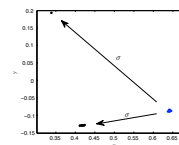
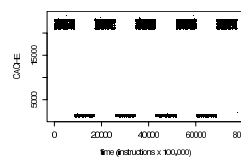
The effect of noise is to add isolated points to the set and a shoulder to the $C(\epsilon)$ curve:



So if you know that the object is connected — like the attractor of a flow — you can reasonably assume that any isolated points are noisy, and remove them by pruning with $\epsilon = \epsilon^*$

Robins et al., *Intelligent Data Analysis* 8:505, *Chaos* 14:305

Continuity and filtering



Idea:

- deterministic, differentiable dynamics (maps & flows) are *continuous*

Conjecture:

- if the image of a connected set is not connected, more than one dynamics is at work

Approach:

- track connectedness over time

Applications:

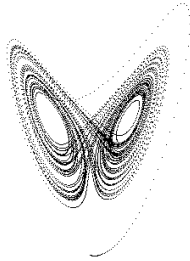
- pulling apart interleaved dynamics, removing noise...

Alexander et al., *CHAOS*, 2012

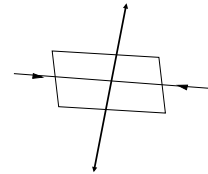
Chaos and control

key concepts:

- dense attractor coverage
- exponential trajectory separation
- un/stable manifold structure
- local-linear control

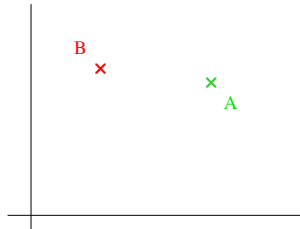


Recall: local-linear control of a saddle point works successfully in a region whose geometry is defined by the λ_i and the W^s/W^u , together with the sensor & actuator capabilities...



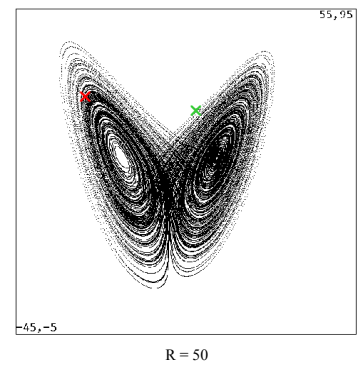
Control:

getting from A to B, minimizing some cost functional

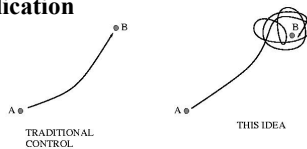


Lorenz System:

denseness, reachability, and control



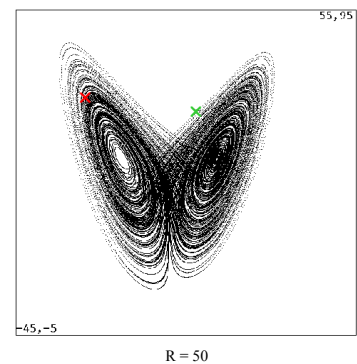
Denseness & reachability in a real engineering application



- can control position/volume/density of attractor — *within limits*
- possibly not reachable any other way
- not for time-critical applications (that “eventually”)

Using Chaos to Broaden the Capture Range of a Phase-Locked Loop
Elizabeth Bradley, Member, IEEE

Now what?

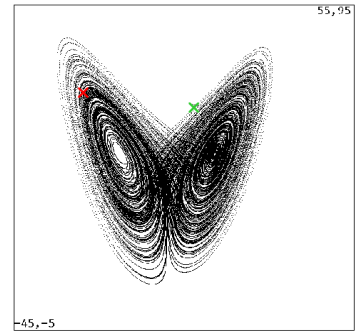


OGY control

- dense attractor coverage $\rightarrow$ reachability
- un/stable manifold structure + UPO denseness + local-linear control $\rightarrow$ controllability

Ott et al., PRL 64:1196

Use local-linear control, designed using the eigenvalues and eigenvectors at that point $\times$ to balance a chaotic system on a UPO passing through that point.



But you're relying on denseness to get you into the controllable region, and that may take a while...

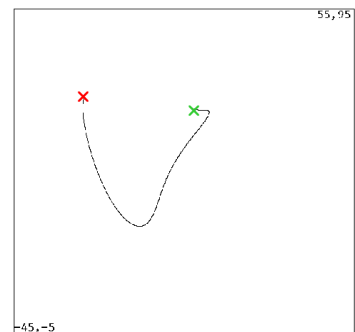
- dense attractor coverage $\rightarrow$ reachability
- un/stable manifold structure + UPO denseness + local-linear control $\rightarrow$ controllability
- exploit sensitive dependence, too???
 $\Rightarrow$ "targeting"

Lorenz System:

SDOIC-based targeting

OGY & co. have been used in *tons* of systems; see Shinbrot review paper.

Alfred Hubler has done a lot of cool stuff in this area as well.



Four R switches; 240X faster

Bradley, Cybernetics & Systems 26:299

Program in
Applied
Mathematics



Erik Bollt

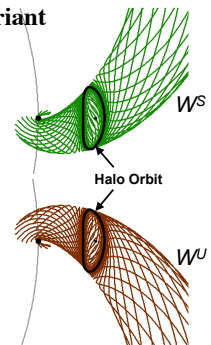
University of Colorado at Boulder
Boulder CO 80309-0526
(303) 492-4668

Other cool ways to use invariant manifolds

Want to get a spacecraft onto a "halo orbit," which is a UPO of the dynamics.

Unstable Periodic Orbits (UPOs) have invariant manifolds:

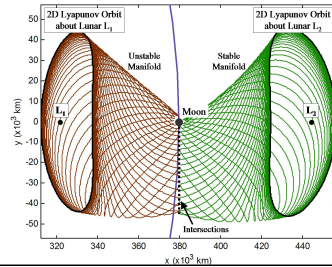
- Stable Invariant Manifold (W^s)
 – The set of all trajectories a particle could use to arrive onto the UPO.
- Unstable Invariant Manifold (W^u)
 – The set of all trajectories a particle could take after a small perturbation from the UPO.



Jeff Parker, PhD thesis, UColorado 2008

Low-energy (cheap) orbit transfers

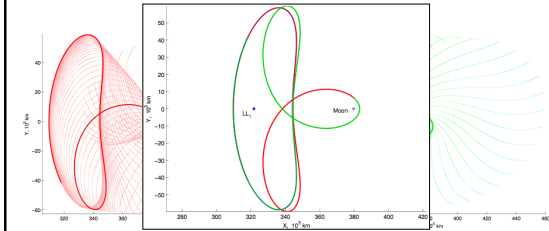
- Depart along W_{L1}^U & arrive on W_{L2}^S



Jeff Parker, PhD thesis, UColorado 2008

Homoclinic orbits - The best case

- If a trajectory in Stable and Unstable intersect ("homoclinic connection")



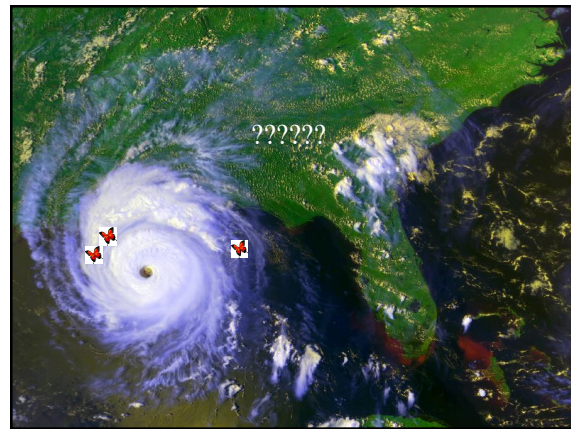
Unstable Manifold of an LL_1 Lyapunov Orbit

Stable Manifold of an LL_1 Lyapunov Orbit

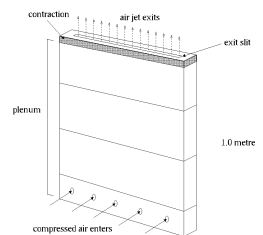
Jeff Parker, PhD thesis, UColorado 2008

Can we do any of that in spatially extended systems?

(i.e. harness the butterfly effect, exploit un/stable manifold geometry?)

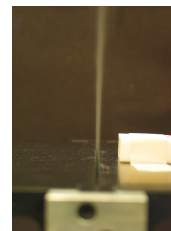


A 2D jet

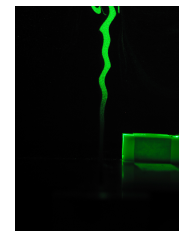


Peacock et al., *Exp. Fluids* 37:22

End view



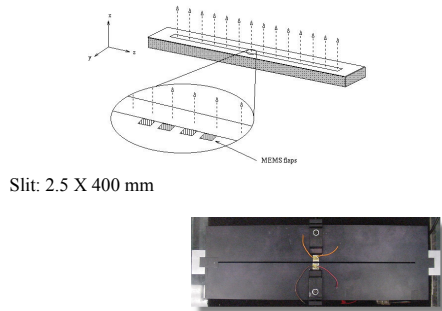
room lighting



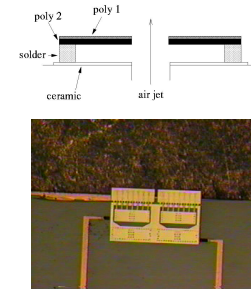
stop-action laser "slice"

aerosolized canola oil

Forcing the jet flow



MEMS actuators

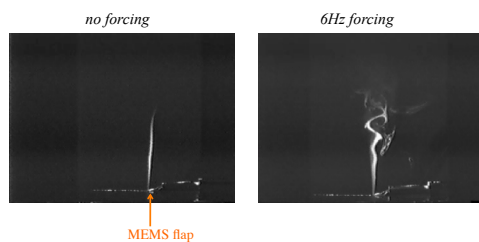


Video : overhead view at 2Hz, 10V

Area of individual flap is 1.0 x 0.25mm

Ma et al., IEEE Trans. Adv. Packaging 26:268

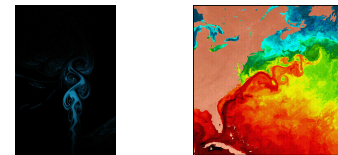
The Butterfly effect in action...



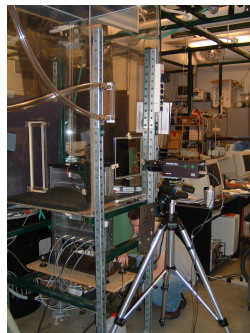
Forcing generates coherent structures that enhance entrainment and mixing

Peacock et al., Exp. Fluids 37:22

Does this have anything to do with reality?



Measurement & isolation:



Communication and chaos:

- Two coupled Lorenz systems will synchronize
- Robust to a small amount of noise
- Use this to transmit & receive information

$$\begin{aligned} x' &= a(y-x) \\ y' &= rx - y - xz \\ z' &= xy - bz \end{aligned}$$

$$\begin{aligned} x' &= a(y-x) \\ y' &= r(x+ex) - y - xz \\ z' &= xy - bz \end{aligned}$$

- Chaotic carrier wave, so hard to intercept or jam

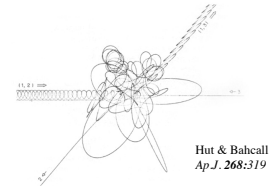
Pecora & Carroll Phys. Rev. Lett 64:821

Another interesting application: chaos in the solar system

- orbits of Pluto, Mars
- Kirkwood gaps
- rotation of Hyperion & other satellites
- ...

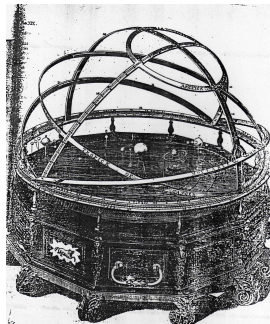
Solar system stability:

- recall: two-body problem not chaotic
- but three (or more) can be...

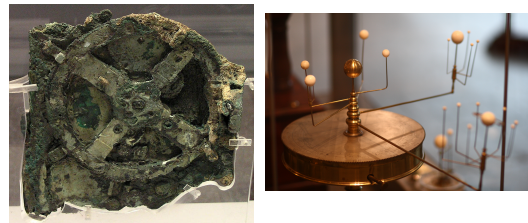


Exploring that issue, circa 1880:

An *orrery*, which is a *mechanical computer* whose gear ratios and circular platters simulate the orbits of the planets



Exploring that issue before the digital computer age...

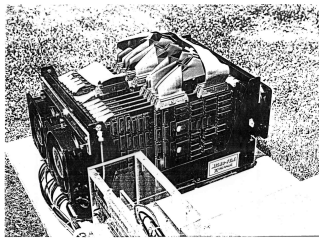


An *orrery*, which is a *mechanical computer* whose gear ratios and circular platters simulate the orbits of the planets

Exploring that issue, circa 1980:

- write the n -body equations for the solar system
- solve them using symplectic ODE solvers on a special-purpose computer

The *digital orrery*
(Wisdom & Sussman)



Numerical Evidence That the Motion of Pluto Is Chaotic

GERALD JAY SUSSMAN AND JACK WISDOM

The Digital Orrery has been used to perform an integration of the motion of the outer planets for 845 million years. This integration indicates that the long-term motion of the planet Pluto is chaotic. Nearby trajectories diverge exponentially with an e -folding time of only about 20 million years.

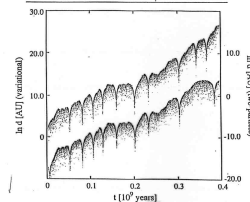
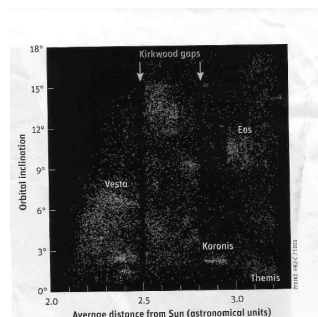


Figure 2 The exponential divergence of nearby trajectories is indicated by the average linear growth of the logarithms of the distance measures as a function of time. In the upper frame we see the growth of the variational distance around a reference trajectory. In the lower frame we see how two Pfitter diverge with time. The distance estimates are 80AU when the Pfitter are on opposite sides of the Sea. The variational method of studying neighboring trajectories does not have the problem of saturation. Note that the two methods are in excellent agreement until the two-trajectory method has nearly saturated.

Should we worry?

- No.

Kirkwood gaps:



From *Sky & Telescope*

Chaos and the Kirkwood gaps

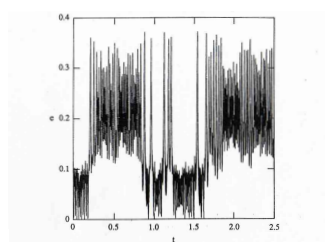


FIGURE 3. Eccentricity of a typical chaotic trajectory over a longer time interval, the time is now measured in millions of years. Bursts of high eccentricity behavior are interspersed with intervals of irregular low eccentricity behavior, broken by occasional spikes.

Wisdom, *Nuclear Phys. B* 2:391

Evidence in favor of the conjecture:

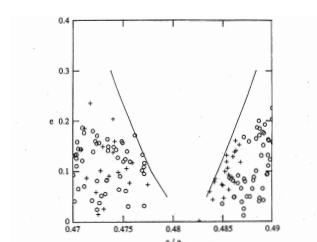
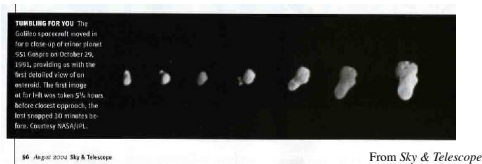


FIGURE 3. Comparison of the actual distribution of asteroids with the outer boundaries of the chaotic zone. There is both a chaotic region and quasi-periodic region in the gaps, but trajectories of both types are planet crossing.

Wisdom, *Nuclear Phys. B* 2:391

Chaotic tumbling of satellites:

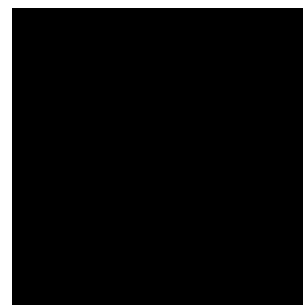
Voyager and Galileo saw this...



14. August 2001 Sky & Telescope

From *Sky & Telescope*

Ap. J. 97:570
Ap. J. 98:1855



www.nasa.gov/mission_pages/cassini/multimedia/pia06243.html

Chaotic tumbling of satellites:



This happens for **all** satellites at some point in their history, unless they are perfectly spherical and in perfectly circular orbits (pf: KAM theorem; see Wisdom paper on syllabus)

Some of them are still tumbling chaotically because of their geometry, but most (like the earth and its moon) have settled down into tidal equilibria

More chaos in the solar system:

- obliquity of Mars (Touma & Wisdom, *Science* 259:1294)



www.solarviews.com

- etc.

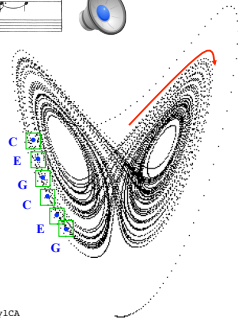
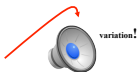
Musical Variations from a Chaotic Mapping



Dabby Chaos 6:95

Pitch sequence:
C, E, G, C, E, G, C, E...

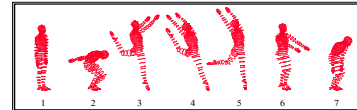
C symbol dynamics



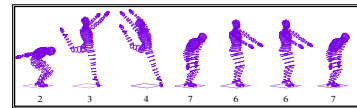
Also fun: <http://www.youtube.com/watch?v=B2XtE97y1CA>

Chaotic variations on movement sequences

original piece



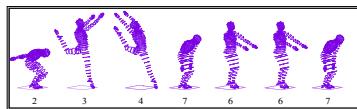
chaotic mapping



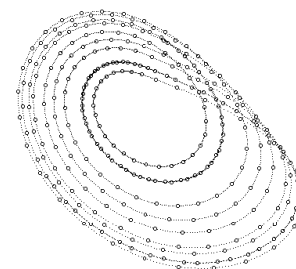
chaotic variation

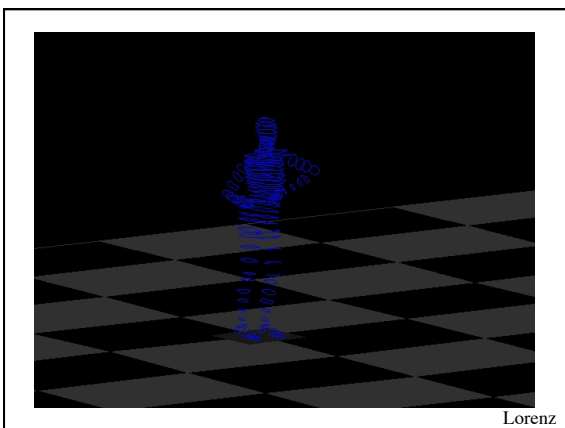
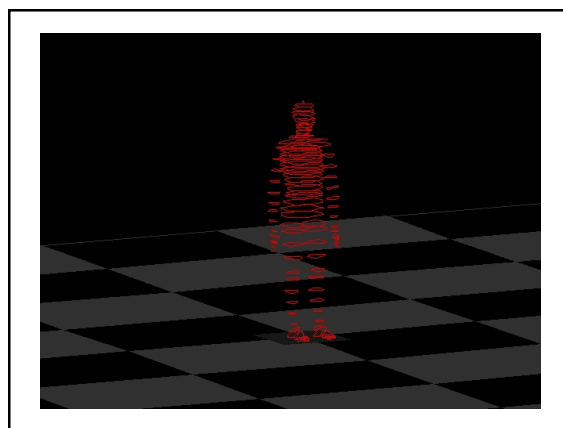
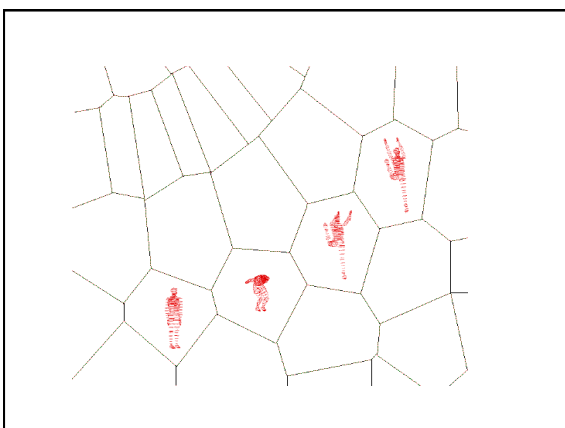
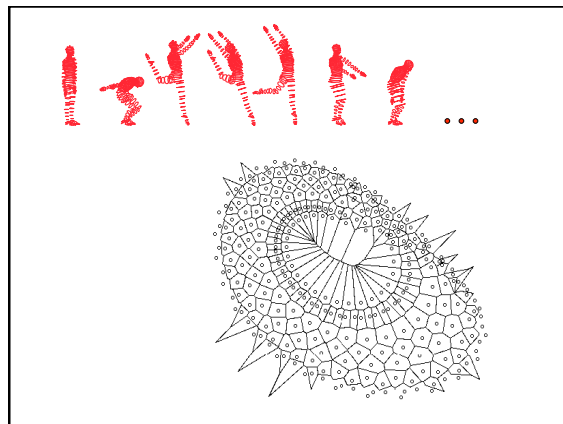
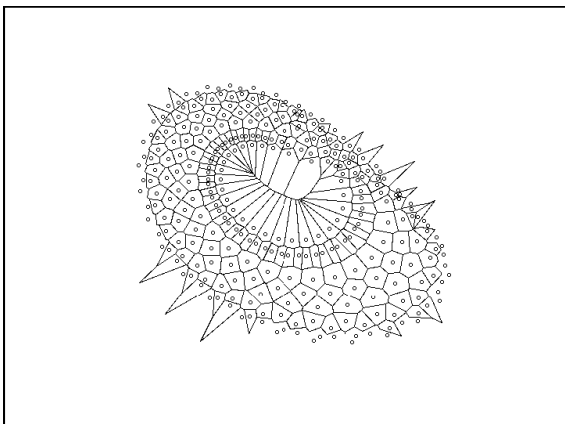
Bradley & Stuart, *Chaos* 8:800

original piece

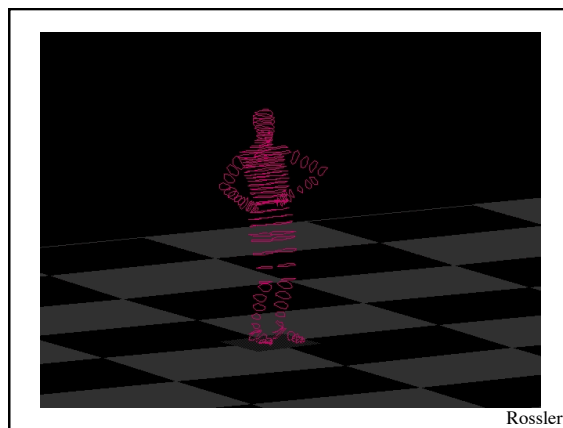


chaotic variation

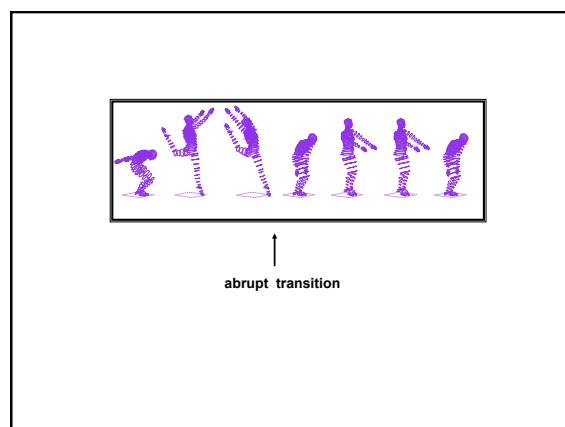
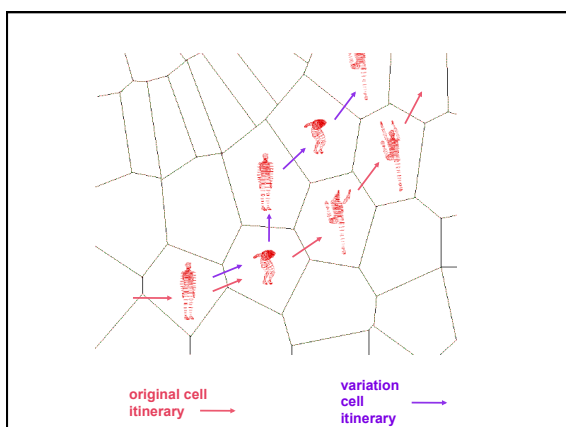
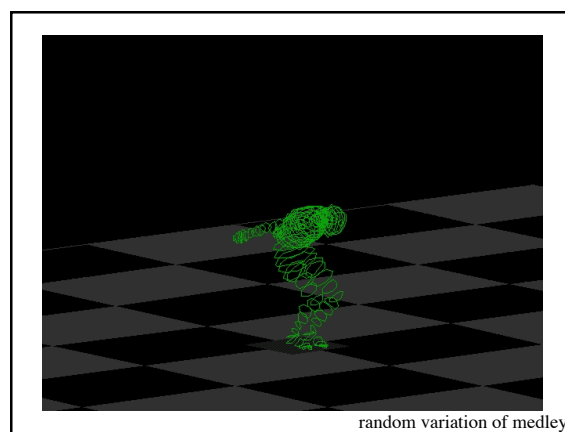
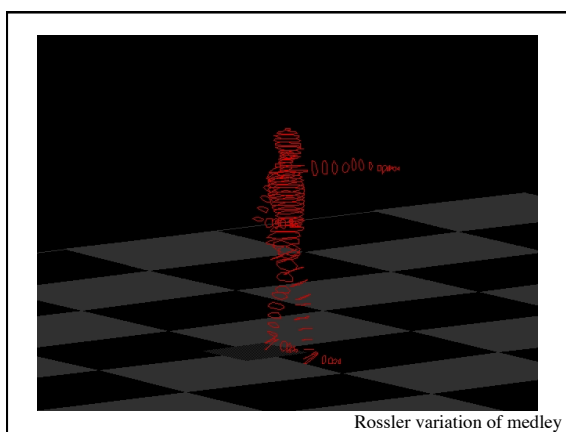
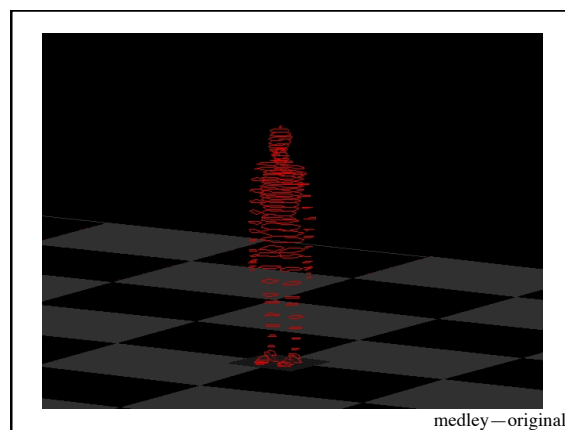
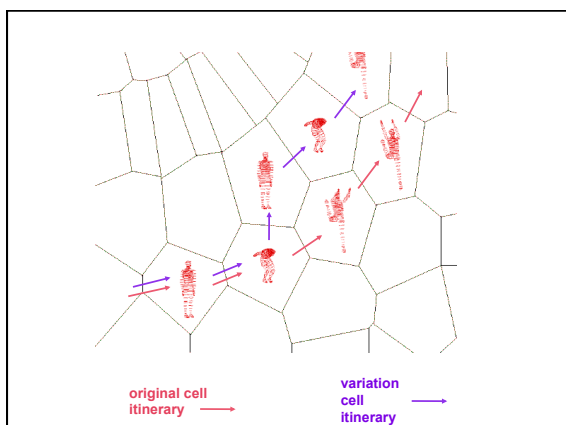


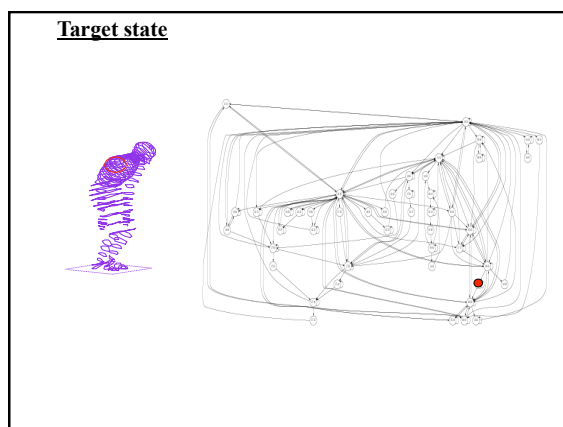
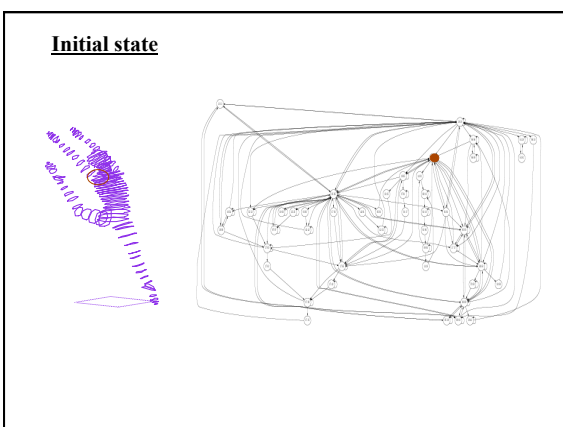
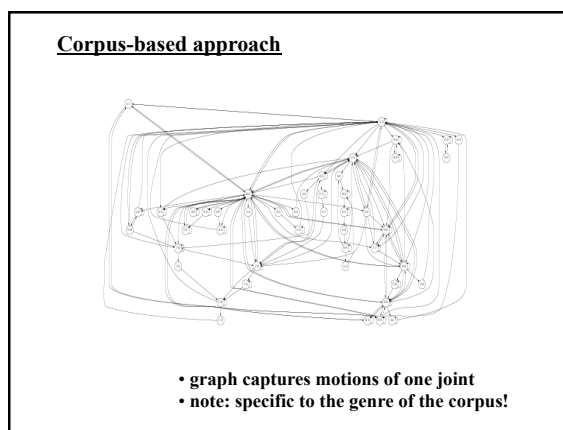
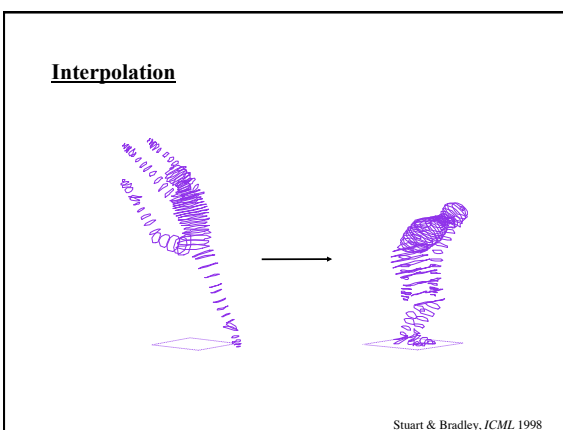
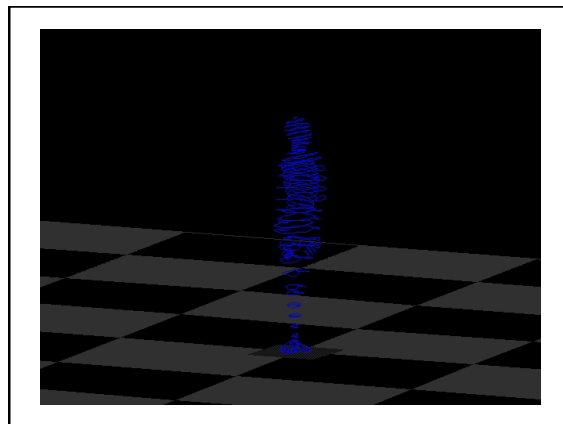
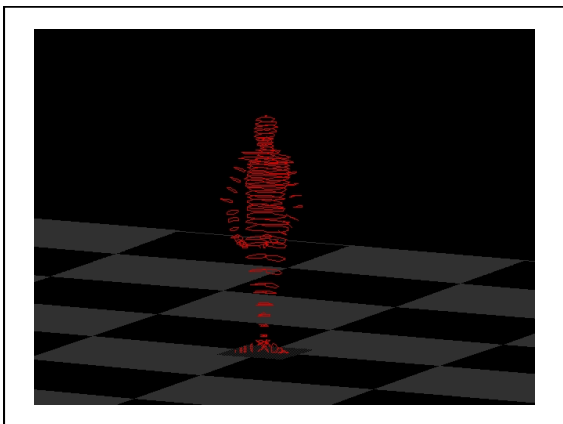


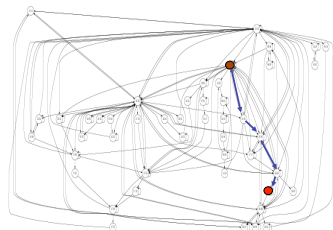
Lorenz



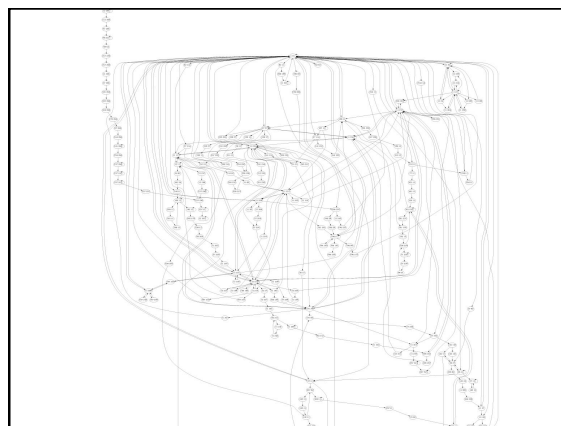
Rossler



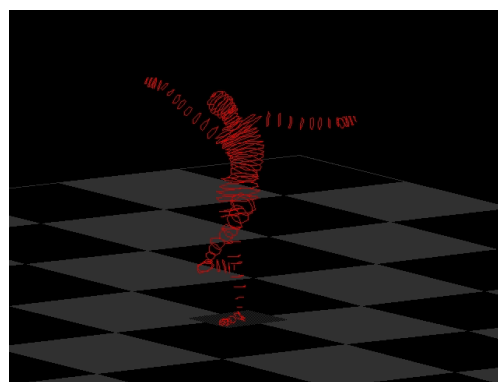


Graph search

...for 44 joints in parallel!



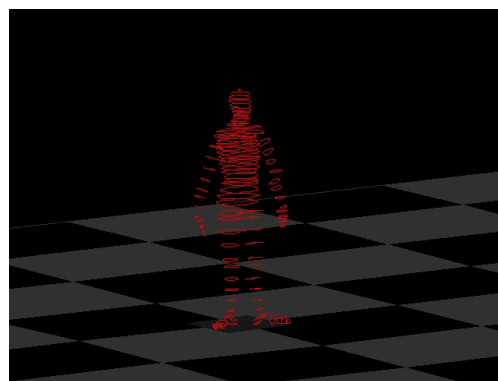
initial



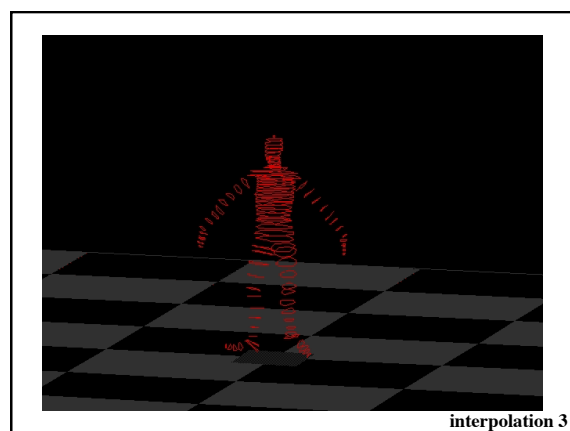
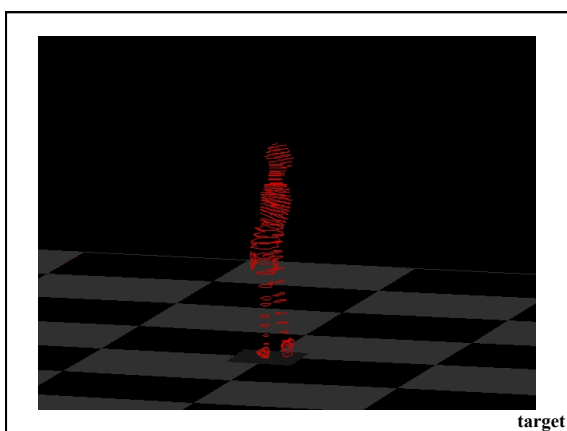
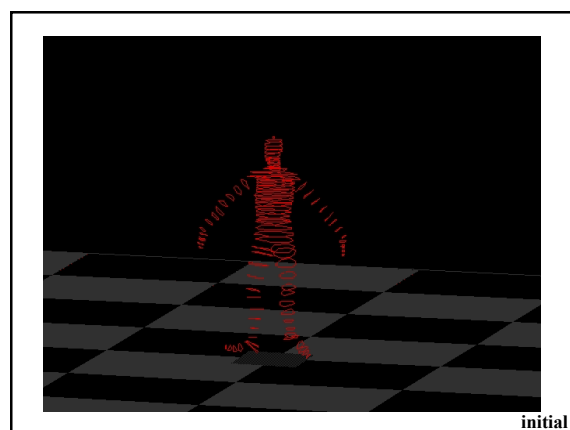
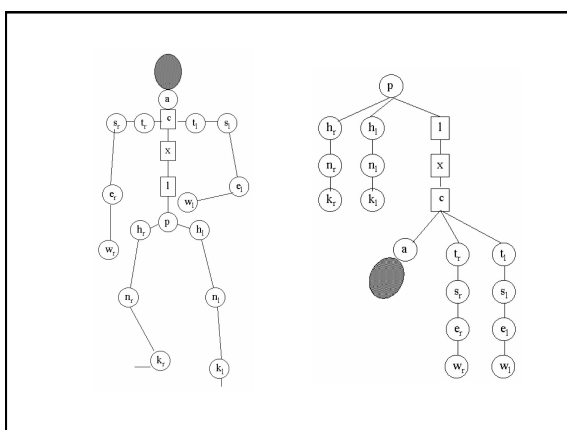
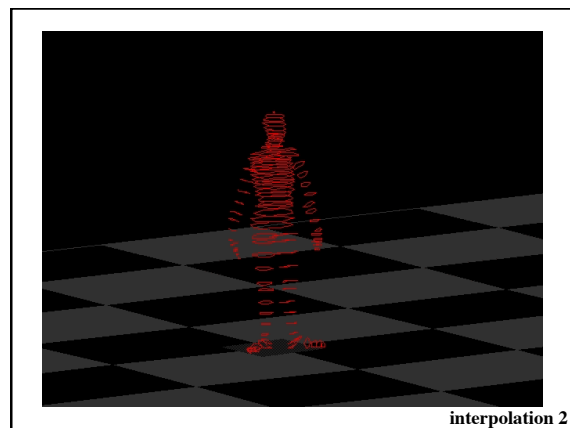
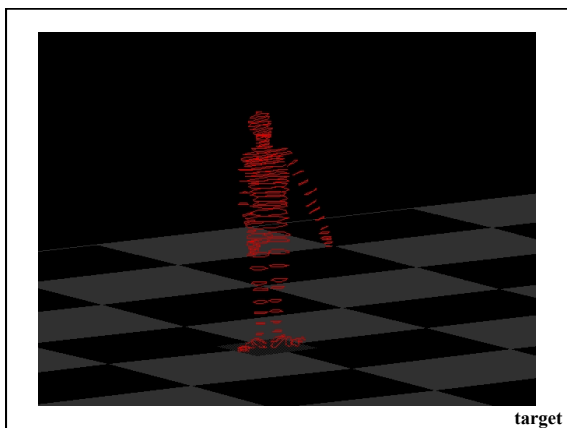
target

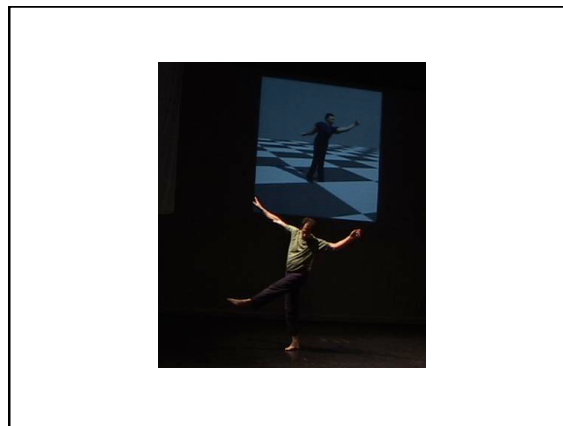
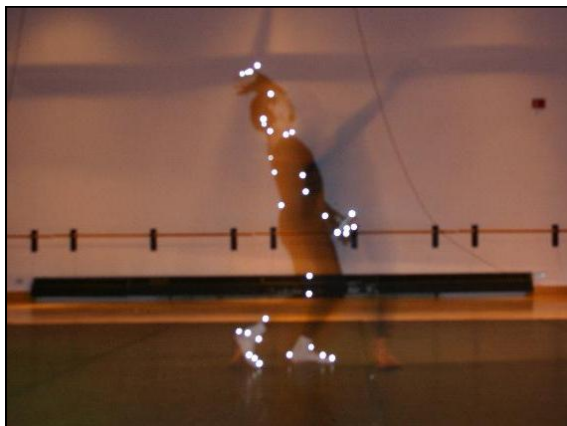


interpolation 1



initial





Con/cantation: (chaotic variations)
A computer-assisted theme and variations performance project

Radcliffe Institute for Advanced Study
Tuesday, April 17th
5pm
Radcliffe Gym
Radcliffe Yard
10 Garden Street
Cambridge, MA 02138
Free Admission

Created by David Capps and Liz Bradley
Video and layout: Angelika von Chamier

Music and algorithms: Josh Stuart
Motion capture and animation: Carnegie Mellon Graphics Laboratory
(Professor Jessica Hodgins, leader; Justin Mastry, motion-capture technician; Mi Mahler, animation and character design)
Code: David Townsend and Evan Sheehan
Inspiration: Diana Dibby

Made possible with support from the Radcliffe Institute for Advanced Study, the National Science Foundation (05-4024322), the David and Lucile Packard Foundation, and the Graduate Council on Arts and Humanities at the University of Colorado.

