
Name

Complex Systems Summer School

of the Santa Fe Institute

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2013 Lab Manual

Experiment 1: Chaotic Water Wheel (30 min)

Background:

- Water Wheel Equation [1,2]:

$$y_{n+1} = (y_n + 2 / \sin y_n) \text{ modulo } (2\pi / B),$$

where y_n is the angle between the vertical and the line through the center of the top bin and the center of the wheel after the n -th turn. B is the number of bins (here $B=8$). This dynamics is deterministic since y_{n+1} is a function of y_n . The dynamics is chaotic since the average slope of this function is greater than 1.



If static friction forces are larger than noise forces, the water wheel has a finite size attractor:

$$y_{n+1} = 0 \quad \text{if } |y_n| < y_a$$

$$y_{n+1} = (y_n + 2 / \sin y_n) \text{ modulo } (2\pi / B), \quad \text{if } |y_n| > y_a$$

where y_a is the size of the attractor. The dynamics of this system is very similar to the dynamics of the water wheel equation with “rational” initial condition.

Objectives:

- If the above mapping function is iterated with a digital computer the limiting state is a fixed point or a periodic attractor, because computers use decimals with finite precision. If the mapping function is iterated with real numbers, the limiting state is chaotic. This experiment illustrates that an iteration with decimals with finite precision is closer to the real system, because static friction and rounding have a similar effect on the dynamics.
- The water wheel equation is a complicated mapping function. Therefore reconstructing the mapping function requires a large number of data [3,4].
- The mapping function is both deterministic and incomputable, because the branches of the mapping function are arbitrary close, but computers have a finite precision.
- The data collected represent extremes of the trajectory. We show that these “outliers” are most valuable in modeling the dynamics.
- The slope of the mapping function gives an estimate of the Lyapunov exponent. The scatter in the mapping function gives an estimate of the noise level. The Lyapunov exponent and the noise level can be used to estimate how many turns are predictable.

Experiments:

- (1) **What kind of motion does the wheel perform as a function of the flow rate?** At which flow rate(s) does the behavior change? Estimate this flow rate with the beaker.
- (2) **Adjust the flow so that the turns appear irregular and determine the return map.** Record the angle where the wheel stops twenty times. Plot the y_n -values versus time step n and plot y_{n+1} versus y_n . Plot the theory. Do they match? Is the water wheel deterministic? Is the water wheel dynamics chaotic?
- (3) **Is there a range of y_n -values where the wheel never turns?** Tilt the axis of water wheel by about 30° with respect to the horizontal. Use the small rock in the container. This increases the

static friction and reduces the maximum amount of water in the bins. Now the torque that static friction can produce is comparable in size to the torque due to the weight force of the water in the bins. How large is range of angles y_n where the wheel does not turn? What are the consequences of a finite size attractor?

- (4) **EXTRA CREDIT:** Use Newton second law to derive the water wheel equation. Use the following simplifying assumptions: (i) the bucket empties at the bottom (i) kinematic friction force equals maximum static friction force.

Dynamics of a Water Wheel

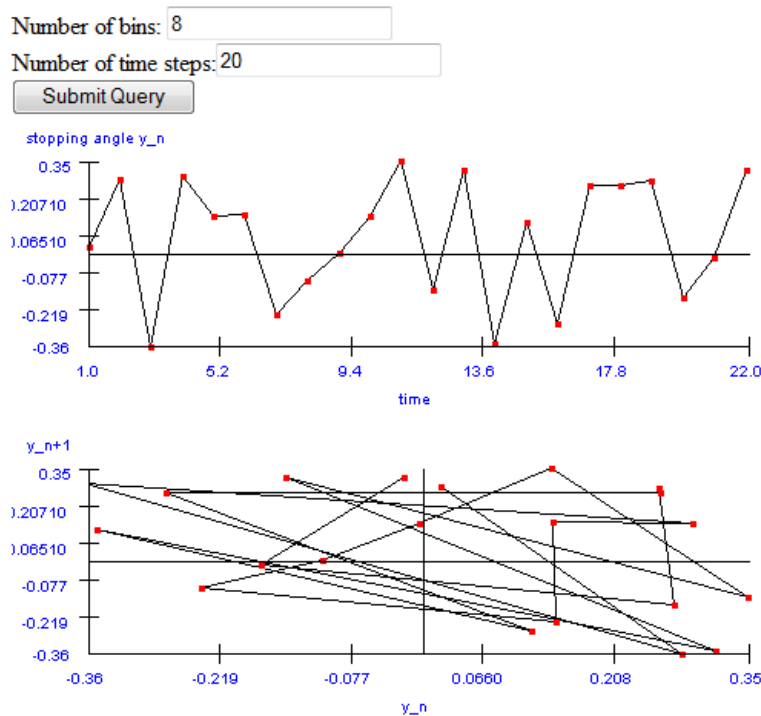


Figure 1. The top figure shows a typical time series y_n and the bottom figure the corresponding return map y_{n+1} versus y_n for $n=0, 1, \dots, 20$. Because the data set is small, the return map looks irregular (<http://www.how-why.com/cgi-bin/cyberprof/os.exe?home=&document=http://www.how-why.com/phys194/Notes/MapDynamicsActivity.html>, user: guest, password: guest)

MatLab Code:

```
clear all;
turns=20;
bins=8;
x=zeros(turns);
xnext=zeros(turns);

x(1)=rand(1)*.1; % random initial angle
for n=1:turns-1 % time loop
    x(n+1)=mod(x(n)+2/sin(x(n)),pi/bins);
    xnext(n)=x(n+1);
end
figure;
plot([1:turns],x);
figure;
plot(x,xnext,'sk');
```

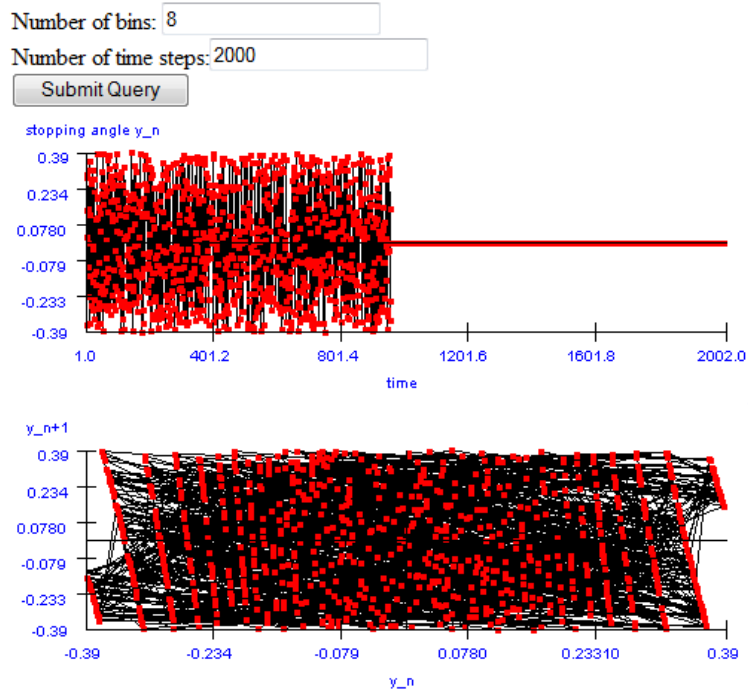


Figure 2. The top figure shows a typical time series y_n and the bottom figure the corresponding return map y_{n+1} versus y_n for $n=0, 1, \dots, 2000$. Because the data set is large, the return map looks more regular. The limiting state of the time series is a stationary state $y_n=0$, because computers use decimal numbers with a finite number of digits. Rounding and static friction have a similar impact on the dynamics.

References:

- [1] E.N. Lorenz, J. Atmos.Sci. 20, 130-141(1993)
- [2] N. Sachdeva, A. Hubler, A phase transition between chaos and a set of stationary states within a fractal basin of attraction in waterwheels with static friction and noise, [preprint](#)
- [3] J. Cremers, A. Hübler, *Construction of Differential Equations from Experimental Data*, Z. Naturforschung **42a**, 797-802 (1987), [reprint](#)
- [4] J. Breeden, A. Hübler, *Reconstructing Equations of Motion from Experimental Data with Unobserved Variables*, Phys.Rev. A **42**, 5817-5826 (1990), [reprint](#)
- [5] Alfred Hubler, *A noise induced phase transition to chaos in systems with static friction*, [Complexity](#) **18(1)**, 7-9(2012)

Experiment 2: Mixing and Segregation in Open Dissipative Systems (10 min)

Background:

- Experiment: Segregation of materials in a rotating drum (K. Hill, G. Gioia, D. Amaravadi, PRL **93**, 224301 (2004))
- Theory: According to the second law of thermodynamics, disorder, measured in term of entropy, increases in closed systems. Only processes which increase entropy occur spontaneously. This is different in open dissipative systems. In an open dissipative systems order can increase or decrease, depending of the system parameters.



Objective:

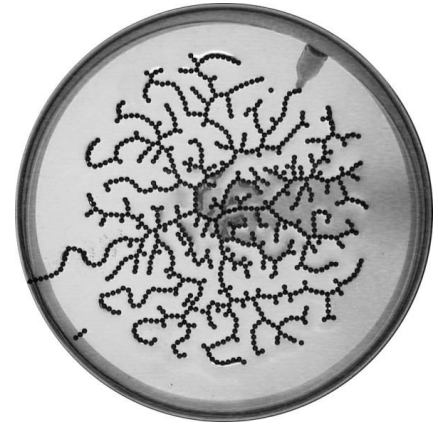
The objective of this experiment is to illustrate that in open dissipative systems there is a strong tendency to order and segregate.

Experiments:

- (1) Circular drum with small and large glass beads (fill level < 50%). What kind of patterns do you observe as a function of the angular frequency?** At which angular frequencies does the behavior change? Estimate the angular frequencies in rotations per second. Sketch the emergent pattern.
- (2) Circular drum with small and large glass beads (fill level >50%). What kind of patterns do you observe at small frequencies (turn manually)?** Sketch the emergent pattern.
- (3) Square drum with small and large glass beads. What kind of patterns do you observe as a function of the angular frequency (turn manually)?** At which angular frequencies does the behavior change? Estimate the angular frequencies in rotations per second. Sketch the emergent pattern.
- (4) Design a system which mixes rapidly.** (Hint: complicated boundary => complicated segregation pattern, friction between particles helps to reach this pattern quickly)

Experiment 3: Arbortrons (30min)

Background: Ilya Prigogine received the Nobel Prize in 1976 for his observation that stable stationary states of certain open dissipative systems are minimum-entropy-production states. An example is the aggregation of conducting particles in Castor oil under the influence of an electric current .



Objective: This experiment shows the growth of ramified transportation networks with minimum entropy production (resistance) for three different types of initial conditions. Many natural and man-made systems grow networks, for example rivers , blood vessels, the internet, and road networks. However there are very few laboratory experiments on network growth. This experiment is currently probably the only lab experiment on network growth.

Some observables, such as the number of end points, depend only on the system parameters such as the number of particles and therefore easily reproducible. But other observables, such as the number of trees, depend sensitively on the initial conditions (chaos) and the number of particles.

Experiment:

- (1) **Produce a random distribution of beads, then increase the voltage and observe the growth process?** (a) Record the limiting structure with the copy machine (cover the dish with a piece of blank paper). Repeat the experiment and record the limiting structure with the copy machine. (b) Are there any closed loops? Make a closed loop and check if it opens. (c) Are the resulting structures stable? Break one of the chains (voltage off) and watch it repair (voltage on). (d) Do the trees repel each other? (e) Does the network disintegrate when the voltage is off? How does it decay? (f) Count the number of tips. Count the number of branching points. Count the number of particles. Count the number of trees (with more than 5 particles). How reproducible is the number of tips and branching points?
- (2) **Place all the beads in the center, lower the high voltage electrode to about 1 inch above the oil, and repeat the experiment 1 part (a).** How many trees emerge?
- (3) **Place all the beads near the perimeter, raise the high voltage electrode to about 1.5 inches above the oil and repeat the experiment 2.** How many trees emerge?

[1] [M. Dueweke, U. Dierker, A. Hubler, *Self-assembling Electrical Connections Based on the Principle of Minimum Resistance*, Phys.Rev.E **54**, 496-506 \(1996\).](#)

[2] [M. Sperl, A Chang, N. Weber, A. Hubler, *Hebbian Learning in the Agglomeration of Conducting Particles*, Phys.Rev.E. **59**, 3165-3168 \(1999\).](#)

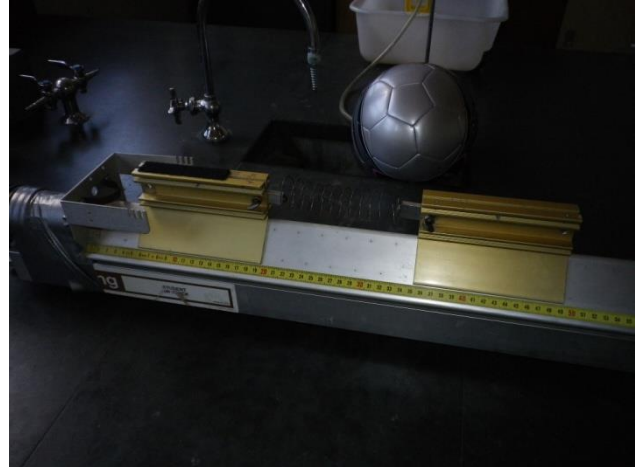
[3] [D. Smyth, A. Hubler, *A Conductivity-Dependent Phase Transition from Closed-Loop to Open-Loop Dendritic Networks*, Complexity **9**, 56-60\(2003\).](#)

[4] [Joseph K. Jun and Alfred W. Hubler, *Formation and structure of ramified charge transportation networks in an electromechanical system*, PNAS **102**, 536–540 \(2005\).](#)

- [5] V. Soni, P. Ketisch, J. D. Rodriguez, A. Shpunt, and A. W. Hubler, *Topological similarities in electrical and hydrological drainage networks*, J. Appl. Phys. 109, 036103 (2011)
- [6] M. Singleton, G. Heiss, and A. Hubler, *Optimization of Ramified Absorber Networks Doing Desalination*, Phys. Rev. E 83, 016308 (2011)
- [7] Alfred Hubler, Cory Stephenson, Dave Lyon, and Ryan Swindeman, *Fabrication and Programming of Large Physically Evolving Networks*, [Complexity 16\(5\), 7-8\(2011\)](#)
- [8] A. Hubler and J. Crutchfield, *Order and disorder in open systems*, [Complexity 16\(1\), 6-9\(2010\)](#)
- [9] Martin Singleton, Gregor Heiss, Alfred Hubler, *The optimal shape of roots*, [Complexity \(15\)4, 9-11\(2010\)](#)

Experiment 4: Chaos of a Bouncing Ball - Time Discrete Models for Time Continuous Systems

Background: Stephen Wolfram promotes the idea that very simple discrete model provide good models for real systems [1]. Hubler and Gerig show that difference equation can be more accurate in modeling the dynamics of a macroscopic systems than differential equations [2].



Newton second law,

$$F = m a \quad (1)$$

for the center of mass provides a time continuous model: *weight force = mass * acceleration, i.e. $m g_e = m d^2x/dt^2$* (free fall), and $v = -v$ if $x=0$ (collision), where m is the mass of the two-cart system and x is the center of mass. x'' is the acceleration of the center of mass. $g_e = g \sin(A) \approx g/100$ is the effective gravitational constant. A is the angle between the air track and the horizontal. The continuous model predicts that the collision is infinitely short and that the peak positions of the center of mass decrease monotonically.

However the equation of motion for the center of mass derived from a more microscopic model is different: It is a difference equation which can produce irregular motion. But which model is correct, Newton's second law for the center of mass or the difference equation derived from a more microscopic model? In this experiment we will take a look at the experiment to decide which model is more accurate.

But first, we derive the equation of motion. The experimental setup consists of two rigid carts on an air track which are coupled by a soft spring. The air track is slightly inclined. We use Newton's second law to model the motion of both carts with the position x_1 and velocity v_1 of the first cart and the position x_2 and velocity v_2 of the second cart. Then we change to center of mass variables to $x_c = (x_1+x_2)/2$, $v_c = (v_1+v_2)/2$ and vibrational variables $x_v = (x_1-x_2)/2$, $v_v = (v_1-v_2)/2$ and average the transformed equation of one full period T of the vibrational variables. The integration eliminates the vibrational variables and converts the differential equation to a difference equation with time step T .

This difference equation for the position and velocity of the center of mass predicts that the duration of the collision is of length $T/2$ and that the dynamics can be irregular very much like the dynamics of the experimental system. This is consistent with the experiment.

In contrast, the time-continuous model (Eq. 1) predicts that collision is instantaneous and that the amplitude of the bounces decreases gradually and monotonically. This is **not** consistent with the experiment.

In summary: This experiment shows that the time discrete model (difference equation) for the center of mass motion describes the experimental dynamics more accurately than the time-continuous model.

Objective: This experiment illustrates the dynamics of a bouncing ball in slow motion. It illustrates that a time discrete map is a good model if the spring is not very stiff. If the spring is stiff, then the two-cart object is a rigid body, and both the time discrete model and the time continuous model are accurate. However the time continuous model is easier to solve analytically. In conclusion, time continuous models of the dynamics of mechanical systems work well for rigid bodies, but are inaccurate for others, including a bouncing soccer ball.

Experiment:

- (1) **Place the two carts at the center of the air track and observe a sequence of collisions.** How often does the lower cart collide with the bumper? Record the peak positions of the upper car. Are they monotonically decreasing?

- (2) **Study the motion when the amplitude is small.** There are two types of motion: (i) both cars are moving and (ii) only the upper car is moving. The time discrete model predicts the complicated two-car motion and the transition to the one-car motion, whereas the time continuous model predicts a simple motion.

References:

- [1] Stephen Wolfram, A New Kind of Science, [reprint](#)
- [2] [Hubler and A. Gerig, Are Discrete Models more Accurate? Complexity 16\(2\), 5-7\(2010\)](#)

Experiment 5: Disappearance of Wave Chaos as an Example for Adaptation to the Edge of Chaos

Background: The concept of adaptation to the edge of chaos has been a common theme in Complex Systems Research

Objective: This experiment illustrates adaptation to the edge of chaos in a single system, a water droplet.



When the shape of the water droplet leads to an aperiodic wave dynamics (quantum chaos = wave chaos) then the shape of the droplet is changing. If the wave pattern is stationary, the shape of the droplet stays the same. Hence wave chaos disappears eventually. This is an illustration of adaptation to the edge of chaos.

However, even if the wave pattern is stationary, a small perturbation can push the system back into chaos. This is called a chaotic outbreak.

Experiment:

- (1) Place water droplet on the disc, attach the gray function generator, and vibrate it with 0.123 kHz. Observe the rearrangements of surface waves and evolution of the drop perimeter.** Does the wave pattern become stable? Do you see chaotic outbreaks? What is the impact of the size of the droplet and the shape of the droplet? Watch what happens. Sketch a droplet with its wave pattern.
- (2) Wipe the plate dry. Place Lycopodium powder on the disk, and change amplitude and frequency. Vibrated powder is like a liquid without surface tension. Observe the emergent patterns.**
Do you see some regions where the powder form a flat layer? Can you find a frequency and amplitude where the emerging patterns look like erosion patterns? Can you produce volcanic eruptions?

Settings:

Wavetek function generator: Frequency Range = 200 Hz, Function = “~” (sine wave), Attenuator = Not Pressed, Symmetry = in the middle of the range, Amplitude Modulation = OFF, DC Offset=0, Amplitude = small, Frequency = 123 Hz, Sweep = in the middle of the range, PUSHED, Rate = Sweep = in the middle of the range, PUSHED;

Carver power amplifier: Sequence = OFF; Left= 0dB; Right = 0dB

[1] [P. Melby, J. Kaidel, N. Weber, A. Hubler, *Adaptation to the Edge of Chaos in the Self-Adjusting Logistic Map*, Phys.Rev.Lett 84 5991-5993 \(2000\).](#)

[2] [P. Melby, N. Weber, A. Hubler, *Robustness of Adaptation in Controlled Self-adjusting Chaotic Systems*, Phys. Fluctuation and Noise Lett. 2, L285-L292 \(2002\).](#)

[3] [P. Melby, N. Weber, A. Hubler, *Dynamics of Self-Adjusting System with Noise*, CHAOS 15, 33902 \(2005\).](#)

[4] [Wotherspoon and A. Hubler, *Adaption to the Edge of Chaos with Random-Wavelet Feedback*, Journal of Physical Chemistry A 113\(13\), 223-3226\(2009\)](#)

Experiment 6: Oscillating Chemical Reactions (15 min)

Background: The BZ reaction is one of the most well known chemical oscillators.

[J.M. Bodel, C. Vidal, A. Pacault, F. Argoul, in Non-Equilibrium Dynamics in Chemical Systems, ed. C. Vidal and A. Pacault, Springer Series in Synergetics, 102-107(1984)]



Objective: This experiment illustrates spatially inhomogeneous chemical reactions. Get an intuition on the dynamics of chemical waves.

Procedure: Use safety goggles and gloves. Start with a small empty Petri dish. Add 1.8 ml of 0.6 molar H_2SO_4 . Add 1.2 ml of 0.55 molar Malonic Acid. Add 3 drops of Ferroin. Add 1.2 ml of 1 molar NaBrO_3 . Mix the liquids by gently swirling the dish until a homogenous mixture is obtained. Make sure that the liquid covers the bottom of the container and place in a shady spot. After the initial mixing be careful not to disturb the liquid.

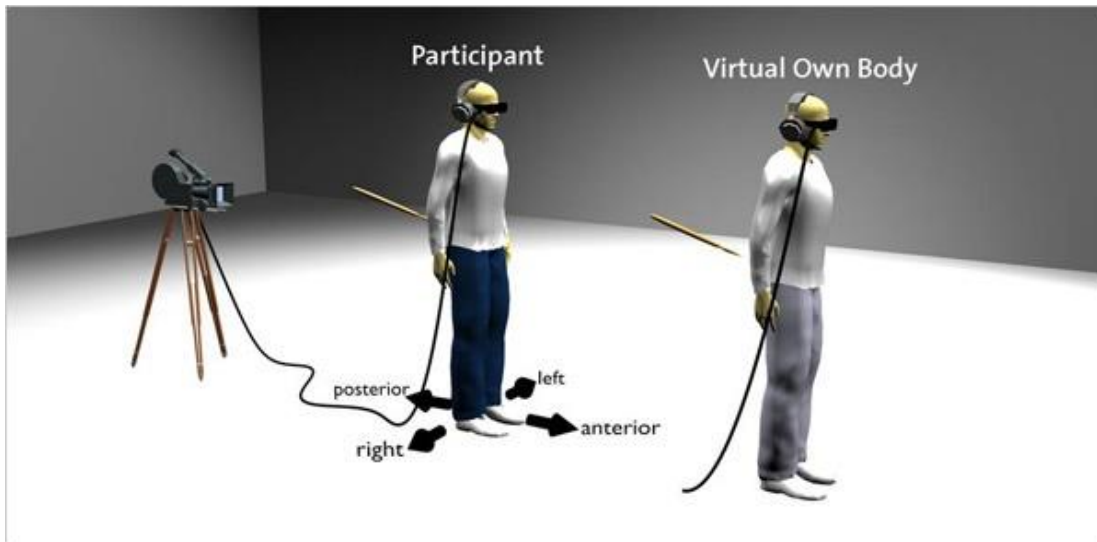
Data collection:

- (1) What happens during the first few minutes? Describe your observations.
- (2) When do the first wave fronts occur?
- (3) How fast do the wave fronts move?
- (4) The system creates chemical waves for about 2 hours. Do other experiments now, but check every 15min on the status of your BZ system.

Experiment 7: Exploring Mixed Reality States in an Interreality System (15 min)

Background: An interreality system comprises a real world system (the driven physical pendulum on the desk) and a similar virtual system (the virtual pendulum on the computer). The two systems interact through a small bi-directional coupling. The magnitude of the coupling is fixed [1].

Objective: Show that a real system enters a mixed reality state, if the dynamics of the virtual counterpart is close enough.



Experiment: In this experiment we investigate human-computer interactions.

- A video camera is placed in the third-person perspective, i.e. behind the person looking at the head and shoulder of the person at an angle of 45 degrees from the horizontal.
- The subject sees video image of itself on a monitor.
- A stick strokes person's shoulder gently at a place where the camera can see it.



People reported the sense of being outside their own bodies, looking at themselves from a distance where the camera is located.

- While people were experiencing the illusion, the experimenter pretended to smash the virtual body by waving a hammer just below the cameras. Immediately, the subjects should register a threat response as measured by sensors on their skin. They sweat and their pulse may race.

Conclusion: If the participant has a sweat response the person is in a mixed reality state, otherwise the person is in a dual reality state.

[1] [V. Gintautas and A. Hubler Experimental evidence for mixed reality states in an interreality system. Phys. Rev. E 75, 057201 \(2007\).](#)

Experiment 8: Thermo-Acoustic Resonator (5 min)

Background: A thermo-acoustic resonator converts thermal energy into sound. A flame is placed near the end of a vertical circular tube. A sound wave in the tube makes the flame flicker. The flickering flame strengthens the sound wave. The sound wave is particularly strong if the flame is placed just barely inside the tube. However if the sound wave is too strong the draft of the air may blow out the flame.

Objective: This experiment illustrates the paradigm “the whole is more than the sum of the parts”. The sound wave and its effect on the flame depend mostly on macroscopic quantities, such as the shape of the container and the location of the openings, and much less on the microscopic properties of the tube material or the gas inside the tube.

Procedure: Ignite the grill lighter, hold it vertically, and insert the flame into the tube. Avoid heating the glass!

Data collection:

- (1) Move the flame slowly in and out. Describe your observations.
- (2) Move the flame fast in and out. Describe your observations.

[1] [Alfred W. Hbler, Predicting Complex Systems with a Holistic Approach, Complexity 10, 11-16 \(2005\)](#)



Experiment 9: Solitons (10 min)

Background: In spatially extended nonlinear systems, such as a chain of pendulums, small amplitude waves move at a constant speed and become smaller as a function of time. Large amplitude waves are complicated. Some of them can be solitons. A soliton is a wave where the amplitude stays constant. In a chain of pendulums, there are solitons and anti-solitons. They are called Sine-Gordon solitons. Sine-Gordon solitons are used as a model for matter and anti-matter.



Objective: Study the creation and dynamics of Sine-Gordon solitons.

Procedure: Use the soliton machine to watch the creation, motion, and annihilation of solitons.

Data collection:

- (1) Create a soliton and an anti-soliton in the center of the chain, by rotating the center of the chain horizontally by 360 degrees. Describe your observations. Is there an attractive **force between the soliton and the anti-soliton**.

- (2) Create another soliton and an anti-soliton pair in the center of the chain, by rotating the center of the chain horizontally by 360 degrees again. Describe your observations. Is there an attractive **force between two solitons**?

- (3) Move a soliton or an anti-soliton towards the open end of the chain. What happens to the soliton/anti-soliton? Is there an attractive force between a soliton and the end of the chain?

- (4) Move a soliton close to an anti-soliton. What happens to the soliton and the anti-soliton?

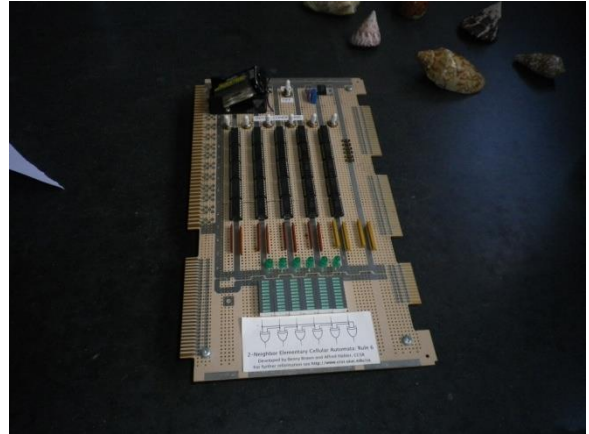
Further reading:

[1] Dene Farrell, Alfred Hubler, Joseph Brewer, Ines Hubler, *Acceleration beyond the wave speed in dissipative wave-particle systems*, [Complexity 15\(5\), 8-11 \(2010\)](#)

Experiment 10: Elementary Cellular Automata (10 min)

Background: Elementary Cellular Automata are dynamical systems which are discrete in time, discrete in amplitude, discrete in space, and local. $x_i(t)=0,1$ is the amplitude at time step t at location i , where $t=0,1,2,\dots,T$ and $i=1,2,\dots,N$. T is the number of time steps and N is the number of cells. Since CA are local they obey the following rule:

$$x_i(t+1)=f(x_{i-1}(t), x_i(t), x_{i+1}(t))$$



For more details see: S. Wolfram, A New Kind of Science.

Objective: This experiment illustrates an electronic device which is well described by a CA.

Experimental Procedure: Use the hardware implementation of the CA. The CA has $N=6$ cells, $T=10$ time steps and circular boundary conditions: $x_1(t+1)=f(x_N(t), x_1(t), x_2(t))$ and $x_N(t+1)=f(x_{N-1}(t), x_N(t), x_1(t))$.

Data collection:

(1) What are the patterns for the following initial conditions:

111111 100000 100001 111000

(2) Which initial condition produces the pattern

??????

111111

000000

000000

000000

...

(3) What is the function f ?

[1] [A. Hubler, Digital wires, Complexity 14\(5\), 7-9 \(2009\)](#)

Experiment 11: Video Feedback (10 min)

Background: Nonlinear mapping functions have a rich solution set in particular if they can store information. In video feedback systems a camera is pointed toward a TV screen and records the image on the screen, which is then displayed on the screen. The image on the screen changes 60 times per second. However, the fluorescent surface of the screen memorizes previous images for a fraction of a second. Video feedback systems can be modeled as nonlinear mapping functions with thousands of variables and memory.



Objective: This experiment illustrates an analog electronic device which iterates a high dimensional mapping function with memory.

Experimental Procedure: Point the camera toward the center of the screen and rotate the camera by 30 degrees. Adjust the focus to obtain sharp images. Choose a large contrast. Zoom in to the bright area in the center of the screen.

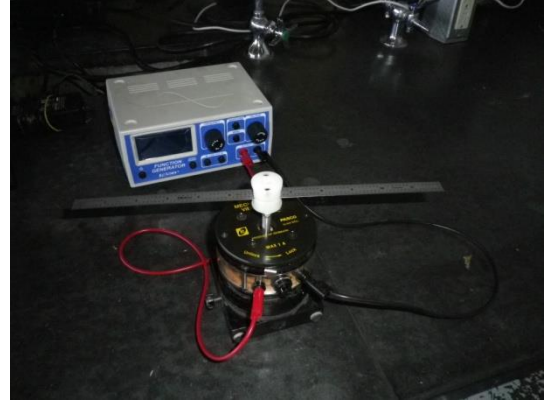
Data collection:

Try to create colorful spirals and meandering patterns.

[1] James P. Crutchfield, *Space time dynamics in video feedback*, Physica 10D, 191-207 (1984)
<http://csc.ucdavis.edu/~cmg/papers/Crutchfield.PhysicaD1984.pdf>
<http://www.youtube.com/watch?v=B4Kn3djJMCE>

Experiment 12: Quantization Phenomena in dissipative Wave- Particle Systems (10 min)

Background: In classical mechanical systems forces are created by static fields. In classical systems with drag forces and noise, the limiting dynamic is near the minimum of the potential energy. Waves can push around particles too. In systems with noise and waves, the limiting dynamic is most of the time near a preferred state. There are often many preferred states. The system “jumps” between preferred states, similar to a quantum system jumping between eigen states. The formalism of quantum mechanics can be used to describe the dynamics of such dissipative wave-particle systems.



Objective: This experiment illustrates preferred states in dissipative wave particle systems.

Experimental Procedure: Center the measuring stick and choose a frequency of 0.013 kHz or 0.080 kHz.

Data collection:

Push the stick away from the center and watch it return to the center. What happens to the vibration of the stick during this process?

Tilt the device with and without vibrations by 60 degrees. What is the difference?

[\[1\] D. Sivil, A. Hubler, Quantized motion of a particle pushed around by waves, Complexity 15\(2\), 10-12\(2009\)](#)

Experiment 13: Synthetic Atoms: High energy density and a record power density (10 min)

Background: Energy storage in molecules and capacitors is both due to interaction of positive and negative charges. But capacitors can store much less energy, because they arc if the electric field exceeds about 1 million V/m (see Fig. 1), whereas in molecules there is no charge recombination even if the field is 10,000 times larger. Quantization prevents arcing. Recently it was found [1-4] that in nano-capacitors arcing is suppressed by quantization as well (see Fig. 2). Therefore the energy storage capacity (limiting energy density) of molecules and nano-capacitors is similar. However the discharge time of nano-capacitors is much shorter than the discharge time of chemical systems. For instance in a Li-ion battery energetic molecules have to diffuse to the electrode release their energy and then diffuse back into the middle of the electrolyte, which is a very slow process on the order of minutes and hours. In contrast in nano-capacitors the energy release occurs as soon as electrons have crossed the nm-gap at a speed which is close to the speed of light, which is in the sub femtosecond range. Therefore nano-capacitors have a record power density, more than 20 orders of magnitude larger than Li-ion batteries and about 1 million times larger than nuclear devices (see diagram on the right). This means 50 milligram (3 table spoon) of nano capacitors can deliver the same power as a nuclear chain reaction of 50kg (100 pounds) of highly enriched nuclear material. Currently, 50 milligrams of nano capacitors can store roughly the same amount of energy as a digital camera battery, but can potentially store 100-1000 times more energy.

Objective: This experiment illustrates preferred that in capacitors arcing occurs at about 1 million V/m. Further it illustrates that the 4nm natural aluminum oxide layers on aluminum foils have a high resistance, but arc if the voltage exceeds about 1 V/nm. The plot on the right illustrates the arcing in a 4 nm aluminum oxide layer.

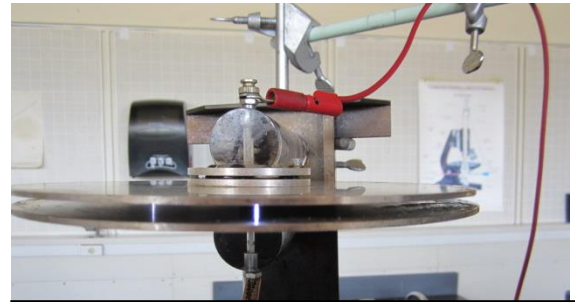


Figure 1. Arcing parallel plate capacitor. The dielectric strength is about 1 million V/m.

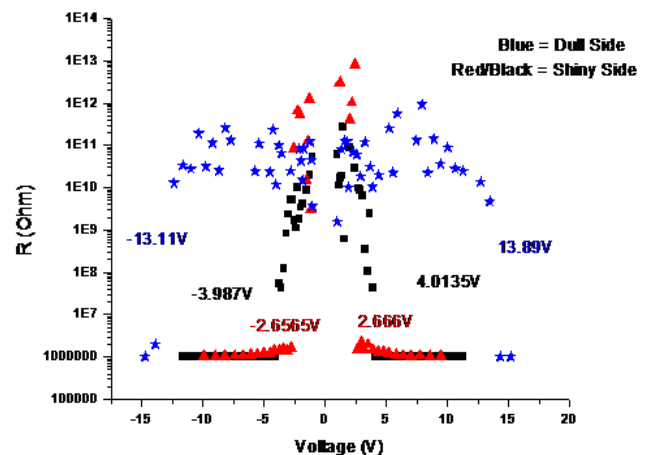
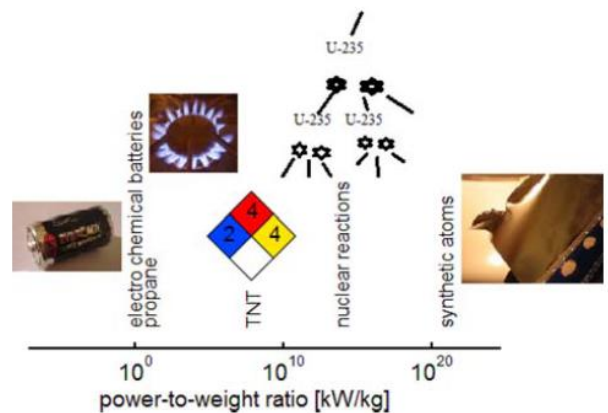
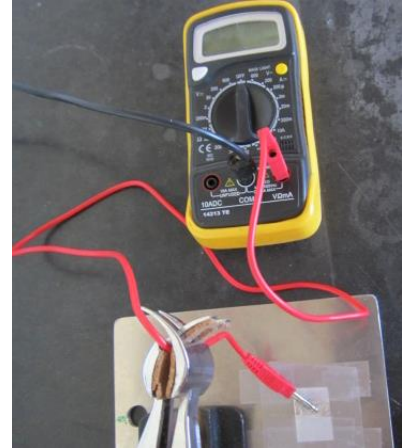


Figure 2. Dielectric Strength of a 4nm natural aluminum oxide layer is about 1 billion V/m: Resistance of the oxide layer versus applied voltage.



Experimental Procedure:

- (1) Separate the plates of the parallel plate capacitor by 1cm. Increase the voltage until arcing between the plates occurs. Determine the dielectric strength: (arcing voltage) / (gap size).
- (2) Pour a small amount of castor oil on the lower plate. Connect the parallel plate capacitor, and raise the voltage to 10kV. Slowly decrease the gap size between the plates, by raising the lower plate with a plastic stick. You should see the emergence of an array of oil columns, which repel each other and compete with each other.
- (3) A) Switch the yellow multi-meter to the 2M Ohm range. Place the electrode at new position. The reading should be 1 (out of range, too large)
B) Switch the yellow multi-meter to the 200k Ohm range. The reading should be 1 (out of range, too large)
C) Switch the yellow multi-meter to the 20k Ohm range (10V). The reading should be 1 (out of range, too large)
D) Switch the yellow multi-meter to the 2k Ohm range. The reading should be 0 (out of range, too small). This means arcing has occurred and destroyed the aluminum oxide layer.
E) Switch the yellow multi-meter to the 2M Ohm range. The reading should be 0 (out of range, too small).



Summary: At about 1V/nm arcing destroys the aluminum oxide layer as shows in Fig. 2. The layer will self-repair within one hour. Oxygen in the air will create a new 4nm aluminum oxide layer at the location where the arcing occurred.

- (4) Cut off a short segment of the bicycle lock. This illustrates that very little energy is necessary to break a lock. High power = moderate amount of energy applied at a very small area (knives, sissors) or for a very short period of time (impulse of a hammer) cuts and breaks things.

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