Introduction to Nonlinear Dynamics

Santa Fe Institute

Complex Systems Summer School

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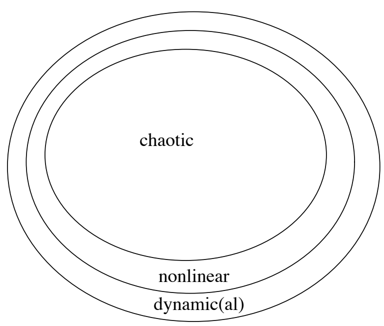
Chaos

Complex behavior, arising in a deterministic nonlinear dynamic system, which exhibits two special properties:

- sensitive dependence on initial conditions
- characteristic structure…



Chaos

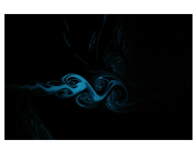
Complex behavior, arising in a deterministic nonlinear dynamic system, which exhibits two special properties:

- sensitive dependence on initial conditions
- characteristic structure…

Systems that exhibit chaos are ubiquitous; many of them are also simple, well-known, and "well-understood"

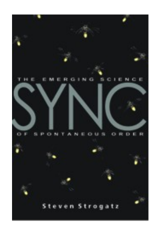
Where nonlinear dynamics turns up

- Flows (of fluids, heat, …)
  - Eddy in creek
  - Weather
  - Vortices around marine invertebrates
  - Air/fuel flow in combustion chambers



Where nonlinear dynamics turns up

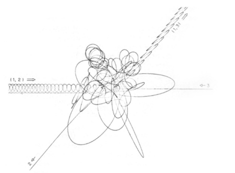
- Driven nonlinear oscillators
  - Pendula
  - Hearts
  - Fireflies



- and lots of other electronic, chemical, & biological systems

Where nonlinear dynamics turns up

- Classical mechanics
  - three-body problem
  - paired black holes
  - pulsar emission
  - ….
- Protein folding
- Population biology
- And many, many other fields (including yours)



Hut & Bahcall *Ap.J.* ***268***:*319*

- continuous time systems:
  - time proceeds smoothly
  - "flows"
  - modeling tool: differential equations
- discrete time systems:
  - time proceeds in clicks
  - "maps"
  - modeling tool: difference equation

A useful graphical solution technique

- "cobweb" diagram
- *aka* return map
- *aka* correlation plot

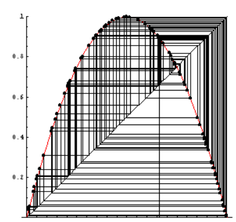
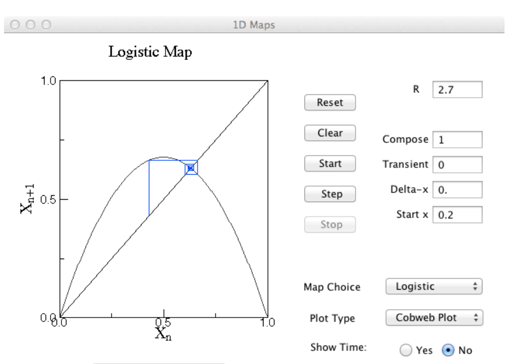


Image from Doug Ravenel's website at URochester



Bifurcations

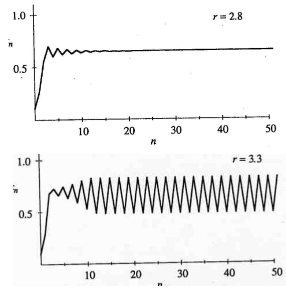
Qualitative changes in the dynamics caused by changes in *parameters*

Bifurcations

Qualitative changes in the dynamics caused by changes in *parameters:*

- Heart: pathology
- Eddy in creek: water level
- Olfactory bulb: smell
- Brain: blood chemicals
- etc. etc.

Bifurcations in the logistic map

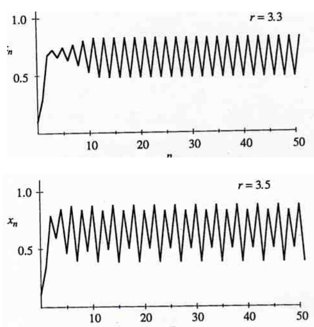


R=2.8

R=3.3

Note: in discrete time plots, it makes no sense to connect dots!!

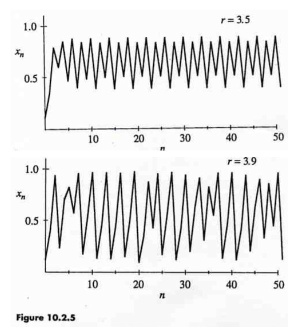
Plots from *Strogatz*



R=3.3

R=3.5

Plots from *Strogatz*



R=3.5

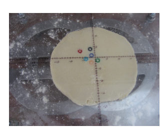
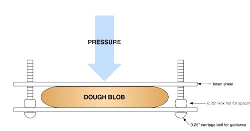
R=3.9

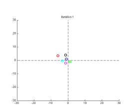
Plots from *Strogatz*






Courtesy of Allison Brown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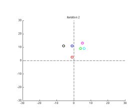
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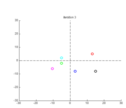
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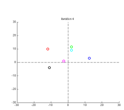
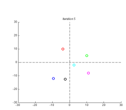
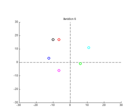
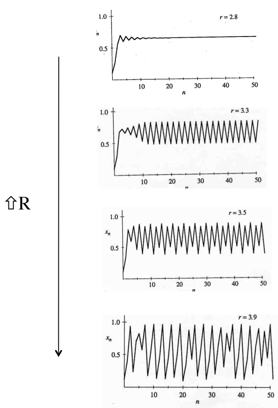
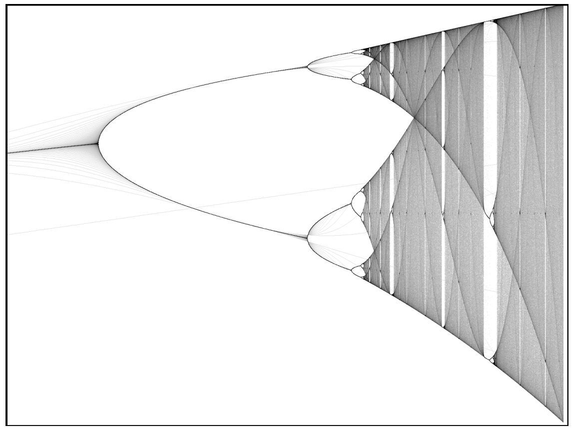

n=3

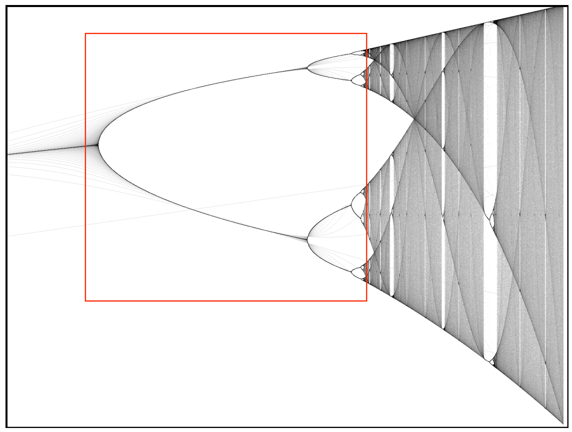

n=4


n=5



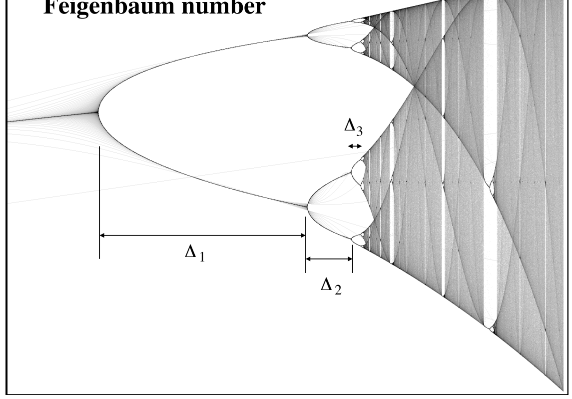


- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)



- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- *period-doubling cascade @ low R*

Feigenbaum number

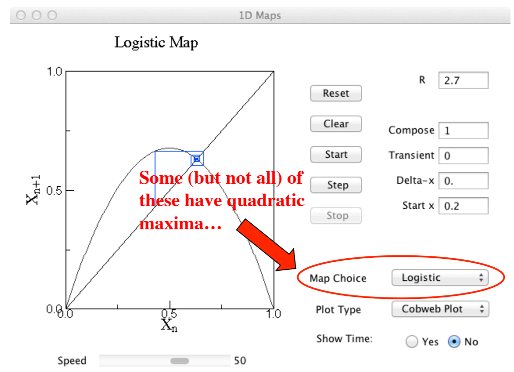


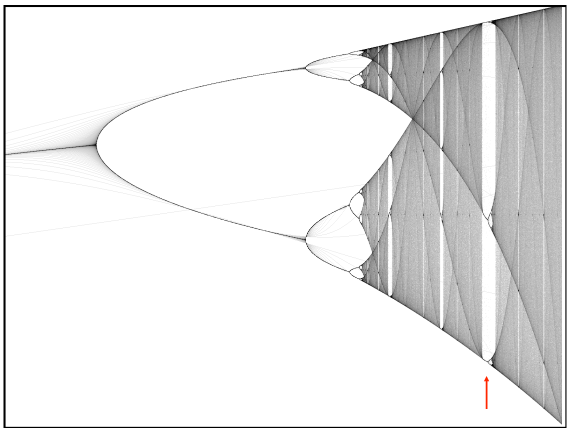
Universality!

Feigenbaum number and many other interesting chaotic/dynamical properties hold *for any 1D map with a quadratic maximum*.

Proof: renormalizations. See Strogatz §10.7

Don't take this too far, though…

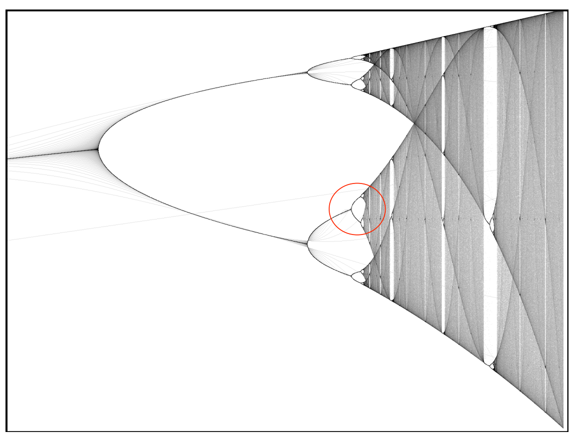




- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- period-doubling cascade @ low R
- *windows of order within the chaos, complete with their own period-doubling cascades (e.g., 3 to 6 to 12)*

A bit more lore on periods and chaos

- Sarkovskii (1964)
  3, 5, 7, …3x2, 5x2, …3x2^2, 5x2^2, … 2^2, 2, 1
- Yorke (1975)
- Metropolis *et al.* (1973)

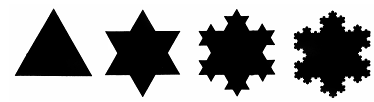


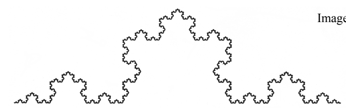
- chaos
- veils/bands: places where chaotic attractor is dense (UPOs)
- period-doubling cascade @ low R
- windows of order within the chaos, complete with their own period-doubling cascades (e.g., 3 to 6 to 12)
- *small copies of object embedded in it (fractal)*

lots of other interesting stuff, too — e.g., Misiurewicz points

Fractals

- non-integer Hausdorff dimension
- self-similar

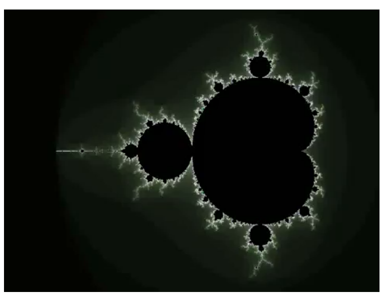




Images from Gleick

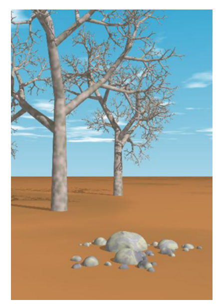
Examples: Cantor set, coastlines, trees, lungs, clouds, drainage basins, …

The Mandelbrot set



www.youtube.com/watch?v=G_GBwuYuOOs

Fractals in computer graphics



Matthew Ward, WPI
davis.wpi.edu/~matt/courses/fractals/trees.html

Fractals in the wild

http://paulbourke.net/fractals/googleearth/

Fractals in maps

Newton's method on $x^4 - 1 = 0$

From Strogatz

Fractals and chaos…

The connection: *many (most)* chaotic systems have fractal state-space structure.

But **not** "*all*."

The rest of today…

- Lunch (cafeteria downstairs)
- Dynamics Lab I:
  - Meet here at 1:30pm
  - Bring your laptop, if you have one here
  - Make sure it has Java installed, and some browser besides Chrome
  - Lab handouts on the CSSS wiki
- Intro to Santa Fe (3pm, here)
- Public lecture tonight (shuttles at 6:45)