

# A Multi-Agent Simulation Using Cultural Algorithms: The Effect of Culture on the Resilience of Social Systems

**Ziad Kobti**

School of Computer Science  
University of Windsor  
Windsor, ON, Canada N9B-3P4  
[kobti@uwindsor.ca](mailto:kobti@uwindsor.ca)

**Robert G. Reynolds**

Department of Computer Science  
Wayne State University  
Detroit, MI 48202  
[reynolds@cs.wayne.edu](mailto:reynolds@cs.wayne.edu)

**Tim Kohler**

Washington State University  
Pullman, WA 92037  
[tako@wsu.edu](mailto:tako@wsu.edu)

**Abstract-** Explanations for the collapse of complex social systems including social, political, and economic factors have been suggested. Here we add cultural factors into an agent-based model developed by Kohler for the Mesa Verde Prehispanic Pueblo region. We employ a framework for modeling Cultural Evolution, Cultural Algorithms developed by Reynolds [1979]. Our approach investigates the impact that the emergent properties of a complex system will have on its resiliency as well as on its potential for collapse. That is, if the system's social structure is brittle, any factor that is able to exploit this fragility can cause a collapse of the system. In particular, we will investigate the impact that environmental variability in the Mesa Verde had on the formation of social networks among agents. Specifically we look at how the spatial distribution of agricultural land and the temporal distribution of rainfall impacts the systems structure. We show that the distribution of agricultural resources is conducive to the generation of so called "small world" networks that require "conduits" or some agents of larger interconnectivity to link the small worlds together. Experiments show that there is a major decrease in these conduits in early 1200 A.D. This can have a serious potential impact on the networks resiliency. While the simulation shows an upturn near the start of the 14th century it is possible that the damage to the network had already been done.

## 1 Introduction

The model used in this study was initially designed by Tim Kohler [2000] and a team of developers from Washington State University to simulate the early Anasazi settlement and farming practices using environmental and archeological data. The object of the model was to present an approach to understand the reasons leading to their eventual disappearance from the Mesa Verde region based upon modern archaeological knowledge of the region.

In our work we extend Kohler's original model by establishing a framework using Cultural Algorithms, in which to embed a population of social agents, and to document the impact that their social interaction and

cultural learning has on the system performance. In this paper we focus on establishing the framework for social interaction and the acquisition of cultural knowledge. The Cultural Algorithm framework is described in the following section.

In section 3 we discuss how Kohler's model is extended to include social parameters. In particular, we are interested in the nature of the social networks that are produced by various combinations of social parameters. While the Cultural Algorithm provides a framework in which the agents can learn to select various combinations of co-adapted parameters, our goal here is to see how varying these parameters causes certain aspects of the social networks to appear and or dissipate. Specifically, we look at one parameter, search or move radius. This determines how far away the original family a new family will settle, or if a family decides to leave its current location how far away it can look for a new location.

In section 4, we describe the emergent social network. In all cases they are variation on the small world network. However, the parameters of the network vary as the search radius changes. A small move radius produces dense networks, while a larger move radius produces a sparser one. Section 5 gives our conclusions.

## 2 Adaptation in the Cultural Framework

Holland developed a formal framework for any generic adaptive systems [Holland, 1975]. His framework for adaptation concerns a system that is able to alter its structure and/or behavior based on the experience in some set of performance environments [Reynolds, 1979]. Adaptability is the capacity to function in an uncertain or unknown environment, and to use information to evolve and learn and it can take place at three different levels: population, individual and component. [Angeline, 1995]. Flannery [1968] proposed a model of cultural adaptation and evolution. These ideas form the basis for the Cultural Algorithm framework used here.

Cultural Algorithms consist of a social population and a belief space [Reynolds, 1979]. Selected individuals from the population space contribute to the cultural knowledge by means of the acceptance function. The knowledge resides in the belief space where it is stored and manipulated based on individual experiences and their

successes or failures. In turn, the knowledge controls the evolution of the population by means of an influence function. A Cultural Algorithm thereby provides a framework in which to accumulate and communicate knowledge within a population so as to allow self-adaptation in an evolving model.

There are five basic categories of knowledge that are important in the belief space of a cultural evolution model: situational, normative, topographic, historical or temporal, and domain knowledge [Reynolds, 2003]. In our cultural model all of these knowledge sources are present as well. For example, in our current model we assume that agents can get access to knowledge about the distribution of agricultural land (topographic knowledge), the distribution of rainfall as it occurs over time (history or temporal knowledge), agricultural planting and harvesting techniques (domain knowledge).

We will concentrate on the acquisition or learning of just two types of knowledge by agents here, situational and normative knowledge. Situational knowledge is a “snapshot” of the state of the world. The world can be viewed as a sequence of situations linked by social behaviors [Russell, 1995]. Examples of specific individual experiences correspond to a set of situational knowledge or relationships between individuals and their physical and social environments. Normative knowledge on the other hand describes how a rational agent should act in terms of ranges of acceptable behavior [Russell, 1995]. In other words, normative knowledge defines a standard or ideal that can be used to judge which behavior is desirable or undesirable [Valente, 1994]. E.g. Acceptable speeds on an expressway can range between 45 as a minimum and 70 as a maximum. Norms provide standards for interpreting and determining individual behaviors by providing guidelines within which individual adjustments can be made. Usually older individuals tend to acquire more normative knowledge while new individuals tend to acquire more situational knowledge.

Our first pass at the Cultural Algorithm here will assume that the agent is given knowledge about agricultural practices, and can make observations about land distribution, and rainfall. We will focus on the learning of agents in terms of situational and normative knowledge. In the next section we describe the basic Cultural Algorithm framework. We will describe the characteristics of individual agents and the aspects decision-making knowledge that might be learned by these agents in the population.

### 3 Cultural Algorithm Configuration

The evolutionary process can occur at the individual and population level. The agents in the Village simulation are households scattered across the Mesa Verde study area. Each household contains a number of individuals, with gender and age characteristics. The age variable is important in terms of determining whether an individual is old enough to establish a new household, as well as the

determination of the amount of food consumption. The gender variable relates to household formation due to marriage and knowledge that is needed in generating kinship relations. Mate selection choices may be random (current model) or based on an individual’s preferences, possibly influenced by household proximity and wealth (determined by productivity).

The social network contributes to the development of a cultural system. Each household in the network, as it survives overtime, develops its own “culture” outlined by accumulating situational knowledge of their environment, productivity planning, and other behavioral characteristics that enhance their survival. Knowledge is communicated across the social network forms a global belief space; a set of beliefs shared across households, normative knowledge. Experience from individual households influences the global belief space and the culture can evolve accordingly. In turn, households can use this cultural knowledge in the belief space to improve their own situation.

Each agent has a local set of knowledge comprised of an array of parameters that the individual household has. Overtime, these parameters are adjusted by the current beliefs held by the general population. In turn, currently popular beliefs are influenced by the outcomes of individual experiences. The basic pseudocode for the Cultural Algorithm is shown in figure 1.

```

Begin
  Year = first_year;
  Initialize POP(Year);
  Initialize BLF(Year);
  Repeat
    Evaluate POP(Year);
    Vote(BLF(Year), Accept(POP(Year)));
    Adjust(BLF(Year));
    Evolve(POP(Year), Influence(BLF(Year)));
    Year = Year + 1;
    Select POP(Year) from POP(Year - 1);
  Until (Year == final_year);
End

```

Figure 1: Cultural Algorithm Pseudocode

In our Cultural Algorithm model, the knowledge is maintained by each household is as follows:

- Move Radius: Distance a household uses to decide how far to search for a new location to reside in when it decides to move. We view this as an index of social aggregation of the agent.
- Move Frequency: How often should an agent move (in years). The current model hardcode this parameter to 1 (i.e. Every year an agent considers moving).<sup>1</sup> = 2.54 cm.
- Move Trigger: Whether or not a household should move. This is a probability value that can be adjusted.

#### 3.1 Adjusting Knowledge in the Belief Space

Chung [1997], showed how the normative knowledge can be expressed in the form of ranges, or intervals, of

acceptable values for each parameter and how these ranges can be updated or learned based upon experience. A current acceptable range for each parameter is calculated from a set of acceptable individuals by listing them in ascending order. The minimum value would be the lower bound and the maximum value is the upper bound. The belief range is adjusted based on new limits found. If a new limit is outside of the old interval, then the new range is widened (generalization). If the new limit is inside of the old interval and the score of the new interval is better than that of the old limit, then the new range is narrowed (specialization). If the score of the new interval is not better than that of the old limit, then the new range is not narrowed. The adjusted range in the belief space, normative knowledge, becomes the current acceptable standard for the culture.

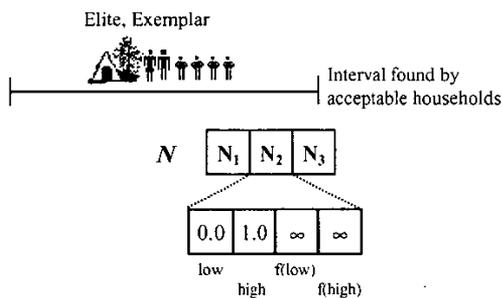


Figure 2: Cultural Influence and Sample Normative Knowledge

Within this context, individual households can gather information through their social interactions. The acceptance function selects a subset of these experiences to update the belief space. Knowledge there can be generalized and used to direct the future of agents. As such, it is a knowledge-based adaptive framework. Thus, agents can potentially learn new cultural knowledge over time and use it to adapt to changes in their environment. In the base model described in this paper we examine the behaviors of the population component on its own, without any cultural learning take place. We will focus on understanding the impact of the various parameters on the emergence of the social network in the population component only. Future work will then add the belief space component into the simulation and allow for the group to acquire and adapt their cultural component.

### 3.2 Preliminary Modifications: Establishing Social Networks

The current population model in the Cultural Algorithm operates at the household level (Village 6.1.3). In order to establish a household model that can be actively involved in the cultural framework the following current assumptions must be addressed and modified accordingly. The key issue is that in Kohler's preliminary model, kinship issues were not originally addressed. Thus, when agents formed new families through marriage, the social context of the new family was not kept. If a female from a

family wished to marry, they were probabilistically allowed to marry and the spouse given the age of the first. No genealogical records were kept to link the new family to its ancestors. The original model likewise did not distinguish gender for children or any individual. Here we want to know the attributes of the parties, such as gender, age, and genealogy in order to establish social links between them and other agents. These kinship relations are the basis for the social networks that emerge.

### 3.3 Marriage/Re-marriage Rules Modifications

In order to establish the concept of kinship and create links between households across a network, the marriage rules have to be controlled and individuals accounted for. The following modifications were amended to the existing system:

1. A global knowledge base of "eligible mates" seeking marriage is maintained and shared among households (agents). This will serve as a knowledge base for individuals seeking to marry and keep track of search areas and household tags.
2. Newborns are assigned a specific gender.
3. When a marriage event is selected to occur based on the existing model probabilities and age rules, the eligible individual will add its name to the pool and a search for a match is conducted based on the following rules:
  - a. An individual cannot marry another from the same household.
  - b. An individual cannot marry another from outside its catchment area (Defined as a distance radius constant from the current household location).
  - c. When an individual is married, its tag is retracted from the eligibility list, along with its mate.

Other rules based on spatial or economic consideration, can be implemented in order to select a suitable mate. However, in the current model we use a random search rule to select a mate from the eligibility list.

### 3.4 New Household Formation

When a new household is formed as a result of a marriage relation, it is initially located in the same cell as one of the two parent agent families. However, since there are two parents and hence two possible initial locations, an election procedure has to be implemented as follows:

1. Set the initial location of the household in the same cell as the parent whose cell is known to be more productive. (In case of a tie we would randomly elect one).
2. Check if the household limit per cell has been exceeded; if so then attempt to settle in the other parent's cell. If that cell is also full then seek a

neighboring cell starting with the vicinity of the parent whose cell was more productive and if exhausted search the neighbors of the cell of the other parent.

The local structure that establishes this network formation is based on the addition of the direct lineages from the perspective of an individual agent (household). It follows that an agent maintains a link to the parent(s) by storing the unique tag identifying the parents. Also, a link to the children is maintained in order to flow the relations in reciprocal directions. Also, consider one more case: household from single parent and another single parent.

### 3.5 Social Network Structure

The initial objective is to merge the independent agents into a collective social unit and allow them to form a community of social relations. The first step in developing a form of relation between households is to develop the social network. In order to establish a communication network between agents a network model is setup based upon kinship relations. Each agent is a household which in turn is composed a husband, wife and children. Household members live together in the same location, survive from the family farm and are affected by the same environmental conditions in the region. They are laborers, farm the land and feed from its crops. Their cooperation as one workforce allows them to produce a crop yield that could sustain them along with any young children.

The structure of the social network needs to support the first level of kinship. A given household would need to know who the parental households are, from either side of the family. Children who grow up, marry, and move away need to remember who their parents are, and furthermore the parent needs to keep a link to its child. The layout of the social network from the perspective of a household is:

ParentHHTagA	a link to the parent from the mother's side,
ParentHHTagB	a link to the parent from the father's side
ChildHHTag	one link to each child that moves away from this household and form its own household
RelativeHHTag	one link to each extended family member

Table 1: Stored Links Descriptions

Each household (agent) is identified by a unique TAG in the system. One could think of this tag as the household's name that uniquely identifies it. In the initial step, agents are formed without any links (or relations). Subsequent steps, that is each year, an agent updates its links based on simple initial rules: If a new child marries and moves on its own, the parent household will then have a link to it.

The child household in turn will maintain a link to the parent household which it just moved away from. This effort is duplicated since each child comes from a different household. So cumulatively, a child household would have links to two parent households as minimum; one link to the parent household from the wife's side while the other is to the parent household of the husband. A child household can also remember its siblings. A household member can remarry if the spouse dies. In that case, the household would keep a link to the initial household, except now as a relative, and updates a new parent link to the new spouse's household. Kinship relation rules can be extended to other family members. Other concepts such as friendships and neighbors could be also modeled that are necessarily based on kinship relations, but could be done based on household's physical locations and proximity. The overall social network is maintained dynamically as it is updated every time step.

### 3.6 Network Properties

The set of relations between agents cumulatively form a directed graph. Each time step, we update and store this graph in the format of an adjacency list. Then we can plot the graph and examine the distribution of structural properties for the vertices (agents) and edges (relations) shown in figure 3.

```
Social Link = { < Parent Household from Wife's side>
                <Parent Household from Husband's side>
                <Child Household>1, <Child Household>2,
                ... <Child Household>c
                <Relative Household>1,
                <Relative Household>2, ...
                <Relative Household>r }
```

```
GraphSocial network = { <Social Link>1, <Social Link>2, ...
                       <Social Link>s }
```

Figure 3: Social Network

The format of the output file containing the adjacency list generated for one time step (each year) is:

TAG	Agent Tag whose links we are describing (unique)
X, Y	Position of the agent
ParentTagA, ParentTagB	Links to each parent
ChildHHTag <sub>1</sub> ...	Link to each child
RelativeHHTag <sub>1</sub> ...	Link to each relative

Table 2: Stored Network Structure

This kinship model can set the stage for knowledge propagation in a cultural framework. One goal is to see what social networks this base model produces in response to changes in these social parameters. Here our experiments will specifically adjust one of the several social parameters, move radius, and look at its impact on

the social network. In follow up studies we can modify other combinations of parameters but more importantly allow the agents to modify them and see how it impacts the resultant networks.

## 4 Results

The Cultural Algorithm framework discussed above is implemented in the Swarm simulation environment [Langdon, 1995]. This environment provides a framework to facilitate development and experimentation with a large number of agents in a dynamic environment. Currently the development is entirely in Objective-C and the Swarm 2.1.1 libraries. The model is a graphical multi-agent simulation that allows us to probe individual agents in a dynamic environment. Agents reside in their simulated world that is in turn composed of cells. At every model step in the simulation, the cells are updated from the database of collected environmental data using GPS technology [Van West, 1994]. Agents may be observed by means of probes that examine their internal properties or by accumulating output files for later examinations. Dynamic plots are supported to show specific details over each time step.

In the current model, an agent is represented as the head of a single household. The agents in the original model (Village 6.1.3) perform calculated tasks that are essential for survival; specifically: Farm, Harvest, Eat, and natural propagation functions such as marriage, birth and death. Agent knowledge is static and determined at the beginning of the simulation, no learning is supported in the base model.

The simulation was run 5 times over the period from 900 A.D. to 1300 A.D. (1281 A.D.). For each time step the network is generated and stored into a file named "links<YEAR>.out". This file is used to examine the properties of the emerging network and to be able to examine any characteristic we may find relevant to network resiliency. Visualization of the network is enabled in a separate package, using MatLab and Visual Basic (shown in figures 4 & 5). The program we developed allows us to plot the graph and to closely examine the distribution and densities of the links between agents. As the graph becomes more and more dense especially as more agents develop more social links over time, we can visually filter out weak links and display only those edges attached to a node with a certain associated out-degree. In other words, we can identify the agents with the highest connectivity in the social network social network in terms of the number of associated links that it maintains.

In the effort to set the course for properly selecting the parameters that can affect the structure of the social network and help us understand its resiliency, we analyze the network output files looking for instances where the network showed evidence of emergent changes in its structure. In the experiments we conducted, the search radius that the agent uses when looking for a new plot to

move into was varied from 10 to 30 pixels. For the different values of the move radius tested, the network volume and the number of links over time were generated. The network volume is the sum of all of the links associated with each individual household (agent) present. The idea behind this is that with a larger move radius the agents can relocate farther away from where they were located before. Since relocation occurs frequently when a group fissions as a result of population growth, a larger radius means that they relocate farther away from a previously successful area. Whether the strategy of increasing search radius is successful or not depends upon the distribution of productive land in the region.

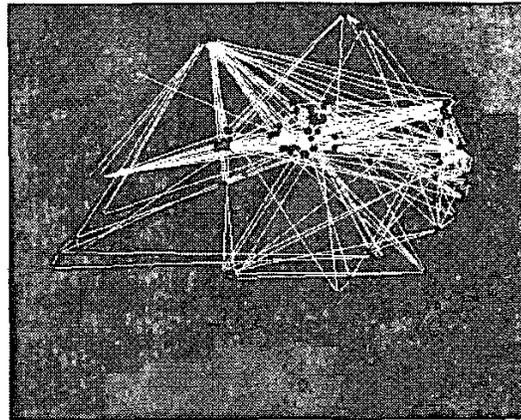


Figure 4: Using a search radius of 10 (MOV\_RAD 10) the network is plotted for 950 A.D. showing initial clustering.

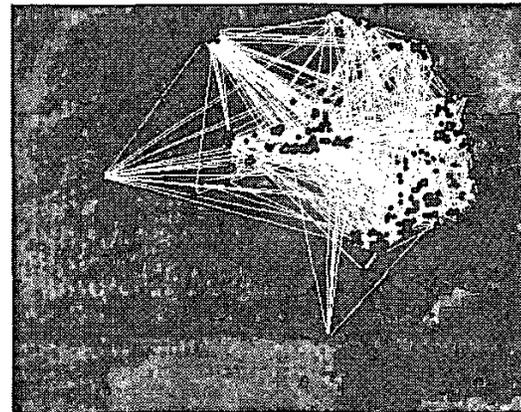


Figure 5: Using a search radius of 10 (MOV\_RAD 10) the network is plotted for 1280 A.D. showing highly dense nodes.

In the figures below we compare the results for runs with the smallest (10) and largest (30) move radius used in terms of the total population of agents supported over time, the total volume of the network formed in each year (figure 7), and the characteristics of the links for individual agents (figure 6). For the individual agents in each year we identify the average number of links per

agent, the maximum number of links for an agent, and the minimum number of links. In addition, figure 8 gives a comparative graph of the network volumes and the agent population for each of the move radius tested.

Regardless of the move parameter value used, small world networks were emergent within the population. A small world network is defined as a network that despite large number of edges, there is a relatively short path between any two vertices. For example, note in figure 6 that the average number of links per agent is small, but there are some nodes that have many more links than the average. These are the hub nodes.

The parameter of these small world networks can be different. For example, small move radii produced denser more compact networks while larger move radii produced sparser more distributed networks as detailed below.

Also, there is an inverse correspondence between social network volume, and population size with search radius as shown in figure 8. That is, the farther individuals need to relocate the more sparse the settlement network is. They are less likely to collect around areas of high productivity. As search area is reduced, agents are more likely to relocate in areas that they have been successful at. This results in larger population sizes and larger social network volume.

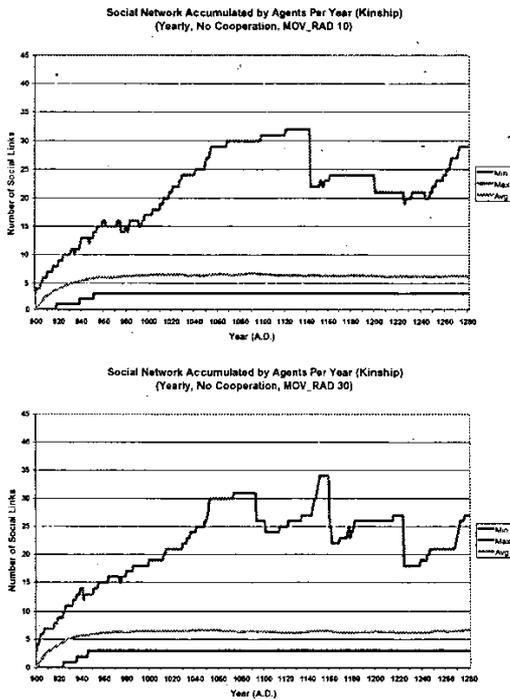


Figure 6: Plots showing the number of links as maintained by agents over time. The agents holding the maximum number of links with a search radius of 10 (Left) showed larger losses than those with a search radius of 30 (right) in the same time period (around 1180AD).

However, the overall result of reduced search radius is to put too much stress on those high performing areas such

that when an environmental perturbation comes along, as evidenced in figures 6 and 7 around 1180 A.D. While both of the systems produced by small and large move radius are effected, the network produced by the small move radius exhibits a much greater decrease in population size and network connectivity, network volume than the network produced by a large move radius. One reason is that the small move radius system exploited the increase in precipitation that occurred prior to the reduction much more than the sparse system did and the subsequent drought provided more additional stress than it did to the sparse system.

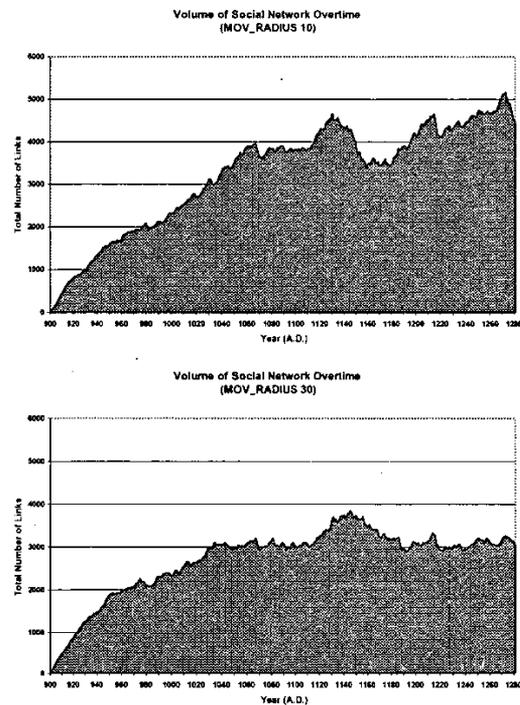


Figure 7: Volume of the social network with a move radius of 10 (left) and 30 (right). The volume of the social network is calculated by adding together the total number of links held by all living agents during each year.

Although the population size is markedly reduced in the more aggregated system, it does not approach that for the sparser settlement system generated by a larger move radius. In other words, a sparse settlement system can survive such perturbations but may not provide the quality of social life required by the agents who are used to a more aggregated settlement system. The precipitous drop in the ability to support the population at a given level of aggregation may have been enough to cause the observed social collapse. In fact, the collapse may be nothing more than a transition to a sparsely settled system. It will be of interest to see whether our cultural model will make such an adjustment when learning is allowed.

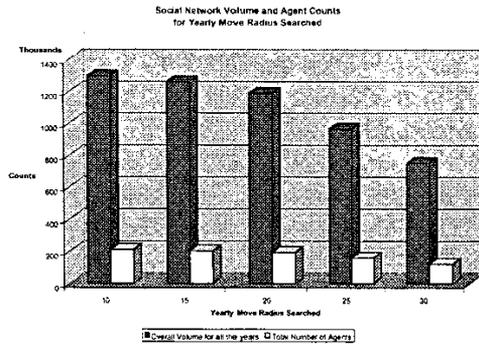


Figure 8: Social network volume and agent counts for a range of move radius search values.

If one observes the average number of links between agents in figure 6 for the 10 and 30 move radius, notice that they are in fact about the same, averaging around 5 links each. The question is how can the social network support such a larger population. The answer lies in the maximum number of links associated with individuals in the two systems. Notice that in the system with low search radius the maximum number of connections can become much larger than for the sparse network. White and Houseman [2003] suggest that such phenomenon might be explained in terms of strong and weak ties. Each individual has a small world composed of a number of strong ties. These individuals are connected to the rest of the social network by individuals with whom they have weak ties. These latter individuals have a number of weak ties with a larger number individuals and serve as centralized hubs or conduits that allow the individual agents worlds to be small but localized. The central hubs are the ones characterized by the maximum number of social relations or arcs in the graph. Spatial distribution of the networks reveal their presence early on from a spatial perspective.

Thus, we postulate that based on figure 6 the biggest impact of the drought is on the number of these central hubs, and their ability to keep the network connected. With the reduction in population produced by the drought the system exhibits an almost catastrophic reduction in the complexity of these weak ties and associated central hubs. Thus, while there is still more population than in the sparse setting the small world social phenomenon has been altered substantially. Perhaps enough to cause a major shift in population and a restructuring of the remaining population in order to adapt to the environmental stress. This might involve changing move radius as well as other parameters, ones that we have not yet tested. In addition to the distance radius that the agent uses to search for a new location when they want to relocate, other parameters should be also considered including: the move frequency, and a related move trigger. These parameters along with others are being tested and considered for their effects on the network collapse in future studies.

The bottom line is that the timing of the drought around 1180 A.D. came on the heels of an increase in population and social complexity in the valley, an increase that made the system especially vulnerable to stress at that point. The reduction in precipitation itself was not monumental but came at a time when the system was in a transitional period and not prepared to handle it. The question is, then what would happen if we allow the agents to utilize the network connections to exchange resources? Would that help to mediate the impact of the environmental changes? That is a question we will study in future papers.

## 5 Conclusions

Here, we used our Cultural Algorithm framework to test the impact that a single variable, move or search radius had on site settlement aggregation and the emergent social network. The emergent structure has many properties of a small world network based upon strong and weak ties between agents. In a small world network each many agent has a few strong local ties, and is able to navigate the entire network using agents who act as central hubs, supporting a larger number of weak ties. We have shown that increased move radius resulted in reduced population size and a sparser social network. A small move radius produced a more aggregated distribution, with a denser network.

Our experiments suggest that hub nodes of the small world network are located in positions that are most sensitive to drought conditions. Thus the drought periods produced reductions in hub complexity and a concomitant reduction in network connectivity. While rainfall conditions improved after that, it is suggested that the quality of life may have evolved to the point where migration to another region was deemed necessary. In order to investigate this we will examine the impact that these changes in network structures have on the flows of materials and information within the system in future work.

## Acknowledgments

This research is funded by NSF Biocomplexity Grant #4-46511.

## Bibliography

Angeline, Peter A. "Adaptive and Self-Adaptive Evolutionary Computation," in *Communication Intelligence*, Eds. Marimuthu Palaniswami, et.al., IEEE Press, New York, 1995, pp. 152-163.

Chung, Chan-Jin, 1997, *Knowledge-Based Approaches to Self-Adaptation in Cultural Algorithms*. Doctoral dissertation. Wayne State University.

Holland, John H., *Adaptation in Natural and Artificial Systems*, First edition, Ann Arbor: The University of Michigan Press, 1975; First MIT Press edition, 1992.

Kohler, T., 2000, "The Final 400 years of pre-Hispanic Agricultural Society in the Mesa Verde Region", *Kiva*, V. 66:191-264.

Langton, Chris et.al. "The Swarm Simulation System A Tool for Studying Complex Systems". Santa Fe Institute, 1995.

Reynolds, Robert G., *An adaptive computer model of the evolution of agriculture for hunter-gatherers in the valley of Oaxaca, Mexico*, Doctoral dissertation, University of Michigan, Ann Arbor, 1979.

Reynolds, Robert G., "An Overview of Cultural Algorithms", *Advances in Evolutionary Computation*, McGraw Hill Press, 1999.

Reynolds, R., Brewster, J., and Jacoban, R., "Cultural Swarms: The Impact of Culture on Social Interaction and Problem Solving", *IEEE Swarm Intelligence Symposium*, Indianapolis Indiana, April 24, 2003.

Russel, Stuart and Peter Norvig, *Artificial Intelligence- A Modern Approach*, Englewood Cliffs, NJ, Prentice Hall, 1995.

Tainter, J., 1988, *The Collapse of Complex Societies*, New York: Cambridge University Press.

Valente A, and J. Breuker, "A Commonsense Formalization of Normative Systems," *Proceedings of the ECAI'94 Workshop on Artificial Normative Reasoning*, 1994.

Van West, Carla. *Modeling Prehistoric Agricultural Productivity in Southwestern Colorado: A GIS Approach*. Department of Anthropology Reports of Investigations 67. Pullman, WA: Washington State University. 1994.

White, D., and Houseman, M, "The Navigability of Strong Ties: Small Worlds, Tie Strength, and network Topology", *Complexity*, Wiley Interscience, Vol. 8, No. 1, September/October, 2002, pp: 72-81.