Cooperation networks

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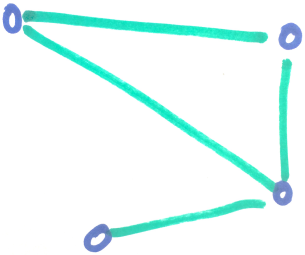
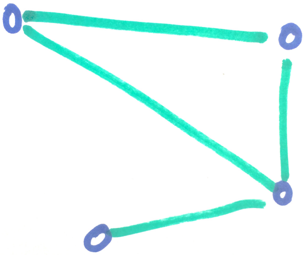
Nodes

Nodes and edges.



Nodes and edges.



Why edges, structure, cooperation?

Node: something alive.

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H. Morowitz:

the disordering and degradative influences of thermal energy as reflected in the second law of thermodynamics. Living systems self-replicate, that is, they give rise to organisms like themselves, thus accounting for the continuity of life. Living organisms must

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Simplest model: noisy multiplicative growth, *e.g.* of biomass, group size, wealth *etc.*

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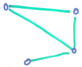
Edge: resource-pooling, support, sharing.



…but

- Expectation value of resources unaffected by pooling (conservation law).
- Richer entity loses when resources are pooled.

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- Small/simple/early networks have no new function.

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Key insight

Growth is not ergodic, *i.e.* expectation value does not reflect what happens over time.

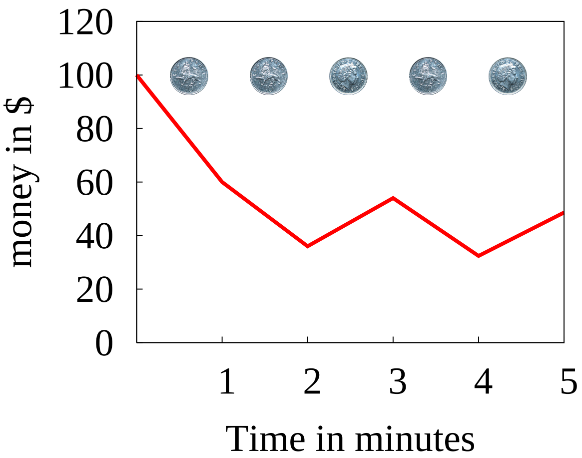
Not in expectation, but over time, pooling leads to faster growth.

Game

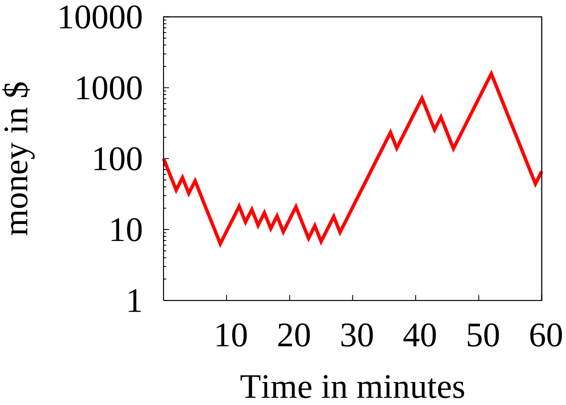
Heads: win 50%.

Tails: lose 40%.

Toss coin once a minute

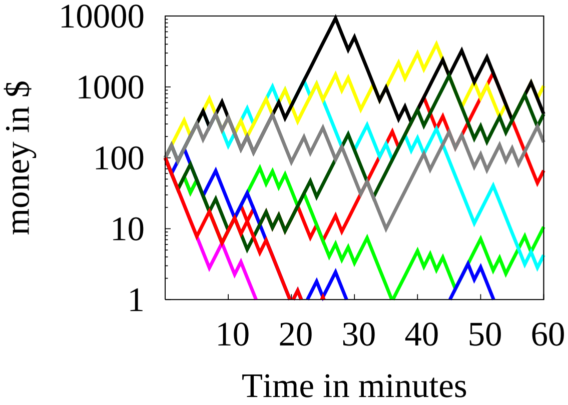


One sequence

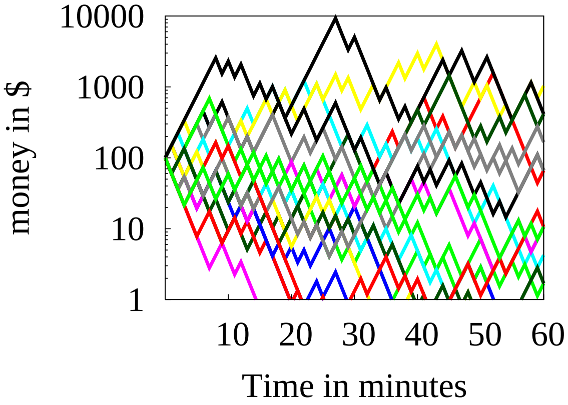


Fermat and Pascal 1654:
Imagine all possibilities and average over them.
Call this average the "expectation value."

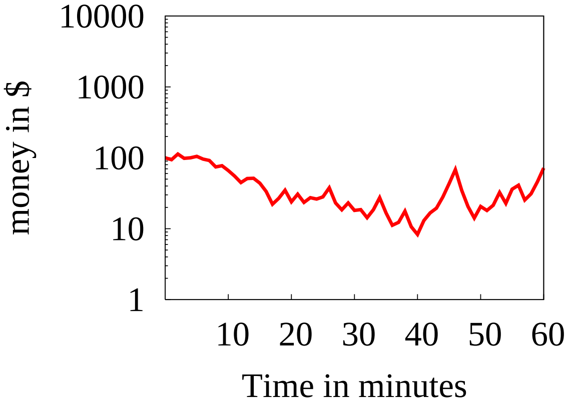
Fermat and Pascal 1654:
10 sequences



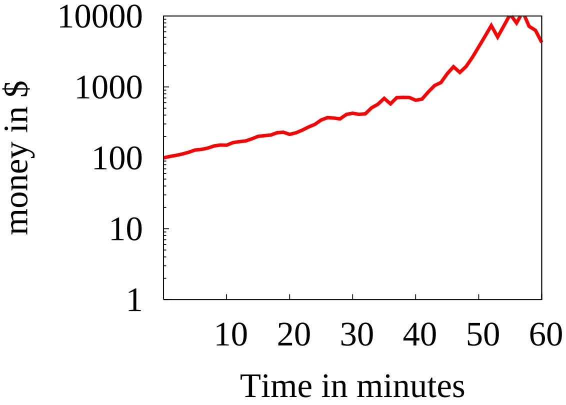
FERMAT and PASCAL 1654:
20 sequences



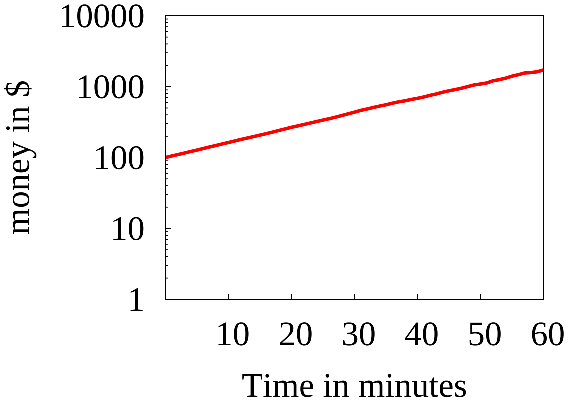
Fermat and Pascal 1654:
Average of 20 sequences



Fermat and Pascal 1654:
Average of 1000 sequences



Fermat and Pascal 1654:
Average of 1,000,000 sequences



Conclusion

Game worth playing, on average.

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Puzzle

But people won't play – why?

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Solution: psychology, irrationality, utility theory.

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Alternative: What average?

Game worth playing if averaging across parallel universes.

Boltzmann 1872:

"Is expectation value also average over time?"

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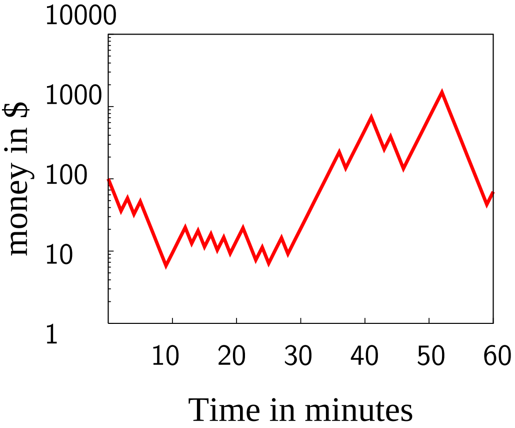
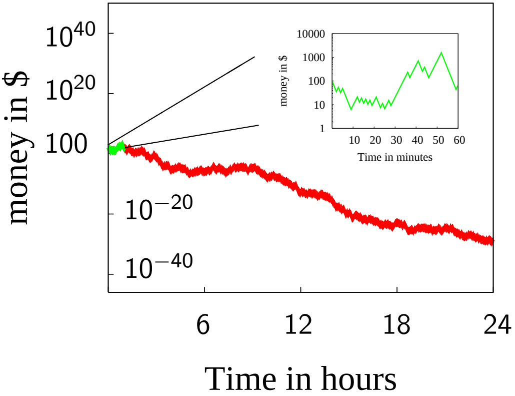
$\rightarrow$ Play for a long time, and see what happens.

BOLTZMANN 1872:
Play for one hour...

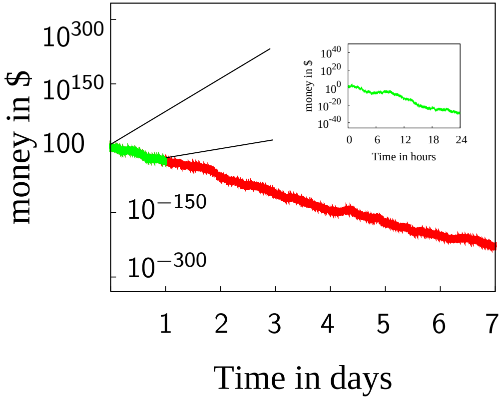


BOLTZMANN 1872:

..continue one day (note scales)...

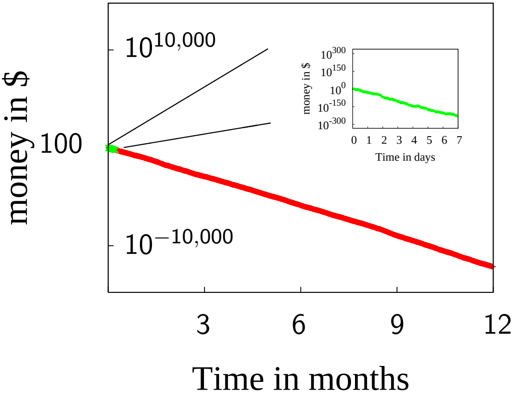


Boltzmann 1872:
..continue one week (note scales)...

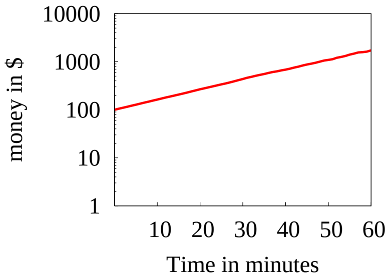


Boltzmann 1872:

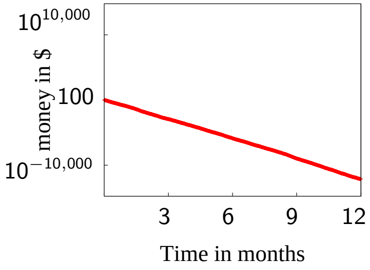
..continue one year (note scales)...



1654
Ensemble perspective



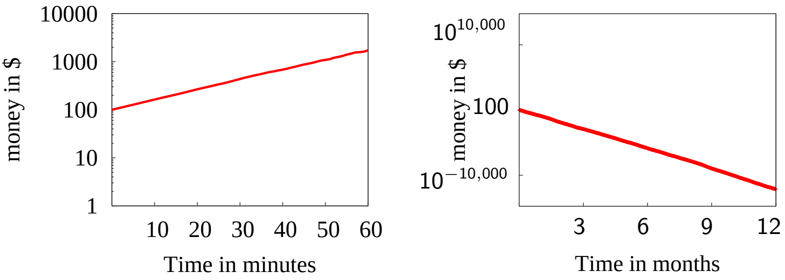
1872
Time perspective



Non-ergodic

1654 ≠ 1872

Ensemble perspective ≠ Time perspective



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Because they would lose *over time*.

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We care about the future (time), not the multiverse (expectation values).

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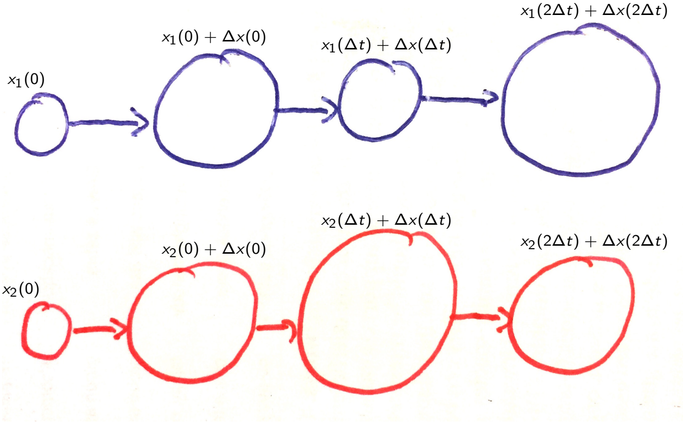
Shameless ad

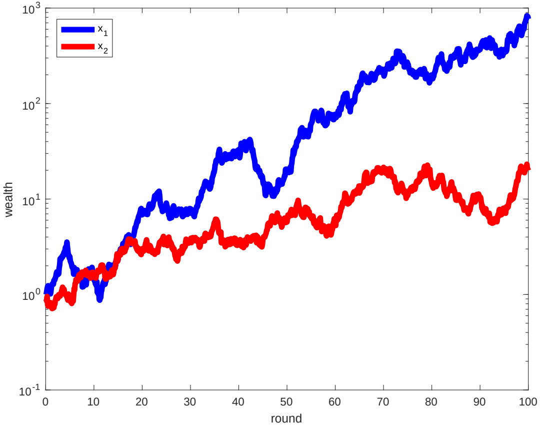
Formal economics built on expectation values.

My program

Re-develop all of economics.

Two living things, following noisy exponential growth
$dx_i = x(\mu dt + \sigma dW_i)$





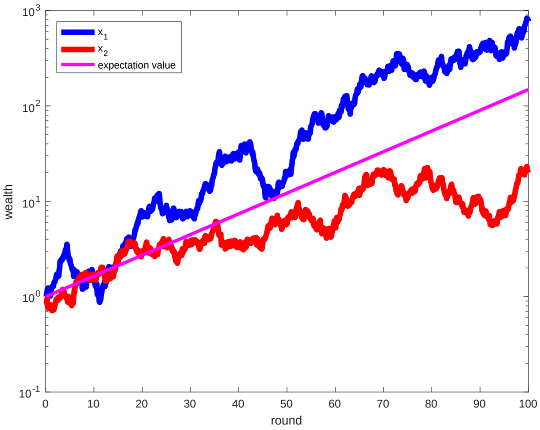
x1
x2
wealth
round
10^3
10^2
10^1
10^0
10^-1
0
10
20
30
40
50
60
70
80
90
100

Expectation value (Fermat 1654)

$$\langle x(t) \rangle = \exp(\mu t)$$

Expectation-value growth rate

$$g_{\langle\rangle} = \frac{1}{t} \ln\left(\frac{\langle x(t) \rangle}{x(0)}\right) = \mu$$


x1
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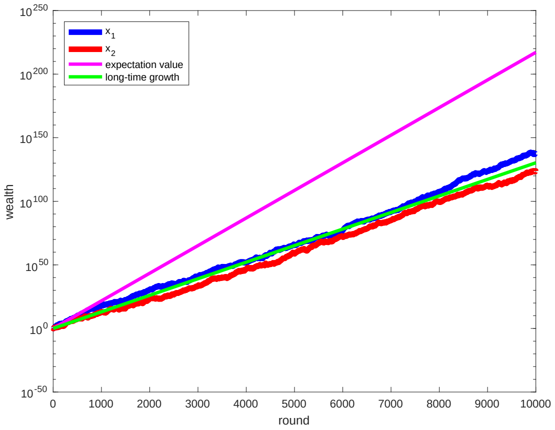
Time-average growth rate (OP 2011)

$$g_t = \lim_{t\to\infty} \frac{1}{t} \ln\left(\frac{x(t)}{x(0)}\right) = \mu - \frac{\sigma^2}{2}$$

(building on Whitworth 1870, Itô 1944, Kelly 1956 *etc.*)



x_1
x_2
expectation value
long-time growth
10^{250}
10^{200}
10^{150}
10^{100}
10^{50}
10^{0}
10^{-50}
wealth
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
round

Fluctuations reduce growth.

- reducing fluctuations increases growth (risk management).

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cooperation reduces fluctuations

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💡 cooperation reduces fluctuations 💡

→ cooperation increases growth.

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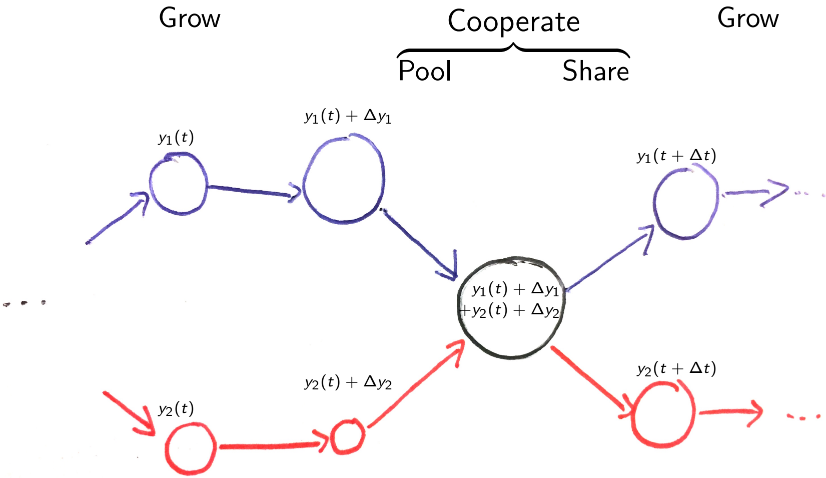
- reducing fluctuations increases growth (risk management).

cooperation reduces fluctuations

→ cooperation increases growth.

→ cooperators outcompete non-cooperators.



Grow
Cooperate
Pool
Share
Grow
$y_1(t)$
$y_1(t) + \Delta y_1$
$y_1(t) + \Delta y_1$
$+y_2(t) + \Delta y_2$
$y_1(t + \Delta t)$
$y_2(t)$
$y_2(t) + \Delta y_2$
$y_2(t + \Delta t)$

No cooperation:

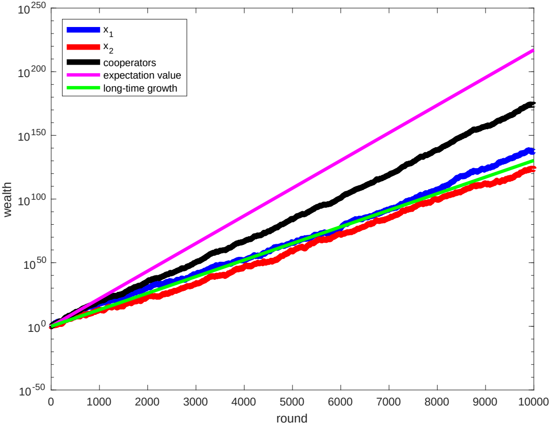
$dx = x(\mu dt + \sigma dW)$

$g_t(x) = \mu - \frac{\sigma^2}{2}$

Cooperation reduces σ:

$dy = y(\mu dt + \frac{\sigma}{\sqrt{2}} dW)$

$g_t(y) = \mu - \frac{\sigma^2}{4}$



x1
x2
cooperators
expectation value
long-time growth
wealth
round
10^250
10^200
10^150
10^100
10^50
10^0
10^-50
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

Big thoughts

- Emergence: cooperating whole (network) is more than sum of non-cooperating parts (nodes).
- In multiplicative growth, pooling is not just adding – different mathematics.
  Conservation law broken by growth $\rightarrow$ non-linear.
- Much of biology is risk management.

Explained the link – atom of network design.

Extension to network design

- Cooperating with inferior entity, $\mu_1 > \mu_2$, still beneficial, if $\mu_1 - \frac{\sigma_1^2}{2} > \frac{\mu_1+\mu_2}{2} - \frac{\sigma_1^2+\sigma_2^2}{8}$.
- Fully connected graph with partial sharing (weighted edges): taxation $dx_i = x_i[(\mu - \tau)dt + \sigma dW] + \langle x \rangle_N \tau dt.$
- n cooperators grow at $\mu - \frac{\sigma^2}{2n}$.

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- Fully connected graph with partial sharing (weighted edges): taxation

$$dx_i = x_i[(\mu - \tau)dt + \sigma dW] + \langle x \rangle_N \tau dt.$$
$$= x_i[\mu dt + \sigma dW] - (x_i - \langle x \rangle_N)\tau dt.$$

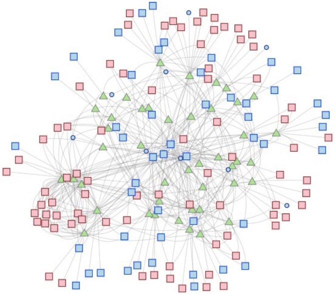
- n cooperators grow at $\mu - \frac{\sigma^2}{2n}$.

Conditions

- Important for early-growth networks, prior to new function (Geoff West's economies of scale, specialization *etc.*).
- Correlation between cooperators reduces cooperation benefit.
- Fluctuations increase cooperation benefit.

Business world

- Used for big financial risk management systems.



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Thank you.