FOUNDATIONS OF COMPLEX SYSTEMS

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An approach to Complexity from the perspective of fundamental science is outlined, drawing on the cross-fertilization of concepts and tools from nonlinear dynamics, statistical physics, probability and information theories, data analysis and numerical simulation. Emphasis is placed on the intertwining between different levels of description and on the probabilistic dimension of complex systems, in connection with the issue of prediction.

It is by now established that large classes of systems composed of interacting subunits give rise to certain characteristic behaviors perceived by the observer as "complex ", encountered in a variety of contexts of great interest from the materials we manipulate in everyday life to the seemingly capricious fluctuations of the weather or of the stock market [1-3]. For a long time the idea prevailed that this perception reflected essentially the practical difficulty to gather detailed information on such systems, following the presence of often prohibitively large numbers of parameters and variables masking the underlying regularities. This view which, if true, would leave no room for a scientific approach to complexity, is now obsolete. Theoretical and experimental developments spanning the last two decades show that complexity is on the contrary an authentic phenomenon rooted into the laws of nature and constitutes in fact, in many respects, the most exciting and innovative facet of systems composed of interacting subunits – undoubtedly, the vast majority of systems encountered in nature and interfering with man's everyday experience. This realization leads to a drastic reshaping of the scientific landscape and is at the origin of a new scientific paradigm qualified in this context as a "post-Newtonian" paradigm[4].

The way is now open for a research on complex systems as a branch of fundamental science. On the one side one witnesses the encounter and cross-fertilization of concepts and techniques from nonlinear dynamics, chaos theory, statistical physics, information and probability theories, data analysis and numerical simulation, in close synergy with experiment. And on the other side, insights from the practitioner confronted with large scale systems as encountered in nature, technology or society, many of them outside the strict realm of traditional mathematical and natural science, where issues eliciting the idea of complexity show up in a most pressing manner, are increasingly being integrated into the general framework. Concepts that till recently were not even part of the established scientific vocabulary are now occupying a central place forcing a reassessment of principles and practices. This synthesis, this conjunction of complementary views, this multilevel approach confers to complex systems research its unique status, high relevance and added value beyond the traditional disciplinary approach to the understanding of nature. In addition to being one of the most active and fastest growing branches of science, complexity research is today a forum for the exchange of information and ideas of an unprecedented diversity cutting across scientific disciplines, from pure mathematics to biology to finance.

It is often stated that fundamental science is tantamount to the exploration of the very small and the very large. This assertion becomes, simply, unfounded in the light of progress in complex systems research. There exists, indeed, a huge class of phenomena of the utmost importance between these two extremes waiting to be explored, fundamental as well as practical, in which the system and the observer – the external world and ourselves – co-evolve on comparable time and space scales.

Complex systems display a phenomenology of their own

A system perceived as complex induces a characteristic phenomenology the principal signature of which is the multiplicity of possible outcomes, endowing it with the capacity to choose, to explore and to adapt. This process can be manifested in different ways [1].

- The emergence of traits encompassing the system as a whole, that can in no way be reduced to the properties of the constituent parts. Emergent properties are manifested by the creation of self-organized states of a hierarchical and modular type, where order and coherence are ensured by a bottom-up mechanism rather than through a top-down design and control. Classical laboratory scale examples of this behavior are found in fluids under stress (e.g. Rayleigh-Bénard cells in a fluid heated from below) and in open chemically reacting systems (e.g. bistability, oscillations, Turing patterns and wave fronts in the Belousov-Zhabotinski reaction and related systems). Further examples, in naturally occurring systems, are provided by the communication and control networks in living matter, from the genetic to the organismic to the population level.

- The intertwining, within the same phenomenon, of large scale regularities and seemingly erratic evolutionary trends. This coexistence of order and disorder raises the issue of predictability of the future evolution of the system at hand on the basis of the record available. Typical examples are provided by the atmosphere in connection with the familiar difficulty to issue reliable weather forecasts beyond a horizon of a few days, as well as by earthquakes, floods and other extreme geological and environmental phenomena. Human systems such as traders in stock markets influencing both each other and the market itself are also confronted to unexpected crises and collapses, despite the rationality supposed to prevail at the individual level.

If the effects generated by the underlying causes were related to these causes by a simple proportionality there would be no place for multiplicity. Nonlinearity is thus a necessary condition for complexity.

In addition to its macroscopic level manifestations complexity is also ubiquitous at the microscopic level. Systems with build-in disorder like glassy materials give rise to a rich variety of evolutionary processes driven by microscopic level interactions. Many systems operating on the nanometer scale exhibit complex behaviors like energy transduction and anomalous transport, arising from the interplay between microscopically generated spontaneous fluctuations and systematic environmental constraints. The very origin of irreversibility is related to the intrinsic complexity of the dynamics of the atoms constituting a macroscopic system under the effect of their mutual interactions [5].

Complex systems lie at the crossroads of the deterministic and probabilistic views of nature

As alluded already in the preceding section, nonlinear dynamics provides a natural setting for a systematic description of key properties of complex systems and for sorting out generic evolution scenarios [1]. In most situations of interest nonlinearity coexists with <u>constraints</u>, a set of actions reflecting the influence of the environment on the system of interest and manifested at the level of the evolution laws through the presence of a set of <u>control parameters</u>. As the constraint is gradually applied several qualitatively different regimes are generated. To these regimes correspond well-defined mathematical objects, the attractors, each of which is reached irreversibly from a set of initial states that is specific to it, referred to as basin of attraction. The evolutionary landscape is thus partitioned into cells in which different destinies are realized, and yet, this is not in any sort of contradiction with the deterministic character of the underlying evolution laws. The nature, number and accessibility of the attractors can be modified by varying the constraints. These variations are marked by critical situations where the evolutionary landscape changes in a qualitative manner as new kinds of behavior are suddenly born beyond a threshold value of the constraint, notably through the mechanisms of instability and of bifurcation. Criticalities and bifurcations confer to the system the possibility to choose, to adapt and to keep the memory of past events since different pathways can be followed under identical ambient conditions, as illustrated in Fig. 1. Systems close to criticalities also display an enhanced sensitivity to the parameters and to the initial conditions, since minute differences in their values will generate evolutions towards different regimes. As a rule there is no universal, exhaustive classification of all possible evolution scenarios: the evolution of complex systems is an open-ended process that remains, nevertheless, compatible with the causal and deterministic character of the laws of nature.

Figure 1

Among the regimes that can be realized by nonlinear systems the regime of deterministic chaos is of special interest, as robustness and sensitivity are here in permanent coexistence: while the attractor descriptive of chaos is reestablished once perturbed, initially nearby states on the attractor diverge subsequently in an exponential fashion. This sensitivity to the initial conditions highlights further the issue of predictability of certain phenomena associated with complex systems, even if these are governed by deterministic laws. It also provides yet another generic mechanism of evolution in which the future remains largely open [1, 6].

On the basis of the foregoing it appears that the conjunction of multiplicity and sensitivity confers to complex systems an intrinsic randomness that cannot be fully accounted for by the traditional deterministic description, in which one focuses on the detailed point wise evolution of individual trajectories. The probabilistic description offers the natural alternative. The evolution of the relevant variables takes here a form where the values featured in a macroscopic, coarse grained description are modulated by the random fluctuations generated by the dynamics prevailing at a finer level. This accentuates further the variety of the behaviors available and entails that the probability distribution functions, rather than the variables themselves, become now the principal quantities of interest. They obey to evolution equations like the master equation or the Fokker-Planck equation which are linear and guarantee (under mild conditions on the associated evolution operators) uniqueness and stability, contrary to the deterministic description which is nonlinear and generates multiplicity and instability.

Thanks to its inherent linearity and stability the probabilistic description of complex systems is the starting point of a new approach to the problem of prediction, in which emphasis is placed on the future occurrence of events conditioned by the states prevailing at a certain time as provided by experimental data. This approach finds nowadays intensive use in operational weather forecasting, where it is known as ensemble forecasting (cf. Fig. 2). Of great interest is also the prediction of extreme events such as earthquakes or floods, of the recurrence of states sharing certain characteristic features and of the crossing of thresholds [1,7].

Figure 2

Complex systems imply the emergence of levels of description obeying to their own laws

There is an apparent paradox accompanying the transition to complexity. On the one side complexity seems to follow its own rules reflecting the emergence, at some level of description, of new qualitative properties not amenable to those of the individual subunits. But on the other side, since the laws of nature are deterministic these properties are bound to be deducible from the interactions between lower order hierarchical levels.

A first case where this apparent conflict can be resolved thereby allowing one to establish a connection between different hierarchical levels pertains to the macroscopic description, in which individual variability and more generally deviations from a globally averaged behavior are discarded. Let $\{X_i\}$, i=1..n be a set of macroscopic observables, where n can be as large as desired, λ the control parameters and F_i ({X_j}, λ) the laws governing the evolution:

$$\frac{dX_i}{dt} = F_i(\{X_j\},\lambda) \tag{1}$$

Suppose that the system operates in the vicinity of a criticality, in the sense specified in the preceding section. An important result of nonlinear dynamics is that for certain (generic!) types of criticalities there exists a limited number of collective variables, to which one refers as order parameters, obeying to universal evolution laws characteristic of the criticality at hand, to which one refers as normal forms. All other variables follow passively the evolution of the order parameters [8,9]. As an example, near the bifurcation depicted in Fig. 1 – referred to as pitchfork bifurcation – there exist just two order parameters (or a single complex valued one z), obeying to the universal equation

$$\frac{\partial z}{\partial t} = (\lambda - \lambda_c)z - u\,|z|^2\, z + D\nabla^2 z \tag{2}$$

where λ_c is the critical value of the control parameter λ. The specific nature of the original evolution laws (eqs (1)) is immaterial as long as it gives rise to the relevant bifurcation and enters only to specify the values of the parameters λ_c, u and D in the normal form. We here have a first instance of how the concept of emergence can be quantified, as a new level of description following its own rules is being generated. Notice that the essential property sought here is closure, namely the existence of an autonomous set of laws for the relevant variable pertaining to the level of description considered.

Let us next switch to the probabilistic description afforded by master or Fokker-Planck type equations, as discussed in the preceding section. Since a probability distribution is specified by the infinite set of its moments, each of these stochastic evolution equations is equivalent to an infinite hierarchy of moment equations. Now, under well defined conditions on the stochastic evolution operators it can be shown that this hierarchy can be truncated to a finite order and, in particular, to the first order in which the relevant variable is just the observable appearing in the macroscopic description. In other words under certain conditions the macroscopic description can be viewed as an emergent property, starting from a probabilistic description [1].

A most exciting point is that under certain (generic!) conditions the probabilistic description itself acquires the status of an emergent property, free of heuristic approximations, starting from a deterministic microscopic level description. This passage from the Liouville equation to the master or Fokker-Planck equations depends crucially on the unstable, chaotic character of the microscopic dynamics. A second important ingredient is a judicious choice of "states", through an adequate partition of the full phase space spanned by the variables descriptive of the elementary subunits into cells. As the microscopic trajectory unfolds in phase space (Fig. 3) transitions between cells – states – are induced, which are isomorphic to a probabilistic process. Such considerations are also instrumental for building a microscopic theory of irreversibility [4,5].

Figure 3

Despite the appealing character of the foregoing it should be realized that there are limits to the hierarchical view, reflecting the failure of the decoupling between levels of description. This is what happens, in particular, in nanoscale systems, in systems subjected to strong geometric or nonequilibrium constraints, or in phenomena associated with the occurrence of extreme values of the relevant variables. A full scale description becomes then necessary, in which the fine details of the structure of the probability distributions begin to matter. Universal laws governing some key observables can still be extracted, examples of which are given by fluctuation type or more generally large deviation type theorems [10].

Characterization of complex systems

The conjunction of the probabilistic and deterministic descriptions as well as of the macroscopic and microscopic views opens the way to a multilevel approach at the heart of present day complexity research as summarized briefly below.

(i) Scales, correlations, self-similarity. The most familiar characterization of complex systems is in terms of correlations, a set of quantities utilized extensively in statistical physics and in data analysis to describe, in an averaged way, how a system keeps in time and space the memory of a perturbation inflicted initially on one of its parts. As a rule the onset of complex behaviors is marked by the generation of long range correlations, which in some extreme situations are scale free in the sense of displaying no privileged characteristic time or space scale. The associated probability distributions display then, in turn, power law tails. These features are referred as self-similar, or fractal laws [11].

(ii) Complexity, entropy and generalized dimensions. A probabilistic process to which, as seen above, the evolution of a complex system can be mapped under certain conditions can be characterized by a hierarchy of entropy-like quantities, describing the amount of data needed to identify a particular state of the system (Shannon entropy) or a sequence thereof (block or dynamical entropies) with a prescribed resolution [12]. The Kolmogorov-Sinai entropy is the infinite resolution limit of block entropies and characterizes the degree of dynamical randomness of the system. Entropy-like quantities generate also a hierarchy of dimension-like quantities, generally fractal, providing a useful geometric characterization of complexity.

(iii) Complexity and information. The probabilistic description of complex systems offers a representation in terms of sequences of states than can be regarded as symbols, or letters of an alphabet (cf. Fig. 3). In this view, complex systems are regarded as sources and processors of information. Symbolic sequences can be characterized by the length of the minimal algorithm that allows the observer to reconstitute them, referred to as algorithmic complexity or Kolmogorov-Chaitin complexity [13]. Fully random sequences are the most complex ones in this perspective as reflected by a linear scaling of their block entropies with the length of the sequence, with a proportionality coefficient equal to the maximum Shannon entropy. Natural complexity lies between full order and full randomness adding, in a sense, a dynamic, "nonequilibrium" dimension to the concept of algorithmic complexity. The scaling of the associated block entropies is here more involved and contains, in particular, sublinear contributions.

(iv) Simulating complex systems. Direct simulation of a process of interest, rather than the integration of a set of underlying evolution equations, is an indispensable element in the study of complex systems. Starting from a minimal amount of initial information deemed to be essential, different scenarios compatible with this information are explored. Generic aspects of complex behaviors observed across a wide spectrum of fields (in many of which the detailed structure of the constituting units and their interactions may not be known to a degree of certainty comparable to that of a physical law) are captured in this way through models governed by simple local rules. In their computer implementation these models provide attractive visualizations and deep insights, from Monte Carlo and multi-agent simulations to cellular automata and games [14].

Concluding remarks

We have seen that the evolution of complex systems is an open-ended process of marked multiplicity, flexibility and diversity, thereby setting a prototype for modeling evolutionary and regulatory processes in naturally occurring systems. This approach provides new insights in among others life sciences, in which emphasis is placed on the need to complement the traditional molecular biology based analysis by accounting for global, "holistic" features not amenable to molecular-level properties[15,16].

Complex systems possess an irreducible random element, forcing a reconceptualization of the concept of prediction and of the strategies to be followed in order to monitor – or more generally to communicate with – a complex system. This opens the way to a novel approach to meteorology, global climate and environmental sciences in general complementing the traditional approach based on statistical data analysis and the integration of large numerical models often comprising tens of millions of variables [1, 7].

Complex systems can be regarded as information sources and processors capable of selecting within a finite time span, by means of the underlying dynamics, sequences of states that would a priori be highly improbable. This suggests the possibility of a "nonequilibrium" extension of traditional information and computation theories [1].

Finally, complex systems provide a privileged interface between fundamental mathematical and natural sciences on the one side and social sciences, finance and management on the other [17]. Here, the laws governing the evolution are not known to any comparable degree of detail as in a physico-chemical system. Still, many of the behaviors observed turn out to be part of the characteristic phenomenology of complex systems as outlined in this article, the additional distinctive feature being the intrinsic ability of the individual "actors" to adapt and to respond. One is thus often led to proceed by analogy, and this requires special care in order to apply properly the methodology of complex systems approach while respecting the specificities of the system at hand.

Complexity research has the privilege to attract audiences of an unprecedented diversity. There is a need to establish adequate communication pathways between the different parts of this highly heterogeneous community, and this is likely to necessitate training procedures of a new kind. At the same time, it is essential to highlight further the status of complexity research as part of fundamental science and to keep investing on mathematically and physically motivated issues and on the sharpening and further development of the associated techniques.

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Figure captions

Fig. 1 The acquisition of historical dimension as a result of the phenomenon of bifurcation, illustrated here on the example of the Rayleigh-Bénard convection. Left panel: beyond the instability point the system must choose between two new solutions b_1, b_2 that become available (two different directions of rotation in the case of a Rayleigh-Bénard convection cell). The upper smooth line depicts a particular evolution pathway. Right panel: mechanical analog of the process, where a ball rolling in the indicated landscape may end up in valley b_1 or valley b_2 beyond the bifurcation point λ_c.

Fig. 2 Illustrating the nature of ensemble forecasts. The ellipsoids represent three successive snapshots of an ensemble of nearby initial conditions (left) as the forecasting time increases. The full line represents the traditional deterministic single trajectory forecast, using the best initial state as obtained by advanced techniques such as data assimilation. The dotted lines represent the trajectories of other ensemble members, which remain close to each other for intermediate times (middle ellipsoid) but subsequently split into two subensembles (right ellipsoid), suggesting that the deterministic forecast might be unrepresentative.

Fig.3 Coarse-grained description in terms of transitions between the cells C_1… of a phase space partition, as the trajectory of the underlying deterministic dynamical system unfolds in phase space.

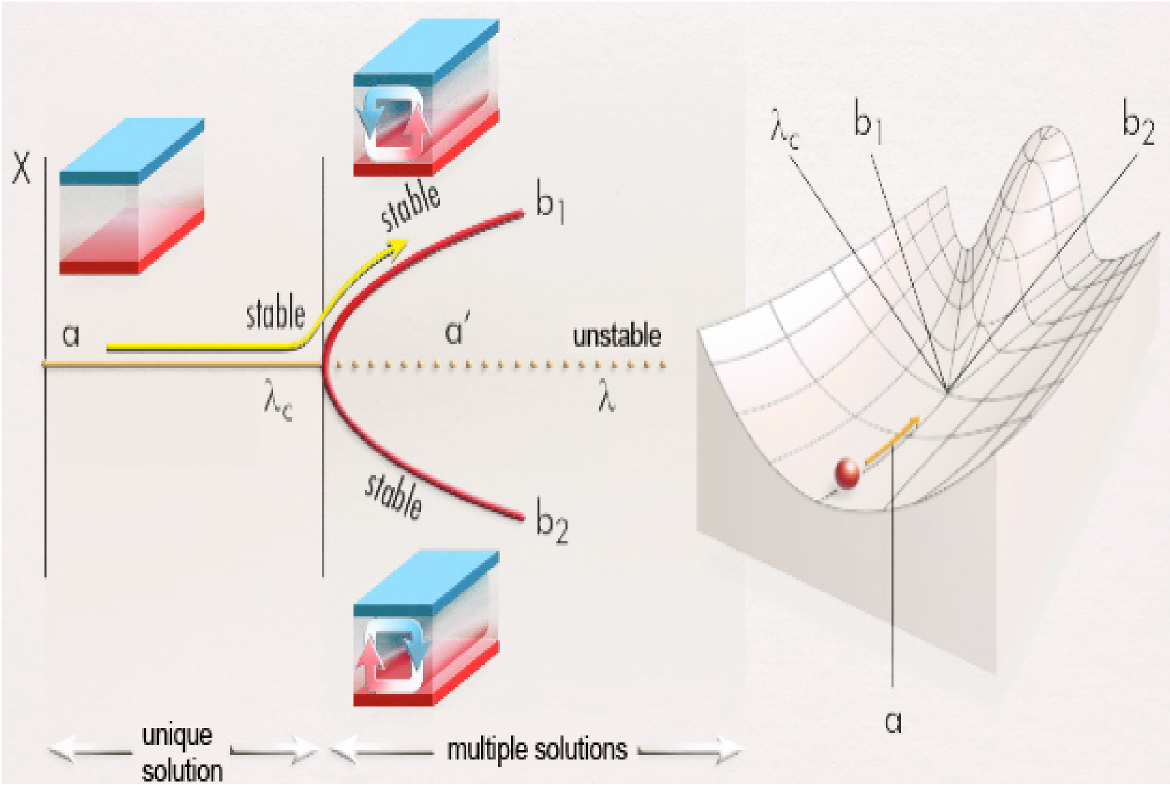
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Figure 1

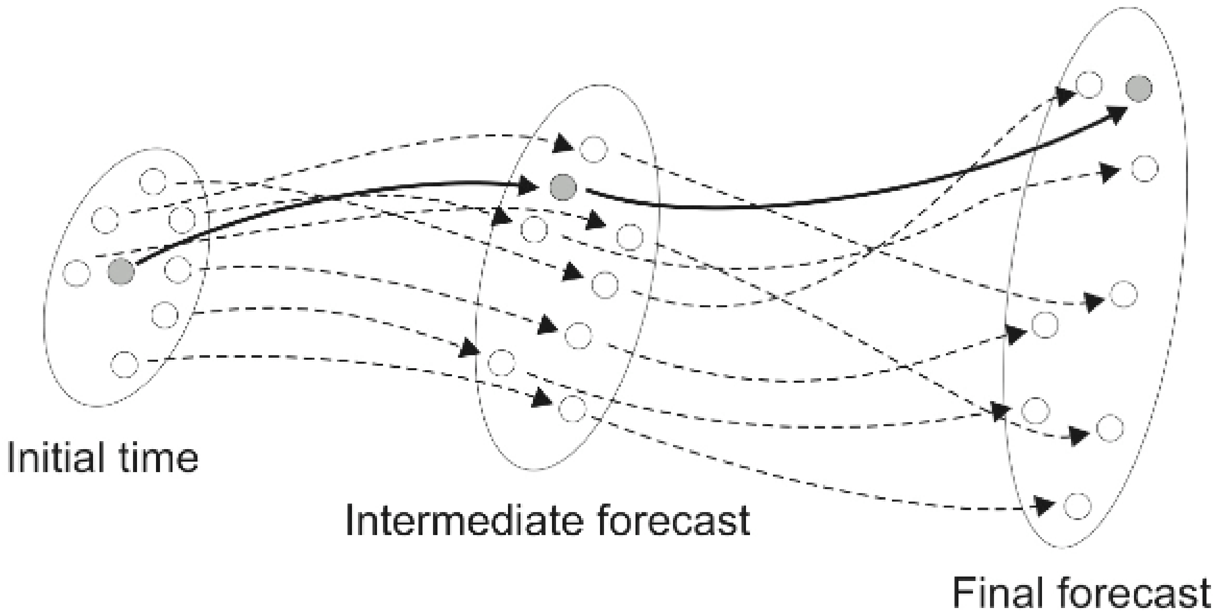

Figure 2

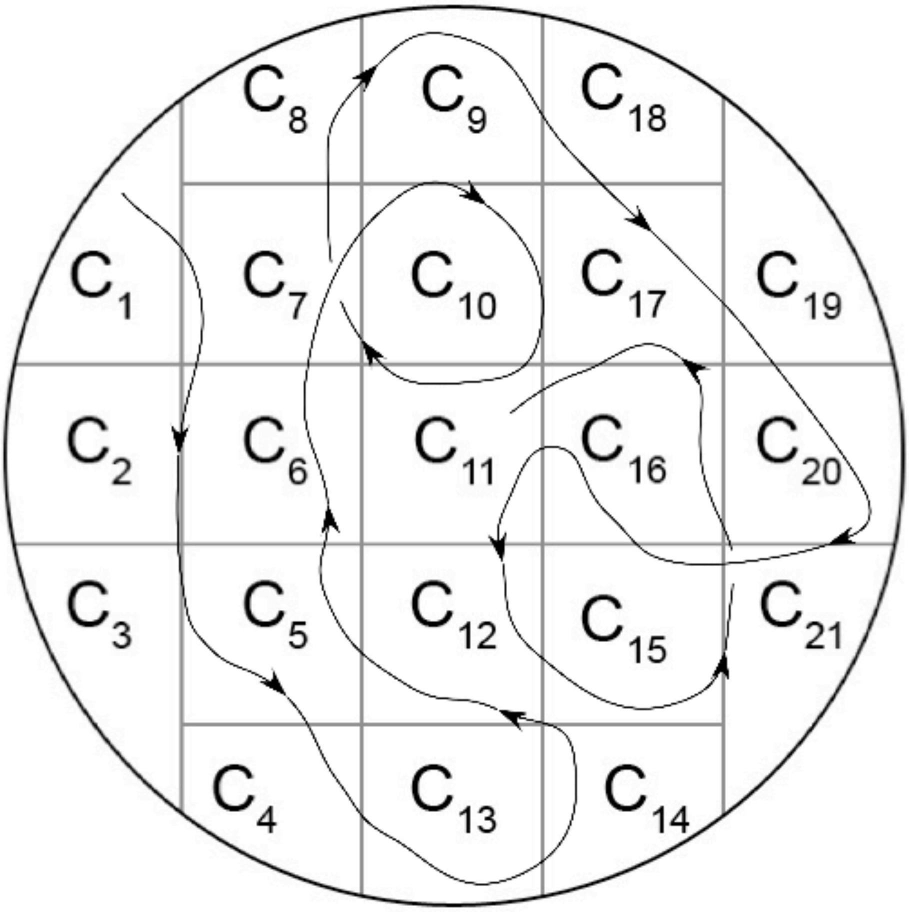
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Figure 3