International Journal of Modern Physics D
Vol. 15, No. 8 (2006) 1–17

World Scientific
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HIERARCHIES IN THE LARGE-SCALE STRUCTURES OF THE UNIVERSE

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Received 2 September 2005
Revised 5 May 2006
Communicated by D. V. Ahluwalia-Khalilova

We study the common relationships that exist between the various structures in the Universe, and show that a unifying description appears when these are considered as emerging from dynamical critical phenomena characterized by complex exponents in the two-point correlation function of matter density fluctuations. Since gravity drives their formation, structures are more likely to form where there is maximal correlation in the matter density. Applying this simple principle of maximal correlation to the two-point correlation function in a scaling regime with complex exponents leads to a hierarchy of structures where: (1) the structures can be classified according to an integer and (2) there is a common real exponent for the two-point correlation function across the range of structures. This in turn implies the existence of both universal size and mass hierarchy-order relationships. We show that these relationships are in good agreement with observations, and that sizes and masses for the known structures, from Globules in the Interstellar Medium to Clusters of Galaxies, can be classified (essentially to within one order of magnitude out of more than 10 orders of magnitude) in terms of just three constants.

Keywords: Complex phenomena; cosmology; astrophysics.

1. Introduction

Matter in the Universe clumps in a variety of structures, which are contained within larger structures, which, in turn, are contained within even larger structures. This observational property leads us to think of the Universe as “hierarchical;”

i.e., as made up of structures contained within structures.[a] For astrophysical objects, this property manifests itself for at least 10 orders of magnitude in their typical longitudinal size and more than 20 orders of magnitude in mass. Such regularity, from molecular clouds to superclusters of galaxies and beyond, is remarkable. [5] It is also known that *within each class* in this hierarchy, the two-point density correlation function, as a function of object separation, displays power-law (or scaling) behavior with an exponent value which is numerically similar for all the classes and, therefore, almost "universal," in spite of the different mechanisms that may be at work in the formation of each of the classes.[6–12]

In this paper we show that the above features of the Universe can be related and that, because of this, sizes and scales can be determined (within an order of magnitude) in terms of just a few constants. This unification follows from *assuming* only: (a) that the exponent in the power law of the two-point correlation function for matter density fluctuations is a complex number, and (b) that gravitational clumping will most likely occur in those regions where matter will have a higher statistical correlation and better chances of evolving into stable states.

Assumption (a) must be viewed as phenomenological and is inspired by the physics of dynamical critical phenomena,[13] where it is known to be associated with the presence of hierarchical behavior (see, for example, Ref. 14); it also appears in critical systems (and therefore with power law behavior for their free-energy and other thermodynamical variables) of finite size.[2,3] It turns out that this assumption brings together the hierarchical nature of the Universe and the measured "common value" for the exponent of the two-point correlation function for the various structures[b]; at the same time, it relates an additional number of observational facts.

The differentiated structures that we consider here, and some properties relevant to our purposes, are summarized in Fig. 1 and Table 1. For a discussion regarding the nature of the entries and database used, see Sec. 3. In Table 2 we have collected the structures of Table 1 into groups of structures according to criteria also discussed in Sec.3. The resulting sizes are tabulated in the column labelled "Typical size. The column labelled "Typical mass" lists the corresponding value for the masses of the same grouping of structures. Finally, "Hierarchy index" refers to the integer value to be used for that structure in the size-hierarchy equation, Eq. (7), derived later.

[a]Simon, in Ref. 1, defines a hierarchical system as "composed of interrelated subsystems each of the latter being in turn hierarchic in structure until we reach some lowest level of elementary subsystem." The objects in each of the levels of the hierarchy can be identified on the grounds of their morphological similarity or of their basic physical properties such as size and mass. Today, hierarchies in general can be understood in terms of complex exponents of the appropriate correlation functions. [2–4] We restrict ourselves in this paper to the structures listed in Table 1. In particular, we will not be concerned here with protoplanetary clouds or planetary systems.

[b]Recent observations[15] seem to indicate that even within a class of structures, for example galaxies, there is some evolution with redshift in the power law exponent. This can be accomodated by dynamical critical phenomena, where in the neighborhood of a critical point the exponent acquires a correction as a function of size.

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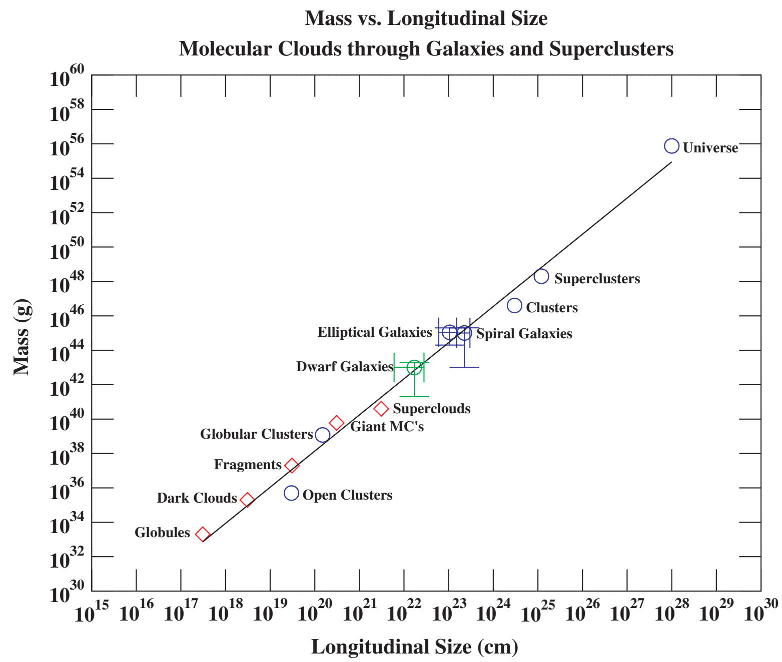

Fig. 1. Plot of the data in Table 1. Mass vs. longitudinal size distribution of discrete structures in Universe from molecular clouds through galaxies to superclusters of galaxies and power law fit (slope $= 2.10 \pm 0.07$).

Table 1. Mass and longitudinal size data for the known structures displayed in Fig. 1. Data are taken from Refs. 10, 11 and 17. "Bound IHS" stands for "Bound Interstellar Hierarchical Structure." The entry labelled "Universe" represents the current estimate for the size of the Universe with $H = 100$ km/s/Mpc. *Precise* values of these quantities for each class of object are not available; the figures quoted here are considered as typical of the objects in the class.

Type of object	Designation	Label	Typical size (cm)	Size Δ (cm)	Typical mass (g)	Mass Δ (g)	References
Bound IHS	globules	A	3.086×10^{17}	1.0×10^{17}	1.989×10^{33}	1.0×10^{33}	6–9
Bound IHS	Dark clouds	B	3.086×10^{18}	1.0×10^{18}	1.989×10^{35}	1.0×10^{35}	6–9
Star clusters	Open	C	3.0×10^{19}	1.0×10^{19}	5.0×10^{35}	1.0×10^{35}	10
Bound IHS	MC fragments	D	3.086×10^{19}	1.0×10^{19}	1.989×10^{37}	1.0×10^{37}	6–9
Star clusters	Globular	E	1.5×10^{20}	1.0×10^{20}	1.2×10^{39}	1.0×10^{39}	10
Bound IHS	Giant MC's	F	3.086×10^{20}	1.0×10^{20}	5.967×10^{39}	1.0×10^{39}	6–9
Bound IHS	Superclouds	G	3.086×10^{21}	1.0×10^{21}	3.978×10^{40}	1.0×10^{40}	6–9
Galaxies	Dwarf	H	1.7×10^{22}	1.1×10^{22}	1.0×10^{43}	9.8×10^{42}	11, 16, 17[a]
Galaxies	Elliptical	I	2.25×10^{23}	7.5×10^{22}	11.0×10^{44}	9.0×10^{44}	10, 17
Galaxies	Spiral	J	1.05×10^{23}	4.5×10^{22}	1.01×10^{45}	1.0×10^{45}	10, 17
Clusters of galaxies	Clusters	K	3.0×10^{24}	1.0×10^{24}	4.0×10^{46}	1.0×10^{46}	10, 17
Clusters of	Super clusters	L	1.2×10^{25}	1.0×10^{24}	2.0×10^{48}	1.0×10^{48}	10, 17
Universe galaxies		M	1.0×10^{28}	1.0×10^{27}	7.5×10^{55}	1.0×10^{55}	10, 17

[a]See the contribution of V. Trimble to Ref. 17.

Table 2. Values in cm and g of the typical longitudinal size and mass for the data presented in Table 1. See main text for explanation of the criteria used in preparing this table.

Label	Typical size (cm)	Size Δ (cm)	Typical mass (g)	Mass Δ (g)	Hierarchy index
A	3.086×10^{17}	1.5×10^{17}	1.989×10^{33}	1.0×10^{33}	1
B	3.086×10^{18}	1.5×10^{18}	1.989×10^{35}	1.0×10^{35}	2
C + D	3.0×10^{19}	1.5×10^{19}	1.0×10^{37}	9.5×10^{36}	3
E + F	2.3×10^{20}	1.2×10^{20}	3.58×10^{39}	2.3×10^{39}	4
G	3.086×10^{21}	1.5×10^{21}	3.978×10^{40}	1.0×10^{40}	5
H	1.7×10^{22}	1.1×10^{22}	1.0×10^{43}	9.8×10^{42}	6
I + J	1.65×10^{23}	8.6×10^{22}	1.05×10^{45}	5.0×10^{44}	7
K	3.0×10^{24}	1.5×10^{24}	4.0×10^{46}	1.0×10^{46}	8
L	1.2×10^{25}	6.0×10^{24}	2.0×10^{48}	1.0×10^{48}	9
M	1.0×10^{28}	5.0×10^{27}	7.5×10^{55}	1.0×10^{55}	12

The hierarchical nature of the universe is, of course, an old question and is found frequently in the literature. These discussions include, for example, Refs. 5 and 18–20. More recently, see Ref. 21, there has been some discussion of the subject in the context of understanding cosmic evolution using ideas based on free energy processing rates and the evolution of complexity in self-organized systems. Although de Vaucouleurs noticed what amounts to some form of log-periodic behavior in his discussion of the hierarchical Universe, he did not enter into a description of its phenomenology. To our knowledge no phenomenological understanding of this problem of the kind proposed here is available.

As should be clear from Appendix A, nothing we present here obviates conventional physics explanations for the mass-scale of different structures in the universe using non-gravitational physics, as well as features of gravity, particular to each case. Nor do we suggest that any of those analyses are mere coincidence. Rather, we make use of complex scaling phenomena common to all of the particular cases and that encompasses all of their results in a common framework. What we identify here is a commonality that, in each case, absorbs the physics of smaller scales into a noise function that has common statistical mechanical properties overall. The conventional explanations all start from the *assumption* of investigating a particular morphology on a particular scale. By ignoring the particular morphological characteristics, based on conventional non-equilibrium physics, we are able to identify the set of scales on which the different morphologies appear. The specific analyses for each morphology associated with each specific range of scales is what determines the specific characteristics and properties associated with that identified morphology.

2. Motivation for a Power Law with Complex Exponents in the Two-Point Density Correlation Function

2.1. *Theoretical motivation and analysis*

The episodes of structure formation in the Universe are among the series of major events that have taken place in the history of the Universe[12] during its evolution from an initial state of relative disorder (the primæval gas clouds of light elements) into a state with a different order (the various *discrete* structures that we now observe).

Events or transitions where a system changes its state, or in which the order of the system is qualitatively altered occur in many physical systems. For systems in equilibrium and away from equilibrium they are, respectively, known as "phase transitions"[22] and "dynamical phase transitions."[13]

A striking feature of phase transitions is their association with power law (or "scale invariant") behavior for physical quantities considered as functions of , e.g., system size or deviation in temperature from a critical value.[22] In particular, it is well known that this power law phenomenology is due to the collective involvement of the full system when near a critical regime. Mathematically, it translates into the scale invariance of the n-point correlation functions of some order parameter suitable for the description of the physics of the system (see, e.g., Ref. 22).

For systems where gravity plays a (dominant) rôle, as in the evolution of structures in the Universe, an appropriate order parameter[24–28] is the local fluctuation in matter density, $\delta\rho(r,t)$, from an average value $\rho_0(t)$:

$$\rho(r,t) = \rho_0(t) + \delta\rho(r,t). \tag{1}$$

Then we can ask the question "Commencing with a region of local matter density $\rho(r',t')$, what is the probability of finding another region with local density $\rho(r,t)$?" This probability is, of course, determined by the two-point correlation function for the density,[12]

$$\langle \rho(r,t)\ \rho(r',t')\rangle = \rho_0^2(t) + \langle \delta\rho(r,t)\ \delta\rho(r',t')\rangle, \tag{2}$$

where the angular brackets denote averaging over the sample. Fits (see e.g., Refs. 16 and 6–9) to data of the two-point correlation function for *different* known structures lead to power law behavior as in

$$\langle \delta\rho(r,t)\ \delta\rho(r',t')\rangle|_{t\to t'} \equiv \langle \delta\rho(r)\ \delta\rho(r')\rangle \propto |r-r'|^{2\chi}, \tag{3}$$

where the exponent χ is a *real* number (cf. Eq. (10)).[c]

Real-valued exponents appear in most equilibrium phase transitions,[22] as well as in many dynamical critical phenomena.[23] In *some* equilibrium phenomena,[14,29,30] in dynamical critical phenomena,[13] and in the physics of networks[31–35] or hierarchical systems,[2,3] it is known that *complex* exponents do appear because of the existence of complex fixed points in the renormalization group equations (RGEs)[36] that describe the scaling behavior of many-component systems.

For example, in fluid-dynamic phenomena associated with large-scale structure formation in the presence of gravity[37] or in phenomena modeled by stochastic

[c] The limit $t \to t'$ has been taken above, indicating that the temporal separation between the two correlated fluctuations within the angular brackets is neglible compared with the persistence time for the structure. In the case at hand, this is equivalent to saying that the time of formation and relaxation of the structures is small compared with how far back in time we are looking. See further discussion in Sec.3.

partial non-linear differential equations, complex fixed points are more of the rule than the exception when there is a finite length scale *and* the stochastic forcing term contains colored noise.[37] (See the Appendix for a brief description of how this takes place in the evolution of large- scale density perturbations.) Among the latter are the ubiquitous KPZ equation[38] and its related family of equations.[39]

In the case of large-scale structure formation, the length scale is provided by Hubble's constant.[27,28] The noise term models the various types of fluctuations that inevitably appear in the course of the evolution of the large scales, including the contribution from degrees of freedom whose typical size or time scales are smaller than the resolutions at which we study the sytem.

In the physics of networks, complex exponents appear due to the existence of some basic length scale or finite size. When combined with the scaling behavior of basic quantities such as the free energy density of the system, log-periodic behavior follows automatically.[2,3]

2.2. *Complex exponents and hierarchies*

Let us *assume*[d] then that the above 2-point correlation function for density perturbations scales as

$$\langle \delta\rho(r')\delta\rho(r)\rangle \propto |r - r'|^{2\chi} \tag{4}$$

with χ now *complex* and given by $\chi = \alpha + i\zeta$.

Since densities (and probabilities) are real, instead of Eq. (4) one must actually have that

$$\langle \delta\rho(r')\ \delta\rho(r)\rangle = \mathbb{R}e\, \tilde{c}\, e^{i\beta} \left|\frac{r - r'}{r_0}\right|^{2\alpha + i2\zeta}\ , \tag{5}$$

where $\tilde{c}$ is a scaling factor, β an arbitrary phase and r_0 an (arbitrary) reference scale. Then[e] $c = \tilde{c}\, r_0^{-2\alpha}$)

$$\langle \delta\rho(r')\ \delta\rho(r)\rangle = c \cdot |r - r'|^{2\alpha} \cdot \cos\left[\beta + 2\zeta \log\left(\frac{|r - r'|}{r_0}\right)\right]. \tag{6}$$

Accumulation of material, and therefore "gravity assisted" structure formation, will be favored (and happen faster[f]) in those regions where *maximal correlation*

[d] In this paper we will not commit ourselves to any specific model of the dynamics leading to the fixed point, concentrating only on the phenomenology implied by the existence of a complex exponent for the two-point mass–density fluctuation correlation function. It is possible, see for example, Refs. 27 and 37, to construct such models, and with them the values of α and ζ are calculable from first principles.

[e] Galaxy-to-galaxy correlation functions of the form in Eq. (5) lead to log-periodic corrections in the galaxy power-spectrum. Such effects seem to be present in the Sloan Digital Sky Survey data as well as in previous surveys,[40] but to our knowledge this is the first time they are mentioned in the literature. See, e.g., Ref. 15.

[f] If we think of this as an out-of-equilibrium phenomenon, this will be the case because out-of-equilibrium phenomena reach their most stable configurations faster than equilibrium systems.

between regions of density $\rho(r) = \rho_0 + \delta\rho(r)$ and $\rho(r') = \rho_0 + \delta\rho(r')$ occurs. This will happen whenever Eq. (6) has a maximum, i.e., for *discrete* distance (=typical longitudinal dimension) intervals given by

$$|r_{(n)} - r'| = r_0 \cdot \exp\left\{\frac{1}{2\zeta}\left[\tan^{-1}(\alpha/\zeta) - \beta\right]\right\} \cdot (e^{\frac{\pi}{2\zeta}})^n$$
$$\equiv A \cdot b^n \tag{7}$$

with $n = 0, \pm 1, \pm 2, \ldots$ and $b = \exp(\pi/2\zeta)$. (We note in passing that Eq. (7) is similar in form to the famous Titius–Bode law of planetary distances[41]).

Equation (7) tells us that there is a *hierarchy* of regions of average linear sizes $R_{(n)} \equiv |r_{(n)} - r'|$ such that accumulation of material on that scale is favored. Note that this implies that $R_{(n+i)}/R_{(n)} = b^i$. Hence if $b > 1$ structure size diverges as i grows; on the other hand, it leads to smaller and smaller structures if $b < 1$. To determine the stability properties when $b = 1$, one must consider higher order terms. However, as we will see below (Sec. 2.3), here $b \neq 1$ and no higher order terms need be considered. That $b > 1$ or $b < 1$ is determined by whether n increases as one goes up or down in the size sequence of hierarchical structures, and is related to which size is chosen as the reference size. (In our fit to size, $n = 8$.)

Furthermore, because of Eqs. (6) and (7), $R_{(n)}$ represents the typical size of a region condensed within another region and for which density fluctuations *on the average* have absolute values of $\delta\rho$ larger than the background. Hence the mass enclosed within this clump can be estimated by calculating the mass contained in a spherical clump of matter of *local* density fluctuation $\delta\rho(r)$ and radius $R_{(n)}$,

$$\Delta M_{(n)} \propto \int_0^{R_{(n)}} dr \cdot r^2 \cdot \delta\rho_{(n)}(r)\,. \tag{8}$$

This implies the existence of a mass hierarchy *in addition* to the size hierarchy just discussed (see later).

The hierarchies can be classified according to the *integer* n. They are completely determined once A, a mass normalization, and b are known. With a simple formula we can describe *all* the members in the hierarchy (see also the last remark in Footnote d).

2.3. *Phenomenological analysis*

2.3.1. *Size hierarchy*

The size hierarchy of Eq. (7) applied to the Universe is shown in Fig. 2. Here all the *distinct* structures that are known to occur in the Universe (beyond planetary systems), from the smallest molecular clouds to the horizon "size" of the Universe, are presented and compared with what Eq. (7) *produces* for $A = (3.92 \pm 0.67) \times 10^{16}$ cm, $\zeta = 0.714 \pm 0.008$ (or $b = 9.02 \pm 0.24$) obtained by a chi-squared fit to

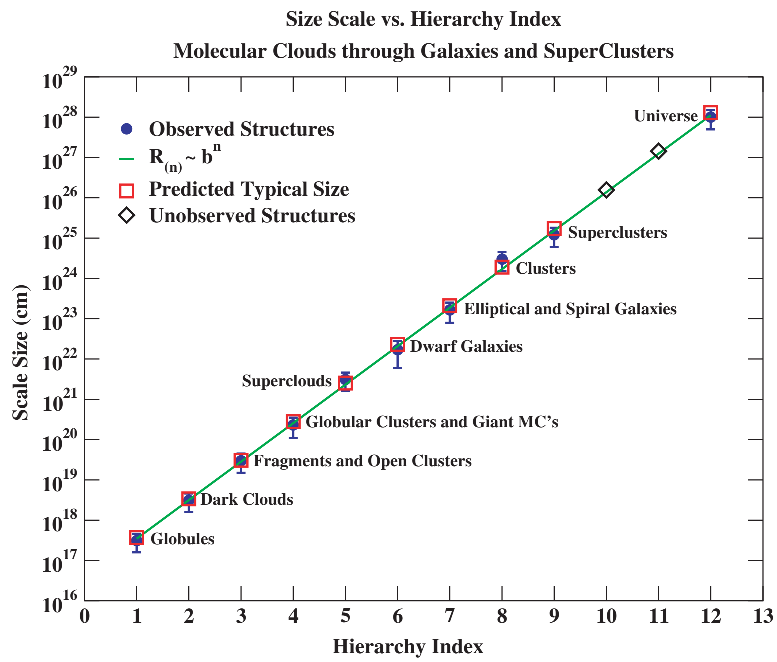

Fig. 2. Longitudinal size scale vs. hierarchy index n for discrete structures in Universe from molecular clouds through galaxies to superclusters of galaxies and predictions from Eq. (7). The value of b is 9.02 ± 0.24.

the data in Table 2. This fit agrees with observations to well *within one order of magnitude* for each of the members of the hierarchy.

From Eq. (6) we infer that [g]

$$\delta\rho(r) \sim \mathbb{R}e \left\{ |r - r'|^{\alpha} \quad . \quad \sqrt{\cos\left(\beta + 2\zeta \log \frac{|r - r'|}{r_0}\right)} \right\}. \tag{9}$$

Two interesting implications of this equation are that: (i) there are *gaps* between consecutive structures in the hierarchy, and (ii) there is a *range of allowed sizes* for the members of a particular class of structures. The gaps occur when the cosine inside the square root becomes negative, i.e., where $\delta\rho$ cannot remain real and becomes pure imaginary. Structures can form in the regions where the cosine is positive, although they are most likely to form around the scale where the correlation is maximal, as already noted.

Some comments are now in order. First, we see from Fig. 2 that *all* the known structures have longitudinal sizes which *cluster* around what is predicted by Eq. (7).

[g] This is equivalent to a procedure frequently invoked for turbulent fluids in the inertial range. There one deduces from $\langle \delta v(\ell) \delta v(0) \rangle \sim \ell^{2/3}$ for the velocity fluctuations that, phenomenologically, $\delta v(\ell) \sim \ell^{1/3}$; see for example Ref. 42.

That is, according to this equation, structures do not have sizes which fall into a continuous distribution encompassing *all* the known structures; instead they tend to form in a discrete hierarchy, the levels of which correspond to each of the known classes of structures and which can therefore be classified according to integers. However, for each hierarchy level its members can be distributed according to some continuous distribution. No stable class of structures is *expected to exist between* two consecutive integers. Second, we see from the figure that there are two integer values (at $n = 10$ and $n = 11$) which do not seem to correspond to any observed structures. The existence of two additional classes of structures, at specific scales larger than any currently known but less than that of the Universe as a whole, *may* represent additional predictions.

2.3.2. *Mass hierarchy*

Next we turn to a discussion of some additional features which emerge when we combine the size-hierarchy Eq. (7) with other known observational properties of the two-point correlation function for density fluctuations. Specifically, we now make use of the known value of the exponent in the scaling regime of the two-point density correlation function within each class of structures in the observed astrophysical hierarchy.

It is known that (e.g., for galaxies or clusters of galaxies [h]) the two-point correlation function for the density contrast of a *particular class* j of objects, $\delta\rho_{(j)}/\rho_0$, obeys[16]

$$\left\langle \frac{\delta\rho_{(j)}(r')}{\rho_0}\,\frac{\delta\rho_{(j)}(r)}{\rho_0} \right\rangle \propto |r' - r|^{2\alpha} \tag{10}$$

with $-2\alpha = 1.65 \pm 0.15$. (As remarked earlier, data for galaxies indicates that α has some evolution with redshift. This value for α must be taken as an approximation.)

By restricting (10) to sizes of the order of a typical nth hierarchy member we can write $\delta\rho_{(n)}(r) \sim R^{\alpha}_{(n)}$. This will allow one to estimate the mass of a typical object (see Table 1) in a given hierarchy class. Carrying out the integration in Eq. (8) we get

$$\begin{aligned} \Delta M_{(n)} &\propto R^{3+\alpha}_{\max}\left(1 - \frac{R_{\min}}{R_{\max}}\right)^{3+\alpha} \\ &\approx R^{3+\alpha}_{\max} \end{aligned} \tag{11}$$

where we have written $R_{(n)} = R_{\max} - R_{\min}$ and eliminated the subscript (n) to avoid clutter in the formula.

[h] A similar result is found to apply for molecular clouds although it is less firmly established and there is less general agreement on the value of α than for galaxies.[6–9]

For the value of α quoted above, we *predict* that

$$\Delta M_{(j)}^{\mathrm{Pred}} \sim R_{(j)}^{2.175\pm 0.075}. \tag{12}$$

This is the mass contained within a region of size $R_{(j)}$ and should be compared with the fit to the *data* plotted in Fig. 1. The observational data for *all* the structures is best fit by $\Delta M_{(j)}^{\mathrm{Obs}} \sim R_{(j)}^{2.10\pm 0.07}$, which agrees, within the estimated uncertainties, with what is predicted in Eq. (12).

Putting together Eqs. (7) and (12), we are then able to predict the typical mass of a member of a particular structure-class; the hierarchy relation is

$$M_{(n)}^{\mathrm{Pred}} \propto [b^{2.175\pm 0.075}]^{n}. \tag{13}$$

Given a normalization and the value of b we can predict what the mass is for each of the structures in the hierarchy. Using the value of $b = 9.02 \pm 0.24$ obtained above, and setting the overall scale by matching to the $n = 8$ mass–point, we obtain that the average mass of the nth structure is

$$\begin{aligned} M_{(n)}^{\mathrm{Pred}} &= (9.02 \pm 0.24)^{(2.175\pm 0.075)n} \times [(2.46 \pm 0.62) \times 10^{30}]\,\mathrm{g} \\ &= (123 \pm 27)^{n} \times [(2.46 \pm 0.62) \times 10^{30}]\,\mathrm{g}. \end{aligned} \tag{14}$$

In Fig. 3, we plot the values predicted by Eq. (14) for the masses together with the observed values. The observed data is best fit by

$$M_{(n)}^{\mathrm{Obs}} = (98.8 \pm 11.1)^{n} \times [(7.64 \pm 4.82) \times 10^{30}]\,\mathrm{g}. \tag{15}$$

The agreement between this quantity and the prediction of Eq. (14) is also well within the observational uncertainties.

3. Methods

In determining the entries for Table 1, we found that uncertainties for those quantities were not provided in Refs. 6–10 and 16. To obtain an accurate estimate of such uncertainties requires a complete analysis of the number vs. size and number vs. mass distributions in the raw observational data. In the absence of availability of such information, we have used the means and ranges of Trimble in Ref. 17 to estimate uncertainties, and have assumed similar percentage uncertainties for all other data. In making these estimates, we have assumed that when the reported mean is nearer to one end of the range than the other, that the uncertainty is characterized by the nearer end of the range. (The distribution in such cases clearly cannot be Gaussian; our approach is consistent with the known Shechter function[43] for such distributions.) Finally, we have presumed that the range as a whole represents several standard deviations from the mean rather than simply the single σ of a full-width half-maximum Gaussian distribution. In general, however, we have been conservative and have fixed uncertainties at the 50% level in the absence of other information.

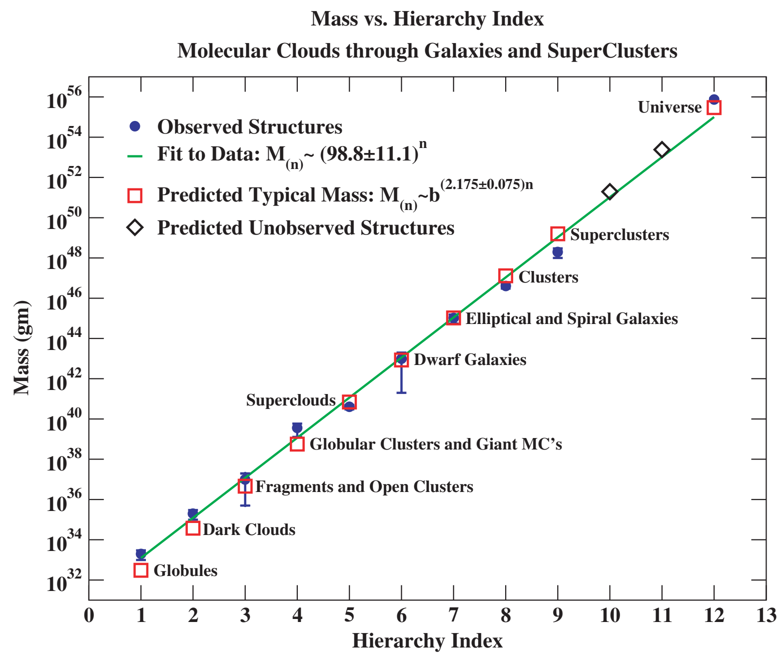

Fig. 3. Mass vs. hierarchy index n for discrete structures in Universe from molecular clouds through galaxies to superclusters of galaxies and predictions from Eq. (14). The value of b is 9.02 ± 0.24.

These assumptions were made on the basis that some reasonably peaked distribution of number vs. size (or mass) must exist in the data; i.e., not flat, although not necessarily Gaussian. Of course, one expects a Shechter function[43] for the distribution, which is decidedly non-Gaussian but certainly peaked. The complex fixed point behavior for the mass correlation function indicated by Eq. (6) also implies that a peaked structure for such number distributions should be expected, as we shall discuss in detail elsewhere.

In Fig. 4, we indicate the nature of the formation and evolution of the correlated matter and size distributions that we expect for each morphological class observed.

The structure may be understood by first considering a time relatively early in the evolution of the Universe, but late enough so that fluctuation seeds for density perturbations have developed under the influence of the relevant dynamics. Our conjecture in this paper is that there will be a hierarchy of preferred scales, defined by the recurrences inherent in complex fixed point scaling dynamics for the size and mass of the fluctuations and associated with the generic features of dynamical critical phenomena. Fluctuations on these preferred scales will develop and intermediate ones will dissipate. The scales are not precise, but allow for some distribution around each value.

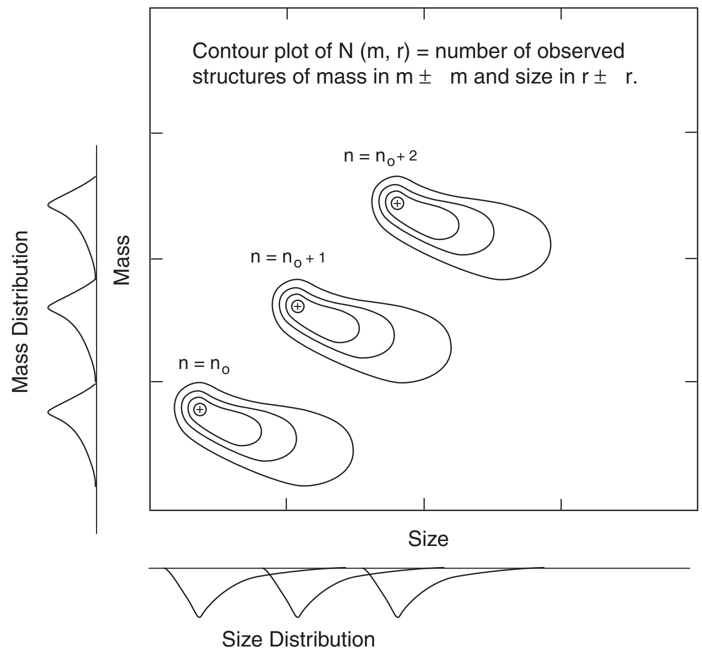

Fig. 4. Expected structure of contour plot of number distribution vs. size and mass scales for three different hierarchical elements corresponding to three distinct observed morphologies, when the observations for different z-values are not distinguished. The peak (marked by the "+" sign) is shifted by evolutionary effects since the logarithmic distribution is: (a) dynamical, in principle, hence evolutionary and (b) the log-periodic distribution intrinsically peaks asymmetrically within each particular emphasized band of sizes and masses.

In terms of Fig. 4, imagine a narrow initial distribution centered in the lower right of each of the lobes drawn as the initial configuration. Over a sufficiently long time scale, and as the Universe evolves, two developments will occur: (1) Under the influence of gravity, local overdensities will shrink in overall size (if they are bound), despite the expansion of the Universe, and (2) they will continue to accrete additional mass from the surrounding underdense medium as it dissipates. Thus, the peak of the correlated mass-size distributions in each hierarchical element will evolve towards smaller size and larger mass. Over the entire Universe, however, the correlated hierarchical distributions in size and mass are as indicated *schematically* by the contours plotted in Fig. 4.

Celestial surveys of objects (of any given morphology), which do not distinguish the epoch (z-value) in which they reside, include objects with masses and sizes at different evolutionary stages. Hence the separate size or mass distributions have the features shown by the projections on each axis. Because of the evolution, the shape is necessarily asymmetrical and broad and one expects some overlap between different morphological classes, especially at the small size and mass tails of the distributions. We surmise that this is related to the origin of the observed structure of Schechter distributions.

4. Further Discussion

Comment is also called for with regard to the entry for dwarf galaxies. This was taken from Trimble in Ref. 17, but has been corrected for a systematic difference. This is a factor of 2.7 in size scale between the Trimble values and those in Ref. 16 for spiral and elliptical galaxies. Such systematic differences are not uncommon, due to differing methods for defining size measurements; all that is important here is that the two methods have been brought into registration by requiring agreement in the well-known case of normal galaxies.

As noted earlier, in Table 2 we collected some of the structures of Table 1 into groups of structures. This was done according to the criterion that various structures may be coalesced into a single data point for a "structure class" if their typical sizes are such that their dispersion in size (as given in Table 1) is comparable to or within the difference between their respective sizes.

The only problematic case is the combination of Open Star Clusters and Fragments of Molecular Clouds, where the typical sizes are quite comparable but the masses differ significantly. Of course, variations in size scale of only a factor of 3 correspond, without changes in density, to variations in mass as large as a factor of 27, which is almost as large as the difference in mass between these two morphologies. To accomodate this, we represent the combined mass data point as having an uncertainty covering the entire range, as indicated by the large error on the arithmetic average value taken for the mass of these objects.

Lastly, we would like to suggest a different approach to data assembly which has the potential to resolve many questions related to the intrinsic data. We suggest that the common practice of grouping by morphology may not lead to the best possible representation of the entire range of data. Instead, we propose consideration of a collection method that counts the number of observations of systems within a given range of mass and size, independent of their individual morphology.

On the basis of what is already known and our analysis above, we expect this number distribution to be peaked in regions within the mass-size plane. The projections onto the mass or size scale axes are the distributions for which the averages reproduced in our tables arise. Hence, the different regions should be associated with different morphologies, or sets of morphologies. And finally, since structures tend to shrink and accrete some mass during formation and relaxation, the shape and distribution within the regions could provide information regarding the physics and time scales of the formation of the structures.

5. Conclusions

To summarize we recapitulate our results.

We have explored the phenomenology of a complex exponent for the two-point correlation function of density perturbations in the Universe. The assumptionof a complex exponent is motivated by the well-known theoretical observation that the coarse-graining of many-component systems undergoing dynamical phase transitions leads to fixed points of the renormalization group, and that some of these correspond to complex scaling exponents for the correlation functions in their scaling regimes. The formation of structure in the Universe and its evolution under gravity being one example of this kind of phenomena.

We have shown how a complex exponent for the two-point correlation function, together with the requirement of maximal spatial correlation of density perturbations, leads to a *discrete* hierarchy of structures of finite size and mass. These structures can be classified according to an integer which labels its place in the hierarchy and a *common* base, $b = 9.02 \pm 0.24$, given in Eq. (7). Our analysis implies the existence of a *common* real exponent (α) for the two-point correlation function which applies separately to objects in each class within the hierarchy. Using the value $\alpha = -0.825 \pm 0.075$ known from observations of the various classes of structures, we obtain a *universal* mass–size relationship, Eq. (12). Finally, combining Eqs. (7) and (12) produces a scaling prediction, Eq. (14), for the mass of a typical object in the nth class. Agreement with observations is remarkably close in both cases as is shown in Figs. 2 and 3.

We are able to understand *why* there are mass and size hierarchies in the Universe, and *where the elements are of each of the hierarchies.* We can also see *when the hierarchical organization described here does not apply*, namely for systems where competition between gravity-dominated clustered and clustering due to other forces invalidate either of our two basic assumptions; this is expected to happen as we go to smaller scales, where electromagnetic effects (magnetic fields, chemical reactions, electrostatic adhesion, etc.), shocks and other microscopic (such as nuclear reactions) or macroscopic phenomena invalidate our assumptions.

Acknowledgments

The authors thank Murray Gell-Mann, Alvaro Giménez and Daniel Holz for discussions, useful comments and encouragement, and an anonymous referee for helpful comments.

Appendix A

The equations describing the evolution of density perturbations $\delta\rho(x,t)$ in the Universe

$$\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot((1+\delta)\mathbf{v}) = 0, \tag{A.1}$$

$$\frac{\partial\mathbf{v}}{\partial t} + H\mathbf{v} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{w} - \frac{1}{a\rho}\nabla p, \tag{A.2}$$

$$\nabla\cdot\mathbf{w} = -4\pi Ga\rho_0(t)\delta \;\;,\;\; \nabla\times\mathbf{w} = 0, \tag{A.3}$$

where $\mathbf{v}(x;t)$ and $\mathbf{w}(x;t)$ are the peculiar velocity and acceleration fields of the evolving matter fluid, respectively. In the equations above $H = \dot{a}(t)/a(t)$ is the

Hubble parameter, $a(t)$ is the scale factor for the underlying cosmological background and $\rho_0(t)$ is the (time-dependent) average cosmological density. Thus, the local density is $\rho(x;t) = \rho_0(t)(1 + \delta(x;t))$ and δ is the dimensionless density contrast. These equations can, for very general conditions[44,37] be recast in terms of conformal time, τ, as

$$\frac{\partial \Psi}{\partial \tau} = \nu \nabla^2 \Psi + \frac{1}{2}\lambda(\nabla\Psi)^2 - m^2(\tau)\Psi + \tilde{\eta}(x,\tau)\,, \tag{A.4}$$

where ν and λ are parameters describing various aspects of the physics, $m^2(\tau)$ takes the expansion of the Universe into account and the function $\Psi(x,\tau)$ is related to the velocity potential $\psi(x,t)$, the Laplacian of which is, in turn, directly proportional to the local density contrast $\delta\rho(x,\tau)$ in comoving coordinates. (For details and references, see Schulman *et al.*[37]) The noise term, $\tilde{\eta}(x,\tau)$ includes the effects of phenomena occurring at scales smaller than the ones probed by $\delta\rho(x,\tau)$ as well as the presence of random processes not actually modeled by Eq. (A.4). The noise is described in terms of its correlation function and, as in many non-equilibrium phenomena, is assumed to be colored and gaussian, since this is the most general class.

Since the stochastic effects are scale dependent and there exist conservation laws and divergences associated with Eq. (A.4), one needs to introduce a procedure to deal with them. This has been done in condensed matter and in elementary particle physics for many years by means of the renormalization group (RG). This leads to scale-dependent parameters, such as ν and λ, which inherit a size-dependence emerging from the form-invariance of Eq. (A.4). This then guarantees the applicability to wider time and space scales than would otherwise be expected.

The two-point correlation function for $\Psi(x,\tau)$, under these conditions, can be shown to be of the form

$$\langle \Psi(x,\tau)\Psi(x',\tau')\rangle \propto |x - x'|^{2\chi_\Psi} f\left(\frac{|\tau - \tau'|}{|x - x'|^{z_\Psi}}\right), \tag{A.5}$$

where the two exponents, χ_Ψ and z_Ψ can be calculated in perturbation theory using the RG, and the function $f(u)$ has the following properties:

$$\lim_{u\to\infty} f(u) \to u^{2\chi_\Psi/z_\Psi} \tag{A.6}$$

and

$$\lim_{u\to 0} f(u) \to \text{constant}. \tag{A.7}$$

In the case of the density perturbations these exponents can also be calculated and, in the appropriate limit, one obtains Eq. (4) of the text, $\langle\delta\rho(r')\delta\rho(r)\rangle \propto |r - r'|^{2\chi}$. For the vast majority of noise functions, see Ref. 37, one discovers that χ is complex, i.e.,

$$\chi = \alpha + i\zeta \tag{A.8}$$

where α and ζ can be calculated in terms of *generic* properties of the noise.

In this paper, we use the knowledge that complex fixed points (and therefore exponents) *are possible* in evolutionary astrophysical processes and we have sought to study the range of exponent values that could accommodate the observations. In a future publication, we plan to actually narrow down the exponent values and the classes of noise that can best accommodate the observations.

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