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# Inequality and Segregation

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Despite declining group inequality and the rapid expansion of the black middle class in the United States, major urban centers with significant black populations continue to exhibit extreme racial separation. Using a theoretical framework in which individuals care about both the affluence and the racial composition of neighborhoods, we show that lower inequality is consistent with extreme and even rising levels of segregation in cities in which the minority population is large. Our results can help explain why segregation continues to characterize the urban landscape even though survey evidence suggests that individuals favor more integration than they did in the past.

## I. Introduction

Several decades have elapsed since the landmark Civil Rights Act of 1964 outlawed discrimination in employment and public education, and the 1968 Fair Housing Act extended these protections to the sale and rental of housing. Over this period, racial disparities in educational attainment and household income have narrowed, and a significant population of middle-class African Americans has emerged. Approximately one-half of black Americans now live in middle- or upper-income

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households as compared with about one-fifth in 1960. The black-white gap in high school completion rates for 25–29-year-olds dropped from 20 percentage points in 1967 to seven points in 1996. Median black household income rose by 41 percent between 1967 and 1999, whereas median white household income rose by 24 percent over the same period (Council of Economic Advisors 1998; U.S. Census Bureau 2000).[1]

Despite the decline in group inequality and the rapid expansion of the black middle class, residential segregation remains a striking feature of the urban landscape in many large metropolitan areas with significant black populations. Major urban centers such as New York, Chicago, Detroit, Newark, and Milwaukee continue to be characterized by extreme levels of racial separation, even as areas with smaller black populations have begun to show signs of greater integration. Moreover, the levels of segregation experienced by black households are uniformly high across all income categories, and the relative segregation of different cities has remained remarkably stable for over a century.[2]

The easiest way to reconcile the narrowing racial income gap with persistent segregation is to argue that households prefer more segregated neighborhoods than they did in the past. If racial identities have hardened and the racial composition of a neighborhood has become more important than its wealth in determining residential location, the effects of income gains of blacks may be more than offset by these changes in preferences. Survey evidence, however, suggests quite the opposite: attitudes of white Americans toward integrated schools and neighborhoods seem to have softened considerably. In a recent review of the literature, Bobo (2001, p. 269) maintains that the "single clearest trend shown in studies of racial attitudes has involved a steady and sweeping movement toward general endorsement of the principles of racial equality and integration." For instance, in 1963, only 39 percent of white respondents disagreed with the statement that whites had a right to keep blacks out of their neighborhood; by 1996 this had risen to 86 percent (Schuman et al. 1997).[3] What, then, accounts for the continuing hypersegregation of major metropolitan areas with significant black populations?

[1] The extent of convergence in the racial wage gap is not uncontroversial. Chandra (2003) points out that estimates based on the Current Population Survey may be flawed since they do not account for the selective nonparticipation of blacks in the labor force. He argues that as much as 85 percent of the observed convergence in earnings may be accounted for by selection problems caused largely by the relatively high black incarceration rates. While this may be true, it is mainly the nonincarcerated who make decisions on residential location, and it is the racial income convergence for this group that is relevant here.

[2] See Denton and Massey (1988), Massey and Denton (1993), Farley and Frey (1994), and Cutler, Glaeser, and Vigdor (1999) for evidence and interpretations, and Glaeser and Vigdor (2001) and Lewis Mumford Center (2001) for an analysis of the most recent data.

[3] Additional evidence on attitudes is discussed in Sec. II below.

We show in this paper that narrowing racial income disparities can, under certain circumstances, result in *increasing* residential segregation in cities with significant black populations. This can happen even when all households have a strong preference for integrated over segregated neighborhoods. Hence the intuitive and widely held view that declining segregation is a natural consequence of reductions in group inequality is not supported by our analysis. The conventional view is rooted in the intuition that if households sort themselves across neighborhoods on the basis of income, then racial income disparities will be mirrored in segregated residential patterns. This intuition fails when households make location decisions based on multiple neighborhood characteristics. When individuals care also about the racial composition of their communities, the relationship between inequality and segregation is more complex and depends in subtle ways on both intraracial and interracial disparities in income.

Our analysis is conducted within the framework of a model in which incomes vary both within and between groups, and individuals care about both the level of affluence and the racial composition of the communities in which they reside. This concern with racial composition may be pro-integrationist, in that households prefer some degree of mixing to homogeneous neighborhoods of either type. Individuals are able to locate in any neighborhood, provided that they are willing and able to outbid others to do so.[4] We focus on equilibrium allocations that are *stable* in the sense that small perturbations in the neighborhood of the equilibrium are self-correcting under the dynamics of decentralized neighborhood choice.

We present two main results. The first result demonstrates that when the share of minority households is substantial, extreme levels of segregation can be stable if racial income disparities are either sufficiently large or small, but unstable in some intermediate range. Hence racially integrated equilibria are most likely to be observed for moderate degrees of racial inequality. Continued narrowing of racial income disparities can give rise to *resegregation*, and, from a cross-sectional perspective, one ought not to expect cities with the smallest racial income disparities to be the ones with the lowest levels of segregation. The relationship between inequality and segregation that is suggested by our analysis helps account for the fact that income convergence has not typically been found to have statistically discernible effects on changes in segregation over time (Farley and Frey 1994).

Our second result shows that when racial income disparities are small,

[4] This may not be possible in practice because of racial steering by real estate agents or discrimination in mortgage lending markets (Yinger 1995). We abstract from such overt discrimination because its effects on segregation are reasonably well understood and because doing so allows us to better focus on the questions at hand.

multiple equilibria exist, and these equilibria can differ dramatically in their levels of segregation. The existence of multiple equilibria suggests that although integration may become viable as racial income disparities lessen, history may trap a city in a segregated equilibrium. This is where social policy may be most effective: temporary incentives for integration may give rise to permanent effects. Integration comes at the cost of higher stratification by income, however, so integrationist policies need not be unambiguously welfare enhancing even when preferences are strongly pro-integrationist.

To obtain some intuition for our findings, consider first the extreme case in which the two income distributions are identical and the black minority is substantial. Under complete segregation, households are faced with a choice between racially homogeneous neighborhoods of comparable mean income. Even if all households prefer some integration, segregation will be stable as long as individuals prefer racially homogeneous neighborhoods populated by their own group to racially homogeneous neighborhoods populated by other groups. As income disparities between groups widen, so do mean neighborhood incomes under segregation, and at some point affluent blacks will outbid the less affluent whites to live in a higher-income, predominantly white neighborhood. At this point segregation becomes unstable. At the other extreme, when income disparities are large, segregation by race is almost equivalent to stratification by income, and segregated allocations are again stable. Hence the relationship between inequality and segregation is not monotonic. With small income disparities, the allocation in which households sort themselves purely by income will be stable as long as preferences for integration are strong enough. Since segregation is also stable under these conditions, we have multiple equilibria.

Our work is closely related to two literatures that deal with the decentralized dynamics of neighborhood choice. The idea that extreme levels of segregation can arise under a broad range of preferences over neighborhood composition was developed in seminal work by Schelling (1971, 1972). This analysis contains the important insight that even when all individuals prefer integrated neighborhoods to segregated ones, integration may be unsustainable in that a few random shocks can tip the system to a segregated equilibrium. It is difficult, therefore, to deduce anything about individual preferences from aggregate patterns of residential location.[5] While Schelling's analysis neglects the role of prices in rationing housing demand, broadly similar conclusions hold in models that take full account of adjustments in rents (Yinger 1976;

[5] "People who have to choose between polarized extremes … will often choose in a way that reinforces the polarization. Doing so is no evidence that they prefer segregation, only that, if segregation exists and they have to choose between exclusive association, people elect like rather than unlike environments" (Schelling 1978, p. 146).

Schnare and McRae 1978; Kern 1981). This literature neglects the fact that individuals consider both race and income when making location choices and that forces acting to produce income stratification can substantially mitigate the amount of racial segregation that results. While extreme levels of segregation are consistent with pro-integrationist preferences in our model, it is also the case that, under certain circumstances, stable equilibria can entail greater *integration* than any individual, black or white, considers ideal.

There is also an extensive literature on neighborhood sorting when individuals differ with respect to their incomes and sort themselves across jurisdictions on the basis of neighborhood characteristics such as local taxation, redistribution, public education, or peer effects (de Bartolome 1990; Epple and Romer 1991; Benabou 1993; Durlauf 1996; Fernandez and Rogerson 1996; Epple and Platt 1998). Stratification by income occurs in many such models. What is missing from this body of work is the possibility that individuals care about certain intrinsic characteristics of those with whom they share their neighborhoods and that such preferences are themselves related to group membership. When there is inequality both within and between groups, adding these components to the analysis yields significant new insights that appear neither in the segregation literature descended from Schelling nor in the literature on neighborhood sorting in the Tiebout tradition.

The paper is organized as follows. Section II provides some discussion and justification for our key assumption that individuals care about both the racial composition and the level of affluence in their communities. The model is developed in Section III, and its equilibrium properties are characterized in Section IV. Section V examines the relationship between racial income disparities and residential segregation, and Section VI presents conclusions.

## II. Preferences

Extensive survey evidence on the racial attitudes of Americans has been collected for more than half a century. Several studies have specifically attempted to ascertain the preferences of respondents over neighborhood racial composition (Farley et al. 1978, 1993; Bobo, Schuman, and Steeh 1986). The best recent evidence comes from a "Multi-city Study of Urban Inequality" funded jointly by the Ford Foundation and the Russell Sage Foundation. Subjects drawn from the Los Angeles and Boston metropolitan areas were asked to construct an "ideal neighborhood that had the ethnic and racial mix" that the respondent "personally would feel most comfortable in." They did so by examining a card depicting three rows of five houses each, imagining their own house to be at the center of the middle row, and assigning to each of the remaining houses an ethnic/racial category using the letters *A* (Asian), *B* (black), *W* (white), and *H* (Hispanic). The study found evidence that "all groups prefer both substantial numbers of co-ethnic neighbors and considerable integration" (Charles 2001, p. 257). On average, the ideal neighborhood consisted of a plurality of the respondent's own type (ranging from 40 percent for black respondents to 52 percent for whites) together with significant representation from other groups. Only 2.5 percent of blacks and 11.1 percent of whites considered homogeneous neighborhoods populated only with their own type to be ideal. Overall, this reflects a clear desire for some degree of integration on the part of all groups, with a bias toward members of one's own group. This is consistent with prior studies of attitudes toward racial composition and motivates the specification used in this paper.

Why might individuals care about the racial composition of their neighborhoods? As noted by Cornell and Hartmann (1997, pp. 23–24), "race still wields monumental power as a social category" despite the fact that "racial categories are historical products that are often contested." Farley et al. (1994) trace white attitudes to negative racial stereotypes and black attitudes to anticipated hostility from whites. Ellen (2000) argues that white households hold an exaggerated view (relative to black households) of the association between changes in racial composition and structural decline in neighborhood quality. Whites are consequently less willing than blacks to settle in neighborhoods that have recently experienced increases in the share of black residents. O'Flaherty (1999) has argued that interracial transactions of many kinds are rendered difficult because the signals blacks and whites send each other through their actions and words "are garbled by stereotypes and the possibility of animosity" (p. 4). The fact that communication is easier and less ambiguous when it does not cross racial lines could account for a desire to live among one's own group. Signals also play a key role in the search-theoretic model of Lundberg and Startz (1998), where signals from members of one's own group are interpreted with less noise than signals from others. Again this can lead endogenously to a desire to associate primarily with one's own group. While we take preferences over neighborhood racial composition to be exogenously given, our specification is consistent with these interpretations. In addition, we allow for the possibility that there may be a preference for some degree of integration on the part of both blacks and whites, as suggested by the survey evidence.

We assume that, in addition to neighborhood racial composition, individuals also care about the level of affluence in their communities. There are a number of reasons why this might be the case. The quality of public schools is liable to be better in more affluent neighborhoods even if government expenditures per pupil are uniform across the city.

The reason is that voluntary contributions to parent-teacher associations increase with income, and human capital transfers that occur in the home have spillover effects in school. The presence of positive role models (and the absence of negative ones) is correlated with the degree of affluence of a community. Living in a more affluent community provides entry into social networks that can be lucrative. And if the external upkeep of one's residence is a normal good with positive external effects, more affluent communities will be more desirable. Each of these effects has been discussed extensively in the literature (Bond and Coulson 1989; de Bartolome 1990; Benabou 1993). Although the desire to live in a more affluent community can be endogenously derived on the basis of any of the above concerns, it is treated here as a primitive of the model.

## III. The Model

Consider a city with a continuum of households represented by the interval [0, 1]. Households differ along two dimensions, income and race. There are two groups, black and white, and the share of black households in the city is denoted $\beta < \frac{1}{2}$. Within each group the income distribution is represented by absolutely continuous distribution functions $F^b(y)$ and $F^w(y)$, with $f^b(y)$ and $f^w(y)$ denoting the corresponding densities. The supports of the two income distributions are given by the intervals $[y^b_{\min}, y^b_{\max}]$ and $[y^w_{\min}, y^w_{\max}]$. It is assumed that $y^b_{\min} < y^w_{\min} < y^b_{\max} < y^w_{\max}$, and for any $y \in [y^b_{\min}, y^w_{\max}]$, $F^b(y) > F^w(y)$. Taken together, these assumptions imply that whites are wealthier than blacks as a group, although the wealthiest black households are better off than the poorest white ones. What distinguishes blacks from whites in this model is simply the fact that the former are members of a socially identifiable minority group with an income distribution that is dominated by that of the majority group.

The city is divided into two disjoint neighborhoods of equal size.[6] Any subset $A \subset [0, 1]$ with measure one-half represents an allocation of households across neighborhoods, with the interpretation that any household in $A$ resides in neighborhood 1, and the remaining households are in neighborhood 2. Any allocation of households across neighborhoods uniquely determines both the racial composition and the distribution of income within each neighborhood. Let $\bar{y}_j$ denote the mean income in neighborhood $j \in \{1, 2\}$ and $\beta_j \in [0, 2\beta]$ the share of neighborhood $j$'s population that is black.

[6] Neither the assumption of equal size nor the restriction to two neighborhoods is critical. These assumptions considerably facilitate exposition and allow clearer statements of our results. Our main results characterizing the relationship between racial income disparities and equilibrium segregation (propositions 1 and 2) can be appropriately modified to hold also for multiple neighborhoods of unequal size.

Housing units are identical, and rents are accordingly uniform within each neighborhood. We normalize the rent in neighborhood 1 to equal zero and let $\rho$ be the (possibly negative) rent in neighborhood 2. All income not spent on rent is used for private consumption. Apart from their private consumption, individuals care about the general affluence and racial composition of their communities. Neighborhoods with higher mean incomes are more desirable than those with lower mean incomes for all members of the population.[7] Additionally, black and white households differ systematically with regard to their preferences over neighborhood racial composition. We shall assume for simplicity that the preferences of blacks and whites are symmetric in a sense to be made clear below. We do not assume, however, that preferences are monotonic in neighborhood racial composition. In particular, we allow for the possibility that households strictly prefer a wide range of integrated neighborhoods to segregated ones.

Preferences are represented by the following utility function:

$$U(c, \bar{y}, r) = u(c, \bar{y}) + v(r),$$

where $c$ is private consumption, $\bar{y}$ is neighborhood mean income, and $r$ is the neighborhood population share of the individual's *own* group. Hence $r = \beta$ for black households and $r = 1 - \beta$ for whites. We assume that $u(c, \bar{y})$ is differentiable, is strictly increasing in both arguments, and satisfies $u_{11} < 0$ and $u_{12} \geq 0$. These assumptions are standard in the sorting literature and together imply that more affluent households have a higher willingness to pay for increases in mean neighborhood income. We assume that utility from the racial composition of the neighborhood, $v(r)$, is differentiable and strictly concave, with a maximum at some $r^* \in (\frac{1}{2}, 1)$. Hence all households would ideally like some degree of racial integration, which is consistent with the survey data. We assume further that $v(1) > v(0)$, which implies that when choosing between racially segregated neighborhoods, households prefer the one inhabited by their own group. This specification is consistent with a variety of attitudes toward neighborhood racial composition, including the possibility that all households prefer being part of a sizable minority to being in an overwhelming majority.

Equilibrium in this model is an allocation of households across neighborhoods and a rent $\rho$ such that no household prefers a neighborhood different from its own. In other words, equilibrium requires that for any household with income $y$ that resides in neighborhood 1, it must be the case that $U(y, \bar{y}_1, r_1) \geq U(y - \rho, \bar{y}_2, r_2)$. The inequality is reversed for any household in neighborhood 2. Note that the relevant argument

[7] All our results remain intact (with proofs essentially unchanged) if households care about median rather than mean income.

in the utility function is the share in the neighborhood of the individual's own group. We shall refer to an allocation in which each neighborhood contains members of both groups as *integrated* and all other allocations as *segregated*.

We say that an allocation is *intraracially stratified* if there exist threshold income levels $\tilde{y}^b$ and $\tilde{y}^w$ such that one neighborhood consists exclusively of all black households with income above $\tilde{y}^b$ together with all white households with income above $\tilde{y}^w$. The neighborhood with this property will be referred to as the *upper-tail neighborhood*. The other (lower-tail) neighborhood then consists exclusively of all black households with income below $\tilde{y}^b$ together with all white households with income below $\tilde{y}^w$. Intraracial stratification is consistent with complete segregation (if $F(\tilde{y}^b) = 0$ or 1), with pure stratification by income (if $\tilde{y}^b = \tilde{y}^w$), and with a variety of other patterns of neighborhood sorting. Without loss of generality, we adopt the convention that at any intraracially stratified allocation, neighborhood 2 is the upper-tail neighborhood. A direct implication of the assumption that more affluent households have a higher willingness to pay to live in wealthier neighborhoods is that any equilibrium allocation in which the higher-income neighborhood has a higher rent must satisfy intraracial stratification.

When an allocation is intraracially stratified, the mean incomes and racial compositions in each neighborhood can all be expressed as a function of the threshold income level for white households. Let $z = \tilde{y}^w$ denote the threshold income for whites. It must be the case that $z \in [z_{\min}, z_{\max}]$, where $z_{\min}$ is defined by the condition $(1-\beta)[1 - F^w(z_{\min})] = \frac{1}{2}$ and $z_{\max}$ by $(1-\beta)F^w(z_{\max}) = \frac{1}{2}$. When $z = z_{\min}$, the upper-tail neighborhood consists exclusively of white households; when $z = z_{\max}$, the lower-tail neighborhood is exclusively white. Given any value of $z \in [z_{\min}, z_{\max}]$, there exists a unique $\tilde{y}^b \in [y^b_{\min}, y^b_{\max}]$ such that

$$\beta F^b(\tilde{y}^b) + (1-\beta)F^w(z) = \tfrac{1}{2}.$$

The threshold $\tilde{y}^b(z)$ identifies the unique level of black income such that blacks with income above this threshold and whites with income above $z$ together constitute half the population. Note that $\tilde{y}^b(z)$ is a continuous, strictly decreasing function on $[z_{\min}, z_{\max}]$.

At any intraracially stratified allocation $z$, the share of black households in neighborhood 1 is given by

$$\beta_1(z) = 2\beta F^b(\tilde{y}^b(z)),$$

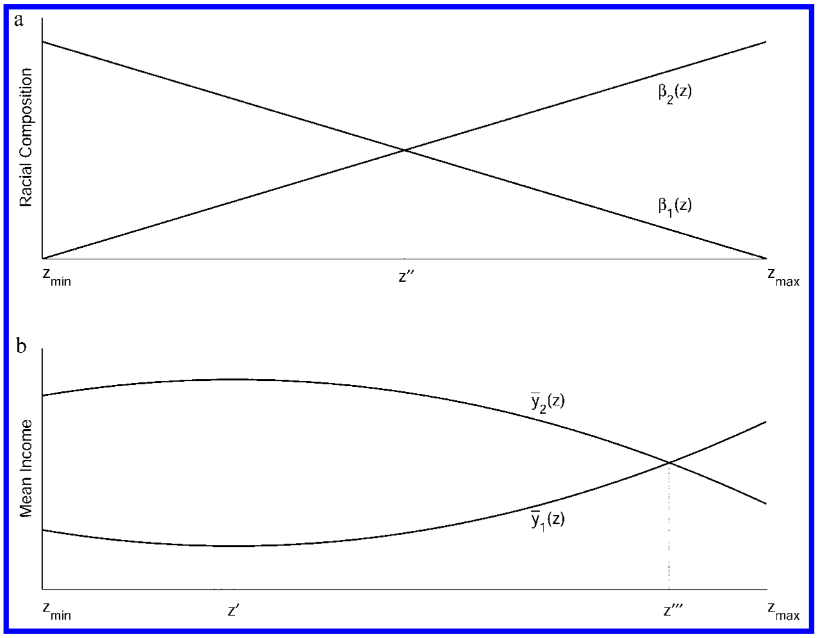


FIG. 1.—Neighborhood characteristics as functions of threshold white income $z$

where $\beta_1(z) \in [0, 2\beta]$ and $\beta_2(z) = 2\beta - \beta_1(z)$. Mean incomes in the two neighborhoods are

$$\bar{y}_1(z) = \frac{\beta_1(z)}{F^b(\tilde{y}^b(z))}\int_{y^b_{\min}}^{\tilde{y}^b(z)} yf^b(y)dy + \frac{1-\beta_1(z)}{F^w(z)}\int_{y^w_{\min}}^{z} yf^w(y)dy,$$

$$\bar{y}_2(z) = \frac{\beta_2(z)}{1-F^b(\tilde{y}^b(z))}\int_{\tilde{y}^b(z)}^{y^b_{\max}} yf^b(y)dy + \frac{1-\beta_2(z)}{1-F^w(z)}\int_{z}^{y^w_{\max}} yf^w(y)dy.$$

Hence all neighborhood characteristics relevant to households are fully determined by the threshold white income $z$.

The manner in which neighborhood characteristics change as the threshold white income varies between $z_{\min}$ and $z_{\max}$ is illustrated in figure 1. When $z = z_{\min}$, there is complete residential segregation by race, with the second (all-white) neighborhood having higher mean income. As $z$ rises from this minimum value, the lowest-income whites in the second neighborhood are replaced by the highest-income blacks from the first, which leads to increasing income disparities across neighborhoods. The point at which neighborhood income disparities are greatest occurs when $z = z'$, where $\tilde{y}^b(z') = z'$. This would be the outcome if sorting were based on income alone.[8] At this point it must be the case that the lower-tail neighborhood has a greater proportion of black households than the upper-tail neighborhood. As $z$ rises beyond this point, overall stratification by income begins to decline. At the point $z = z''$ the two neighborhoods have the same racial composition. If $\beta$ is sufficiently large, at some point the two neighborhoods will have identical mean incomes; this occurs at $z = z'''$ in the figure. Beyond this point the second (upper-tail) neighborhood has lower mean income since it consists of all but the poorest segments of the less affluent group together with a few of the wealthiest members of the more affluent group. Finally, when $z = z_{max}$, the allocation is again segregated, but with the most affluent whites sharing a neighborhood with the city's black population and the lower-income whites living in a racially homogeneous neighborhood.

As noted above, any equilibrium in which the higher-income neighborhood has a higher rent must be intraracially stratified; accordingly, this paper will focus on this class of equilibria. Nevertheless, for the sake of completeness, we note that there can be equilibria that are not intraracially stratified and in which there is no rent differential across neighborhoods. For example, an allocation in which each neighborhood has the same mean income and racial composition as the city as a whole must be an equilibrium in the absence of a rent differential, since all households regardless of race will be indifferent between neighborhoods. There can also be an equilibrium in which the two neighborhoods have equal rents, all black households occupy the same neighborhood, and all white households are indifferent between the two neighborhoods. The difference in mean income across neighborhoods must exactly compensate whites for differences in neighborhood racial composition. Note that all such equilibria require that there be no rent differential across neighborhoods, and the emergence of the slightest such differential will induce households to arrange themselves in a manner that is intraracially stratified. We accordingly confine our attention in the remainder of this paper to intraracially stratified allocations and derive below the conditions for local stability of this class of equilibria.

## IV. Stability

At any intraracially stratified allocation $z$, define the marginal bid rent $\rho^w(z)$ as the maximum rent that a white household with income $z$ is

[8] If the largest black income lies below the median white income and if the black share of the metropolitan population is sufficiently small, there may be no $z \in [z_{min}, z_{max}]$ such that $\tilde{y}^b(z) = z$. In this case sorting by income alone would give rise to a segregated allocation, and the largest income difference between the two neighborhoods would occur when $z = z_{min}$.

willing to pay to live in the upper-tail neighborhood. Similarly, define $\rho^b(z)$ as the maximum rent that a black household with income $\tilde{y}^b(z)$ is willing to pay to live in the upper-tail neighborhood. These functions are defined implicitly by the indifference conditions $G^w(z, \rho^w) = 0$ and $G^b(z, \rho^b) = 0$, where

$$G^w(z, \rho^w) = U(z - \rho^w, \bar{y}_2(z), 1 - \beta_2(z)) - U(z, \bar{y}_1(z), 1 - \beta_1(z)),$$

$$G^b(z, \rho^b) = U(\tilde{y}^b(z) - \rho^b, \bar{y}_2(z), \beta_2(z)) - U(\tilde{y}^b(z), \bar{y}_1(z), \beta_1(z)).$$

There will always exist a finite pair of marginal bid rents that satisfy these indifference conditions provided that $u(c, \bar{y})$ shows enough variation in $c$. We assume that this is indeed the case. Furthermore, this pair of marginal bid rents is uniquely determined since $u(c, \bar{y})$ is strictly increasing in its first argument.

Any integrated allocation at which the two marginal bid rents $\rho^w(z)$ and $\rho^b(z)$ coincide and are positive is an equilibrium allocation with the equilibrium rent being equal to the common marginal bid rents. This is the case because the marginal households (with income $z$ and $y^b(z)$, respectively) are indifferent between the two neighborhoods, whereas all other households have a strict preference for the neighborhood to which they are allocated. Moreover, all integrated equilibria must be such that the marginal bid rents coincide and are nonnegative. At segregated equilibria the marginal bid rents will generally differ, but it must be the case that all black households prefer the neighborhood in which they all reside. Hence if all black households live in the lower-tail neighborhood, we must have $\rho^w(z) \geq \rho^b(z)$; the inequality is reversed if all black households live in the upper-tail neighborhood.

Intuitively, an equilibrium will be stable if small movements of individuals across neighborhoods are self-correcting and hence restore the equilibrium allocation. More formally, for all $z \in (z_{\min}, z_{\max})$, consider the following specification for disequilibrium dynamics:

$$\dot{z} = H(\rho^b(z) - \rho^w(z)), \tag{1}$$

where $H$ is an arbitrary strictly increasing function that satisfies $H(0) = 0$. These dynamics implicitly assume that when individuals relocate, they do so in a manner that maintains intraracial stratification: the marginal households are the first to move. The dynamics at boundary points $z \in \{z_{\min}, z_{\max}\}$ are slightly different, since trajectories must remain within $[z_{\min}, z_{\max}]$. Accordingly, suppose that

$$\dot{z} = \begin{cases} \max\{0, H(\rho^b(z) - \rho^w(z))\} & \text{if } z = z_{\min} \\ \min\{0, H(\rho^b(z) - \rho^w(z))\} & \text{if } z = z_{\max}. \end{cases}$$

Any rest point $z^* \in [z_{\min}, z_{\max}]$ of these dynamics corresponds to an equilibrium. We shall say that an equilibrium allocation $z^*$ is *locally stable*

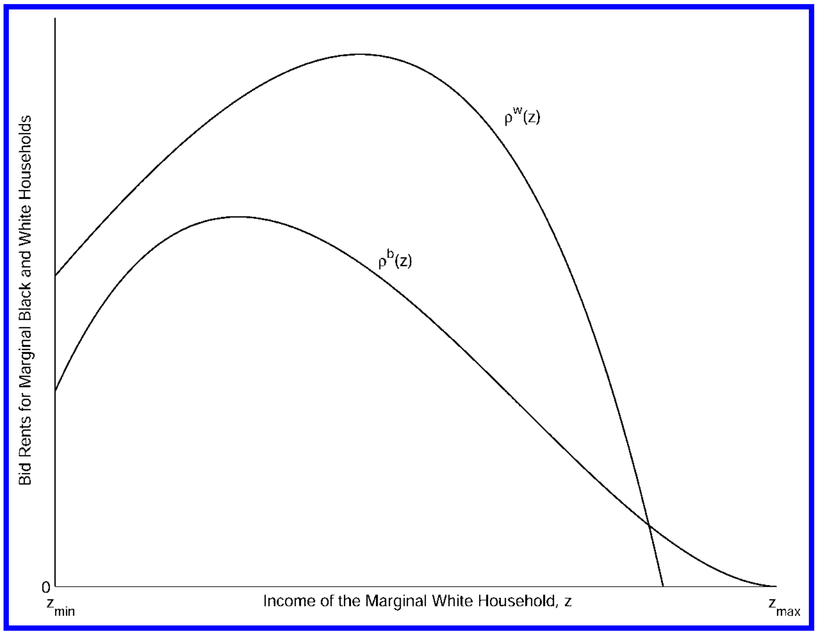


Fig. 2.—Marginal bid rent curves: stable segregation

if it is a locally asymptotically stable rest point of these dynamics.[9] If an equilibrium is not stable, it is *unstable*. An equilibrium allocation $z^*$ is stable if sufficiently small perturbations of $z$ away from $z^*$ in either direction are self-correcting.

The following example illustrates this notion of stability and shows how segregation can be a stable equilibrium outcome even when preferences are quite strongly pro-integrationist.

Example 1. Suppose that the income distributions are uniform with support [0, 0.7] for black households and [0.3, 1] for white households, $u(c, \bar{y}) = \log c + \bar{y}$, $\beta = 0.45$, and $v(r) = r(1 - r + \eta)$, with $\eta = 0.15$. Then there are two equilibria $(z^*, \rho^*)$:

| $z^*$ | $\tilde{y}_b(z^*)$ | $\beta_1(z^*)$ | $\beta_2(z^*)$ | $\bar{y}_1(z^*)$ | $\bar{y}_2(z^*)$ | $\rho^*$ |
|---|---|---|---|---|---|---|
| .36 | .70 | .90 | .00 | .35 | .68 | .11 |
| .83 | .12 | .16 | .74 | .49 | .54 | .02 |

The marginal bid rent curves corresponding to the example are depicted in figure 2. The first of the two equilibria is segregated. At this

[9] The rest point $z^* \in [z_{\min}, z_{\max}]$ is locally asymptotically stable if there exists an open set $N$ containing $z^*$ that has the property that all trajectories that originate at a point $z \in N \cap [z_{\min}, z_{\max}]$ converge to $z^*$.

equilibrium, the lower-income neighborhood is predominantly black and the higher-income neighborhood is exclusively white. Residential segregation is much higher than what *either* group considers ideal.[10] This occurs despite the fact that preferences are very pro-integrationist and the two income distributions have considerable overlap. The equilibrium is sustained by the fact that the marginal white household, despite having a lower income than the most affluent black household, is willing to outbid the most affluent black household to live in the second neighborhood. The reason is that the highest-income black households, despite being able to comfortably afford the higher rents in the more affluent neighborhood, are deterred by the fact that this neighborhood is exclusively white. Similarly, the marginal white household is faced with a choice between an exclusively white neighborhood and a predominantly black one. While neither of these options is particularly attractive, the former is considerably more appealing than the latter. This causes even relatively low-income white households to willingly pay the higher rent in the more affluent neighborhood.

There is a second equilibrium in this example that is integrated and has the seemingly paradoxical property that the higher-income neighborhood consists predominantly of the lower-income group. This equilibrium is locally unstable. To see this, consider a small decline in $z$, brought about by a shift of white households from the first to the second neighborhood, and a movement of black households in the opposite direction. As can be seen from figure 2, this raises the marginal bid rents of both blacks and whites, but the white marginal bid rent rises by a greater amount. Hence at the perturbed allocation the marginal white household will outbid the marginal black household to live in the second neighborhood, leading to an inflow of whites into the second neighborhood, further reducing $z$. Instead of being self-correcting, a small movement of whites to the second neighborhood is self-amplifying, resulting in cumulative divergence away from the equilibrium.

The city in this example therefore exhibits a *unique* stable equilibrium in which the marginal black household is more affluent than the marginal white household ($\tilde{y}^b(z) > z$). Proposition 3 in the Appendix identifies sufficient conditions for the existence of a stable equilibrium with this property. At any such equilibrium, the wealthiest households in the lower-income neighborhood will be black. In other words, there exists a range of incomes lying between $z$ and $\tilde{y}^b(z)$ such that households falling within this range will be in the poorer neighborhood if and only if they are black. White and black households with the same income will there-

[10] The ideal neighborhood racial composition for whites (blacks) in this example entails a neighborhood that is 42.5 percent black (white). In equilibrium, all black households and the lowest-income white households live in a neighborhood that is 10 percent white.

fore experience systematically different levels of neighborhood quality. In the presence of human capital externalities, the income of this group of black households will underpredict the future economic success of their children relative to the income of white households, an effect that could not occur under stratification alone. This is a sobering thought. Even in a world without overt discrimination, and one in which the desire for integration is strong, the advantage of being born to affluence may be magnified if one is also born to an affluent "race."[11]

## V. Income Disparities and Segregation

We now turn to our main results on the relationship between income disparities and the extent of segregation. We approach this question by allowing income distributions to depend on a scaling parameter $\alpha \in [0, 1]$. Let $F^b(y, \alpha)$ and $F^w(y, \alpha)$ represent these distributions. We assume that the distribution functions and their supports are continuous in $\alpha$ and that higher values of $\alpha$ correspond to smaller racial income disparities. Specifically, $F^b(y, \alpha)$ is nonincreasing in $\alpha$ and $F^w(y, \alpha)$ is nondecreasing in $\alpha$.

The scaling parameter $\alpha$ represents racial income disparities, with $\alpha = 0$ corresponding to a completely hierarchical distribution of income and $\alpha = 1$ to identical income distributions. The supports $[y^b_{\min}(\alpha), y^b_{\max}(\alpha)]$ and $[y^w_{\min}(\alpha), y^w_{\max}(\alpha)]$ are bounded and nondegenerate for all $\alpha \in [0, 1]$, are overlapping for all $\alpha \in (0, 1]$, and satisfy $y^b_{\max}(0) = y^w_{\min}(0)$. At all income levels $y$, the distributions satisfy $F^b(y, \alpha) > F^w(y, \alpha)$ for all $\alpha \in [0, 1)$ and $F^b(y, 1) = F^w(y, 1)$. This reflects the hypothesis that the black income distribution is dominated by the white distribution at all levels of $\alpha \in [0, 1)$, with convergence in distributions arising at $\alpha = 1$. Racial disparities in the distribution of income can be tracked by looking at changes in $\alpha$.

We begin by considering conditions under which segregated allocations arise in stable equilibrium. Consider the allocation $z = z_{\min}$ in which the upper-tail neighborhood is exclusively white and the share of the black population in the lower-tail neighborhood is $2\beta$. Since the exclusively white neighborhood has the higher mean income, the marginal white household will have a positive willingness to pay to live there provided that $\beta$ is sufficiently large.[12] In other words, if $\beta$ is large enough,

[11] See Loury (1977) for a pioneering theoretical exploration of the possibility that residential segregation may play a key role in the transmission of racial inequality across generations. These intergenerational effects are not explored in the present paper but clearly constitute an important extension.

[12] On the other hand, if $\beta$ is small enough and preferences are sufficiently pro-integrationist, higher-income whites will outbid poorer whites to live with the (relatively small) black population; so $z = z_{\min}$ will not be an equilibrium allocation.

the marginal bid rent for whites, $\rho^w(z_{\min})$, at the segregated allocation will be strictly positive. This allocation will constitute a stable equilibrium with equilibrium rent $\rho^w(z_{\min})$ if and only if $\rho^b(z_{\min}) < \rho^w(z_{\min})$. Intuitively, this will occur when the highest-income black households are not too much more affluent than the marginal white households. That is, segregation will be stable if racial income distributions are sufficiently unequal. The following result states this formally and also makes the less intuitive claim that segregation will be stable if racial income distributions are sufficiently *equal.*

Proposition 1. There exists $\hat{\beta} < \frac{1}{2}$ with the following property: for any $\beta > \hat{\beta}$, there exist $\alpha_l > 0$ and $\alpha_h < 1$ such that a stable segregated equilibrium exists for all $\alpha \in (0, \alpha_l) \cup (\alpha_h, 1)$.

Proposition 1 states that segregation will be stable in cities with significant black populations provided that racial income disparities are either sufficiently large or sufficiently small. Under these circumstances the lowest-income whites will share the lower-tail neighborhood with the black population and the higher-income whites will live in an exclusively white neighborhood. When racial income disparities are large, even allocations involving pure stratification by income are highly segregated, and preferences over neighborhood racial composition reinforce and exacerbate this effect. Hence the stability of complete segregation in this case is not surprising. Less intuitive is the finding that segregated allocations are stable equilibria when racial income disparities are sufficiently *small.* This occurs because, when the two income distributions are virtually identical, complete segregation does not result in substantial income disparities across neighborhoods. This in turn implies that the benefit to the wealthiest black households from moving to higher-income, predominantly white neighborhoods is small. Even a slight preference for all-black over all-white neighborhoods can overwhelm this effect and lead to stable patterns of extreme segregation. Consequently, the relationship between racial income disparities and the stability of segregated equilibria is not monotonic: segregation may be inconsistent with intermediate values of $\alpha$, whereas it is consistent with values of $\alpha$ lying at either extreme. Note that the result need not hold when $\beta$ is sufficiently small. In other words, segregation will be stable as income distributions converge in metropolitan areas with significant black populations, but need not be stable in areas with small black populations. This is broadly consistent with empirical realities. Speaking of the decline in segregation during the 1980s, Farley and Frey (1994, p. 40) observe that “the largest decreases in segregation occurred in metropolitan areas in which blacks made up a small percentage of the neighborhood of the typical white.” While this finding has commonly been attributed to the hypothesis that whites are threatened by large numbers of black households in their neighborhoods, our analysis suggests an alternative interpretation. When the share of black households in a city is small, whites sort themselves more extensively by income. The difference in income between more affluent white neighborhoods and black neighborhoods is therefore greater, tempting the highest-income black households to move to overwhelmingly white neighborhoods. Thus segregation is less likely to remain stable in cities with small black populations as racial income disparities decline.

Stability of complete segregation does not imply that integrated equilibria cannot also be stable, as the following example illustrates.

Example 2. Suppose that the income distributions are uniform with support [0, 0.9] for black households and [0.1, 1] for white households, with all other specifications as in example 1. Then there exist four equilibria:

| $z^*$ | $\tilde{y}_b$ | $\beta_1$ | $\beta_2$ | $\bar{y}_1$ | $\bar{y}_2$ | $\rho$ |
|---|---|---|---|---|---|---|
| .18 | .90 | .90 | .00 | .42 | .59 | .04 |
| .24 | .83 | .83 | .07 | .37 | .64 | .06 |
| .49 | .53 | .53 | .37 | .28 | .73 | .18 |
| .89 | .03 | .03 | .87 | .48 | .53 | .01 |

Of the four equilibria identified in the example, only the first (involving segregation) and the third (involving substantial integration) are stable. This can be seen in figure 3, where the marginal bid rent functions are depicted. Both stable equilibria are of the kind identified in examples 1 and 2, with the marginal black household having a higher income than the marginal white household.

The above example is robust in that when racial income disparities are sufficiently small and preferences are sufficiently pro-integrationist, a stable integrated equilibrium always exists. To establish this, consider the following parametric form for preferences over neighborhood racial composition:

$$v(r) = r(1 - r + \eta). \qquad (2)$$

The parameter $\eta \in [0, 1]$ measures the degree to which residence with one's own group is desired. When $\eta = 0$, each individual's ideal neighborhood racial composition consists of equal shares of blacks and whites. More generally, the ideal racial composition for an individual is to have a share $\frac{1}{2}(1 + \eta)$ of her own type in the neighborhood. Larger values of $\eta$ therefore correspond to a greater bias toward one's own group. Except in the extreme case $\eta = 1$, such preferences are nonmonotonic: all individuals prefer some degree of integration to complete segregation. For any value of $\eta < \frac{1}{2}$, the range of neighborhood compositions that are strictly preferred to complete segregation includes allocations

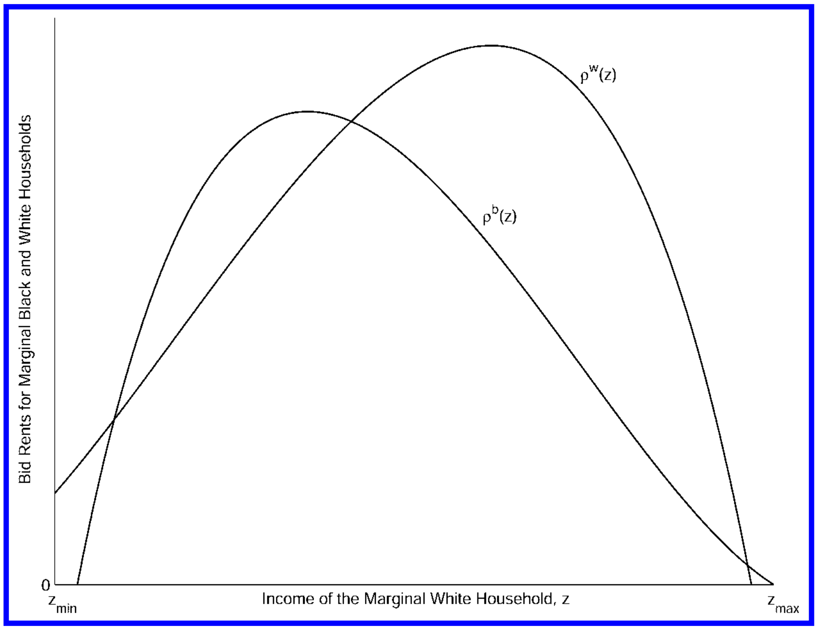


FIG. 3.—Multiple stable equilibria

in which the individual is in a minority. Note that proposition 1 holds for any $\eta > 0$.

The smaller the value of $\eta$, the more pro-integrationist preferences will be. The following result establishes the existence of integrated equilibria when racial income disparities are sufficiently small and preferences are sufficiently pro-integrationist.

PROPOSITION 2. There exists $\hat{\alpha} < 1$ with the following property: for any $\alpha > \hat{\alpha}$, there exists $\hat{\eta} > 0$ such that a stable integrated equilibrium exists for all $\eta < \hat{\eta}$.

In combination with proposition 1, one implication of this result is that multiple stable equilibria exist when preferences over neighborhood racial composition are sufficiently pro-integrationist, racial disparities in the distribution of income are small, and the black share of the metropolitan population is sufficiently large. When $\alpha = 1$, the integrated equilibrium involves identical neighborhood racial compositions and complete stratification by income. In the limiting case in which such disparities disappear, there is a stable equilibrium in which there is effectively no racial segregation in residential patterns.

Propositions 1 and 2 may be illustrated by looking at the special case in which black and white income distributions are both uniform with supports $[0, \frac{1}{2}(1 + \alpha)]$ and $[\frac{1}{2}(1 - \alpha), 1]$, respectively. The manner in

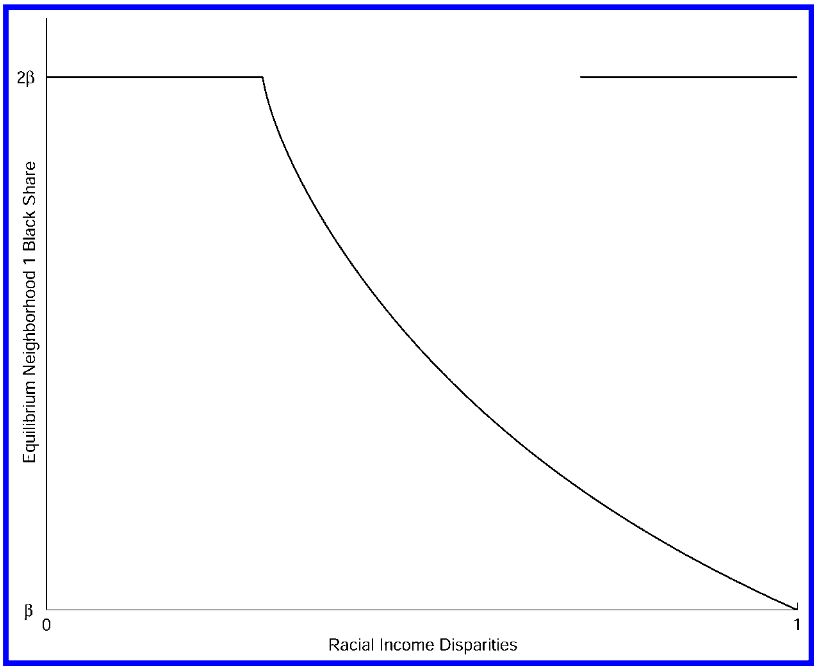


FIG. 4.—Racial income disparities and equilibrium segregation

which the set of stable equilibria varies with racial income disparities $\alpha$ is shown in figure 4.[13] When racial income disparities are extreme ($\alpha$ close to zero), complete segregation is the only stable outcome. As racial income disparities narrow, there comes a point at which the segregated equilibrium loses stability and the unique stable equilibrium involves some degree of mixing. Beyond this point, convergence of incomes goes hand in hand with greater integration. Eventually $\alpha$ crosses a threshold and multiple equilibria arise, with complete segregation becoming stable. Further convergence of incomes can lead to persistent segregation or to increasing integration, depending on which of the equilibria is selected. When the two income distributions are identical ($\alpha = 1$), the two stable equilibria are at polar extremes: one segregated and the other

[13] The figure is based on the following specifications of our parameters: $\beta = 0.45$, $u(c, \bar{y}) = \log c + \bar{y}$, and $\eta = 0.08$. This value of $\eta$ implies an ideal neighborhood racial composition entailing 54 percent of one's own group, which is roughly consistent with the survey evidence reported in Sec. II. The value of $\beta$ approximates the share of black households in central cities such as Philadelphia, St. Louis, and Cleveland. Each value of $\alpha$ corresponds to a particular ratio of black to white mean household income. The critical values of this ratio (corresponding to $\alpha_l$ and $\alpha_h$) in this example are 48 percent and 75 percent, respectively. There is substantial geographic variation in this ratio across the nation, with most major cities with significant black populations falling in the 50–80 percent range.

perfectly integrated, with the neighborhoods having identical racial compositions.

The emergence of multiple equilibria as income distributions converge suggests that even in the face of increasing racial equality, changes in segregation need not be monotonically decreasing. A period of declining segregation can be followed by a process of *resegregation* once a threshold level of racial equality is attained. Whether or not such resegregation arises in practice is likely to depend on the pace at which racial income disparities narrow relative to the speed of adjustments in residential choices. If declines in racial inequality proceed sufficiently slowly, one would expect residential patterns to be close to equilibrium over time and hence to follow a pattern of increasing integration. In the context of figure 4, gradual declines in racial inequality will lead to a movement along the locus of integrated equilibria. On the other hand, if declines in income inequality are sufficiently rapid relative to adjustments in residential choice, it is entirely possible that resegregation can occur. Starting from an integrated equilibrium, a rapid decline in racial inequality can shift the economy into the basin of attraction of a segregated equilibrium. Hence the pace at which racial income disparities converge in a metropolitan area becomes a critical determinant of the steady-state level of segregation.

Another implication of proposition 2 is that stable equilibria can exist that involve higher levels of integration than any household, black or white, considers ideal. For example, if $\eta = \beta = \frac{1}{2}$, the ideal neighborhood for each household requires that it be in a 75 percent majority. Yet if $\alpha = 1$, a stable equilibrium exists in which there is complete stratification by income, and exactly half the households in each neighborhood are black. To see why this is stable, suppose that random perturbations cause $z$ to rise so that the second neighborhood is now majority black. On the basis of preferences over neighborhood composition alone, the marginal black household would outbid the marginal white household for housing in the second neighborhood, leading to cumulative divergence from the equilibrium. But with $z$ exceeding one-half, the marginal black household is less affluent than the marginal white household. On the basis of preferences for neighborhood mean income alone, the latter would outbid the former for housing in the second neighborhood. The combined effect of these two forces determines whether integration is stable. If preferences over neighborhood racial composition are not too strong, the latter effect dominates and integration is stable.

The results in this section suggest that racial disparities in the distribution of income play a subtle and important role in determining patterns of segregation. Even when preferences are strongly pro-integrationist and the ideal neighborhood for all individuals is close to perfectly mixed, complete segregation can result if racial income disparities are negligible or extreme. Multiple equilibria are inevitable when racial income disparities are small. The existence of multiple equilibria suggests that although stable integration may become viable as racial income disparities lessen, a city may remain trapped in the basin of attraction of the segregated equilibrium because of historical patterns of segregation. This is where social policy may be most effective: temporary incentives for integration may give rise to permanent effects. Note, however, that a shift to an equilibrium with greater integration (and correspondingly greater stratification) lowers neighborhood quality in the poorest neighborhood, which consists disproportionately of black households. The movement of upper-income black households to more affluent communities worsens the conditions for those left behind, a point that has been emphasized by Wilson (1987).[14]

Finally, our results imply that one cannot expect a narrowing of racial income disparities to lead inevitably to lower segregation. While the convergence of incomes might imply greater integration at integrated equilibria, it may also cause segregated allocations to become stable. From a cross-sectional perspective, cities with lower levels of racial inequality need not be the least segregated. And from a historical perspective, the march toward greater integration may be halted and reversed in some cities as racial inequality *declines*.

## VI. Conclusions

Given the nation's long history of slavery and *de jure* segregation, race has a degree of salience in American life that perhaps exceeds that of any other socially designated attribute. Significant legislative changes over the past half century have attempted to equalize the racial gap in economic opportunity. Over these years a substantial black middle class has emerged and racial income disparities have narrowed. The effect of these changes on residential segregation has been geographically uneven and, in many major cities with large black populations, marginal, in spite of greater racial tolerance in attitudes.

We have shown in this paper that when households care about both the affluence and the racial composition of their communities, their preferences are reflected in patterns of segregation and stratification in subtle and sometimes unexpected ways. Segregated allocations can be stable when racial income disparities are either very great or very small, but unstable in some intermediate range. And small income dis-

[14] While the effects of greater integration are theoretically ambiguous, Cutler and Glaeser (1997) present evidence supporting the claim that black households overall benefit from declines in segregation.

parities can give rise to multiple equilibria, with segregation and integration both being stable. Taken together, our results help account for substantial cross-sectional variance in segregation levels and the statistical insignificance of reduced income disparities in accounting for changes in segregation over time.

Our analysis is abstract enough to permit alternative interpretations. Instead of race, the dominant attribute governing location decisions might be linguistic preference, religious affiliation, or any other observable trait. It is also not necessary to interpret neighborhoods in a spatial sense: interaction in schools, clubs, or other voluntary associations will be subject to the same kind of dynamics. One could consider wealth rather than income disparities and owned rather than rented housing without substantive modification to the model.

The most obvious significant extension of this work would be to allow income distributions to be endogenously determined in an intergenerational context. It has commonly been argued that residential segregation inhibits the narrowing of racial income disparities over time. Whether this is true when segregation and income are jointly determined is an open question of considerable analytical and policy interest.

## Appendix

*Proof of Proposition 1*

Consider the segregated allocation $z = z_{\min}$. Note that $\beta_1(z_{\min}) = 2\beta$, $\beta_2(z_{\min}) = 0$, $\tilde{y}^b(z_{\min}) = y^b_{\max}$, and $\bar{y}_1(z_{\min}) < \bar{y}_2(z_{\min})$. The marginal bid rents $\rho^w(z_{\min})$ and $\rho^b(z_{\min})$ at this allocation are defined by the unique solutions to

$$u(z_{\min}, \bar{y}_1(z_{\min})) + v(1 - 2\beta) = u(z_{\min} - \rho^w, \bar{y}_2(z_{\min})) + v(1) \qquad \text{(A1)}$$

and

$$u(y^b_{\max}, \bar{y}_1(z_{\min})) + v(2\beta) = u(y^b_{\max} - \rho^b, \bar{y}_2(z_{\min})) + v(0). \qquad \text{(A2)}$$

Define $\tilde{\beta}$ as the unique nonzero solution to $v(1 - 2\beta) = v(1)$, so at any $\beta \geq \tilde{\beta}$, we have $v(1 - 2\beta) \leq v(1)$. Since $\bar{y}_1(z_{\min}) < \bar{y}_2(z_{\min})$, we must therefore have $\rho^w(z_{\min}) > 0$. Consider first the case $\alpha = 0$, which implies $y^b_{\max} = y^w_{\min} < z_{\min}$. In this case a white household with income $y^b_{\max}$ would be willing to pay less than $\rho^w(z_{\min})$ to switch to the second neighborhood. Hence

$$u(y^b_{\max}, \bar{y}_1(z_{\min})) + v(1 - 2\beta) > u(y^b_{\max} - \rho^w(z_{\min}), \bar{y}_2(z_{\min})) + v(1).$$

Note that $v(0) - v(2\beta) < 0 < v(1) - v(1 - 2\beta)$, and therefore the above equation implies

$$u(y^b_{\max}, \bar{y}_1(z_{\min})) + v(2\beta) > u(y^b_{\max} - \rho^w(z_{\min}), \bar{y}_2(z_{\min})) + v(0).$$

Comparing this with (A2) immediately yields $\rho^b(z_{\min}) < \rho^w(z_{\min})$. Hence the pair $(z_{\min}, \rho^w(z_{\min}))$ is a stable equilibrium for all $\beta \geq \tilde{\beta}$ if $\alpha = 0$. For any given $\beta > \tilde{\beta}$, define $\alpha_l \in (0, 1]$ as the smallest value of $\alpha$ at which $\rho^w(z_{\min}) = \rho^b(z_{\min})$, and set $\alpha_l = 1$ if no such value exists. Note that for all $\alpha < \alpha_l$ we have $\rho^w(z_{\min}) > 0$ and, from the continuity of marginal bid rents in $\alpha$, also $\rho^w(z_{\min}) > \rho^b(z_{\min})$. Hence for any $\beta \geq \tilde{\beta}$, $(z_{\min}, \rho^w(z_{\min}))$ is a stable equilibrium for all $\alpha < \alpha_l$.

To prove the latter part of the result, consider first the case $\beta = \frac{1}{2}$ and $\alpha = 1$, which together imply $\bar{y}_1(z_{\min}) = \bar{y}_2(z_{\min})$. From (A1) and (A2) and the fact that $v(0) - v(1) < 0 < v(1) - v(0)$, the marginal bid rents must satisfy $\rho^b(z_{\min}) < 0 < \rho^w(z_{\min})$. If $\beta$ is reduced while $\alpha = 1$ is maintained, then $z_{\min}$ and $\bar{y}_2(z_{\min})$ both increase, $\bar{y}_1(z_{\min})$ declines, and $\tilde{y}^b(z_{\min}) = y^b_{\max}$ remains the same. Let $\hat{\beta}$ denote the largest value of $\beta$ such that $\beta > \tilde{\beta}$ and $\rho^b(z_{\min}) = \rho^w(z_{\min})$, and set $\hat{\beta} = \tilde{\beta}$ if no such value exists. At any $\beta > \hat{\beta}$, we have $\rho^w(z_{\min}) > 0$ and $\rho^w(z_{\min}) > \rho^b(z_{\min})$. Hence the pair $(z_{\min}, \rho^w(z_{\min}))$ is a stable equilibrium for all $\beta > \hat{\beta}$ when $\alpha = 1$. For any given $\beta > \hat{\beta}$, define $\alpha_h \in [0, 1)$ as the highest value of $\alpha$ at which $\rho^w(z_{\min}) = \rho^b(z_{\min})$, and set $\alpha_h = 0$ if no such value exists. Note that, for all $\alpha > \alpha_h$, we have $\rho^w(z_{\min}) > 0$ and, from the continuity of marginal bid rents in $\alpha$, also $\rho^w(z_{\min}) > \rho^b(z_{\min})$. Hence for any $\beta > \hat{\beta}$, $(z_{\min}, \rho^w(z_{\min}))$ is a stable equilibrium for all $\alpha > \alpha_h$. Q.E.D.

*Proof of Proposition 2*

At any allocation $z$ the marginal bid rents $\rho^w(z)$ and $\rho^b(z)$ are defined implicitly by

$$G^w(z, \rho) \equiv u(z, \bar{y}_1) + v(1 - \beta_1) - u(z - \rho, \bar{y}_2) - v(1 - \beta_2) = 0 \tag{A3}$$

and

$$G^b(z, \rho) \equiv u(\tilde{y}^b, \bar{y}_1) + v(\beta_1) - u(\tilde{y}^b - \rho, \bar{y}_2) - v(\beta_2) = 0, \tag{A4}$$

where $\tilde{y}^b$, $\beta_i$, and $\bar{y}_i$ are all differentiable functions of $z$. Both $\partial G^w/\partial\rho$ and $\partial G^b/\partial\rho$ are strictly positive for all $(z, \rho)$, since $u$ is strictly increasing in its first argument. The implicit function theorem may be applied to obtain $\rho^w(z)$ and $\rho^b(z)$ with the properties

$$\frac{d\rho^w}{dz} = -\frac{\partial G^w/\partial z}{\partial G^w/\partial\rho}, \quad \frac{d\rho^b}{dz} = -\frac{\partial G^b/\partial z}{\partial G^b/\partial\rho}.$$

Suppose first that $\alpha = 1$ and consider the pair $(z^*, \rho^*)$, where $z^*$ is defined by $\tilde{y}^b(z^*) = z^*$ and $\rho^* = \rho^w(z^*)$. Since $\alpha = 1$, we have $\beta_1(z^*) = \beta_2(z^*) = \beta$. Furthermore, since $\tilde{y}^b(z^*) = z^*$ and $\bar{y}_2(z^*) > \bar{y}_1(z^*)$, we must have $\rho^w(z^*) = \rho^b(z^*) > 0$; so $(z^*, \rho^*)$ is an equilibrium. (This equilibrium entails pure sorting by income.) To find conditions under which it is stable, note that at $(z^*, \rho^*)$,

$$\frac{\partial G^w}{\partial\rho} = \frac{\partial G^b}{\partial\rho} = u_1(z^* - \rho^*, \bar{y}_2) > 0.$$

Hence $d\rho^w/dz > d\rho^b/dz$ if and only if $\partial G^w/\partial z < \partial G^b/\partial z$. At $(z^*, \rho^*)$ we have

$$\frac{\partial G^w}{\partial z} = u_1(z^*, \bar{y}_1) + u_2(z^*, \bar{y}_1)\frac{d\bar{y}_1}{dz} - v'(1-\beta)\frac{d\beta_1}{dz}$$

$$- u_1(z^* - \rho^*, \bar{y}_2) - u_2(z^* - \rho^*, \bar{y}_2)\frac{d\bar{y}_2}{dz} + v'(1-\beta)\frac{d\beta_2}{dz}$$

and

$$\frac{\partial G^b}{\partial z} = u_1(z^*, \bar{y}_1)\frac{d\tilde{y}^b}{dz} + u_2(z^*, \bar{y}_1)\frac{d\bar{y}_1}{dz} + v'(\beta)\frac{d\beta_1}{dz}$$

$$- u_1(z^* - \rho^*, \bar{y}_2)\frac{d\tilde{y}^b}{dz} - u_2(z^* - \rho^*, \bar{y}_2)\frac{d\bar{y}_2}{dz} - v'(\beta)\frac{d\beta_2}{dz}.$$

From (2), $v'(r) = 1 + \eta - 2r$, so $v'(\beta) + v'(1-\beta) = 2\eta$. Hence

$$\frac{\partial G^w}{\partial z} - \frac{\partial G^b}{\partial z} = [u_1(z^*, \bar{y}_1) - u_1(z^* - \rho^*, \bar{y}_2)]\left(1 - \frac{d\tilde{y}^b}{dz}\right) - 2\eta\left(\frac{d\beta_1}{dz} - \frac{d\beta_2}{dz}\right).$$

Note that $d\tilde{y}^b/dz < 0$ and $d\beta_1/dz < 0 < d\beta_2/dz$. This, together with the assumptions $u_{11} < 0$ and $u_{12} \geq 0$, implies that $\partial G^w/\partial z < \partial G^b/\partial z$ when $\eta$ is sufficiently small. This proves that a stable integrated equilibrium exists for $\alpha = 1$. The result then follows from the continuity of marginal bid rent functions in $\alpha$ and $\eta$. Q.E.D.

Finally, we identify sufficient conditions for the existence of a stable equilibrium in which a set of households experience lower neighborhood quality *conditional on income* if they are black.

PROPOSITION 3. Suppose $v(r) > v(1-r)$ for all $r > \frac{1}{2}$. If $y^b_{\max} > z_{\min}$ and $\beta$ is sufficiently large, there exists a stable equilibrium $(z^*, \rho^*)$ with the property that $\tilde{y}^b(z^*) > z^*$.

*Proof.* If $y^b_{\max} > z_{\min}$, then there exists a unique $\hat{z}$ that satisfies $\tilde{y}^b(\hat{z}) = \hat{z}$. (This allocation involves pure sorting by income.) Note that $\tilde{y}^b(z) > z$ for all $z \in [z_{\min}, \hat{z})$ and furthermore that $\beta_1(z) > \beta_2(z)$ and $\bar{y}_1(z) < \bar{y}_2(z)$ for all $z \in [z_{\min}, \hat{z}]$. If $\beta = \frac{1}{2}$, then $\beta_1 = 1 - \beta_2 > \frac{1}{2}$ for all $z \in [z_{\min}, \hat{z}]$; so we have

$$v(\beta_2) - v(\beta_1) < 0 < v(1-\beta_2) - v(1-\beta_1).$$

Since $v(r)$ is continuous, these inequalities hold also when $\beta < \frac{1}{2}$ but is sufficiently large. The latter inequality, together with the fact that $\bar{y}_1(z) < \bar{y}_2(z)$ for all $z \in [z_{\min}, \hat{z}]$, implies that $\rho^w(z) > 0$ for all $z \in [z_{\min}, \hat{z}]$. The former inequality, together with the fact that $\tilde{y}^b(\hat{z}) = \hat{z}$, implies that $\rho^b(\hat{z}) < \rho^w(\hat{z})$. This leaves two possibilities: either $\rho^b(z) < \rho^w(z)$ for all $z \in (z_{\min}, \hat{z})$, or there exists at least one $z \in (z_{\min}, \hat{z})$ such that $\rho^b(z) = \rho^w(z) > 0$. In the former case, $(z_{\min}, \rho^w(z_{\min}))$ is a stable equilibrium with the required property. In the latter case, let $z^*$ denote the largest $z \in (z_{\min}, \hat{z})$ such that $\rho^b(z) = \rho^w(z)$, and set $\rho^* = \rho^b(z^*) = \rho^w(z^*)$. Clearly $(z^*, \rho^*)$ is an equilibrium with the required property. To see that this equilibrium is stable, note that there exists an open set $N$ containing $z^*$ with the property that $\rho^w(z) - \rho^b(z)$ has the same sign as $z - z^*$ for all $z \in N$. This in turn implies that the equilibrium is locally stable under the dynamics (1).

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