How Topology Affects Security: An Upper Bound of Electric Power Network Security

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Abstract: Electric power network is a fundamental facility in modern society. The importance to ensure and enhance the security of the power network can never be over emphasized. In this paper, we study how the topology of a power network restricts the security. By focusing on the power balanced condition which is necessary for the security after line outage contingencies, we show that a practical power network cannot avoid collapse. Using a method based on ordered binary decision diagram (OBDD) that fast enumerates the line outages causing network collapses, we obtain an upper bound for power network security, and indicate the transmission lines that are critical to power network security. This method is demonstrated on an IEEE 30-bus network. We hope this work brings insights to the understanding of why power networks cannot avoid collapse and how to enhance the security of an electric power network.

1. INTRODUCTION

The electric power network has become a fundamental facility in modern society, and the importance to ensure and enhance the security of a power network can never be over emphasized. A large electric power network is a complex system in the sense that there are always some contingencies not considered when designing the network. For instance, in transmission expansion planning the security of a network is tested against single line outages only. To further reduce the computing budget, only the power flows at each bus (i.e., generators, loads, and transformers) and in the transmission lines in the steady state are calculated. Instead of considering the Alternating Current model (which is a group of nonlinear equations describing the relationship among voltages and currents at the buses and in the lines), a simplified Direct Current model (which is a group of linear equations describing the relationship among real power and angles at the buses and in the lines) is used. This method is known as the $N-1$ method, which is well adopted in practice (Wood and Wollenberg (1996)). The advantage of this $N-1$ method is to fast assess the consequence of single line outages, if the true load pattern (which bus consumes how much electric power) and the true generation pattern (which generator generates how much electric power) in practice are close to the patterns considered in the design stage. Jia et al. (2007) showed that when the load pattern or the generation pattern is different from the one considered in the design stage, networks passing the test of the $N-1$ method may collapse. Furthermore, little can be said about multiple-line outages, leaving alone other types of contingencies, e.g., the dysfunction of power network devices (generators, relays, and breakers). So the fundamental difficulty to assess the security of a power network includes two aspects: the difficulty to think out all possible contingencies, and the large and usually practical infeasible computing budget to consider all these contingencies, even in the design stage.

In practice, we improve the security of the power network by real time adjustment of the generation pattern, by using spinning reserve and backup generators, and by increasing the capacity of the transmission lines. These methods of course increase the security against certain contingencies. For example, Zhao et al. (2007) studied how the tolerance supplied by spinning reserve and backup generators should increase in a power network with growing capacity. In this paper, we address the security issue from another viewpoint. Instead of pursuing a "perfect" power network that resists all or most contingencies, we show that the topology (i.e., how the buses are connected), the load pattern and the generation pattern contribute an intrinsic collapse probability, which cannot be overcome by increasing line capacities. By calculating an upper bound of security, we quantify how much close we can achieve to a "perfect" network if we do not change the topology.

The idea is to consider how the topology, the load pattern, and the generation pattern break the power balanced condition, which is a necessary condition for power network security. The power balanced condition requires that the power generated and consumed in the network should always be balanced, at least within a tolerance level. The tolerance is supplied by spinning reserve, backup generators or small load shedding, which is about 3% in practical power network. When some contingency happens, which

⋆ This work was supported in part by the National Natural Science Foundation of China under Grants NSFC (60574067, 60704008, 60721003, 60736027), in part by the National New Faculty Funding for Universities with Doctoral Program (20070003110), and in part by the Programme of Introducing Talents of Discipline to Universities (the National 111 International Collaboration Project).

causes the power imbalance to exceed certain tolerance, the network enters the emergent status. Load shedding will happen, and the synchronization among the generators may be broken. The network starts to split passively or actively (also called the controlled network partition in Zhao et al. (2003)) into islands. If the power is still not balanced within an island, the island continues splitting into smaller ones, which is known as a cascading failure.

We assume that a power network is originally balanced before the contingency. After some line outages, the network is forced to split into several islands. Only the power balanced islands sustain, and the other islands suffer the blackout. When the blackout area is large enough (say, the total load shedding exceeds 20% of the original power generation), we call it a collapse. We developed a method to fast enumerate all line outages that cause collapses. By calculating the probability of these line outages, we obtain a lower bound of collapse probability (i.e., an upper bound of security probability).

The enumeration method is based on ordered binary decision diagram (OBDD). OBDD was first introduced by Bryant (1986) and has been successfully applied to find proper splitting strategies under a desynchronization contingency of generators in Zhao et al. (2003); Sun et al. (2003, 2005). We briefly present this enumeration method in Section 2, and test its performance on an IEEE 30-bus example in Section 3. A brief conclusion is presented in Section 4.

2. POWER IMBALANCE CAUSED BY LINE OUTAGES

First we formulate the collapse probability of a power network mathematically. We use the node-weighted graph $G(V, E, W)$ to represent a power network as in Zhao et al. (2003), where V is the set of nodes representing the buses, E is the set of undirected edges representing transmission lines, and W is the set of weights on the nodes representing the power generated (with positive weights) or consumed (with negative weights) at the buses. The nodes representing generators constitute the set V_{G}. The rest nodes constitute the set V_{L}. It is obvious that $V = V_{\mathrm{G}} \cup V_{\mathrm{L}}$. A line outage is a cut set E_{c} that consists of the edges that trip off in the line outage, $E_{\mathrm{c}} \subset E$. To assess the upper bound of the security, we assume the transmission lines have sufficient capacity, so that a k-line outage does not cause a $(k+1)$-line outage. When this assumption is violated in practice, the true security is strictly smaller than the upper bound presented in this section. We also ignore the real power loss on the transmission lines. The power balanced condition is then defined as

Definition 1. (power balanced condition). A power network is said to be power balanced if and only if the power generated at the generators and consumed at the loads are balanced within a tolerance, i.e.,

$$\left| \sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}}) + \sum_{v_{\mathrm{l}} \in V_{\mathrm{L}}} W(v_{\mathrm{l}}) \right| = \left| \sum_{v \in V} W(v) \right| \leq d, \qquad (1)$$

where d is the tolerance.

A collapse is defined as

Definition 2. (power network collapse). A power network is said to collapse after a line outage E_{c} if and only if the load shedding after removing the edges in E_{c} is too large, i.e.,

$$\frac{\sum_{v_{\mathrm{g}} \in V_{\mathrm{GS}}(E_{\mathrm{c}})} W(v_{\mathrm{g}})}{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}})} \leq r, \qquad (2)$$

where $V_{\mathrm{GS}}(E_{\mathrm{c}})$ is the set of generators in the sustained islands after line outage E_{c} (the power balanced condition is a necessary condition for an island to sustain), r is the affordable load shedding ratio, $0 < r < 1$. For example, if we set $r = 0.8$, this means we are interested in line outages that lead to a load shedding of over 20% of the overall power generation before the contingencies. A high value of r represents a strict requirement on security performance.

Then the *effect of line outages on network collapse (ELONC)* is quantified by:

$$\sum_{E_{\mathrm{c}} \subset E} \mathrm{Prob} \left\{ E_{\mathrm{c}} : \frac{\sum_{v_{\mathrm{g}} \in V_{\mathrm{GS}}(E_{\mathrm{c}})} W(v_{\mathrm{g}})}{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}})} \leq r \right\}, \qquad (3)$$

where $\frac{\sum_{v_{\mathrm{g}} \in V_{\mathrm{GS}}(E_{\mathrm{c}})} W(v_{\mathrm{g}})}{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}})} \leq r$ specifies that E_{c} causes a collapse, and $\mathrm{Prob} \left\{ E_{\mathrm{c}} : \frac{\sum_{v_{\mathrm{g}} \in V_{\mathrm{GS}}(E_{\mathrm{c}})} W(v_{\mathrm{g}})}{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}})} \leq r \right\}$ is the probability that line outage E_{c} happens. In the rest part of this section, we discuss how to calculate Equation (3) efficiently.

Before introducing the OBDD method, we first introduce a type of problems that OBDD could be applied to solve, i.e., the satisfiability problem.

Definition 3. (The Satisfiability (SAT) problem). Given a Boolean function $f(x)$, where x is an n-dimensional Boolean vector, i.e., $x \in \{0, 1\}^n$, and $f(x) \in \{0, 1\}$. Is there an x, such that $f(x) = 0$ (or 1)?

The SAT problem is a decision problem (i.e., the answer is either "yes" or "no"), and has been proven to be NP-complete (which means this problem is one of the most difficult decision problems). Ordered binary decision diagram (OBDD), which was developed by Bryant (1986), is a description of Boolean function, i.e., any Boolean function can be represented by an OBDD. The OBDD of a Boolean function $f(x)$, where $x = (x_1, x_2, \ldots x_n)$, exclusively depends on the function itself and an order among $x_1, x_2, \ldots x_n$, which can be arbitrarily specified by the user. Basically OBDD can be regarded as a group of if-then-else rules, each of which tests the value of one Boolean variable and decides which rule to use in the next step, or output "0" or "1" as the output of function $f(x)$. So the order among $x_1, x_2, \ldots x_n$ specifies the value of which variable is tested first and which variable is tested later. Given a proper order among $x_1, x_2, \ldots x_n$, Bryant (1986) showed that by exploring the OBDD of Boolean function $f(x)$, we can solve SAT problem in polynomial time. Unfortunately Bollig and Wegener (1996) showed that it is NP-hard to find the optimal order that minimizes this time. Another advantage of OBDD representation of a Boolean function is that this method finds all x's such that $f(x) = 0$ (or 1) at the same time, instead of one per time.

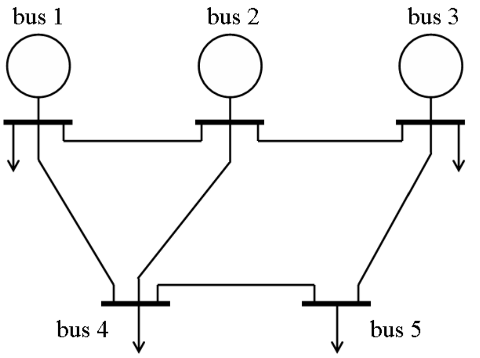

Fig. 1. A five-bus power system (Zhao et al. (2003)). Bus 1, 2, and 3 are generators (represented by circles). Bus 4 and 5 are loads (represented by arrows). Bus 1 and 3 have local loads.

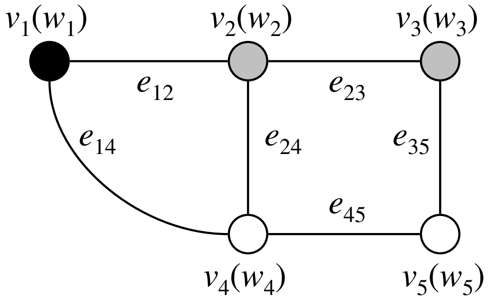

Fig. 2. The node-weighted graph model of the five-bus power network (Zhao et al. (2003)).

Having known that OBDD is a method that solves SAT problem fast (given a proper order among the input Boolean variables) and finds all x's s.t. $f(x) = 0$ (or 1, equivalently), we show how to model the ELONC problem as a SAT problem.

Following the formulation in Zhao et al. (2003), we use a Boolean variable to denote a transmission line. Then the topology of a power network $G(V, E, W)$ can be represented by a semicertain adjacency matrix (SAM) A_{G}, where a_{ij} is a Boolean variable b_{ij} if edge $e_{ij} \in E$ and $a_{ij} = 0$ otherwise. For example, there is a five-bus power system in Fig. 1. The node-weighted graph model of this power network is shown in Fig. 2 (Zhao et al. (2003)).

The SAM (a 5-by-5 matrix) of the five-bus power network is

$$A_{\mathrm{G}} = \begin{pmatrix} 0 & b_{12} & 0 & b_{14} & 0 \\ b_{12} & 0 & b_{23} & b_{24} & 0 \\ 0 & b_{23} & 0 & 0 & b_{35} \\ b_{14} & b_{24} & 0 & 0 & b_{45} \\ 0 & 0 & b_{35} & b_{45} & 0 \end{pmatrix}.$$

To represent line outage E_{c}, we set $b_{ij} = 0$ if $e_{ij} \in E_{\mathrm{c}}$; $b_{ij} = 1$ otherwise, i.e., if the transmission line e_{ij} trips off, $b_{ij} = 0$; otherwise $b_{ij} = 1$.

We define

$$A^*_{\mathrm{G}} = I \oplus A_{\mathrm{G}} \oplus A^2_{\mathrm{G}} \oplus \ldots \oplus A^{n-1}_{\mathrm{G}},$$

where I refers to the identity matrix (1 on the diagonal and 0 elsewhere), $\oplus$ is "OR" operation, $A^{i+1}_{\mathrm{G}} = A_{\mathrm{G}} \otimes A^i_{\mathrm{G}}$, $\otimes$ is "AND" operation, n is the number of nodes in graph $G(V, E, W)$. The matrix A^*_{G} describes the nodes in each island after the line outage. Node i and j are in the same island if and only if $(A^*_{\mathrm{G}})_{ij} = 1$. The generator at bus i sustains if and only if

$$|\,(A^*_{\mathrm{G}})_{i\cdot}\, W| \leq d,$$

where $(A^*_{\mathrm{G}})_{i\cdot}$ is the i-th row of A^*_{G}, and for all the buses connected to bus i after the line outage $|\,(A^*_{\mathrm{G}})_{i\cdot}\, W|$ is the difference between the total power generation and load in that island. Then whether line outage E_{c} causes a line outage is represented by an event

$$\frac{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} \left(|\,(A^*_{\mathrm{G}})_{v_{\mathrm{g}}}\, W| \leq d \right) W(v_{\mathrm{g}})}{\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} W(v_{\mathrm{g}})} \geq r, \qquad (4)$$

where $\left(|\,(A^*_{\mathrm{G}})_{v_{\mathrm{g}}}\, W| \leq d \right)$ is a Boolean formula and outputs 1 if $|\,(A^*_{\mathrm{G}})_{v_{\mathrm{g}}}\, W| \leq d$; outputs 0 otherwise. Then $\sum_{v_{\mathrm{g}} \in V_{\mathrm{G}}} \left(|\,(A^*_{\mathrm{G}})_{v_{\mathrm{g}}}\, W| \leq d \right) W(v_{\mathrm{g}})$ is the total power generation in all the sustainable islands after the line outage. Please note that the elements in A^*_{G} depends on the line outage E_{c}. So Equation (4) is a Boolean function of E_{c}, denoted as $h(E_{\mathrm{c}})$. This Boolean function outputs 1 if Equation (4) is satisfied; and outputs 0 otherwise. In other words, $h(E_{\mathrm{c}}) = 0$ when line outage E_{c} causes a collapse, and $h(E_{\mathrm{c}}) = 1$ otherwise. In this way, the ELONC problem is to find all the E_{c}'s such that $h(E_{\mathrm{c}}) = 0$. Remember that it is a SAT problem to answer whether there is an E_{c} s.t. $h(E_{\mathrm{c}}) = 0$, and the OBDD method can find all the E_{c}'s s.t. $h(E_{\mathrm{c}}) = 0$. This is how we find all the line outages that cause network collapse. As aforementioned, it is NP-hard to find the optimal order of the variables in OBDD so that the calculation speed is maximized. However, as we will see in Section 3, we can achieve a reasonable fast calculation speed by using a natural order of the variables.

By comparing the frequency for each transmission line to appear in a cut set E_{c} s.t. $h(E_{\mathrm{c}}) = 0$, we identify easily the transmission lines that are critical to power network security. In other words, these lines appear most frequently in the line outages that cause collapse. To improve the network security, these critical lines should be improved with high priority, i.e., reducing the probability for these lines to outage improves the network security the most. In the next section, we use an IEEE 30-bus example to first show how the OBDD-based method assess the upper bound of the power network security, and then identify the critical transmission lines.

3. NUMERICAL EXAMPLES

We use an IEEE 30-bus standard power network (Dabbagchi and Christie (1993)) to test the OBDD-based method developed in Section 2. The node-weighted graph of this network is shown in Fig. 3.

There are 30 buses. Without loss of generality, we only consider real-power when testing the power balanced condition. In steady state, the power generation/load at each bus is listed in Table 1. For simplification, we consider integer units of power only.

There are 41 transmission lines in this network. We assume each line trips off independently with probability

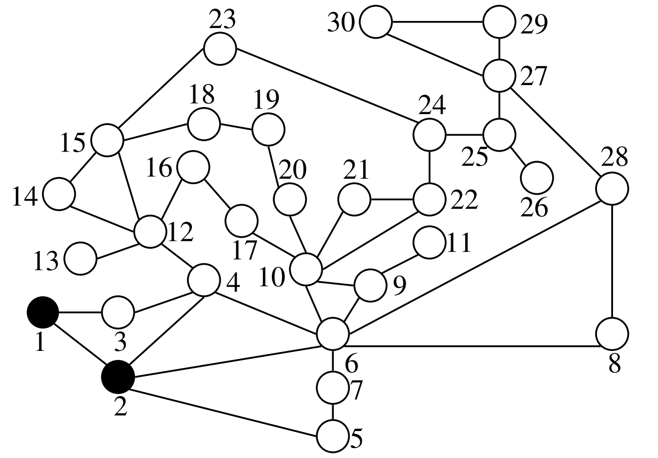

Fig. 3. The node-weighted graph model of the IEEE 30-bus standard power network (the weight of each node is not shown).

Table 1. The real power generation/load at each bus (Unit: MW) (Dabbagchi and Christie (1993)).

Bus #	Power	Bus #	Power
1	245	16	-4
2	18	17	-9
3	-2	18	-3
4	-8	19	-10
5	-94	20	-2
6	0	21	-18
7	-23	22	0
8	-30	23	-3
9	0	24	-9
10	-6	25	0
11	0	26	-4
12	-11	27	0
13	0	28	0
14	-6	29	-2
15	-8	30	-11

$P_s = 0.0018$.[1,2] Since we assume the line capacity is sufficiently large, different lines trip off independently, i.e., $\text{Prob}\{E_c\} = P_s^{|E_c|}(1-P_s)^{|E \setminus E_c|}$, where $|A|$ is the size of set A, $P_s^{|E_c|}$ is the probability that all the lines in E_c trip off, $E \setminus E_c$ is the set of all the lines that do not trip off, and $(1-P_s)^{|E \setminus E_c|}$ is the probability that no line that is not in E_c trips off. So $\text{Prob}\{E_c\}$ is the probability that line outage E_c (and only E_c) happens.

Using the OBDD-based method in Section 2, we find all the line outages that cause this IEEE 30-bus network to collapse. For k-line outage ($k = 1, 2, \ldots 41$), we calculate the ratio of line outages that causes a collapse. We list the collapse ratio for all k's in Fig. 4.

From Fig. 4, we can see that this IEEE 30-bus network is robust to all single line outages (the collapse ratio for

[1] Our method can also be applied when transmission lines are with nonidentical outage probabilities.

[2] In practice, the probability for a line to trip off can be estimated based on historical data. For new lines without a history, we can estimate the outage probability based on lines in similar location, or use the minimal outage probability of the existing lines as an estimate, since we are calculating the upper bound of security.

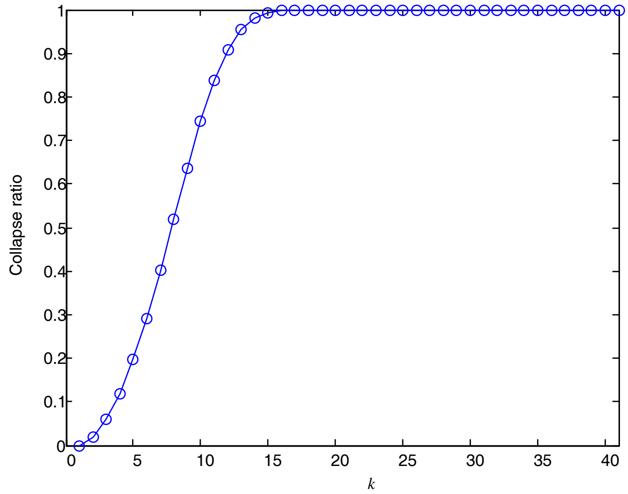

Fig. 4. The collapse ration that a k-line outage causes a collapse, $k = 1, 2, \ldots 41$.

1-line outage is 0). When more lines trip off, the collapse ratio increases fast. For $k \geq 26$, the collapse ratio is 1. This is reasonable, because when more lines trip off the network easily splits into islands. It is obvious that the power allocation in this network is not uniform. Both generators are allocated in the bottom left buses. So the power balanced condition in an island is easily destroyed. We calculate the lower bound of collapse probability of the network:

$$\sum_{k=1}^{41} \text{Prob}\{k - \text{line outage}\}\, CR(k) = 5.1863 \times 10^{-5}, \quad (5)$$

where $\text{Prob}\{k - \text{line outage}\}$ is the probability that a k-line outage happens, and $CR(k)$ is the collapse ratio of k-line outages in Fig. 4. Please note that the probability of a single line outage is obtained through historical data that each line trips off 2 times per year, and each time it takes 8 hours to repair the line, i.e., $P_s = 2 \times 8/24/365 \approx 0.0018$. Then the lower bound in Equation (5) represents that on the average there are 1 collapse in this power network every 26.4 months (2 years 2 months and 12 days), because $\frac{1}{5.1863 \times 10^{-5}}/24/365 \times 12 = 26.4$, where $\frac{1}{5.1863 \times 10^{-5}}$ is the number of hours between two collapses on the average.

Now, we can say that given the topology and power allocation of the IEEE 30-bus network, it collapses at least once every 26.4 months on the average, no matter how we improve the line capacities. From practical viewpoint, this collapse probability is not small. Increasing transmission line capacity does not help to reduce this lower bound on security. To reduce this collapse probability, we need to site new transmission lines (to improve the topology), to reduce the probability for the transmission lines to trip off, or to better design the generation pattern (to make the generators and loads uniformly distributed in the network). In the following we identify the transmission lines that are most critical to the security. By siting new lines identical to these transmission lines, we can effectively improve the upper bound of the security of this power network.

If a line outage leads to a collapse, we call it a fatal line outage. The number of fatal k-line outages in the IEEE 30-bus network is shown in Table 2.

Table 2. The number of fatal k-line outages (NFO), $k = 2, 3, ...41$. NFO=0 when $k = 1$.

k	NFO	k	NFO
2	16	22	2.4×10^{11}
3	628	23	2.0×10^{11}
4	1.2×10^{4}	24	1.5×10^{11}
5	1.5×10^{5}	25	1.0×10^{11}
6	1.3×10^{6}	26	6.3×10^{10}
7	9.0×10^{6}	27	3.5×10^{10}
8	4.9×10^{7}	28	1.8×10^{10}
9	2.2×10^{8}	29	7.9×10^{9}
10	8.3×10^{8}	30	3.2×10^{9}
11	2.6×10^{9}	31	1.1×10^{9}
12	7.2×10^{9}	32	3.5×10^{8}
13	1.7×10^{10}	33	9.6×10^{7}
14	3.5×10^{10}	34	2.2×10^{7}
15	6.3×10^{10}	35	4.5×10^{6}
16	1.0×10^{11}	36	7.5×10^{5}
17	1.5×10^{11}	37	1.0×10^{5}
18	2.0×10^{11}	38	1.1×10^{4}
19	2.4×10^{11}	39	820
20	2.7×10^{11}	40	41
21	2.7×10^{11}	41	1

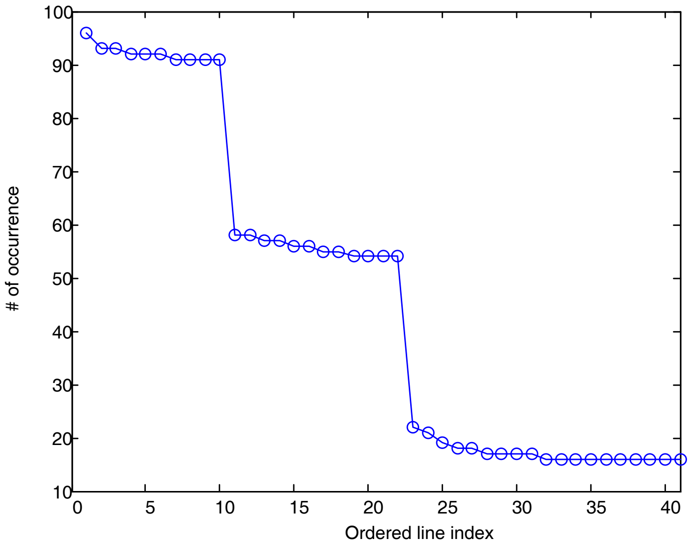

Fig. 5. The number of occurrence of each transmission line in the double/triple-line outages that cause collapses.

Since we assume the transmission lines have sufficiently large capacity, the network sustains all single-line outages. To simplify the discussion, we list only the double/triple-line outages that cause collapse, and counter which lines appear most frequently in these outages. The number of occurrence of each line in these fatal double/triple-line outages are shown in Fig. 5.

From Fig. 5, it is obvious that 41 transmission lines are clearly divided into 3 groups, according to the occurrence in the fatal double/triple-line outages. For the lines in a group, the occurrence is close to each other. We mark the 10 transmission lines in the first group in dashed lines in Fig. 6. The following points should be mentioned:

(1) The direct connection between the generators is critical.
(2) The two branches (branch "bus2-bus5-bus7-bus6" and branch "bus15-bus18-bus19-bus20") are critical. A branch is a sequence of buses, in which the buses in the two ends connect to more than 2 buses, and the other buses in the middle connect to exactly 2 buses. This branch type of connection is not good for security in general, since a single line outage changes the power flow dramatically.

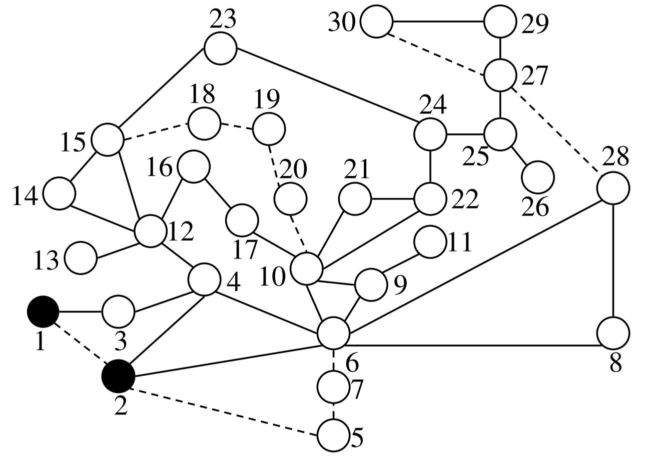

Fig. 6. The top-10 critical transmission lines (in dashed lines).

So in this example, if we consider siting new transmission lines or reducing line outage probability, these dashed transmission lines in Fig. 6 are with the highest priorities.

4. CONCLUSION

In this paper, we study how the topology and power allocation affects the security performance. Roughly speaking, when the buses are more closely connected to each other directly, or when the generators and loads are distributed more uniformly, the upper bound of security performance is more close to 1. However, a practical power network is never a complete graph (i.e., there is a transmission line between each two buses) and the generators and loads are seldom uniformly distributed (One reason to connect small power networks into large ones is to exchange power among different areas, which represents that the generators and loads are not uniformly distributed at all the buses.). So a practical power network cannot avoid collapse. To calculate how large this collapse probability is, we develop an OBDD-based method to fast enumerate all line outages that cause collapse. For a practical power network, this collapse probability can be justified by historical data. In an IEEE 30-bus example, we use numerical examples to show that this collapse probability is not small from practical viewpoint. Then we show how to use the OBDD-based method to identify the most critical transmission lines to the security of the power network. This helps to indicate where to site new transmission lines, or to reduce the outage probability of which transmission lines with high priority. As part of the ongoing work, we are extending the above method to larger power networks. We hope our work brings insights to the understanding of why power networks cannot avoid collapse and how to enhance the security of an electric power network.

ACKNOWLEDGEMENTS

The authors would like to thank the constructive comments from two anonymous reviewers. Of course, all remaining errors are solely the responsibility of the authors.

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