Networks and Games

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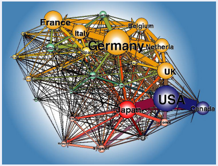
Networks & Economics



Networks play a central role in economics:

- provide access to resources,
- agents affected mostly by the actions of those in their close proximity.

Empirical studies:

- job contact networks (Granovetter, 1974)
- technology transfer (Conley and Udry, 2004)
- R&D alliances (Powell et al., 2005)
- risk sharing (Fafchamps and Lund, 2003)
- network effects (Glaeser et al., 1996; Topa, 2001; Tucker, 2005)

This talk:

1. Networks, economics and strategic interactions

- What type of questions have been studied in the literature?
- What are important open questions?
- Input from other fields
  - physics/ mathematics, sociology, (computer science), ...

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2. Example: Effect of incomplete information in network games.

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2. Example: Effect of incomplete information in network games.

Central tool: Game theory

- take best action *given* opponents' actions

Networks and Economics

Traditionally, economists think of interactions as being mediated by a decentralized *market*.

In a market, all individuals interact directly, and everyone has the same information

Networks and Economics (2)

In reality, individuals interact through a *network*.

- limited interactions
- information flow inhibited

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So what?

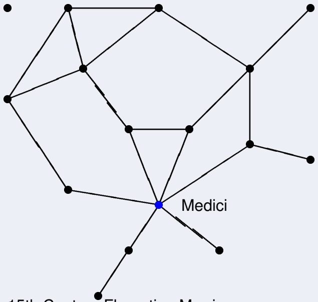
- Different outcomes when individuals interact through a network rather than through a market?
- If so, under what conditions?
- If so, why?
  - endogenous formation of networks
  - information flow
  - limited interactions

Models

- Network formation models
- Network games

Note: typically, these two stages cannot be separated!

Network Formation: Florentine Marriages



15th Century Florentine Marriages
(Padgett and Ansell, 1993)

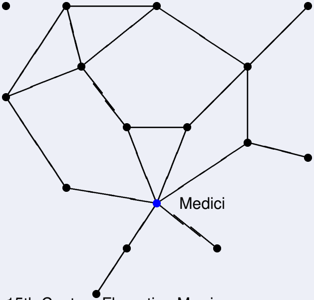
Padgett and Ansel (1993) study the network of marriages between key families in Florence in the 1430s.

- *vertex/node*: family
- *edge/link* between two families if and only if there is at least one marriage between the members of the two families.

Note: these marriages are carefully arranged with an eye to improve the position of the family.

- Game-theoretic analysis applicable

Florentine Marriages (2)



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Central question: How did the Medici family become so powerful?

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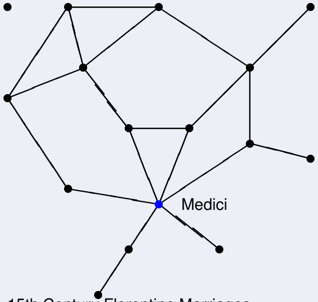

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- wealth
- political clout

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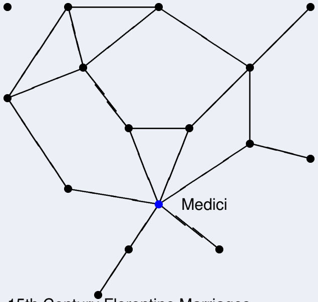

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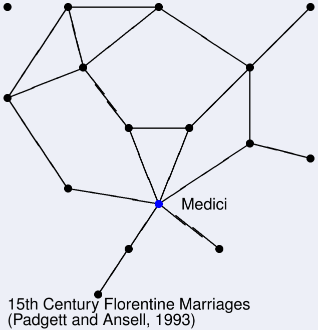
- wealth
- political clout
- network position

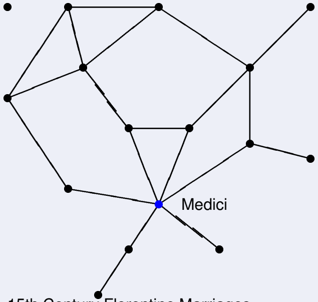
Florentine Marriages (3)



How to quantify a family's network position?

- Centrality

Florentine Marriages (3)



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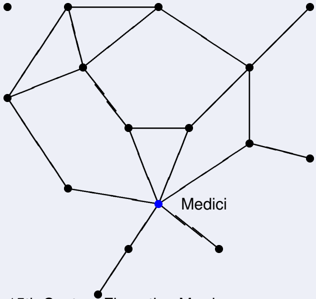
How to quantify a family's network position?

- Centrality

Different measures of centrality (Borgatti, 2005)

- Degree
- Betweenness centrality

Themes



15th Century Florentine Marriages
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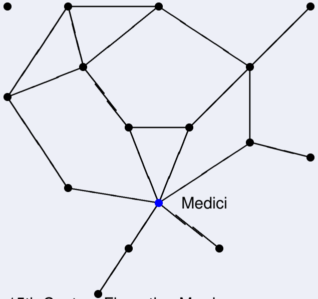
Important themes:

1. Two-way interaction between network formation and interactions on networks;
2. Tension between stability and efficiency
3. Heterogeneity in outcomes for ex ante identical individuals

Two-way interaction



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Self-interested agents form links anticipating the ensuing interactions.

- Strategic interactions on network → network formation

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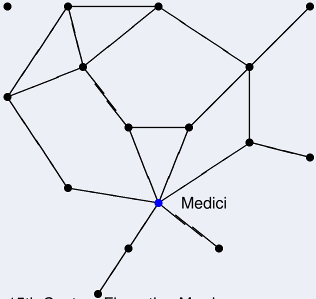

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Self-interested agents form links anticipating the ensuing interactions.

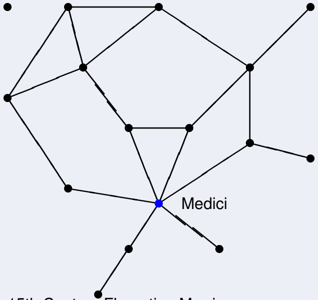
- Strategic interactions on network → network formation

But: the fact that agents can form and break links at will also constrains interactions on network

- Network formation → strategic interactions on network

(also see Ule (2008), Bowles, Iyengar, Kets, Sethi (2008)).

Stability versus Efficiency



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What networks are formed if individuals can choose the links they want to form?

- Stability

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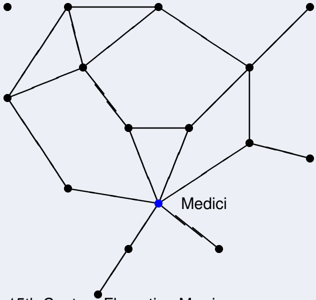

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- Efficiency

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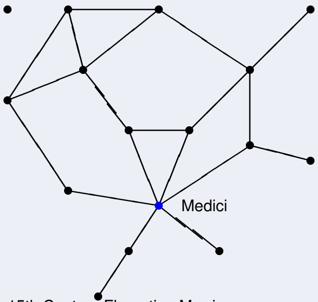

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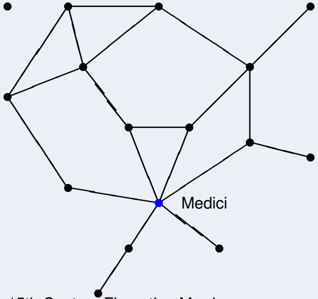
Externalities:

- Classic type
- "Power"

Ex Ante Equality & Heterogeneity in Outcomes



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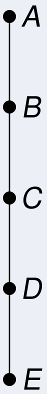
The Medici family did not have a prior advantage over other families.

- Political clout
- Wealth

Why did they rise to power, while other families didn't?

- Establish central position to earn rents.
- Establish central position to prevent *others* from earning rents.

A Simple Model of Betweenness Rents



Goyal and Vega-Redondo (2007).

- Suppose individuals are located on a network. Individuals that are linked (directly or indirectly) create a unit of surplus.

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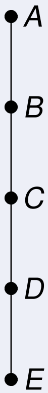

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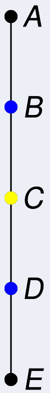

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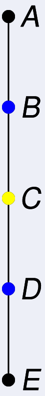

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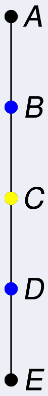

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- The central agents are essential for many transactions. Hence, they will get higher payoffs.
- Individuals therefore form links to create surplus, to earn betweenness rents, *and* to prevent others from obtaining rents.
- Is it possible that "unequal" networks are formed if every agent wants to be central?

Betweenness rents (2)

Result 1: Stable networks are *stars* (no “capacity constraints”) or *wheels* (capacity constraints).

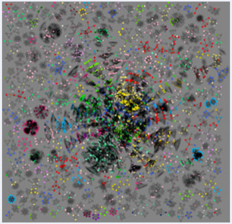
- If everyone competes for a central position, either one may win, or all end up equal, depending on conditions.

Result 2: Stable networks need not be efficient.

- “Classic” externality
- Power

See Jackson and Wolinsky (1996), Jackson (2005).

Betweenness rents (3)



While the results reproduce some features of real networks, this approach cannot account for the complex structures often observed.

General feature: The richness on the strategic side forces one to focus on simple settings

- no initial heterogeneity, no dynamics, complete information. . .

Alternative Approach: Random Networks

Preferential attachment
(Simon, 1955; Price, 1976; Barabási and Albert, 1999)

- Suppose at each point in time, a new agent is born.
- An agent that is born at time t forms m links to “old” agents, with a probability that is proportional to their degree at time t.
  - “Rich get richer”

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- An agent that is born at time t forms m links to "old" agents, with a probability that is proportional to their degree at time t.
  - "Rich get richer"
- This gives rise to networks with a *scale-free* degree distribution

$$f(k) \propto k^{-\alpha},$$

as observed in many real networks

Preferential Attachment: Discussion

- The networks formed in this way can thus reproduce some important features of real networks.

Preferential Attachment: Discussion

- The networks formed in this way can thus reproduce some important features of real networks.
- However, the process is rather mechanical, and incentives are not explicitly modeled.
  - Stability?
  - Efficiency?
  - Why does this mechanism operate in some situations and not in others?

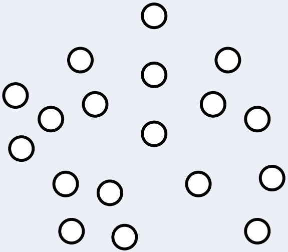
Strategic Interactions on Networks

Network formation models in economics, such as Goyal and Vega-Redondo (2007), typically have a *reduced form* model to determine payoffs from strategic interactions on network.

Literature on *network games* takes the network as given and explicitly models strategic interactions on network.

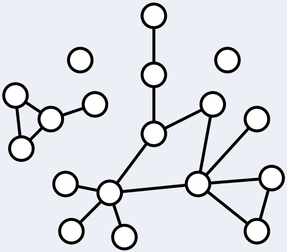
- Complete Information
- Incomplete Information

Network Games



- There is a finite set of agents or **players**.

Network Games



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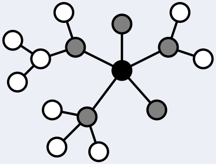
- Players are connected by a **network** and interact only with their neighbors in their network.

Network Games

- There is a finite set of agents or **players**.
- Players are connected by a **network** and interact only with their neighbors in their network.
- Each player is endowed with a set of **actions**.
- The **payoffs** to a player depend on
  - his own action,
  - the actions of his neighbors,

  and are a symmetric function of neighbors' actions.
- Complete or incomplete information on network

Connected local games



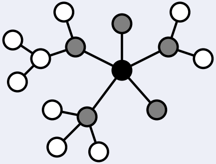
Network games are "local games"

- you only care about the actions of your neighbors

. . . but are interlaced through agents' actions or beliefs.

Connected local games (2)

Complete information: actions



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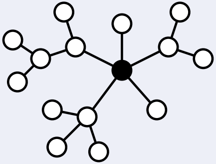
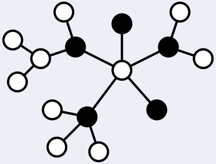


Complete information: actions

- I do *a*

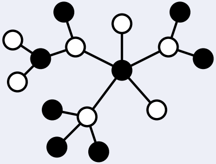
Connected local games (2)



Complete information: actions

- I do *a*
- because my neighbors do *b*

Connected local games (2)



Complete information: actions

- I do *a*
- because my neighbors do *b*
- because their neighbors do *c*...

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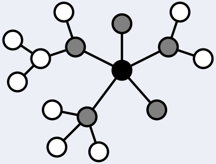

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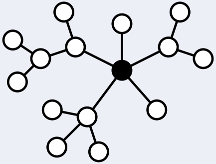

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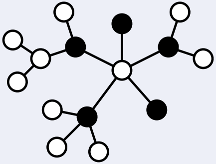
- I believe. . .

Connected local games (2)



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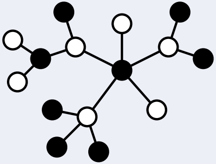
Incomplete information: beliefs

- I believe…
- that my neighbors believe…

Connected local games (2)



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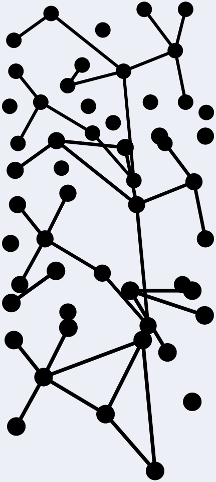
Incomplete Information in Network Games



Networks are often large and complex...

- Goyal et al. (2006)

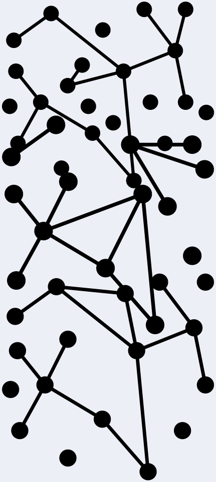
...and evolve rapidly over time...

- Powell et al. (2005)

...so that agents do not know the exact structure of the network they belong to

- Krackhardt and Hanson (1993)

Incomplete Information in Network Games



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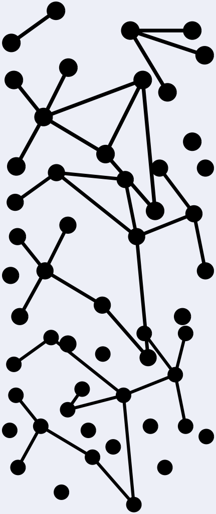

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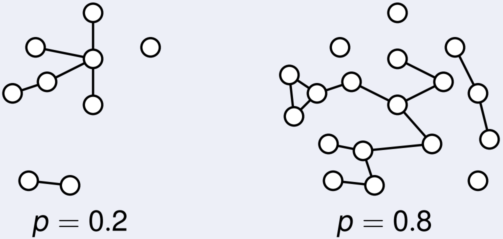
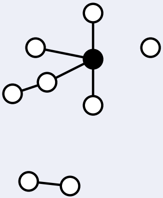
Need explicit assumptions on information and beliefs.

Beliefs and Information

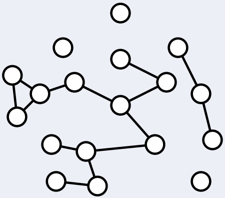


- Consider the set $\mathcal{G}$ of all networks with a finite vertex set.
- Players' beliefs over the network structure are given by a probability measure μ over $\mathcal{G}$
  - **(common) prior**.
  - NB: Uncertainty about network size.

Beliefs and Information (2)



$p = 0.2$



$p = 0.8$

- Each player in the network is informed of the number of neighbors he has, i.e., his **type** is his **degree**.
- Hence, the **type set** is $T := \mathbb{N} \cup \{0\}$.
- This information leads players to update their beliefs (Bayes' rule).

Information and Beliefs

Assumptions on information and beliefs matter!

- Information: local public goods example
- Beliefs: Part 2 of talk

Information and Local Public Goods

Bramoullé and Kranton (2005), Galeotti, Goyal, Jackson, Vega-Redondo, Yariv (2006).

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- Suppose individuals are located on a given network.
- They can contribute to a public good.

Information and Local Public Goods

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- Suppose individuals are located on a given network.
- They can contribute to a public good.
- Contributions are shared equally among neighbors.
- Individual contributions are *strategic substitutes*:
  - if my neighbor contributes, why would I?

Local Public Goods: Results

- If players know the network (*complete information*), then there are equilibria in which players with many connections contribute more and get lower payoffs.

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- **Intuition:** Information on the network structure allows players to *coordinate* on a certain outcome. Hence, if players have more information, more outcomes are possible.
- Hence, outcomes and behavior can be very different, depending on the informational assumptions.

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- **Intuition:** Information on the network structure allows players to *coordinate* on a certain outcome. Hence, if players have more information, more outcomes are possible.
- Hence, outcomes and behavior can be very different, depending on the informational assumptions.
  - How general is this result?
  - What kind of information is important?

Networks & Games: Summary

- Networks can have an important effect on social and economic outcomes.
  - Networks & power in 15th Century Florence
  - Risk sharing, employment, R&D, . . .
- Important themes:
  - Two-way interaction network formation and network interactions
    - changes incentives.
  - Stability vs efficiency
    - new externalities.
  - Heterogeneous outcomes possible.
    - limited interactions and limited information flows.

Networks & Games: Summary (2)

- Difficult to devise a network formation model that both captures the richness on the strategic side and yields realistic network structures.
- Difficult to integrate models of network formation and models of strategic interactions on given network.
- In network games, results may depend heavily on informational assumptions (and on assumptions on beliefs, as we shall see).

Beliefs in Network Games

Kets (2007)

In network games with incomplete information, players only interact with their neighbors.

Yet, these local games are interlaced through players' beliefs.

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Can a

small change

in players' *beliefs* on their network have a

BIG effect

on game-theoretic *outcomes*?

More specifically:

Consider two (common) priors over a class of networks. When are these priors “close” in a strategic sense?

- Sensitivity

That is, what aspects of these prior beliefs are important from a strategic perspective?

Motivation

- We know little about players' beliefs about their network.
  - simple heuristics (Janicik and Larrick, 2005);
  - biased perceptions (Kumbasar et al., 1994; Johnson and Ohrbach, 2002).
- Which different classes of priors to consider?
  - current literature:
    - size of the network is commonly known,
    - players' types are independent.
- Local games interlaced through players' beliefs: local investment game
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Local investment game

Consider the following game. There are two actions, S and R:

- S always gives a payoff of 0,
- payoffs to R depend on player's type t and the actions of his neighbors.

Payoffs to R:

$$\begin{array}{ll} +1 & \text{if } t \neq 1 \text{ and all neighbors play } R, \\ -1 & \text{otherwise.} \end{array}$$

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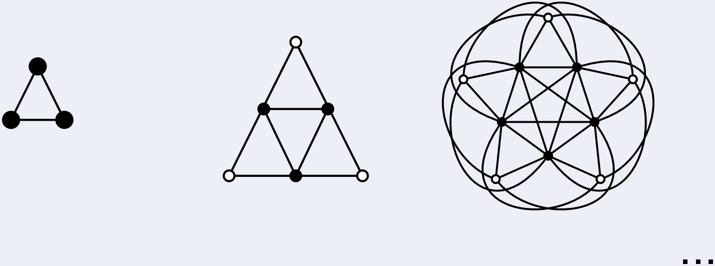
Interpretation: risky investment.

Equilibrium: Each type takes the best distribution over actions given the choices of other types.

Local Investment (2)



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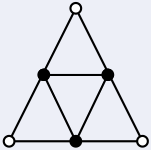

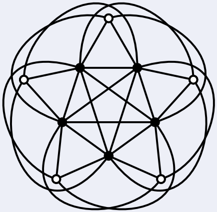

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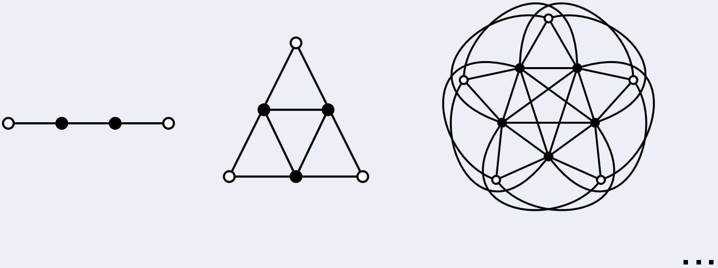
Local Investment (3)

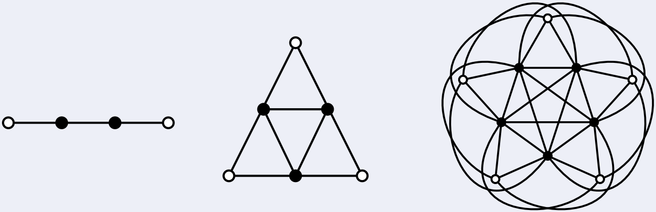
Suppose you have type $t = 2^{35} = 34359738368$.



Local Investment (3)

Suppose you have type $t = 2^{35} = 34359738368$.

- You can be a peripheral player in a network with players with type $t = 2^{36} = 68719476736$.
- You can be a core player in a network with players with type $t = 2^{34} = 17179869184$.



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Strategies

- Because there is uncertainty about the network size, the player set is not commonly known.
- Therefore, players of the same type cannot be commonly perceived to follow different strategies (cf. Myerson, 1998).
- Hence, strategies can only depend on a player's type, not on his identity.

Strategies

- Because there is uncertainty about the network size, the player set is not commonly known.
- Therefore, players of the same type cannot be commonly perceived to follow different strategies (cf. Myerson, 1998).
- Hence, strategies can only depend on a player's type, not on his identity.
- Each *type t*, chooses a probability distribution σ_t on the set of actions.
  - $\sigma_t(a)$: probability that a player of type t plays action a.
- A vector $\sigma = (\sigma_0, \sigma_1, \ldots)$ is a **strategy function.**

Expected Payoffs

- $\varphi_t(a, \sigma; \mu)$: **Expected payoffs** to a player of type t of action a given that the other players follow the strategy function σ under prior μ;

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- $\varphi_t(a, \sigma; \mu)$: **Expected payoffs** to a player of type t of action a given that the other players follow the strategy function σ under prior μ;
- $\Phi(\sigma; \mu)$: **Type-averaged payoffs** of a strategy function σ:

$$\Phi(\sigma; \mu) := \sum_t q_\mu(t) \sum_a \sigma_t(a) \varphi_t(a, \sigma; \mu).$$

where $q_\mu(t)$ is the (prior) probability under μ that a player has type t.

Equilibrium in Network Games

Definition

Let $\varepsilon \geq 0$. An ε-**equilibrium** is a strategy function σ such that for each $t \in T$ such that $q_\mu(t) > 0$, for each action $a \in A$ such that $\sigma_t(a) > 0$,

$$\varphi_t(a, \sigma; \mu) \geq \varphi_t(b, \sigma; \mu) - \varepsilon$$

for all $b \in A$. A 0-equilibrium is an **equilibrium**.

Strategic Convergence

Given a prior μ and a sequence of priors $(\mu^k)_{k\in\mathbb{N}}$, we want to know under what conditions $(\mu^k)_{k\in\mathbb{N}}$ converges to μ *in a strategic sense*.

$$\mu^1 \quad \mu^2 \quad \mu^3 \quad \mu^4 \quad \cdots\cdots \quad \mu$$

Strategic Convergence (2)

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Let $\varepsilon, \delta \geq 0$. The **strategic distance** $SD(\mu, \mu'; \varepsilon)$ between μ, μ' given ε is equal to δ if

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Definition

A sequence $(\mu^k)_{k \in \mathbb{N}}$ **converges strategically** to μ if for each $\varepsilon > 0$, $SD(\mu, \mu^k; \varepsilon) \to 0$ when $k \to \infty$.

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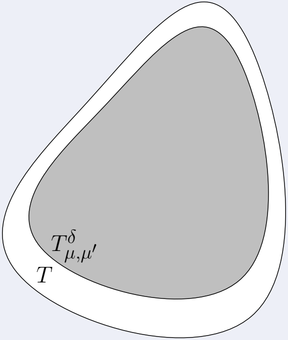
(1) they are close in terms of the prior probabilities they assign to all "local" events,

(2) with high probability, a player has a type such that his conditional beliefs are close, and he believes, given his type, that it is likely that the conditional beliefs of his neighbors are close and that they believe, given their type, ... that the conditional beliefs of their neighbors are close.

Intuition



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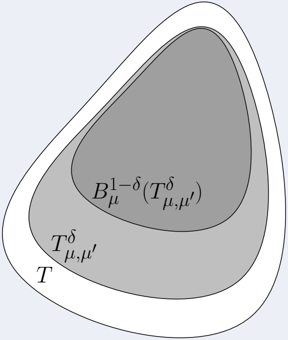

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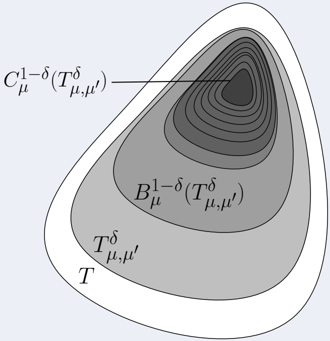

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- The main result shows that two priors are close in a strategic sense if and only if
  (1) they assign similar prior probabilities to all events involving a player and his neighbors,
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Future Work

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- There are many other important questions that can be answered in that way.
  - Sensitivity to the assumptions on players' information.
  - Robustness
    - subclass of payoff functions (Kalai ,2004; Weinstein and Yildiz, 2007)
    - subclass of priors

Background Reading

- M. O. Jackson (2008), Social and Economic Networks, Princeton University Press, forthcoming (July 2008).
  - *broad survey of research on networks, incl. research outside economics, by the leading author in the field.*
- S. Goyal (2007), Connections, Princeton University Press.
  - *Overview geared towards work in economics, with many applications*
- F. Vega-Redondo (2007), Complex social networks, Econometric Society Monographs.
  - *Survey of the work at the intersection of economics and physics.*