Good social science as good physics?

Eric Smith (SFI)

based on work with

Doyne Farmer (SFI)
Supriya Krishnamurthy (Swedish Inst. Comp. Sci)
Laci Gillemot (Budabest U. Technology and Economics)
Giulia Iori (City U. London)
Martin Shubik (Cowles Foundation, Yale)
Paolo Patelli (LANL)
Marcus Daniels (LANL)

Outline

- Some platitudes
- Review of neoclassical microeconomic theory as it is formulated today
- Examples of more tentative and empirical, but also more abstraction-based, approach

The platitudes part

The Scylla and Charybdis of social physics

- Renaming models because we know how to solve them wasn't good physics, and won't be good as any other kind of science either
- Endless regression of data or "digital naturalism" are (at best) barely-science, and they are not theory at all

What distinguishes Theory among yeah-yeah-whatever systems?

- Rich conceptual substructure between empiricism and modeling
- “Abstractions” should attach quantitative consequences to partial problem specifications
- Believe models when the paths that led to them have excluded everything else
- Look for notions of “universality” as the justification for simple models

(This is not intended to be exclusive)

A critique of neoclassical economics as case study in social science or theory

“Marginalist” or “Neoclassical” economic theory today

(A note on terminology) “Rationality” elevated to a red herring; *real* definition is set-theorists’ best attempt to evade ***contingency*** and ***indeterminacy***

- A genuine attempt at Theory -- for all its problems -- but exists only in micro-economics
- Concepts are more borrowed than derived, (often misunderstood), and use of math is bizarrely dissociated from measurement
- *NOT* driven by a humble passion to explain empirical regularities

The generally-accepted goal of theory in micro-economics

- to derive "optimal" allocations (including production decisions) for limited resources in a society/economy *directly and exclusively* from the collection of "properties" of the individuals (and technological constraints) that make up the economy

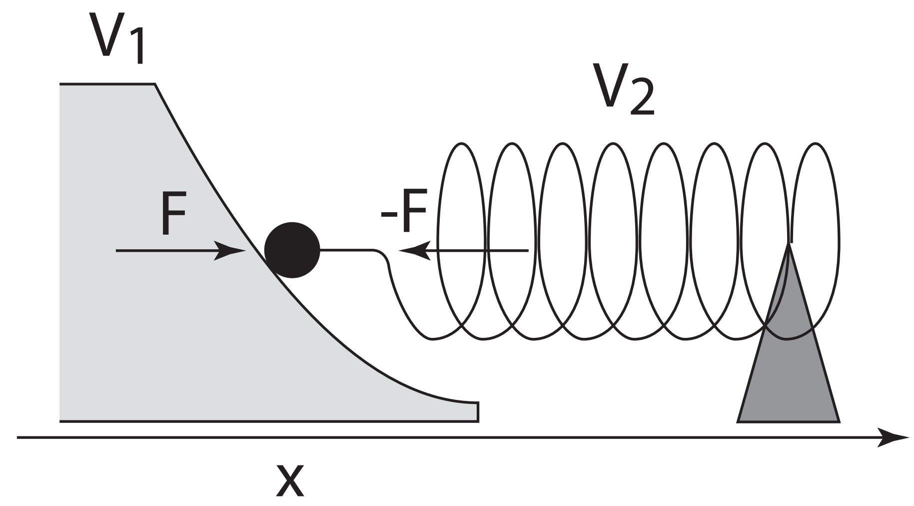
Problems separating individualist, normative motivation from science, of historical origin

*..every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, **led by an invisible hand** to promote an end which was no part of his intention. Nor is it always the worse for the society that it was no part of it. By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it. **I have never known much good done by those who affected to trade for the public good.***

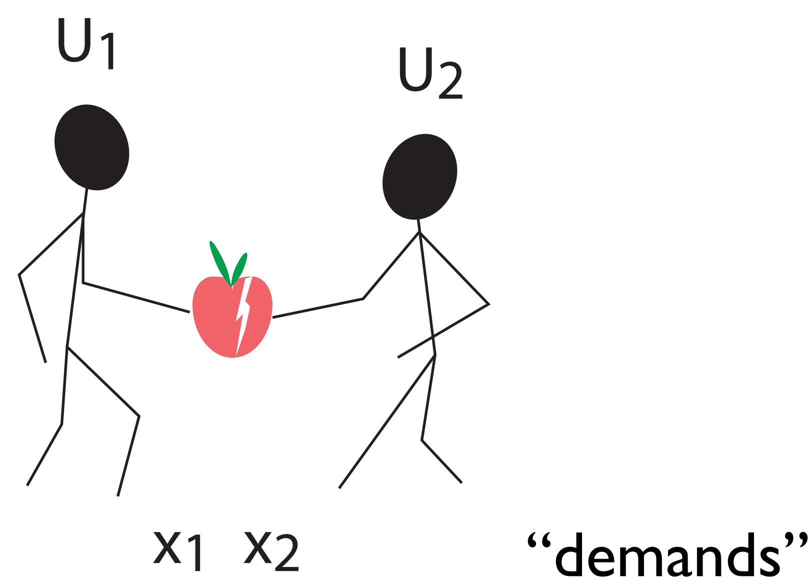
From *An Inquiry into the Nature and Causes of the Wealth of Nations* by Adam Smith (1776)

The jump that made individualism non-dynamical, a-historical, and anti-institutional

Leon Walras (1909)



- Equilibrium as force balance in mechanics



- Equilibrium as balance of "marginal utility" in exchange

In one sense, a fair attempt to map abstractions . . .

Position (x)	Holdings (x)
Potential Energy (V)	Utility (U)
Force (F)	Prices (p)
$F = -\nabla V$	$p = \nabla U$

(Utility is implicitly measurable)

"Ball settles in the bottom of the bowl to minimize energy"

. . . if not drawn from compelling empirical regularities

- Complete, costless contracts
  - All consequences known and bargained for (No pig farms)
  - All contracts, costless, instantaneous, guaranteed (No accountants, courts, gov't)
- "Rationality" := Global self-fulfilling prophecy
  - Strategic actions by everyone "as good as given" (Mutual "best response")
  - Non-constructive existence proof for equilibria (Multiple, uncomputable)
- "Utilitarian" preferences (usually self-regarding)
  - Partial order on all possible states of the world
  - All outcomes over all time have present value

(Known false many ways)

Best understood as a definition of a process-free, institution-free world

What could be different and better?

Heuristics for doing better?

- *Imagine* that institutions, process dynamics, and clear, universal empirical regularities may be important
- Expect to have to justify any use of numbers/arithmetic from coherent measurement systems
- Emphasize the validation of quantitative abstractions as the primary goal

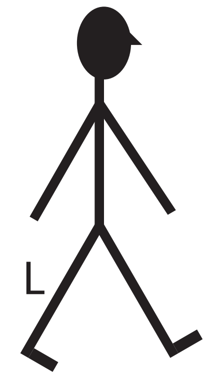
Specific entry paths for physicists in social science

- Institutions are by nature "mechanistic"
  - Based on rules and constraints, which are open to inspection
  - Opportunity for structural and process analysis (analysis of market function, design of auctions, etc.)
- "Zero Intelligence" as a pure model of institutional constraint
  - Constraints can shape both action and reasoning
- Errors in ZI models can be among the best indicators of behavioral regularity
  - Provides better focus for proper behavioral science

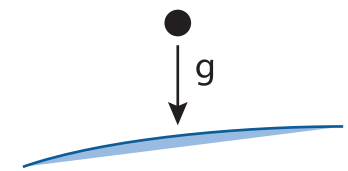
Dimensional Analysis and scaling: a subtle and sometimes tricky, but amazingly versatile framework for abstraction (also the basis for the use of arithmetic)

- Two principles
  - Equations must be homogeneous in dimension
  - Quantities with the required dimensions control scaling
- Two consequences
  - Can guess the sizes of observables
  - Can relate differently-sized cases to a single model

Example: how fast do you walk?

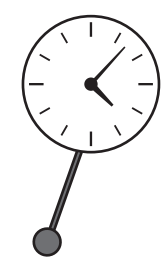


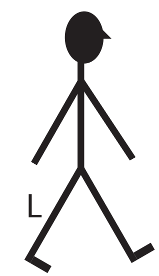
- A pure dimensional analysis
  - You want to know a speed $$[\,\text{speed}\,] = \left[\frac{\text{length}}{\text{time}}\right]$$
  - You have a property of height $$[\,\text{height}\,] = [\,\text{length}\,]$$
  - You are walking in earth gravity $$[\,g\,] = [\,\text{acceleration}\,] = \left[\frac{\text{length}}{\text{time}^2}\right]$$



  - Only one combination has correct dimensions $$[\,v\,] = \left[\sqrt{L\,g}\,\right]$$

- Could break this down in more detail
  - Your leg is a pendulum $$[\,\text{frequency}\,] = \left[\frac{1}{\text{time}}\right]$$

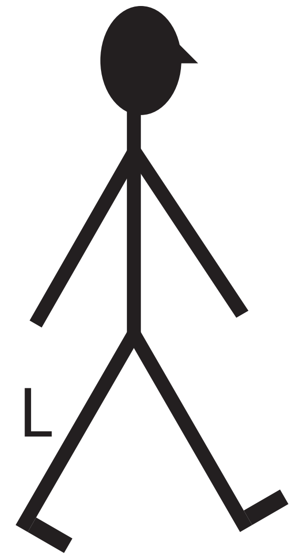




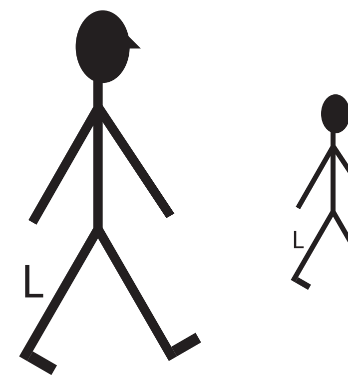
  - Pendulum is characterized by frequency
  - Only one combination works dimensionally $$[\,\omega\,] = \left[\sqrt{\frac{g}{L}}\,\right]$$
  - Speed is "ticking" frequency times length

$$[\,v\,] = [\,L\,\omega\,] = \left[L\sqrt{\frac{g}{L}}\,\right]$$

All walkers great and small

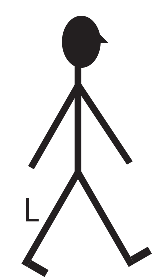


- Walking is a *scale-invariant* process
- All walkers share gravity, they differ mostly in their characteristic lengths
- Dimensional homogeneity can be used to generate scale factors relating different walkers to each other



$$[\omega_i] = \left[\sqrt{\frac{g}{L_i}}\right] = \left[\sqrt{\frac{L_0}{L_i}}\,\omega_0\right]$$

$$[v_i] = \left[\sqrt{L_i\, g}\right] = \left[\sqrt{\frac{L_i}{L_0}}\,v_0\right]$$



Zero-Intelligence (ZI) modeling: a pure formulation of institutional constraint

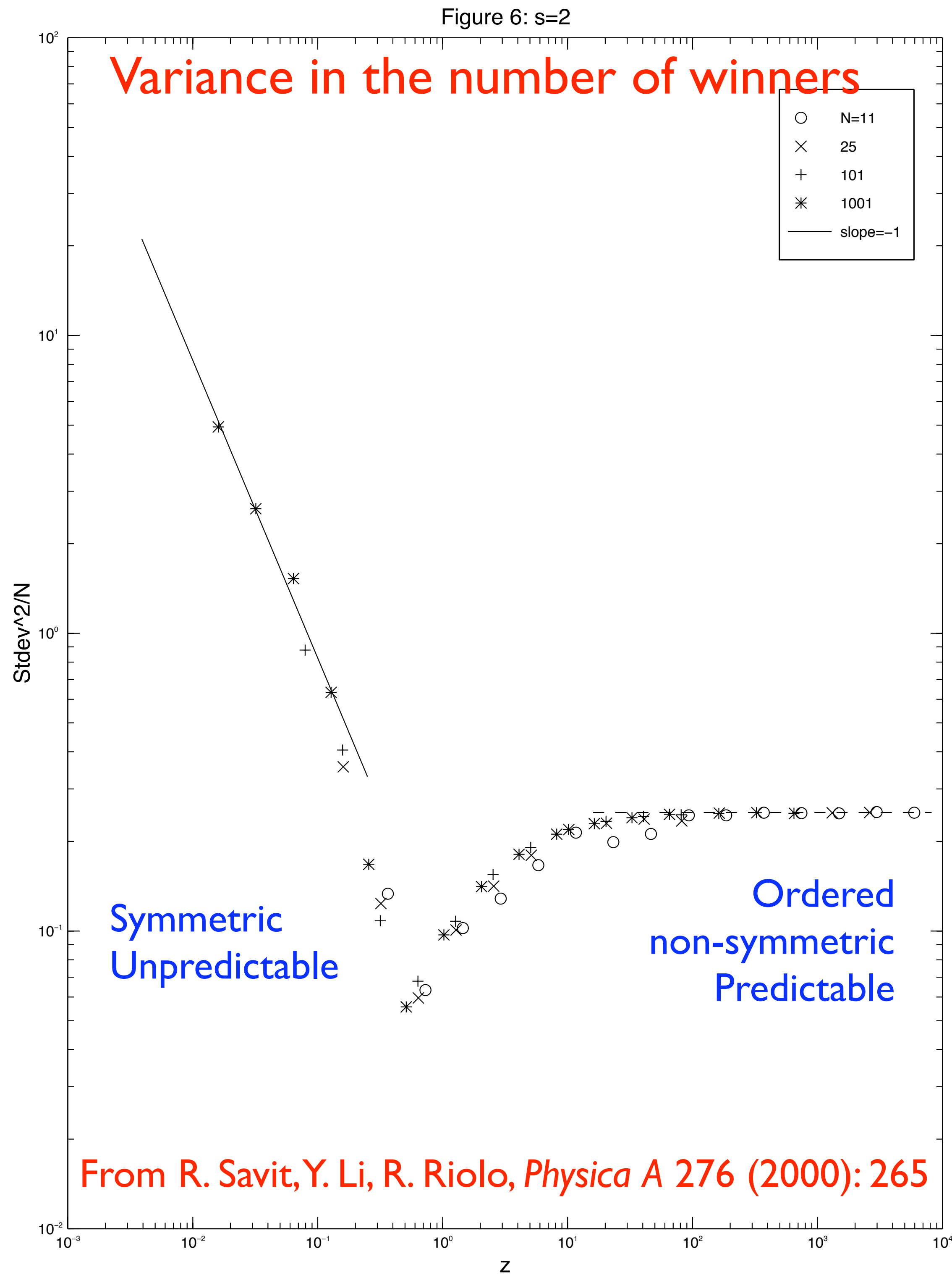
- We don’t know what “intelligence” or “rationality” are; but we do know how to exclude them
- An example: the minority game
- ZI modeling can be a foundation to which more formal models of learning are added

The minimal Minority Game: the original zero-intelligence model

- A population of N players (usually odd) choose one move from the set (0,1), independently and simultaneously.
- Each player whose choice was the minority in the population is awarded a payoff (“wins”).
- Moves have no intrinsic value
- The number of constellations of winners is enormous (frustration)
- A model of the non-rational component of purely speculative stock trading

Minority Games: a basis for learning models

- Each agent remembers outcomes of M prior rounds of play
- A set of random lookup-tables (particular to each agent) provides "next moves" in response to each possible history (2^M)
- Agents "learn" by choosing the table with best performance so far
- A phase transition occurs in $z = 2^M / N$
- Most satisfaction at critical



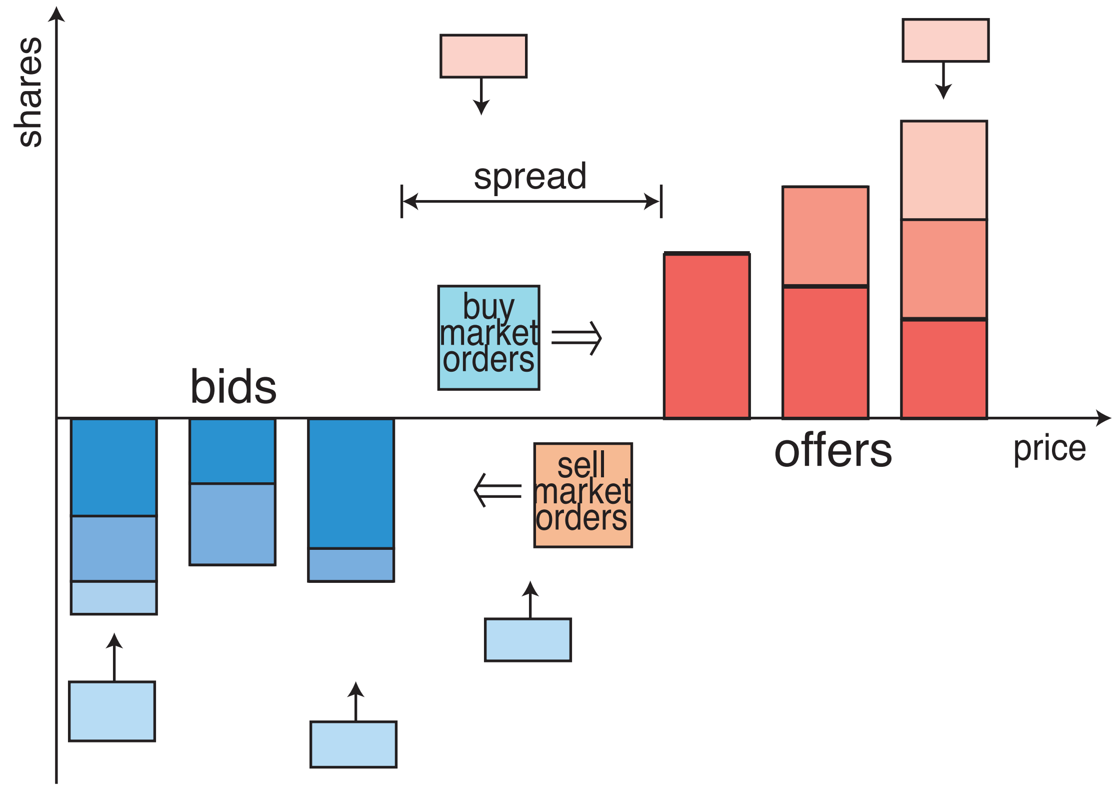
Worked example of dimensional analysis and ZI: the Continuous Double Auction of financial markets

- The basis of all continuous-trading financial markets today
- Solves problem of matching asynchronously-arriving orders to buy and sell
- Heavily institutionalized and high volume
  (~5-7 billion dollars / day in 1999)
- "Complete", good-quality data

Mostly from Smith, E., J. D. Farmer, L. Gillemot, and S. Krishnamurthy. "Statistical Theory of the Continuous Double Auction." *Quant. Fin.* 3(6) (2003): 481-514

The Continuous Double Auction (CDA) mechanism

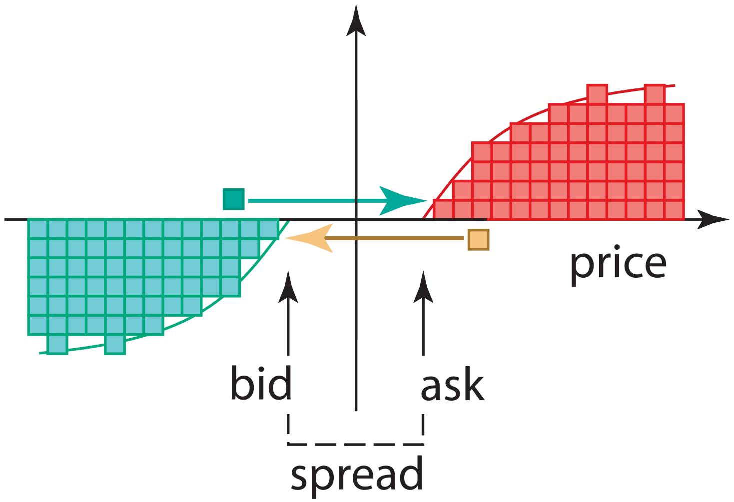
- Two kinds of orders
  - Market (v)
  - Limit (p,v)
- Limit orders (LO) accumulate
- Market orders (MO) clear immediately
- LO at single price clear in order of arrival



Observables of the CDA

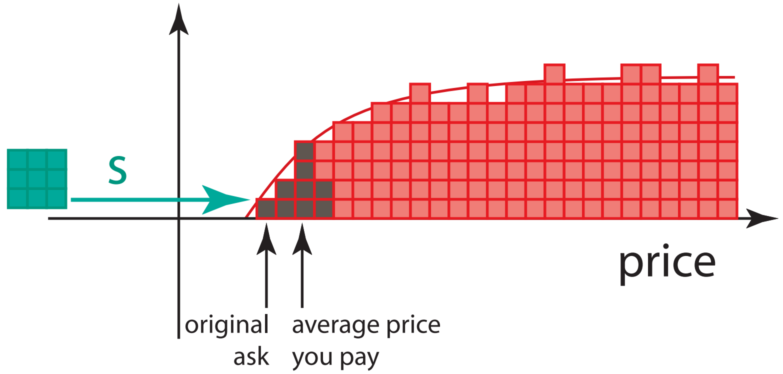
- Bid-ask spread (responsible for costs)
- Volatility (responsible for risk and profit)
- Market impact (responsible for costs and risk)
- Liquidity (resistance to market impact)
- Depth (responsible for stability)

The bid-ask spread



- Bid = best offered buying price
- Ask = best offered selling price
- Spread = ask - bid
- Rapid small buy-sell alterations will pay the average spread per pair of transactions
- Nonzero spread is the leading source of transaction costs
- Nonzero spread is the regulator against divergent volume of trading

Market Impact



- Average price of an order always worse than the starting best price
- Return depends only on price ratios

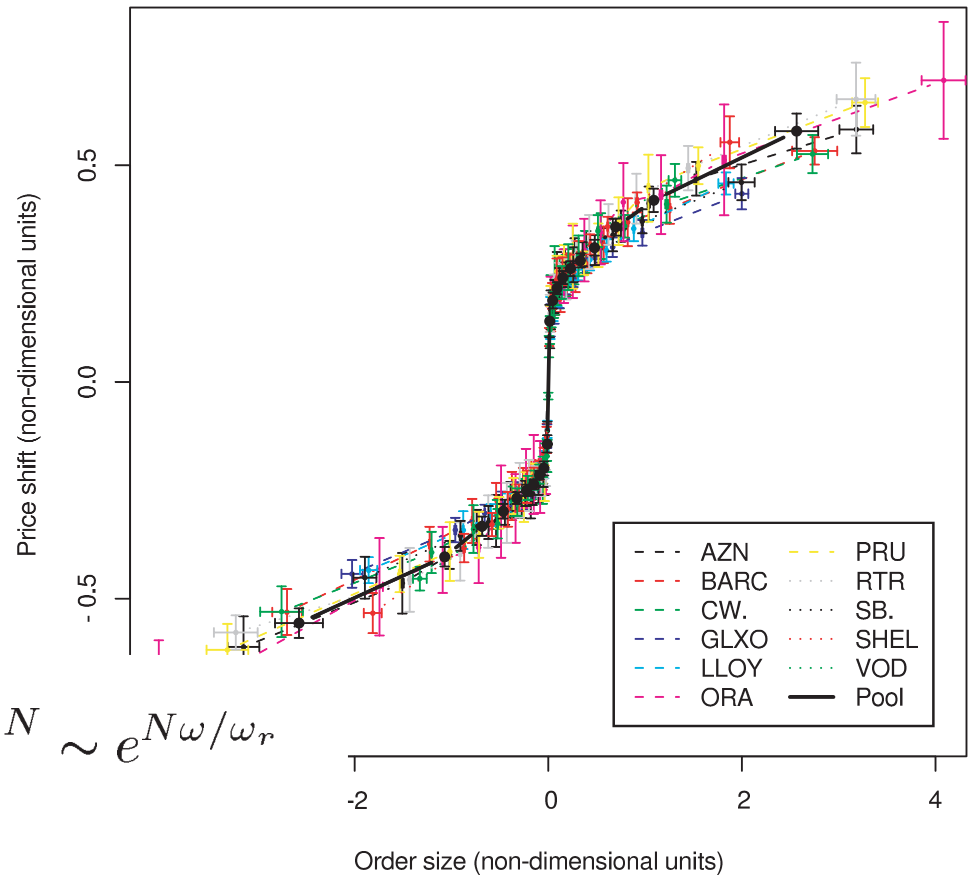
$$\text{return} = \frac{p_{\text{sell}}}{p_{\text{buy}}}$$

- No profit if impact > return

$$(\text{tolerated impact}) \leq \langle \text{return} \rangle$$

- Expect impact to scale exponentially, or else there is an incentive to split or join orders

$$(\text{tolerated impact})_{N\omega} \sim [(\text{tolerated impact})_{\omega}]^{N} \sim e^{N\omega/\omega_r}$$

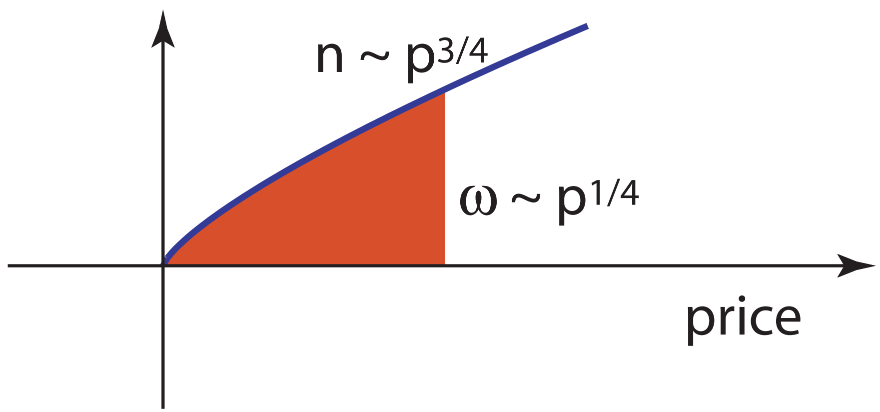


- **Yet it doesn't**

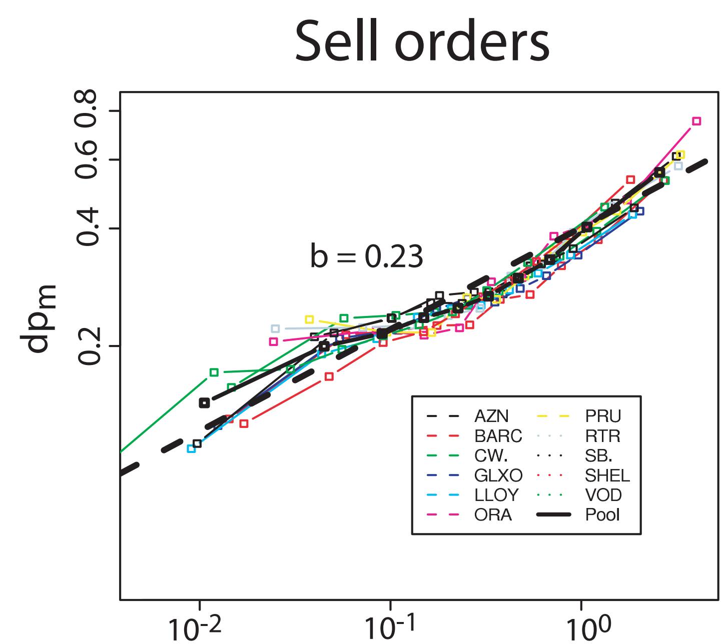
Farmer, J. D., P. Patelli, and I. I. Zovko. "The Predicitive Power of Zero Intelligence in Financial Markets" *PNAS USA* 102(11) (2005): 2254-2259

Observed regularities of the market impact

- Power laws with exponents 0.25 - 0.5 are common
- Corresponds to a power-law distribution of limit orders



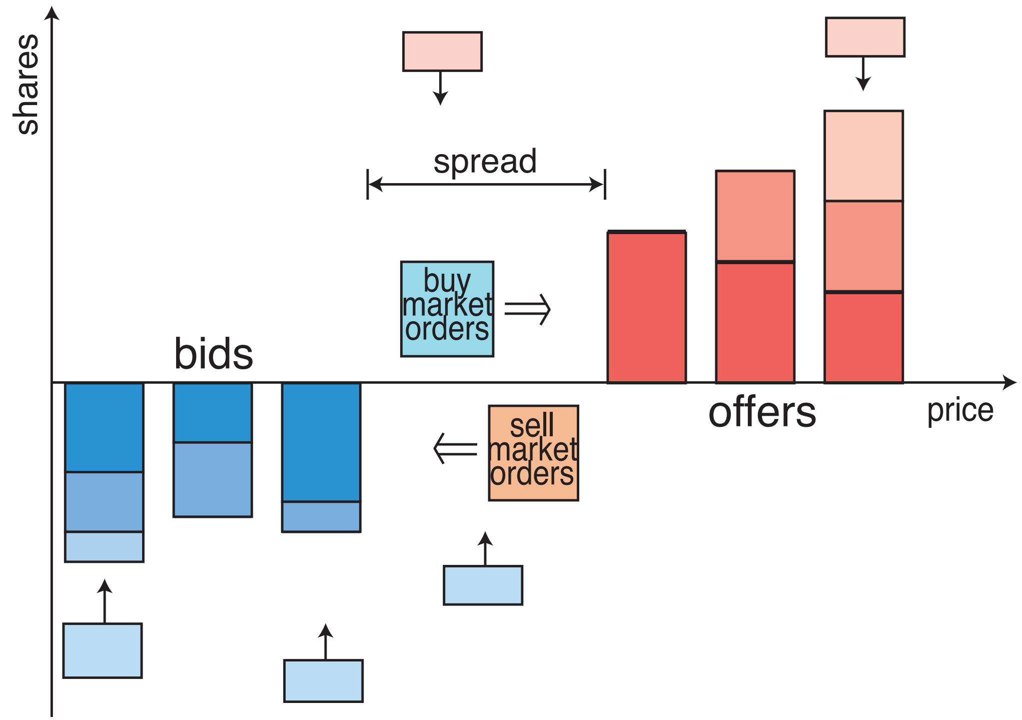
- Market impact is integral of the depth



Courtesy Paolo Patelli

Dimensional analysis of the continuous flows

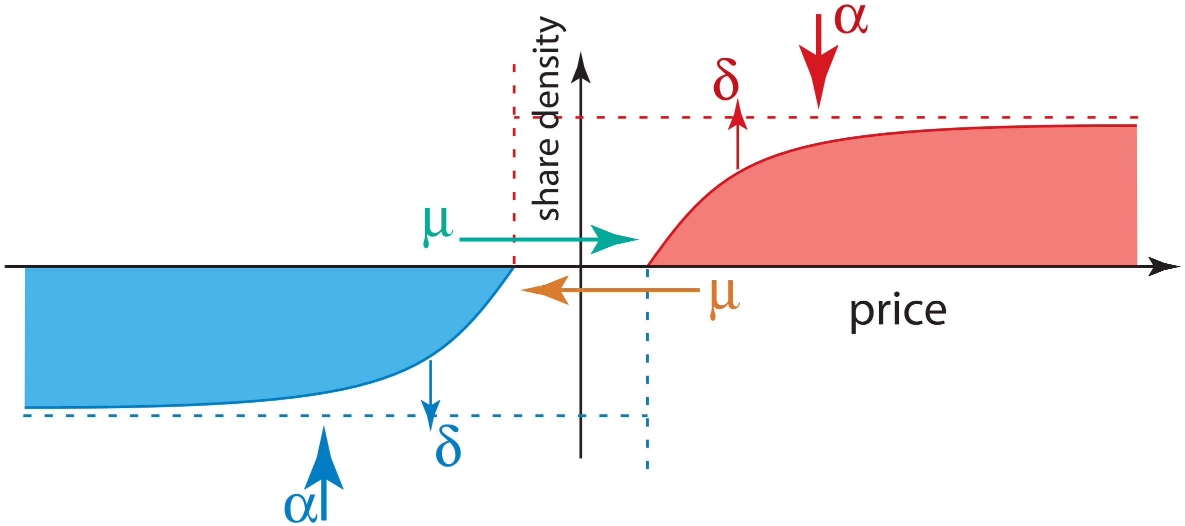
- Summarize the order-placement processes by a collection of continuous rates and rate densities



- Limit order placement α
- Limit order deletion δ
- Market order arrival μ

$$[\,\alpha\,] = \left[\frac{\text{shares}}{\text{price}\cdot\text{time}}\right] \quad [\,\mu\,] = \left[\frac{\text{shares}}{\text{time}}\right]$$

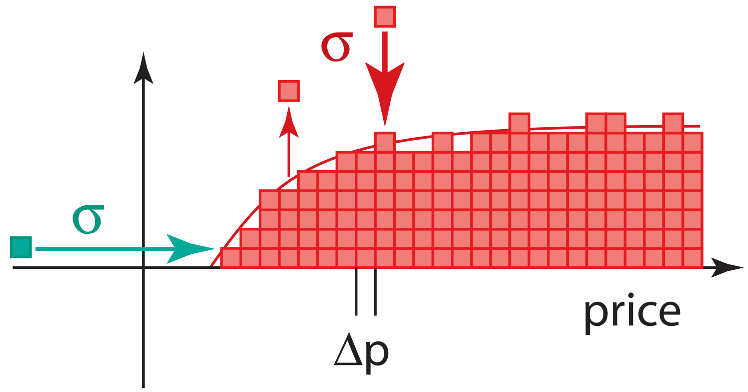
$$[\,\delta\,] = \left[\frac{1}{\text{time}}\right]$$



These are complete for shares, price, and time

Dimensional analysis of granularities

- Orders arrive and are removed in typical-sized chunks σ
- Prices are delimited in "ticks" Δp
- Both units are redundant with dimension provided by flow variables
- Redundancy creates the possibility of functions of non-dimensional variables



$$[\sigma] = [\text{shares}] \qquad [\Delta p] = [\text{price}]$$

Here I will take the flow variables to define the "classical" scaling dimensions, and treat the discreteness parameters as the source of nondimensional corrections

Classical scaling and dimensional analysis

- Flow variables define characteristic scales for fundamental properties

$$[\text{shares}]: \quad N_c \equiv \frac{\mu}{2\delta}$$

$$[\text{price}]: \quad p_c \equiv \frac{\mu}{2\alpha}$$

- Create "guesses" for observables based on their dimensions

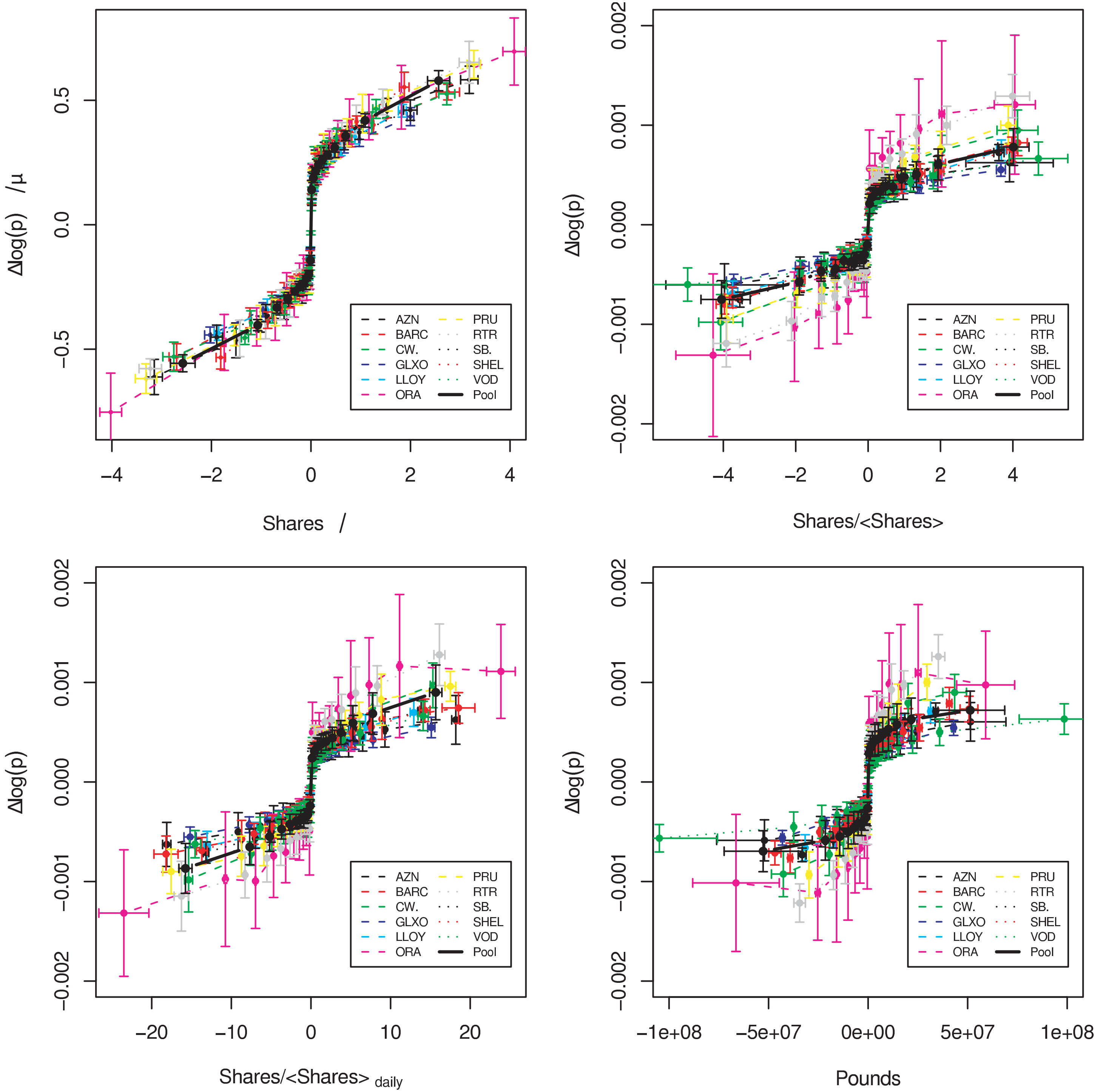
$$[\text{depth}] = \left[\frac{\text{shares}}{\text{price}}\right] \leftrightarrow \frac{N_c}{p_c} = \frac{\alpha}{\delta}$$

$$[\text{spread}] = [\text{price}] \leftrightarrow 2p_c = \frac{\mu}{\alpha}$$

$$[\text{liquidity (slope)}] = \left[\frac{\text{shares}}{\text{price}^2}\right] \leftrightarrow \frac{\alpha^2}{\mu\delta} = \left[\frac{\text{depth}}{\text{spread}}\right]$$

$$[\text{midprice diffusion}] = \left[\frac{\text{price}^2}{\text{time}}\right] \leftrightarrow \frac{\mu^2\delta}{\alpha^2}$$

Data collapse of impact based on classical scaling



Try collapsing the market impact four different ways

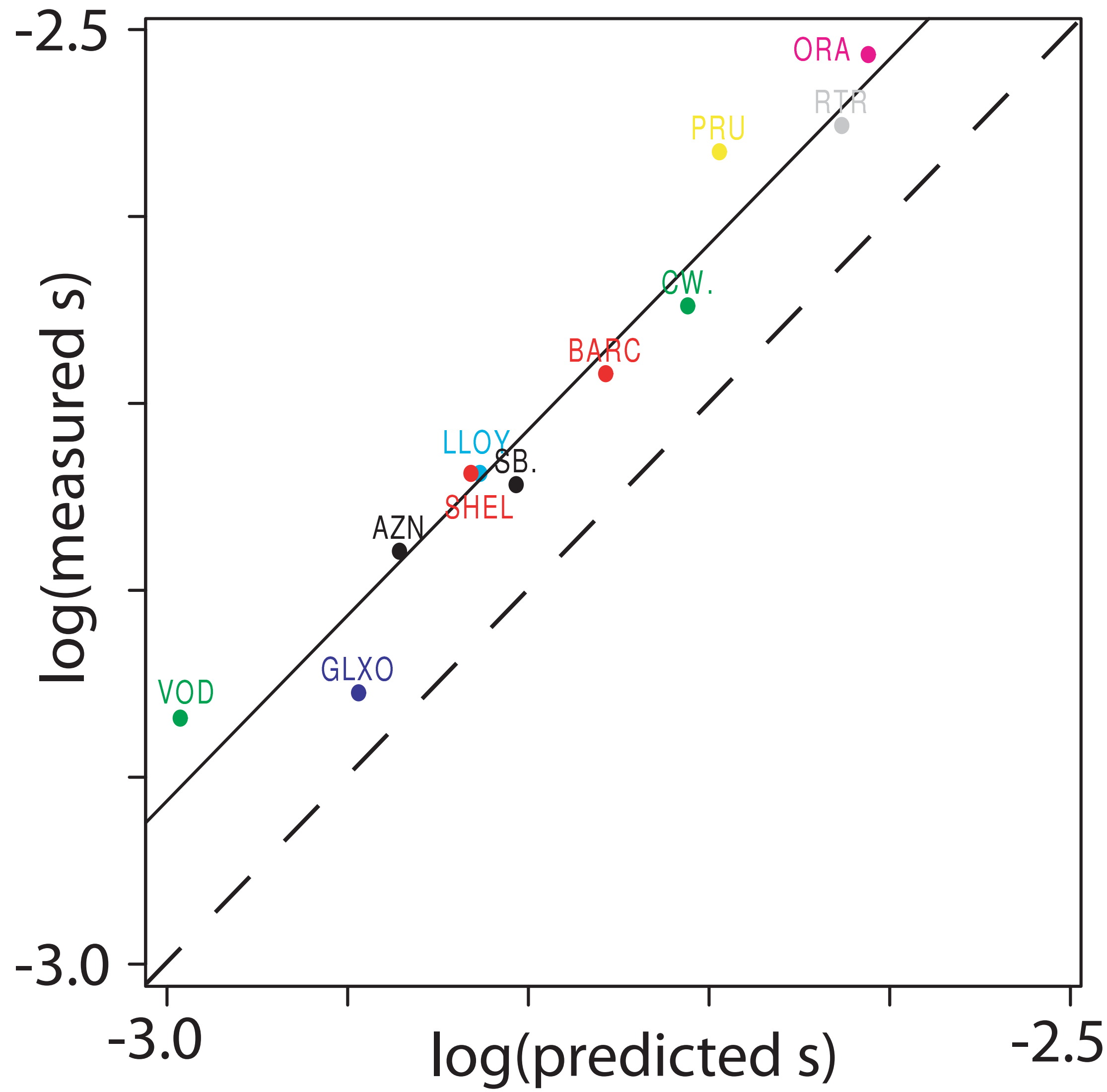
Farmer, J. D., P. Patelli, and I. I. Zovko. "The Predicitive Power of Zero Intelligence in Financial Markets" *PNAS USA* 102(11) (2005): 2254-2259

No-free-parameters predictions for spread

- Prediction for the spread from the flow variables is

$$[\text{spread}] = [\text{price}] \leftrightarrow 2p_c = \frac{\mu}{\alpha}$$

- Broad selection of stocks from the London Stock Exchange agree with this prediction to overall scale



Farmer, J. D., P. Patelli, and I. I. Zovko. "The Predicitive Power of Zero Intelligence in Financial Markets" *PNAS USA* 102(11) (2005): 2254-2259

Corrections from non-dimensional factors

- Recall scale factor and non-dimensionalization of prices
- Classically we would expect non-dimensionalized spread to take value

$$\hat{s} \rightarrow 1$$

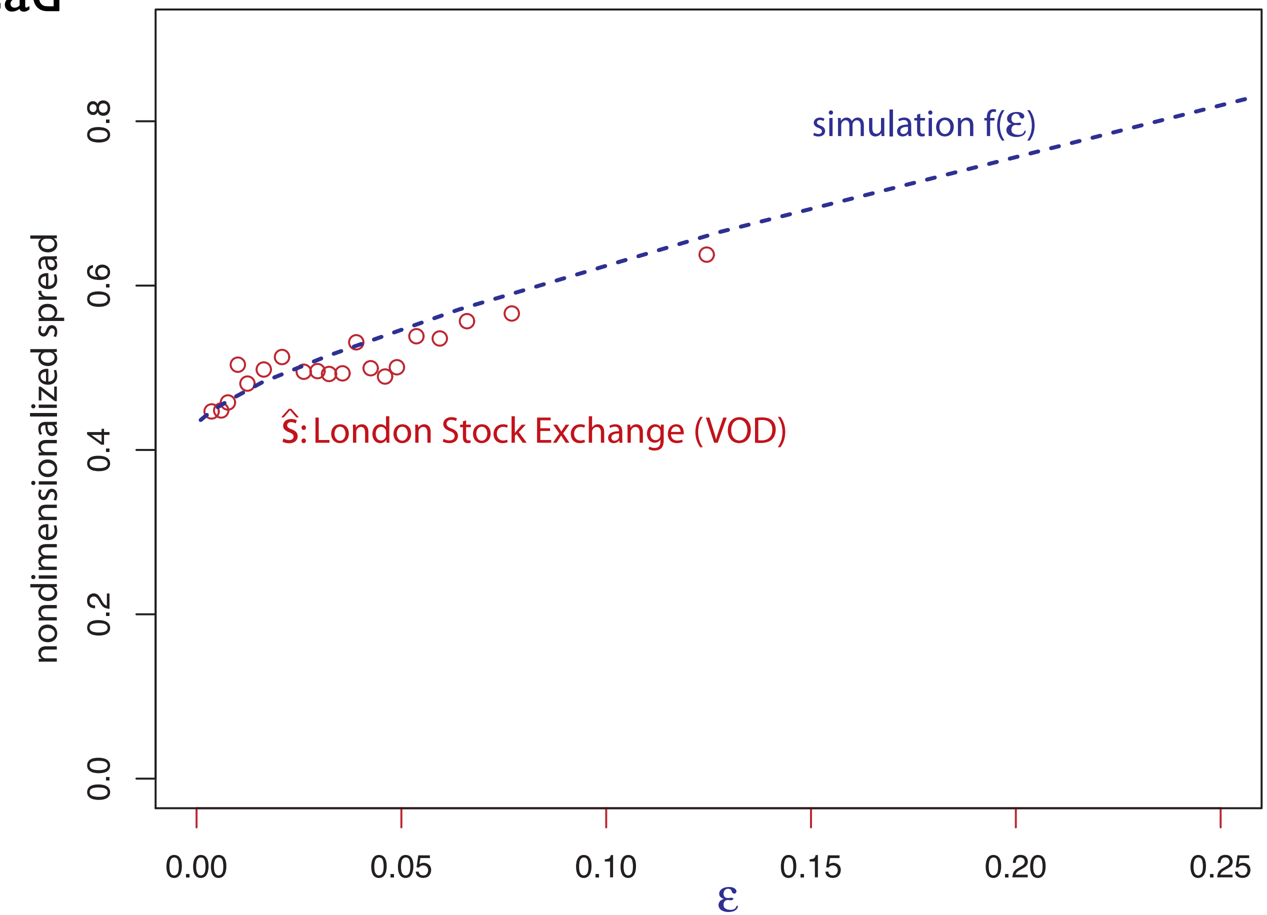
- From simulations get small correction with epsilon

$$\hat{s} \rightarrow f(\varepsilon)$$

- Comparisons to data are remarkably good

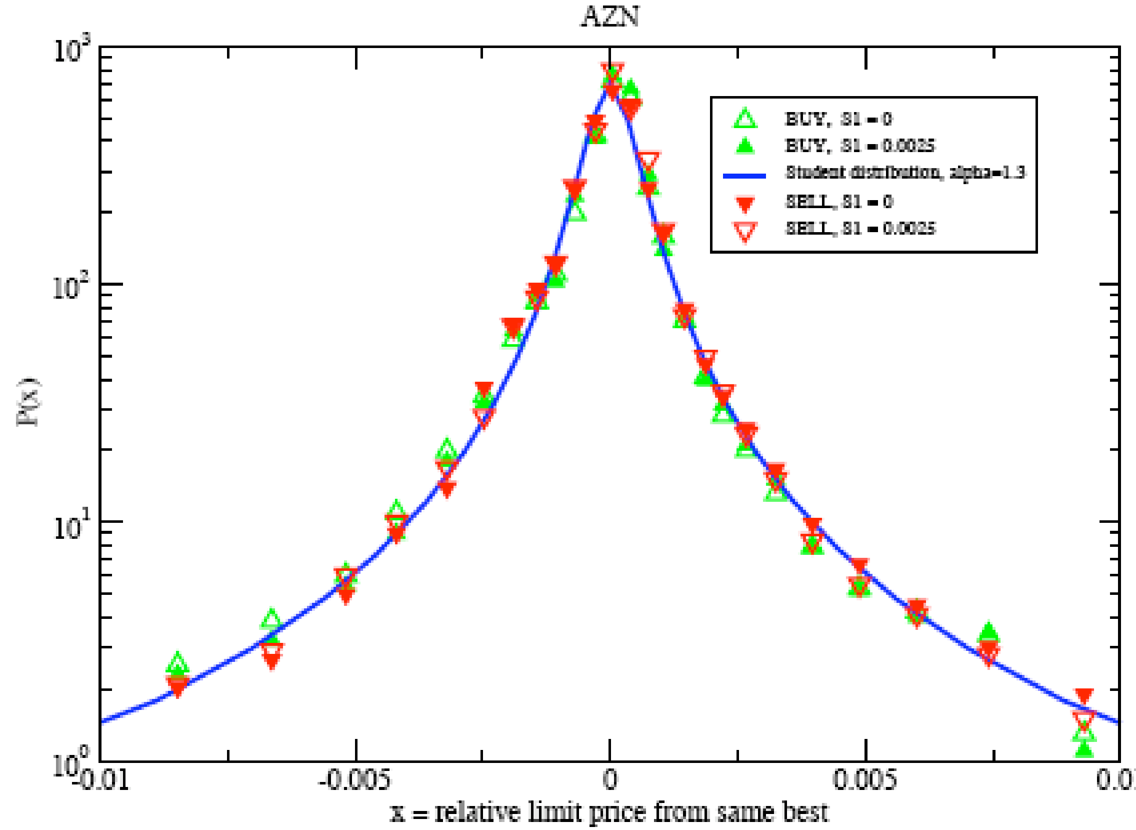
$$[\text{spread}] = [\text{price}] \leftrightarrow 2p_c = \frac{\mu}{\alpha}$$

$$\hat{s} \equiv \frac{s}{p_c} = s\frac{2\alpha}{\mu}$$



Learning from the limitations of ZI models

- Simplest ZI model leads to infinite range of limit prices and number of orders (clearly silly)
- Real order distributions are still very regular, but not by any institutional rule
- Candidate for a regularity of *behavior*



From Mike, S., and J. D. Farmer. "An Empirical Behavior Model of Price Formation." Santa Fe Institute Working Paper 05-10-039

Concluding thoughts

- Neoclassical economics is one of the closer things to a Theory in the social sciences; its major flaw is dissociation from reality
- One can build models, empirically and tentatively, without losing sight of theoretical integrability
- The institution/behavior interface is a fertile one for physical approaches
- Dimensional and scaling analysis and ZI have proved useful for some institutional process analysis

Some further reading

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