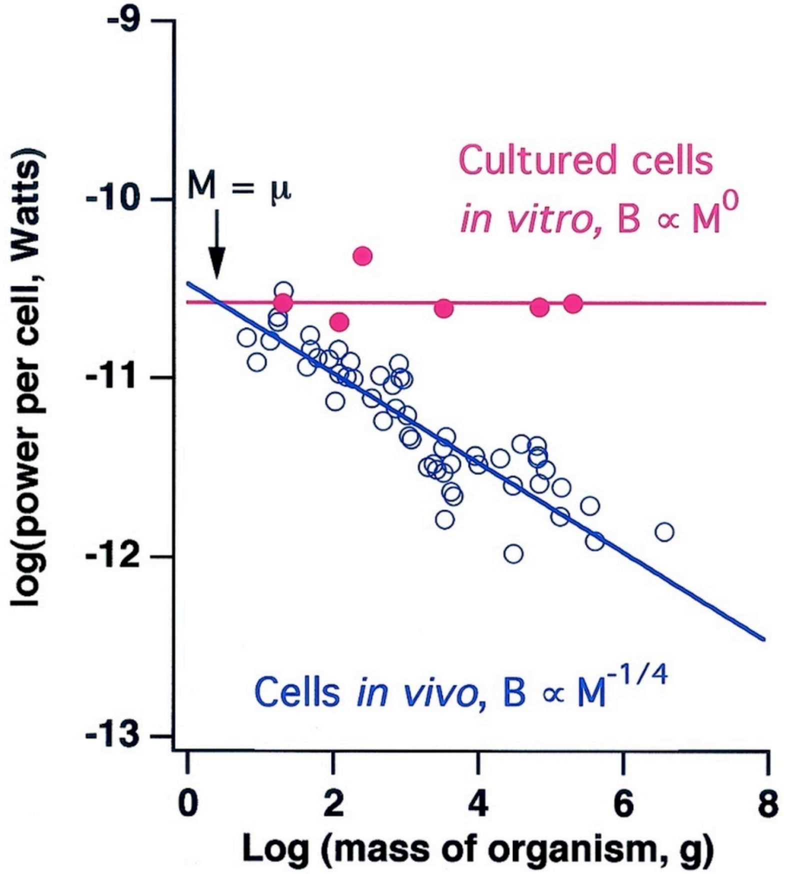
Network effects on game dynamics

Ana, Carlos, Ezequiel, Lucas, and Matt

Motivation



Metabolic power of single mammalian cells as a function of body mass on a logarithmic scale

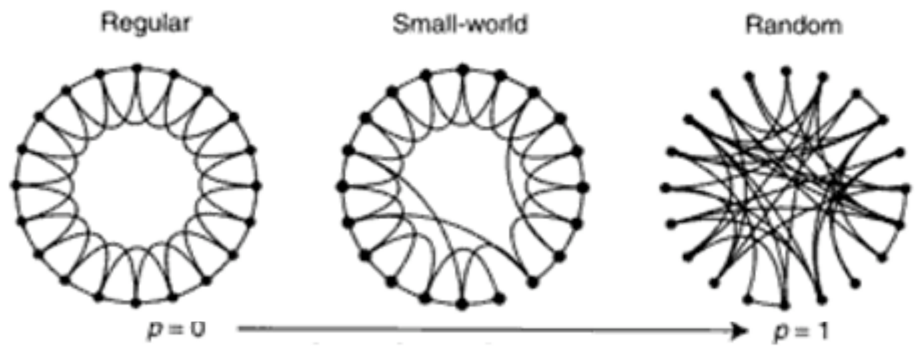
To evaluate the effect of networks on the scaling relationship between an urban index vs. city size

Can we observe scaling behavior with simple dynamics on a network?

West et al. 2002

Model

✓ Modelling a social network with a small-world network:



Watts et al. 1998

✓ Run simple dynamics (like coin games) on the network

<u>The coin game</u>:

✓ Initialize the nodes to start with equal amounts of money (*m*)

✓ Select a node *A* at random and a neighbor of *A* to play the game

✓ The probability to win for each player is p_A and 1-p_A

✓ The winner is determined by a random rule and gets money from the loser

✓ If one player loses and has no money he doesn't pay

✓ Global measures of money distribution:

✓ *Standard Deviation over nodes*

✓ *Shannon Information Entropy*

$$\sigma = \sqrt{\langle m^2 \rangle - \langle m \rangle^2}$$

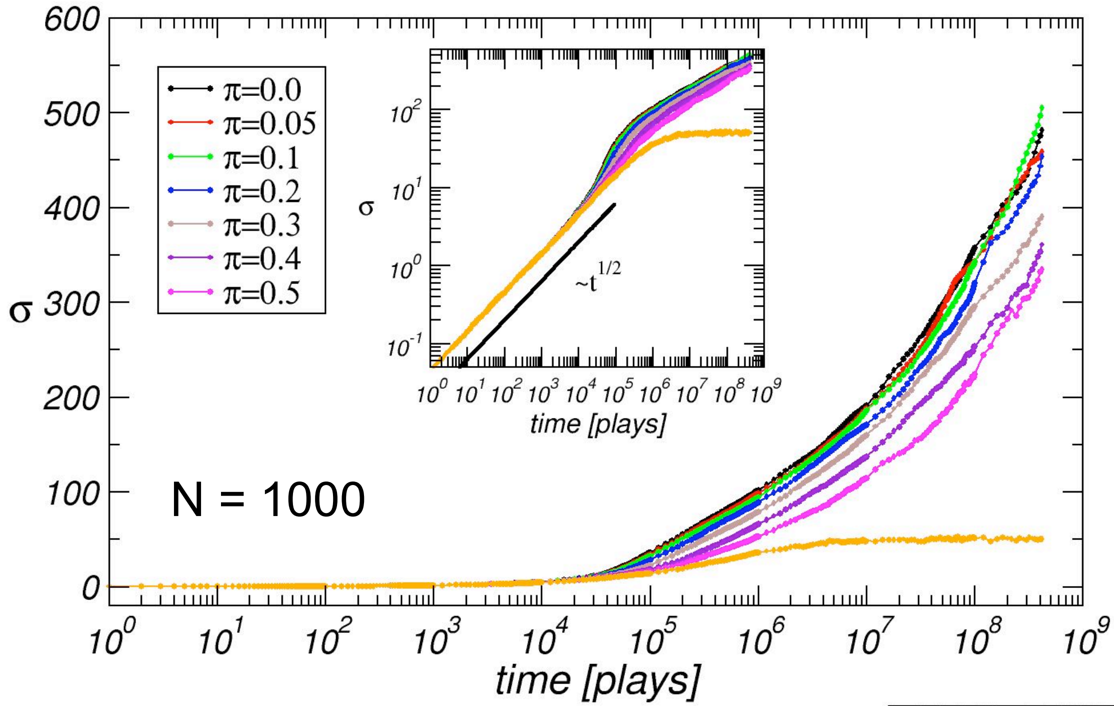
$$S = -\sum_{i=1}^{N} \frac{m_i}{m_{tot}} \log_2\left(\frac{m_i}{m_{tot}}\right)$$

Results I

Games	**Fully connected network**	**Small world network**
Fair coin $p_A = p_B = 0.5$	Random walk for each node $\sigma \sim \sqrt{t}$	Random walk with $\neq D_i$ $\langle\sigma\rangle_i \sim \sqrt{t}$
Rich get richer $p_A = \pi + \left(\frac{m_A}{m_A + m_B}\right)(1 - 2\pi)$	One node wins all the money	Small kingdoms emerge (one rich surrounded by poor neighbors). This state is stable for π=0 and has finite lifetime for π>0.

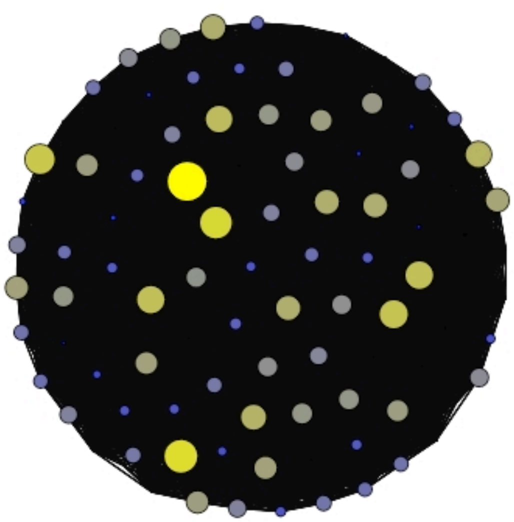
Rich get richer

Results II

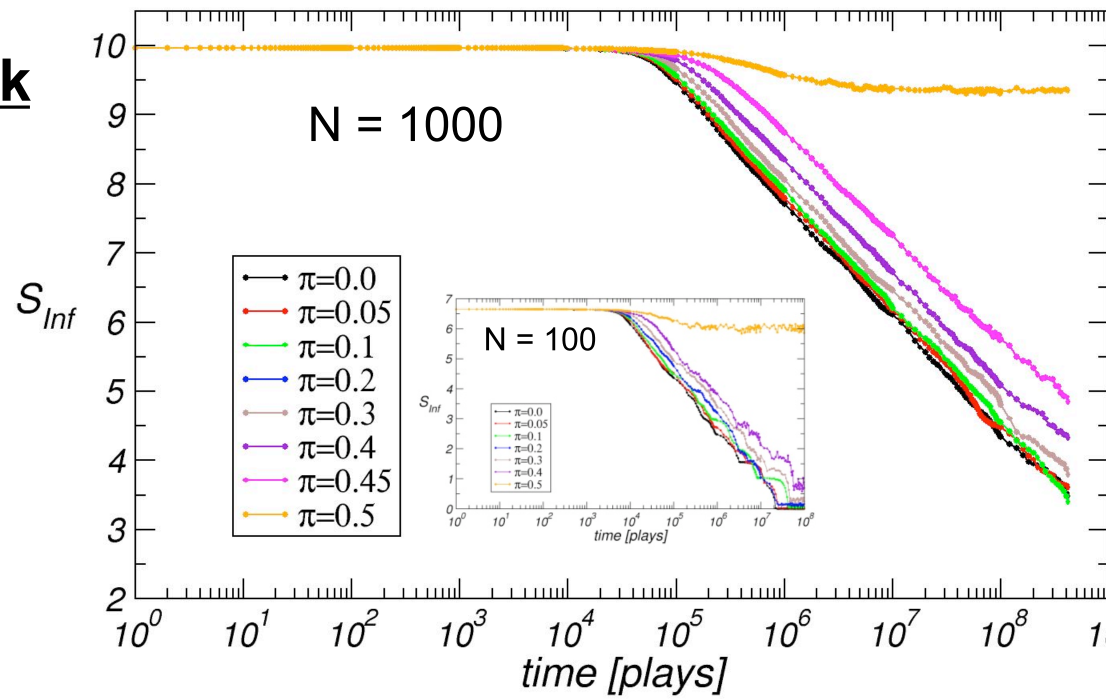


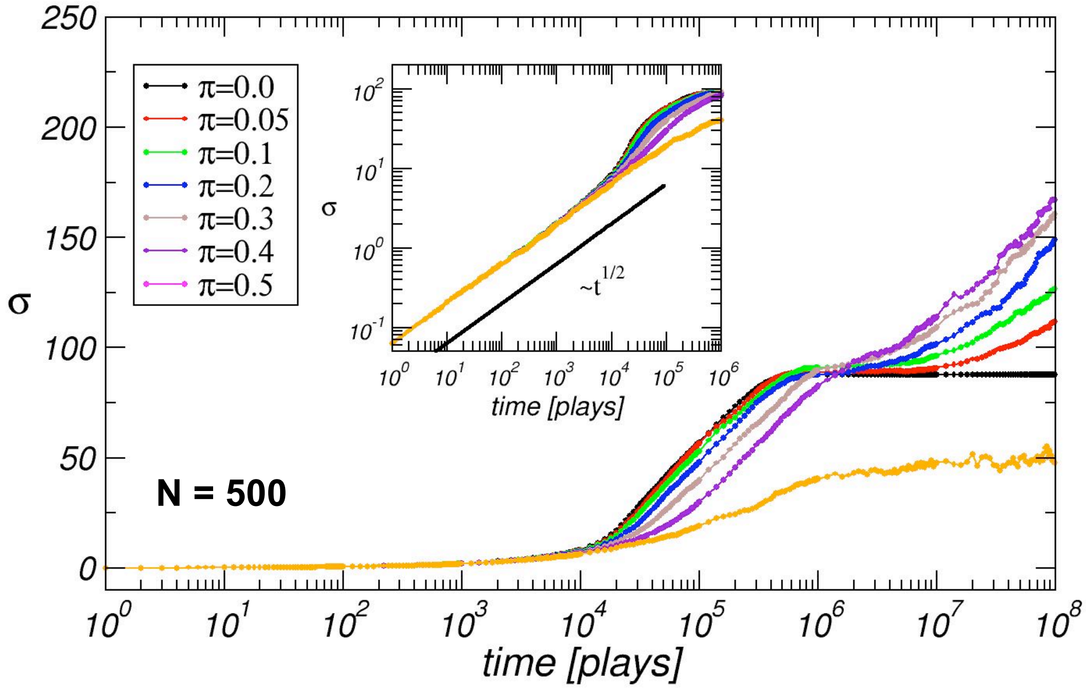
1. Randomly some players start to get richer and dominate the dynamics
2. One node has all of the money at the end of the game

Fully connected network



Nodes: N = 100



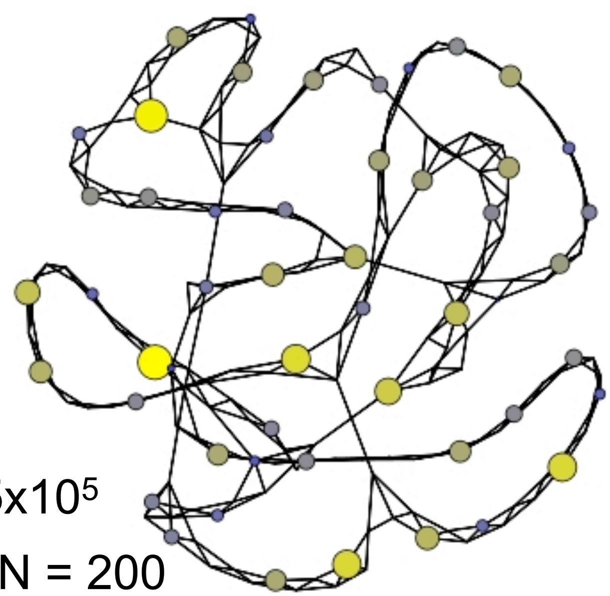


Rich get richer

Results III

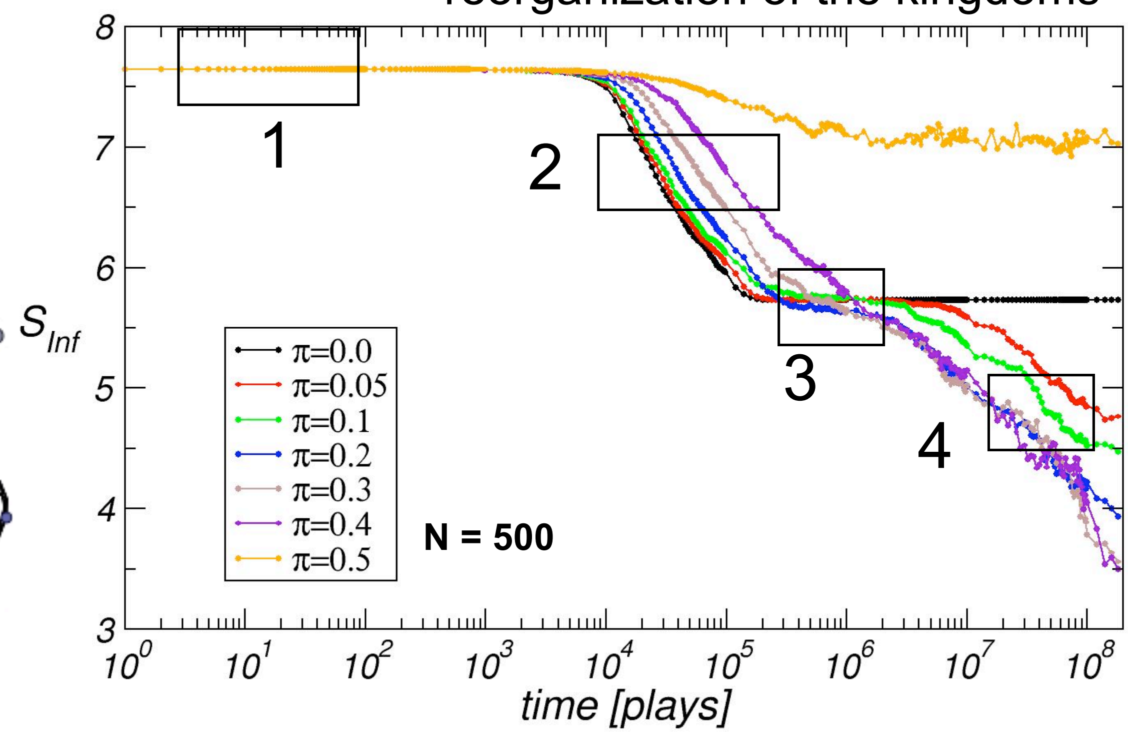
1. Nodes are approximately equal
2. Some nodes start to dominate
3. Rich nodes become isolated so the dynamics slow
4. Flux of money is re-established due to fluctuation and reorganization of the kingdoms

Small world network



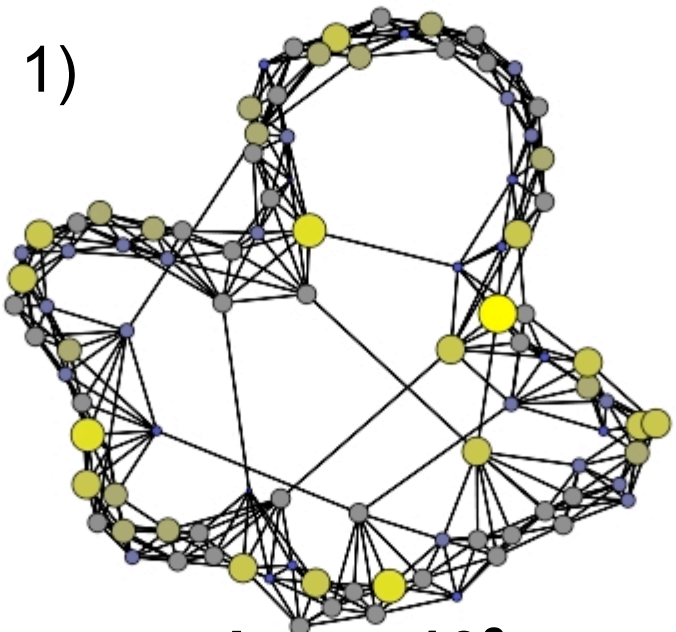
time = 5x10^5

Nodes: N = 200



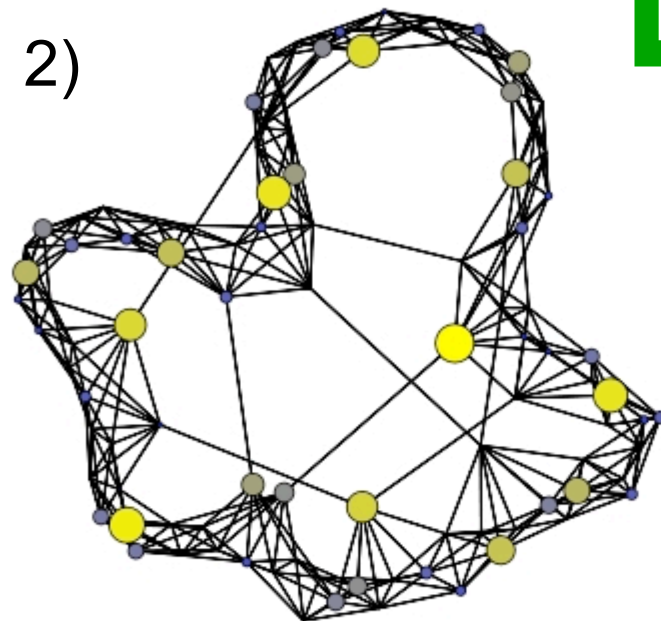
Results IV

1)



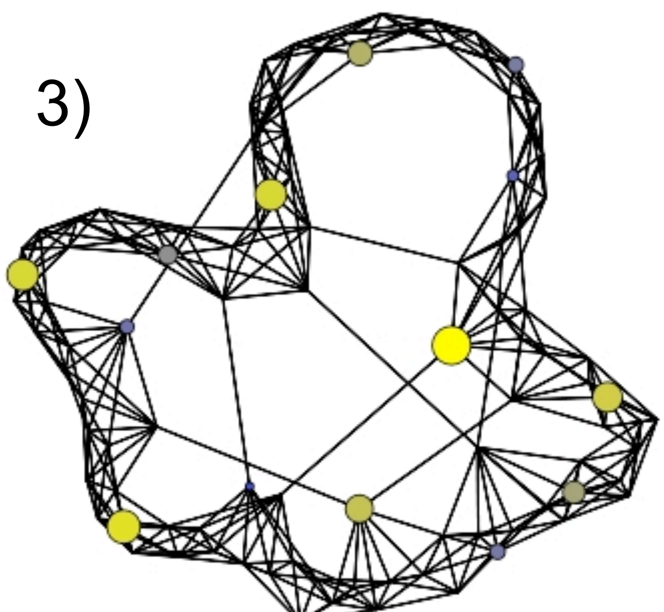
time = 10^2

2)



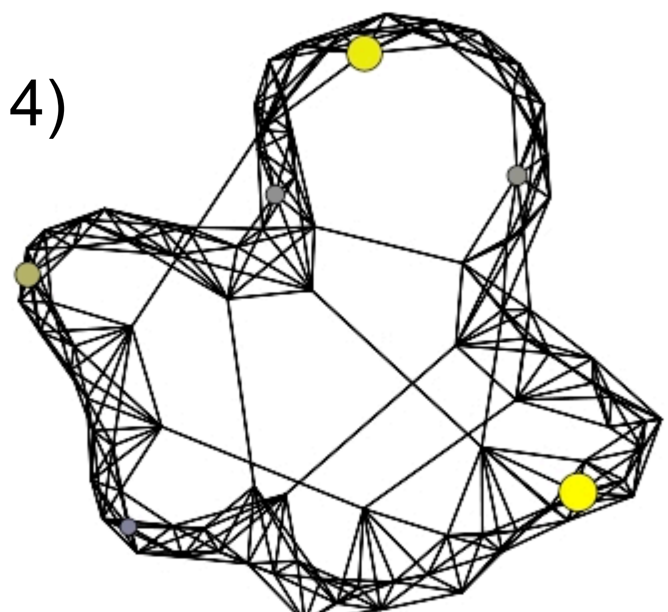
time = $3x10^4$

3)



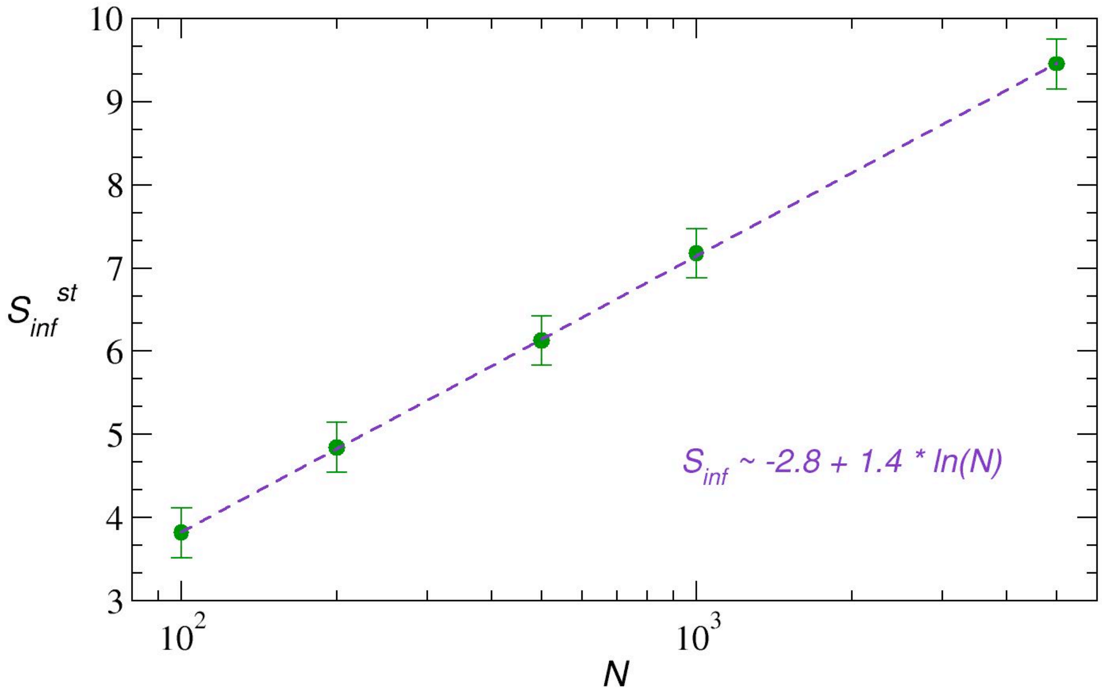
time = 10^6

4)



time = $9x10^7$

Results V



If we suppose that in the final state the N_R rich have approx. the same money then:

$$S \sim log_2(N_R)$$

If we assume a non-trivial scaling of N_R with N:

$$N_R = aN^b$$

Then:

$$S \approx \ln(a) + b \ln(N)$$

Discussion:

✓ A steady state due to network topology slows the dynamics at intermediate times
✓ Information entropy depended non-trivially on network size