Relativity and invariance in information dynamics

John Mahoney[1]
Joseph Lizier[2,3]

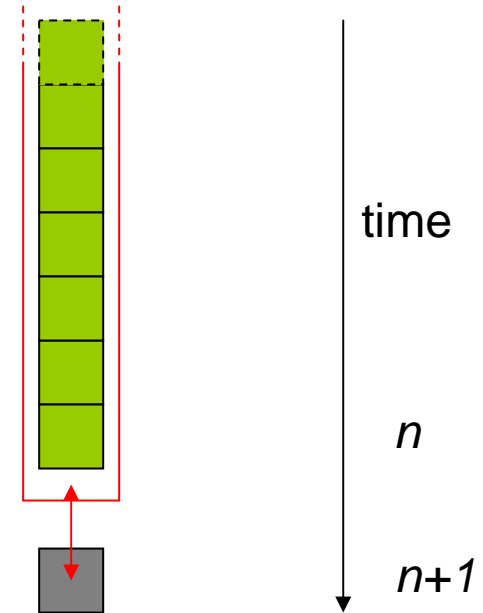[1] The University of California, Davis
[2] The University of Sydney
[3] CSIRO ICT Centre

Framework for Info dynamics

- Computation (information dynamics) has 3 components; defined on a local scale as:
    - Info storage (total = excess entropy), active:

$$a(i, n+1) = \lim_{k \to \infty} \log \frac{p(x_{i,n}^{(k)}, x_{i,n+1})}{p(x_{i,n}^{(k)})p(x_{i,n+1})}$$
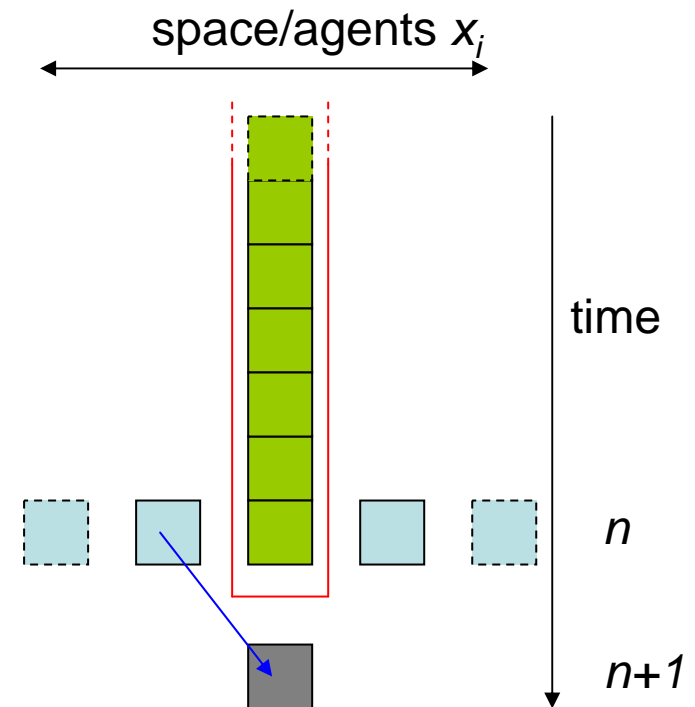
time

n

$n+1$

Framework for Info dynamics

- Computation (information dynamics) has 3 components; defined on a local scale as:

  - Info storage

  $$a(i, n+1) = \lim_{k \to \infty} \log \frac{p(x_{i,n}^{(k)}, x_{i,n+1})}{p(x_{i,n}^{(k)})p(x_{i,n+1})}$$

  - Info transfer (transfer entropy)

  $$t(i, j, n+1) = \lim_{k \to \infty} \log \frac{p(x_{i,n+1}|x_{i,n}^{(k)}, x_{i-j,n})}{p(x_{i,n+1}|x_{i,n}^{(k)})}$$

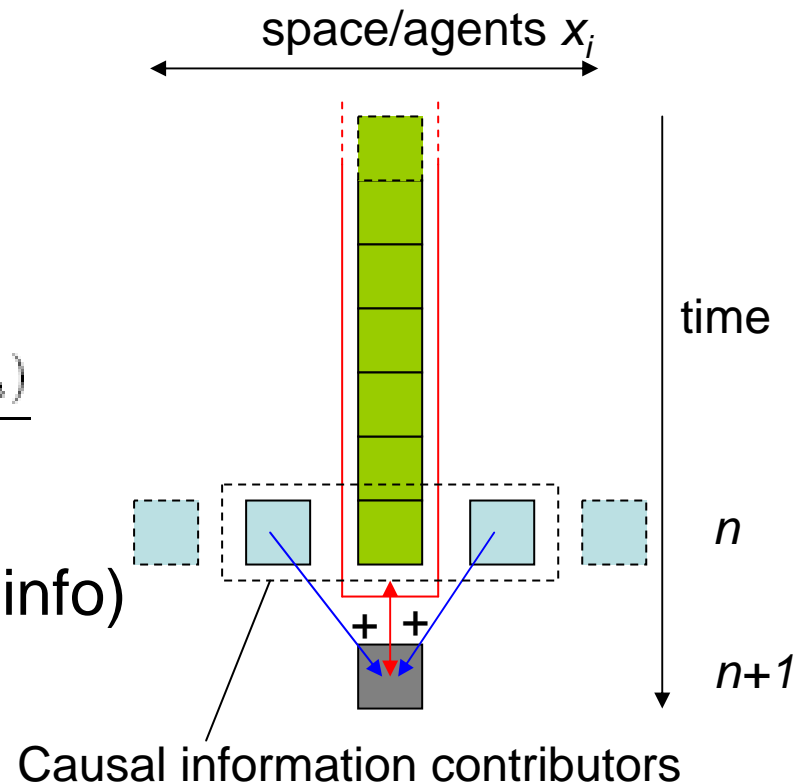space/agents x_i

time

n

$n+1$

Framework for Info dynamics

- Computation (information dynamics) has 3 components; defined on a local scale as:
  - Info storage

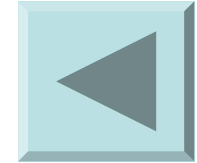$$a(i, n+1) = \lim_{k \to \infty} \log \frac{p(x_{i,n}^{(k)}, x_{i,n+1})}{p(x_{i,n}^{(k)}) p(x_{i,n+1})}$$

  - Info transfer

$$t(i, j, n+1) = \lim_{k \to \infty} \log \frac{p(x_{i,n+1} | x_{i,n}^{(k)}, x_{i-j,n})}{p(x_{i,n+1} | x_{i,n}^{(k)})}$$
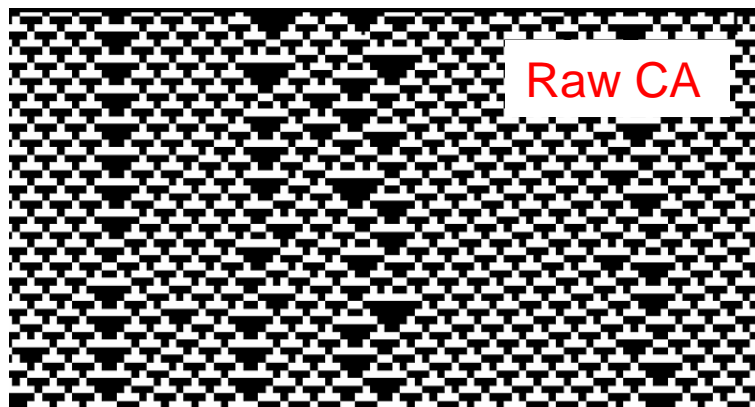
  - Info modification (separable info)

space/agents x_i

time

n

$+$ $+$

$n+1$
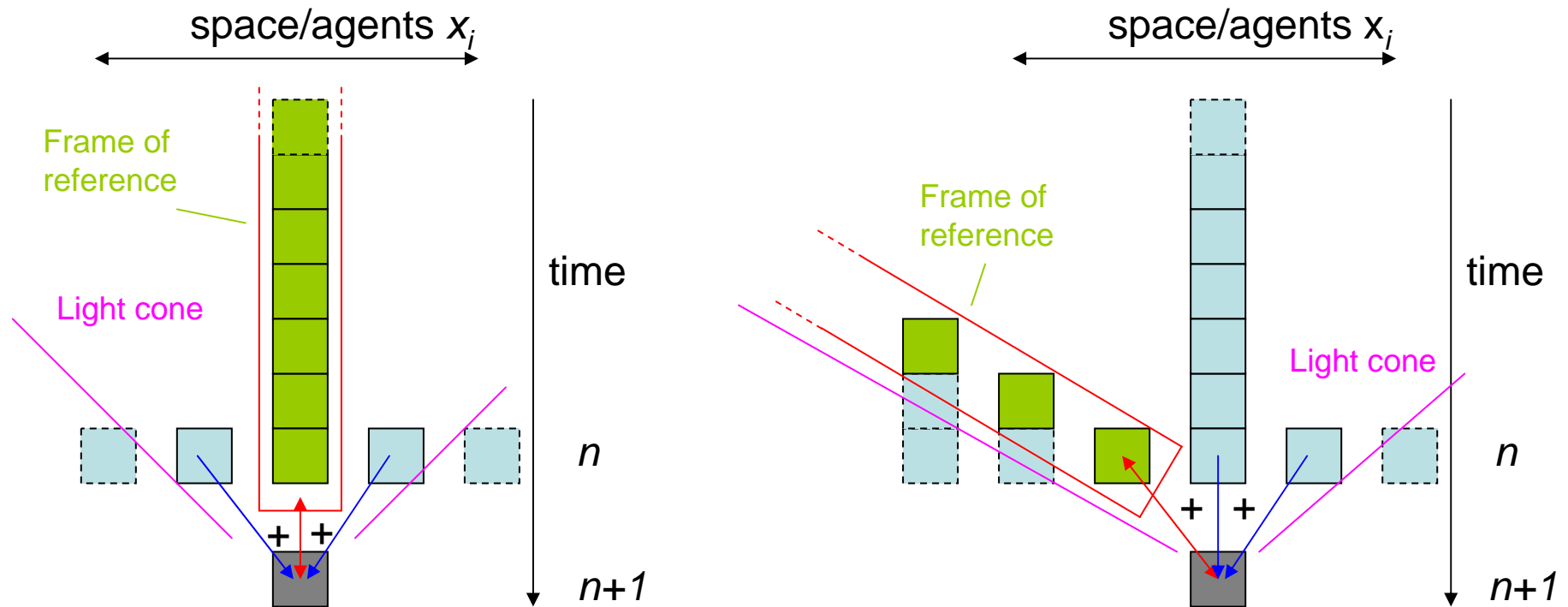
Causal information contributors

Local info metrics provide insights into system dynamics

- e.g. in cellular automata gives evidence:
  - gliders are information transfer agents
  - gliders collisions are information modification events

Raw CA

-ve active info

+ve info tx left

-ve separable info

Can we shift frame of reference?

- Laws of physics hold under (relativistic) shift of frame of reference
- What can we say about information dynamics under shift of information frame of reference?
- Ether relativity = Bounded velocity (Einsteinian) + Preferred frame (classical)

Local information dynamics in CAs under shift of frame of reference

Raw CA

-ve active info

+ve info tx down

-ve separable info
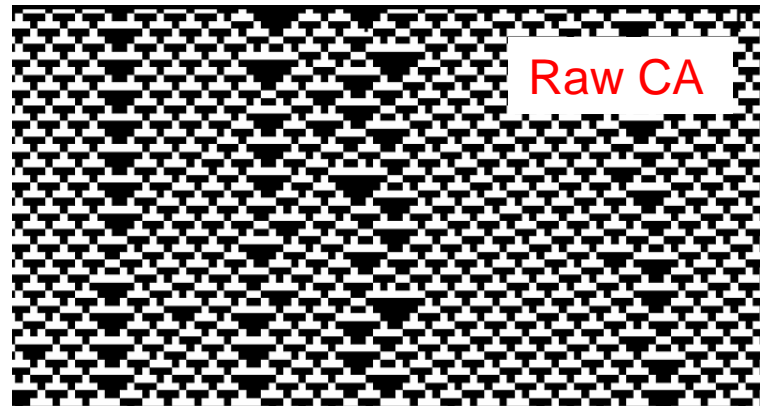
- Frame of reference above travelling one cell to left per unit time.
- Same features are highlighted as different types of information dynamics
  - Gliders moving at speed of frame of reference become information storage
  - Vertical gliders (blinkers) become information transfer
- But magnitudes of information dynamic metrics are different …

Are any quantities invariant under shift of frame of reference?

- Single site entropy $H(x_i)$ is invariant (not defined w.r.t. frame of reference)

- Can compose $H(x_i)$ as a sum of incrementally conditioned mutual information terms:

$$H(X) = \langle a(i, n+1) \rangle + \lim_{k \to \infty} \left(\sum_{j=-r}^{+r} I(X_{i-r,n}; X_{i,n} | d_{(-r,j-1)}(i,n), X^{(k)}) \right)$$

- This sum is independent of frame of reference

- Set of causal information contributors (incoming light-cone) to site remains the same: measures of information dynamics outside this report correlations only.

Future investigations

- Formalise frame of reference change
- Understand transformation laws for information metrics
- Use of frame of reference as a classification tool

References

- Lizier, Prokopenko, Zomaya "Identifying non-trivial computation in complex dynamics", ECAL 2007.