The Individual and the Empire:
The Effects of Agent-Based Emigration Behavior on
the Emergence of Settlement Size Inequality in the Titicaca Basin, Bolivia and Peru

14 September 2009

Wm. Randy Haas, Jr.¹
Jacopo Tagliabue²
Jeremy Barokfsky³

Abstract

Approximately 1000 years ago, from an evenly distributed population in the Andean highlands, a state-level society emerged with settlements ranging in size from hamlets to a large urban center of some 50,000 individuals. This working paper explores a mechanism that can produce inequalities in settlement-size distributions over time and space in the Titicaca Basin. Using agent-based modeling, we show that the cumulative effects of simple, individual-based migration optimization behavior can create a settlement rank-size distribution with the same formal qualities as that observed during the Tiwanaku Empire's apex. The model's general applicability is explored, improvements are suggested, and future directions proposed.

Introduction

Differential population agglomeration is a geographically and temporally pervasive phenomenon amongst agrarian and post-agrarian societies around the globe (Diamond 1997; Johnson 1980; Krugman 1991). Our understanding of important social phenomena such as inequality and collapse hinges in part on our understanding of the contexts in which differentially nucleated settlement patterns develop, persist, and fall. In this working paper, we begin by briefly reviewing a few of the many works that have searched for structural commonalities, variations, and processes that underly the rise of population agglomeration. Log-normal, or Zipf, settlement size distributions and their variants are given special attention. We then turn our attention to the specific case of the Tiwanaku Empire, a state-level society that rose and fell in the Titicaca Basin, Bolivia and Peru between approximately 500 B.C. and A.D. 1100. Unlike the well-known Zipf distribution that characterizes the rank-size settlement patterns of many societies including that of modern cities in a global economy, the settlement rank-size distributions of the Tiwanaku Valley conforms to convex and primo-convex distributions (Johnson 1980; McAndrews et al. 1997; Figure 1).

This paper presents a simple agent-based model that demonstrates how a simple migration mechanism can create the emergence of Tiwanaku Valley's settlement rank-size distributions. In this model, agents simply migrate to other villages in such a way as to minimizes the travel distance while maximizing the destination population.

---

1 School of Anthropology, The University of Arizona, P.O. Box 210030, Tucson, Arizona 85721-0030 U.S.A (wrhaas@email.arizona.edu)
2 Department of Philosophy and CRESA, San Raffaele University, Milan (tagliabue.jacopo@gmail.com)
3 School of Public Health, Harvard University, Cambridge (jeremybarofsky@yahoo.com)
After just a few time steps, the homogeneous distribution of village sizes take a rank-size distribution form that is qualitatively very similar to that of the Tiwanaku Period distribution. The model's predictive power then leads us to ask whether or not “imperfect” emigration rationality could generate the characteristic convex distribution and how well the agent-based emigration model can predict spatial patterning given a geographically realistic model space. Although the latter two attempts were less successful in their fit with empirical data, this paper demonstrates several points. First, primo-convex settlement patterns can emerge from the cumulative effects of individuals making optimum decisions based on a currency of population size and travel cost. Second, our method shows that a step-wise agent-based approach to the modeling of prehistoric settlement dynamics can provide nuanced insights into the emergence of macro-scale patterns that characterize large civilizations. We conclude with a discussion of alternative rule sets and processes that we are exploring as possible behaviors that can generate settlement pattern variation that we see in the historical and archaeological records. Because this paper represents a work-in-progress, we also disclose our ongoing efforts and intentions to improve the current model.

**Rank Size Variation and Population Agglomeration**

In his 1949 work, *Human Ecology and the Principal of Least Effort*, George Kingsley Zipf observed that human settlement patterns in industrial societies tend toward a log-normal, or power law, rank-size distribution. Zipf argues that log-normal distributions, or what he terms rectilinear distributions, arise from human social behaviors that seek least-effort equilibria in their economic goals. In this model, individuals put in a fair share of labor in anticipation of an equal share of \( m \) different kinds of goods. Deviations from log-normality represent inequalities in the relative amount of work and returns, which cause economic, social, and political instability in the system's effort to return to equilibrium. He also argued that downward turns in the lower end (higher rank, lower population) of the graph demarcates rural populations.

Gregory Johnson (1980) later noted that rank-size distributions in societies frequently demonstrate more systematic deviations from the log-normal curves that Zipf described. Johnson called upon six archaeological and historical demographic case studies ranging from 2800 B.C. to A.D. 1850 and from Mexico to China in order to describe and explain two major forms of log-normal deviation—convexity and concavity. In convex rank-size distributions, the largest and smallest settlements are smaller than a log-normal trend would predict, or conversely, the mid-sized settlements are larger than a log-normal trend would predict (McAndrews et al. 1997:70-71). Such deviation can be contrasted with concave distributions for which the opposite is true.

Johnson's review of the socio-political contexts of his case studies reveals that each shares a relative lack of integration, which he defines as the statistical interdependence of population sizes. In two of the case studies—the Susiana Plain and the Colonial United States—the luxury of temporal control and economic data permitted the identification of a strong, positive

*Figure 1: Convex and primo-convex settlement rank-size distributions for the respective Formative and Tiwanaku period settlements of the Tiwanaku Valley. The straight, log-normal distributions are presented for comparison. Data from Albarracin-Jordan and Mathews (1992:Appendix I).*
relationship between rank-size convexity and socio-political integration over time. However, Johnson (1980:239-240) cautions that low integration only forms a constraint on the emergence of log-normal rank-size patterns, and that integrated systems could still exhibit convex rank-size distributions.

On the other hand, primate, or concave, distributions are posited to be a function of vertical integration as in a dendritic system or when a primate center is articulated with a larger, extra-regional system (Johnson 1980:242). In a diachronic analysis of Great Britain and Warka in Southern Iraq, Johnson (1980:242-244) shows a strong correlation between decreasing primacy and horizontal integration. Finally, log-normal distributions are associated with systems that are integrated both horizontally and vertically. Statistical properties of the dataset may also play a role in the identification of rank-size patterns. The pooling of two or more integrated socio-political systems or a sample that includes the edge of a dendritic system would tend to produce a rank-size signature of a non-integrated system (Johnson 1980:239-243).

Understanding how rank-size distribution patterns emerge depends partially on revealing how populations are distributed in geographic space. The differential spatial and temporal dispersion of resources encourages the relative dispersion of populations, and during the early stages of colonization, population growth, and regional packing, human settlement sizes were commensurate relative to the non-linear distributions cited here. Walter Christaller (1966) effectively formalizes this concept when he argues that centralized settlements arise in the context of geographically dispersed goods. More recently, Paul Krugman and colleagues (Krugman 1991; Krugman and Fujita 1994) demonstrate that the differential agglomeration of human population in geographic space is attributable to a combination of centripetal and centrifugal forces, which differentially drive individuals to agglomerate and disperse.

**Settlement Patterns in the Titicaca Basin Tiwanaku Period**

Researchers in the Andean Highlands have explored the distribution and rise of Tiwanaku Period settlements (Albarracin-Jordan 1996; Griffin and Stanish 2007; McAndrews et al. 1997). From a relatively uniform size distribution of Early Formative Period (ca. 2000-1300 B.C.) hamlets less than 1 hectare in size arose the increasingly nucleated regional centers of the Middle Formative Period (ca. 1300-500 B.C.). Stanish (2003) identifies at least 13 such centers in a non-systematic sample. During the Upper Formative Period (ca. 500 B.C. - A.D. 400), two regional centers—Pucara and Tiwanaku/Chiripa—grew an order of magnitude larger than any of the other centers. Finally, by A.D. 400, the urban capital of Tiwanaku emerged as the largest population center occupying approximately 420 hectares and harboring a core population between 30,000 and 60,000 inhabitants. By A.D. 1100, the Tiwanaku Empire, which had become a conquest state, began to collapse.

McAndrews and colleagues (1997) show that the Formative Period settlements of the Tiwanaku Valley fit a fairly typical convex rank-size distribution, but the Tiwanaku Period distribution follows what they term a primo-convex distribution in which the lower portion of the distribution is relatively convex but the upper end exhibits concavity. In the Tiwanaku case, concavity is primarily a function of a steep upturn between the rank 1 site—Tiwanaku—and the rank 2 site. The convexity suggests that functionally discrete settlement clusters were integrated via Tiwanaku institutions but not directly with each other (McAndrews et al. 1997). McAndrews et al. (1997) attribute the convex portion of the distribution to the fact that the site sample consists of multiple settlement clusters, each of which would have been highly integrated internally relative to the level of inter-cluster integration.

Spatial analyses in the Tiwanaku Valley support the inference of independent site clusters. Nearest neighbor and K-means cluster analyses have identified a nested hierarchy pattern in the geographic distribution of site sizes (McAndrews et al. 1997; Figure 2). In general, five site types are recognized—Tiwanaku the primary site at 420ha, secondary sites that exceed 3 ha and have one or more mounds and large architectural components, tertiary sites between 1 and 3 ha with or without mounds and lacking stone architecture, and quaternary sites less than 1 ha consisting of small artifact scatters or a single house mound (Albarracin-Jordan 1996; McAndrews et al 1997). The overall Tiwanaku settlement pattern does not appear to bear

*Working Paper—Do not copy or cite without permission of authors.*
out Christaller's hexagonal lattice model, but elaborate stone architecture at the secondary centers suggests that some aspects of the model may obtain (Albarracin-Jordan 1996:200-204). Albarracin-Jordan argues that secondary settlements represent central places for regional settlement clusters to mobilize labor in diverse agricultural tasks. On the other hand, he shows that settlement clustering is attributed to proximity to agricultural potential and field types, which include terraces, raised fields, and qochas (Albarracin-Jordan

Figure 2: The distribution of Tiwanaku Period settlements in the Tiwanaku Valley (data from Albarracin-Jordan and Mathews 1992). The 2.5km density radius was selected based on the results of a Ripley's K multi-distance cluster analysis. The Tiwanaku capital site is excluded from the density interpolation because it's size overwhelms the density variation.

Working Paper—Do not copy or cite without permission of authors.
of social relations that are analogous to ethnographic Aymara models of social organization in that region. Known as ayllu, this model largely contrasts with previous models, which view Tiwanaku as a highly centralized and bureaucratic apparatus whose imperial colonial centers were created to administer agricultural production throughout the Tiwanaku Valley. Ayllus are the basic socio-economic unit of an ayllu system. Where-as ayllus integrated blood or fictive kin across ecological zones, markas constituted a higher level of integration, incorporating multiple ayllus.

Attendant to geographic clustering and apparent intra-cluster central place patterning were other independent indices of economic and politico-religious inequality. During the Middle Formative Period, ceramic production becomes more specialized in the regional centers. The Yaya-Mama stylistic tradition also emerges and is manifest in the design of stone statues, or stelae, that are concentrated in the regional centers. Stanish (2003) argues that this new tradition is tied to elites and a pan-regional ideology. In the Upper Formative Period, communal architecture such as sunken courts and artificial mountains, or pyramids, appear at regional centers. Janusek (2006) argues that architectural design and iconography at Tiwanaku were designed to co-opt the power of nature and integrate diverse cultural groups. He further suggests that Tiwanaku's success in "out-competing" other regional centers stems from its uniquely integrative religious tradition.

Migration and Differential Population Agglomeration in the Tiwanaku Valley

The aforementioned discussions of rank size and settlement pattern begs the question of how differential population agglomeration occurs. Johnson's (1980) integration model, for example, does not explicitly explain why varying types of integration should necessarily lead to log-normal rank-size distributions or deviations thereof. Moreover, dendritic systems, which Johnson argues should arise from vertical integration, may not always produce a concave rank-size distribution. For example, the stream system in Figure 3 shows a dendritic system for which we understand the underlying processes generating the pattern produces a distribution closer to log-normal.

McAndrews, Albarracin-Jordan, and colleagues draw on Johnson's integration model, and shows that ethnographic labor-pooling ayllus and markas may explain how settlements were integrated in such a way as to create a context for local leaders and communal architecture, but it does not necessarily explain why villages within those clusters harbor different population sizes with convex and primo-convex rank-size distributions. Albarracin-Jordan (1992) shows that site area is linearly and positively correlated with population, and given that all site types consist largely of domestic features and materials, we can assume that larger sites housed larger populations. If the secondary sites served to unify smaller sites within an agricultural production zone, there is no necessary reason to expect, given the integration model, that those sites should contain larger populations than lower order sites. Clearly, then, some set of factors drew residents to inhabit regional centers in a way that generated convex and primo-convex rank-size distributions. Albarracin-Jordan (1996:206) hints at an explanation for why populations at these centers grow when he suggests that some of the aggregation groups seem to integrate specialists in the manufacture of certain crafts. Drawing from economic geography, we might expect that individuals in agrarian societies with sufficient caloric surpluses might be relatively free to behave in such a way as to maximize product and service diversity (Christaller 1966; Krugman 1991).

Griffin and Stanish's (2007) agent based model of the Titicaca Basin offers a mechanism by which differential population agglomeration may have occurred in the Titicaca Basin. Their goal is to use ABM to test a conceptual model of how Titicaca Basin population centers arose and collapsed during the Formative and Tiwanaku periods. They suggest that the interplay of agricultural and pastoral land potential, spatio-temporal rainfall variability, social competition and conflict, inter-settlement migration, and long distance trade all conspired to create the macro-scale patterns that
define the settlement distribution of the Tiwanaku Empire. Their model, which begins with a relatively low population of Terminal Archaic settlements achieves remarkable accuracy with respect to predicting macro-scale patterns and the historical trajectory of the Tiwanaku Empire. Twelve adjustable parameters and many more fixed-value parameters define the model space. Tuning the parameters to find an optimal fit with the empirical record, 540 model runs revealed the ABMs predictive capabilities. They found that 34% of the runs matched six of six key spatio-temporal patterns and consistently satisfied 11/12 empirical validation criteria.

Among the criteria Grifin and Stanish (2007:19-21) sought to validate was the rank-size distribution of sites. As with their model as a whole, the rank-size predictions show remarkable formal similarities with the empirical rank-size distribution. What is unclear, however, is the degree to which any or all of the model variables—land productivity, rainfall, conflict and competition, trade, etc.—are necessary for driving the Formative Period convex distribution or the Tiwanaku Period primo-convex distribution.

Modeling Rank-Size Distributions

In trying to understand what behaviors might underly the Tiwanaku Valley rank size distributions, we drew inspiration from several models. First, we explored Krugman's new economic geography model, which at its most basic level, suggests that competing centripetal and centrifugal forces played a significant role in the emergence of differential population agglomeration. Because centrifugal forces generally include geographically dispersed resources such as agricultural production or raw material quarrying, it was clear that a spatially explicit agent-based model would provide us with an appropriate tool for exploring population agglomeration dynamics. Villages occupying a fixed space would effectively create a centrifugal

Figure 3: The Rio Ramis watershed of the Titicaca Basin and a rank-size distribution plot of the associated stream segments and their drained areas. The stream model is generated using 90m SRTM digital elevation model data. Only stream segments draining greater than 50 km² are included. The segment order is defined using the Strahler stream ordering method. All hydrological processes were performed using ArcGIS 9.3 and associated Spatial Analyst toolkit.

Working Paper—Do not copy or cite without permission of authors.
force. This is consistent with the fact that the Format-
itive Period in the Titicaca Basin (and elsewhere by
definition) is marked by the onset of village seden-
tism.

Next, we ask what simple behavioral mechanism
could produce a skewed rank-size distribution
amongst an initial population of relatively homoge-
neous settlements. Tom Carter's wealth distribution
model serves as our point of departure. It shows that
periodic currency transactions between random indi-
viduals can create a skewed distribution of wealth as
long as there is a lower bound on the wealth of any in-
dividual. Accordingly, we ask what effect simple,
random "transactions" of people would have on a
rank-size distribution.

Figure 4 shows the results of our implementation
of Carter's model in Net Logo 4.0.4. In the featured
model run, 501 villages each containing 25 individu-
als were randomly dispersed on a square playing field.
At a given time step, each village with population
greater than zero migrated one of its members to an-
other randomly chosen village. After approximately
500 time steps, the rank-size settlement sizes arrived
at a logarithmic rank-size distribution. Multiple model
runs under varying random spatial configurations re-
produced the same rank-size pattern.

Although the resultant distribution does not qualit-
atively fit the empirical Tiwanaku data, it does show
that random movement between villages can generate
rank-size convexity that is more attenuated than a
purely distribution of population. Given that the
Tiwanaku Valley rank-size distribution is primo-con-
vecx, we next ask what behaviors could achieve log-
normality and/or concavity. Scale-free network
growth has proven a valuable tool in explaining the
emergence of power-law distributions in human soci-
eties (Bentley 2003), and so that concept provides a
next step in our modeling effort. Specifically, we ask
how preferential attachment would affect our modeled
rank-size distribution. In other words, instead of our
model migrants moving to random locations, we pro-
gram them to bias their migration destination to closer
villages with larger populations. To achieve this beha-
vior, at each time step, a given village determines
population:distance ratio for every other village in the
model world. The emigrant then moves to the village
with the greatest ratio. The resultant rank-size distri-

A more robust implementation of this model would be to use

Figure 4: Simulated rank-size population for a
random model. In this simulation, each of 501
villages consisting of 25 individuals each are required
to migrate 1 individual to another village. If the
village reaches 0 individuals, it waits until it receives
an individual from another village. After
approximately 500 time steps, the rank-size
distribution approximates a logarithmic distribution.
Linear and power-law distributions are provided for
reference.
and simply becomes increasingly exaggerated with each progressive step. These observations suggest that, at least given the behaviors modeled here, we cannot assume that the temporal changes in the Tiwanaku patterns reflect a time-dependent trend.

Given that the primo-convex model shows an extreme degree of primacy and the random model exhibits an extreme degree of convexity, we next ask if intermediary agent behaviors might more consistently produce a rank-size distribution with a formal pattern that resemble the Tiwanaku Formative period distribution. To accomplish such behavior, each agent is programmed to randomly “err” with some probability in its identification of the most rational migration destination. We posit that such error is likely during the Formative Period before communication routes are well established other factors besides distance and population are driving migration decisions. An example of the results of the error-prone-agent model are shown in Figure 6. In that example, agents bypass the optimal migration solution with 20 percent probability. Our observations on repeated model runs suggest that increasing the error probability increases the chances that a given run will settle into a convex distribution without a primate tail. However, we have yet to run a more rigorous test of this observation.

Modeling Geographic Distributions

Finally, we ask how landscape geometry affects rank-size distributions, and how well the model would predict the actual locations of large settlements in the Titicaca Basin. The locations of known large Upper Formative Period settlements are fairly well documented (Figure 7; Stanish 2003). These site locations are taken to represent loci of high population agglomeration. Griffin and Stanish (2007:22-23) observe that landscape geometry could account for much of the patterning behind the rise and fall of dominant polities. They suggest that the Basin and Lake topology affect the number of possible constituent settlements under a chief’s control. Given a fixed distance of potential chiefly influence, the north and south ends of the basin, where the Lake does not clip as large an

Figure 5: Modeled rank-size distributions using a population and distance biased migration behavior. The model on the left shows a single model run over 500 time steps, and the graph on the right shows selected model outputs to reveal the range of variation in 30 runs at time step 100. The results range from primo-convex to bi-convex to convex with 63 percent exhibiting single-site primacy.
area of habitable land, are the most likely areas for dominant polities to form. Similarly, we can imagine that landscape geometry would affect the likelihood of large population agglomerations forming in different parts of the landscape. More expansive areas have a greater population potential and thus under the migration rules outlined here those large areas would be more likely to create relatively large population centers.

To model the geometry of the Tiwanaku Period habitable landscape, we performed the following geoprocessing steps in ArcGIS 9.3: First, 90 m resolution Shuttle Radar Telemetry data were downloaded from the United States Geological Survey's National Map Seamless Server. The extent of Lake Titicaca and the Titicaca Basin watershed was defined using ArcGIS Spatial Analyst's hydrology tool kit. The southeastern end of the basin was arbitrarily cut off. We then defined the habitable space as the area within the watershed, excluding Lake Titicaca, and below 4000 masl, the upper limit of agriculture (Griffin and Stanish 2007:6). This landscape was then imported into NetLogo 4.0.4, where agents were restricted to the resultant geographic model space.

Running the model on the geographic landscape produced some realistic and unrealistic results. Although the model continued to produce primo-convex rank-size distributions, the frequency of single-settlement primacy decreased considerably with more instances of relatively log-normal rank-size distributions. Perhaps the biggest problem with using geometry alone is that large settlements frequently arise in the southeastern end of the Basin where we know them not to exist. Figure 7 shows that, empirically, large sites tend to occur near the lakes edge with no major sites occurring south of Tiwanaku some 15 kilometers southern lake margin. Stanish (2003) reports that Formative Period sites were biased toward the Lake's immediate margin where agricultural productivity is greatest. After the Middle Formative Period, large sites began to develop at greater distances from the lake shores. Accordingly, it is clear that successful prediction requires initial conditions that bias initial site locations toward the Lake. Unfortunately, we have no quantitative data at this point to evaluate the relationship between Lake distance and settlement density, but those data are forthcoming. In the meantime, our model incorporates a simple method of linearly weighting the probability that a site will occur at one of five distance intervals. The resultant model produces a more realistic picture of the distribution of major sites, but it still lacks satisfactory predictive power (Figure 8).

**Summary and Discussion**

The model presented here is a work-in-progress, but even at this stage, it has produced some insights into mechanisms driving the emergence of settlement size inequality. Our agent-based modeling approach demonstrates that the cumulative effects of individual-based optimization strategies predicated on migration behavior can generate the convex and primo-convex rank-size distributions that we see in the Tiwanaku Valley ca. 500 B.C.—A.D. 1100. Yet, the explanatory potential extends beyond the Andean highlands to oth-

---

Figure 6: Rank-size distribution for migrants who decline the optimal solution 80% of the time. The model shows that migration decisions that are only partially optimal can create convex distributions. However, those distributions tend toward a bifurcation and ultimately a power-law distribution with few sites.
er agrarian and industrial societies whose rank-size distributions exhibit similar properties. Many of the rank-size distributions described by Zipf (1944) and Johnson (1980) exhibit convexity, which can arise from migration strategies that do not optimize for distance or population size. We have shown that random migration and occasional “miscalculation” can result in convex rank size distributions.

Similarly, primo-convex settlement size distributions are evident outside the Andean Highlands in Colonial America, Great Britain, Warka Iraq, the Susiana Plain Iran (Johnson 1980), and Austria Zipf (1944). Johnson has explained such distributions as a function of dendritic, or disproportionately vertical integration. However, we show that dendritic systems can, in fact, result in log-normal distributions. Therefore, the observation that dendritic integration is responsible for rank size convexity may not provide a particularly robust explanation. Instead, our model suggests that simple migration optimization behavior may be a special form of dendritic integration in which the agents are not as constrained by hierarchical pathways as in stream or vascular systems. Like the water in a watershed that flows from low-volume areas to spatially adjacent high-volume areas, the agents tend to move along a gradient from low population to high population. But, the agents in this model—unlike naturally flowing water—can make large leaps past neighboring villages to centers whose population pull is relatively large. Accordingly, our model may still be considered dendritic in the sense that there is a relatively high degree of vertical flow,

Figure 7: Upper Formative major site locations from Stanish (2003). The larger dots represent the two largest sites--Tiwanaku and Pucara.
but it differs from the vertically integrated flow of a water drainage network, which is more constrained.

In the Titicaca Basin, it may be that a lack of material diversity fostered little incentive for horizontal integration, such that migration decisions were more heavily dependent on distance minimization and population maximization. If that is the case, then we would expect regions with more heterogeneous material sources to exhibit more of a log-normal or concave distribution. Similarly, as with dendritic stream model presented here, geographic constraints in the movement of people, such as in more densely packed or territorial societies, may result in more log-normal distributions.

The observations made in this model clearly prompt many more questions for further exploration. At the most basic level, we have to ask whether or not post depositional processes, or the differential preservation of sites, affect the observed distributional patterns. Some of the observed convexity at the right side of the distribution could be a result of the fact that smaller sites would tend to be obliterated more easily or because smaller sites are more easily overlooked in surface reconnaissance. Future work should try to account for such sampling effects.

In the spirit of our migration model, we will continue to explore the effects of emigration behavioral variations on rank-size and geographic distributions. In particular, we will implement a gravity model in place of our settlement size:distance ratio approach. This alternation will have the effect of replacing an arbitrarily defined relationship between the relative

---

Figure 8: An example of a model output. The white dots show predicted site locations size graded by population, and the black dots show actual Upper Formative Period site locations from Stanish (2003).
weights of population and distance with one that is explicit. Finally, we are currently developing a network-based method for producing more-realistic path distances between villages. This, we anticipate, will assist in developing better geographic predictions for the locations of large settlement formation. Once we are satisfied that we've understood the effects of migration behavior, we will explore the effects of population growth and the combination of migration and population growth. Our hope is that our ABM approach will ultimately explicate a more reliable mechanism for the emergence of convex distributions that more closely resemble empirical observations and offer a clearly defined dynamic for the emergence of log-normal distributions. The latter would prove relevant to distributions in many parts of the world and at many scale of socio-economic complexity ranging from the modern United States to prehistoric Puebloan societies (Figure 9).

Figure 9: The rank size distribution of modern U.S. cities in 2000 (left) and Puebloan settlements over time in Bandelier National Monument, New Mexico.

Acknowledgements

This work was partially supported by the Santa Fe Institute through NSF Grant No. 0200500 entitled "A Broad Research Program in the Sciences of Complexity." Many participants in the 2009 SFI Complex Systems Summer School offered thoughtful comments and valuable insights. James P. Holmlund contributed valuable commentary on an earlier version of this paper. We are grateful to Jamie Civitello and Rory Gauthier, Bandelier National Monument, who made available the Bandelier settlement data.

References Cited

Albarracin-Jordan, Juan
Albarracin-Jordan, Juan, and James Edward Mathews
Bentley, R. Alexander
Diamond, Jared
Christaller, Walter
Fujita, M., Krugman, P. and Venables, AJ.
Griffin, Arthur F., and Charles Stanish
Krugman, Paul
Janusek, John Wayne
Johnson, Gregory A.
McAndrews, Timothy, Juan Albarracin-Jordan, and Marc Bermann
Stanish, Charles
Zipf, George Kingsley